Pension Reform, Assets Returns and Wealth Distribution

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ABSTRACT – Does a pension reform exacerbate inequality in modern economies? We show that the PAYg system generates more inequality and decreases the interest rate. Our paper points out the importance of the heterogeneity of assets earnings and the unequal access that individuals have to them. In this context, the PAYg pension system generates more constrained-poor agents and a lower interest rate. The rich agents benefit from the reform at the expense of poor agents.

Réforme des retraites, rendement des actifs et distribution de richesse

RÉSUMÉ – Une réforme des retraites augmente t-elle les inégalités dans une économie financière ? Cet article montre l'importance de l'hétérogénéité des revenus de placement et de l'inégal accès des individus à ces revenus d'actifs. Dans ce contexte, nous montrons qu'un système de retraite par répartition engendre plus d'inégalités, c'est-à-dire plus d'agents pauvres contraints et un taux d'intérêt plus faible. Les riches bénéficient d'un développement du système par répartition au dépend des pauvres.

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1 INTRODUCTION

Does a pension reform exacerbate inequality in modern economies? Can a pension reform be positive for all agents? What is the effect of the pension system and its size on the credit market interest rate and on the economic dynamics? In the literature, the effects of Pay-As-You-Go (PAYg) pension system and of its reform on inequality is very controversial.

Two types of arguments are in favor of PAYg pension system with respect to its effects on inequality. The PAYg system is viewed as being more inequality-reducing than a privately funded system because in PAYg system the benefits accruing to an individual are not proportional to his contribution to the pension system. Therefore the PAYg system redistributes income across generations, but also within generations from high earnings agents to low earnings agents. The second argument is that in a PAYg system the individual risk in earnings or assets returns is spread between agents which moderates inequality in the economy.

For example, LIEBMAN [2001] and GUSTMAN and STEINMEIER [2000] find that in the US social security system, own benefits are to a significant extent redistributed from those with high lifetime earnings to those with low lifetime earnings. DEATON, GOURINCHAS and PAXSON [2000] show that social security systems limit the transmission of individual risk into inequality. Moreover KRUEGER and KUBLER [2002] present a case of Pareto improvement of social security reform in an OLG model with stochastic production and incomplete markets. They show that by allowing retired households with a claim to labor income the PAYg system serves as an effective tool to share aggregate risk between generations.

However, these views have been challenged in the literature. Different types of arguments point that PAYg system is welfare-reducing and inequality enhancing. CUBEDDU [2000] argues that in the presence of perfect insurance markets, unfunded pension schemes are welfare-reducing by essentially forcing individuals to substitute private assets for social security tax contributions, since in dynamically efficient economies, the return on unfunded pension schemes is less than the return on private savings. When high heterogeneity of wealth, life expectancy or labor ability across agents are taken into account the PAYg system exhibits more inequality than usually admitted. For instance, in the last fifty years in France, the working class lost out in the PAYg system compare to skilled workers because their life expectancy is much lower. HAIRAULT and LANGOT [2002] show that in an economy with a high heterogeneity in wealth a PAYg pension system may be inequality enhancing. They show that when the contribution rate to the pension system is increased, the workers in the bottom of the wealth distribution lose out from the reform because they are less able to smooth their lifetime consumption.

These analyses seem to miss an important aspect of the relationship between pension systems and inequality, namely, the heterogeneity of assets' earnings and the unequal access that individuals have to them. Our paper is a step in this direction as we consider both heteregeneity of wealth and assets returns. We point out the importance of non-homogeneous assets returns in relation to the effects of a pension reform on wealth distribution. We show that in this context, the PAYg pension system increases inequality in the economy through the negative effect on the equilibrium interest rate which is not standard. In the OLG framework, conventional wisdom is that a PAYg pension system generates lower aggregate savings and, therefore a lower capital accumulation and a higher interest rate.

Our framework is an extension of the model of MATSUYAMA [2000]. We add a pension system and an optimizing behavior of agents. Given the initial wealth distribution, Matsuyama's model generates an endogenous wealth threshold depending on the equilibrium interest rate which separates agents into two categories: the poor and the rich. This endogenous formation of inequality persists in the long run under some configurations of the economy and it is possible therefore to give a complete characterization of the steady state equilibrium. The model of MATSUYAMA [2000] is very close to the models of AGHION and BOLTON [1997] and GALOR and ZEIRA [1993]. As in those models, it is the imperfection in the capital market and the non-convexity in the investment technology that generate persistence of inequality and multiple long run outcomes. It is also close to PIKETTY's [1997] model which is a Solow-type accumulation model with an imperfect credit market. However, there are some substantial differences between those models. AGHION and BOLTON [1997] and PIKETTY [1997] consider the effect of an uncertainty on investment returns that leads to an ergodic steady state distribution, whereas GALOR and ZEIRA [1993] model the borrowing and lending interest rates as different from each other and exogenous. This gap between the borrowing and the lending interest rates creates the difference of comparative advantage the agents have in their investment on education and therefore creates inequality in the long run. On the contrary, in Matsuyama's model the interest rate is determined by the credit market equilibrium. Moreover the equilibrum interest rate determines the returns of the different assets in the economy. Since the PAYg system has a direct effect on agents' savings it affects the equilibrium interest rate and, therefore asset returns in the economy. Therefore, Matsuyama's model offers an appropriate framework to analyse the interactions between wealth distribution, savings, interest rates, asset returns and inequality.

In MATSUYAMA's [2000] framework, we consider optimizing behavior of savings and of investment of agents facing different investment possibilities. In this paper, there are two assets yielding two different returns; an agent can lend his savings in the credit market, which gives him the equilibrium interest rate, or invest in an entrepreneurial fund which earns a constant high return. However the entrepreneurial fund requires a minimum investment. We show that there is a unique wealth threshold dependent on the credit market interest rate, that separates the economic agents into poor-constrained agents, and rich-unconstrained agents. The poor-constrained agents are the lenders earning the equilibrium interest rate on their savings and the rich-unconstrained agents are the borrowers earning the high net returns in the economy.

In this context we show that a higher contribution rate to the PAYg system increases the number of constrained-poor agents in the economy and decreases the interest rate. These effects are greater when the initial wealth distribution is more inegalitarian, meaning that the dispersion is greater. The intuitive explanation is that an increase in the contribution rate causes a decrease in disposable wealth and savings of agents, since the contribution to the PAYg system is based on first period revenue. This has a greater effect on some middle-class agents who become constrained. That is, they lose the capacity to invest in the high return assets in the economy. Since there are fewer unconstrained-rich agents, the decrease in capital demand is greater than the decrease in capital supply. This implies that the interest rate decreases in order to bring equilibrium to the credit market. As shown by

PIKETTY [1997], in presence of imperfect credit market, the equilibrium interest rate may not be equal to the marginal productivity of capital.

The pension reform increases intragenerational inequality. Because the interest rate is the rate of return for poor-constrained agents' savings, they lose out from its decrease. Since the interest rate is the borrowing rate of rich-unconstrained agents, its decrease raises the net return of their savings. Therefore the rich-unconstrained agents benefit from the pension reform at the expense of poor-constrained agents.

Section 2 sets out the model and outlines its main properties. Section 3 presents the different cases of steady state equilibrium. Section 4 discusses the effects of a pension reform on interest rates and inequality.

2 THE MODEL

The framework is a two-period OLG model with bequests. Agents live two periods and make bequests to their offspring. The only source of heterogeneity within a group of same-aged individuals is their inheritance. In their youth, all agents receive a bequest (z_t) from their parents, and a wage endowment[1] (w). They leave a bequest (z_{t+1}) to their offspring in their second period of life. The agent contributes to the pension system at the rate θ, and receives a pension (p_{t+1}) when old.

During his youth, an agent allocates his income (net of the contribution to the pension system) between consumption (c_t), and savings (s_t). He then has to decide how to invest his savings. He may invest his savings in the competitive credit market (become a lender), which earns a gross return of r_{t+1}, or he can start an entrepreneurial project which yields a gross return of ρ per unit invested. However, starting an entrepreneurial project requires a minimum amount of capital, k_t. One can interpret the entrepreneurial project as an investment fund with minimum investment required, and yielding a high return. The technology of the entrepreneurial project is defined by:

(1) $$F(k) = \begin{cases} 0 \text{ if } k < \kappa \\ \rho k \text{ if } k \geq \kappa \end{cases}$$

The minimum investment required (k_t) satisfies $w\,(1-\theta) < \mathrm{k}t$ at any time. It is optimal to invest the maximum possible above k_t, since the technology is of constant return. To invest at a level higher than their savings, the entrepreneur becomes a borrower of the funds supplied by the lender.

Following MATSUYAMA [2000] we assume that there is an enforcement problem in the credit market. An agent who borrows an amount (b_t) would refuse to honor its payment obligation $r_{t+1}\, b_t$, if it were greater than the cost of default. We assume that the lender can, at most, seize a fraction $\lambda \in (0,1)$ of the project output ρk_t in case of default. Knowing this, the lender would allow the entrepreneur to borrow only up to $b_t = \lambda \rho k_t / r_{t+1}$. This defines the borrowing limit in the credit market caused by

1. We assume that w is constant and is the same for all agents and generations in order to focus on the effects of the wealth heterogeneity.

the enforcement problem. As in Banerjee and NEWMAN [1993] it is the possibility of default that urges the lender to limit the amount of their loan. This prevents any default in equilibrium. Thus, any agent who wants to invest $k_t > K_t$ must have savings $s_t = (1 - \lambda\rho / r_{t+1})\, k_t$, which serve as a down payment[2] for the loan. Those with bigger savings may undertake bigger entrepreneurial projects.

Since there is a minimum requirement for setting up a project, only those with savings $s_t \geq (1 - \lambda\rho / r_{t+1})\, k_t$ could effectively set up a project. This defines the minimum level of savings required as a down payment for borrowing and investing in a single project, that we denote $\underline{s}_t = \left(1 - \lambda\rho / r_{t+1}\right) \kappa_t$.

The return on savings invested in the entrepreneurial project net of borrowing costs is $R_{t+1} = \frac{(1-\lambda)\rho}{(1-\lambda\rho/r_{t+1})}$, which is higher than r_{t+1} until $r_{t+1} < \rho$, and equal to r_{t+1} when $r_{t+1} = \rho$. This implies that setting up an entrepreneurial project is always preferable when $r_{t+1} < \rho$. The strategy is to use all the savings as collateral to borrow, and invest the maximum compatible with the borrowing constraint, since the return on the project (net of the borrowing costs) is an increasing function of the capital invested k_t.

The pension system is a fully funded Pay-As-You-Go (PAYg) pension scheme. Pensions are financed by a proportional tax on the first period wage. The budgetary equilibrium of the pension system is written:

$$N_t \theta w = N_{t-1} p_t$$

N_t denotes the number of young people in t, and N_{t-1} the number of old people. The pension system is "Bismarckian" since no direct redistribution is intended. Assume that the wage is constant. Let $p_t \equiv \tau_t w$ denote the pensions benefits received by each retired agent at date t, where τ_t designates the replacement rate. Thus one can show that $\tau = (1 + n)\, \theta$ is constant since we assume that the growth rate of the population n is constant.

Each agent chooses the optimal saving that maximizes his inter-temporal value function. Since the return on savings is higher when it is invested in the entrepreneurial project, some agents gain from forcing their savings to reach the entrepreneurial threshold. They will do so until it raises their value function. This is very similar to the optimal investment decisions of GALOR and ZEIRA's [1993] model. But in their model the interest rate is exogenous and constant. To be sure that the inherited wealth determines the agents' investment decisions we assume that $\gamma w (1 - \theta) < (1 - \lambda\rho/r)k$. This guarantees that the wage endowment is never sufficient to allow for borrowing and investing in a single project.

The agents' program[3] is summarized by:

$$\max_{c_t, d_{t+1}, z_{t+1}} \; c_t^{1-\gamma} \left[d_{t+1}^{1-\sigma} z_{t+1}^{\sigma} \right]^{\gamma} \tag{2}$$

2. For simplicity, we exclude the possibility that second period pensions benefits are part of the resources that the lender could seize in case of default. However, allowing agents to use their expected pensions as down payment would not changes the qualitative results of the model, but it would complexify the expressions in the model.

3. For seek of simplicity and continuity, we consider a Cobb-Douglas utility function with limited altruism. The parents value only the wealth level they transmit to their offrspring. Note that this utility function is equivalent to (except for parameters) the more standard log utility function: $\ln c_t + \gamma \ln d_{t+1} + \sigma \ln z_{t+1}$.

(3)
$$s.t.\begin{cases} c_t + s_t \leq w(1-\theta) + z_t \\ d_{t+1} + (1+n) z_{t+1} \leq \hat{R}_{t+1} s_t + p_{t+1} \\ \hat{R}_{t+1} = \begin{cases} r_{t+1} & \text{if } s_t < \underline{s_t} \\ R_{t+1} & \text{if } s_t \geq \underline{s_t} \end{cases} \end{cases}$$

where $\hat{R}_{t+1}$ is equal to r_{t+1} if the agent is a lender and equal to R_{t+1} when the agent is an entrepreneur. Note that $0 < \gamma, \sigma, \theta < 1$ and, d_{t+1} and p_{t+1} are respectively the second period consumption and agent's pension benefits. For a lender or an entrepreneur the optimal interior solution to his problem is:

(4)
$$c_t = (1-\gamma)\left[w(1-\theta) + z_t + \frac{p_{t+1}}{\hat{R}_{t+1}} \right]$$

(5)
$$d_{t+1} = (1-\sigma)\gamma \hat{R}_{t+1}\left[w(1-\theta) + z_t + \frac{p_{t+1}}{\hat{R}_{t+1}} \right]$$

(6)
$$z_{t+1} = \frac{\sigma}{1+n}\gamma \hat{R}_{t+1}\left[w(1-\theta) + z_t + \frac{p_{t+1}}{\hat{R}_{t+1}} \right]$$

(7)
$$s_t = \gamma\left(w(1-\theta) + z_t \right) - (1-\gamma)\frac{p_{t+1}}{\hat{R}_{t+1}}$$

One can easily compute the corresponding value function of a lender $\left(\hat{R}_{t+1} = r_{t+1}\right)$ or an entrepreneur $\left(\hat{R}_{t+1} = R_{t+1}\right)$ as:

(8)
$$V\left(\hat{R}_{t+1}, z_t\right) = (1+n)^{-\sigma\gamma}(1-\gamma)^{1-\gamma}(1-\sigma)^{(1-\sigma)\gamma}\sigma^{\sigma\gamma}\gamma^{\gamma}\left(\hat{R}_{t+1}\right)^{\gamma}\left[w(1-\theta) + z_t + \frac{p_{t+1}}{\hat{R}_{t+1}} \right]$$

Using equation (7) and the expression of $\underline{s}_t$, one can compute the value function of an agent who saves exactly the borrowing limit to set up one project. It is the optimal value function of the corner solution of the borrower problem:

(9) $$\underline{V}\left(R_{t+1}, z_t\right) = (1+n)^{-\sigma\gamma}\left(\frac{\gamma}{1-\gamma}\right)^{\gamma}(1-\sigma)^{(1-\sigma)\gamma}\sigma^{\sigma\gamma}\left(R_{t+1}\right)^{\gamma}\left[w(1-\theta) + z_t - \underline{s}_t \right]$$

Since there is a threshold saving to set up one project, it may be value function-enhancing to over-save to reach this borrowing limit. This means that some agents will force their first period savings to reach this borrowing limit, and attain the threshold to become entrepreneurs. Since the inheritance is the only source of heterogeneity between agents, one can derive an inheritance threshold that separates the population into two categories: the constrained agents, for whom it is optimal to become a lender, and the unconstrained borrower-entrepreneur. Equalizing the respective value function of a lender and of a borrower setting up one project, i.e. $V(r_{t+1}, z_t) = \underline{V}(R_{t+1}, z_t)$, yields the inheritance threshold. The inheritance threshold is the inheritance of the agent be he borrower or lender in terms of value function:

$$\underline{z}_t = \frac{(1-\gamma)(r_{t+1})^{\gamma}\frac{p_{t+1}}{r_{t+1}} + (R_{t+1})^{\gamma}\underline{s}_t}{(R_{t+1})^{\gamma} - (1-\gamma)(r_{t+1})^{\gamma}} - w(1-\theta) \tag{10}$$

Thus, the members of a household whose inheritance is above this level are rich enough to borrow and become entrepreneurs. They will use all their savings as a down payment to borrow the maximum consistent with the borrowing limit. The households whose inheritance is below this level are not rich enough to attain the borrowing limit, and have no interest in stretching their savings to reach it.

Since R_{t+1} is a function of r_{t+1}, the inheritance threshold is both a single valued function and an increasing function of the interest rate. When the interest rate is high, borrowing is costly and therefore the down payment required is higher. As saving is an increasing function of the inheritance, the corresponding inheritance threshold is higher. The wealth threshold is therefore endogenously determined by the evolution of the interest rate. Thus, being rich or poor is a relative position for an endogenous wealth threshold.

The equilibrium interest rate is determined by the credit market equilibrium condition. The aggregate net demand of capital by entrepreneurs must be equal to the aggregate net supply of capital by lenders. It can be shown[4] that the equilibrium interest rate must agree with the following condition:

$$\lambda\rho < r_{t+1} \leq \rho$$

That is, there are two types of equilibrium. First, $\lambda\rho < r_{t+1} < \rho$, and secondly, $r_{t+1} = \rho$. In the first equilibrium, the projects' return is always superior to the interest rate (which is the return for a lender). In the second equilibrium, the equilibrium interest rate is equal to the project returns, meaning that all agents earn the same return on their savings. The first type of equilibrium yields inequality in earnings, while the second type of equilibrium leads to equality in earnings. Since we are interested in the joint evolution of both the interest rate and wealth distribution with inequality, we will focus the analysis on the case where the equilibrium interest rate is strictly lower than the project return.

4. see Matsuyama (2000)

In the case $\lambda\rho < r_{t+1} < \rho$, the aggregate net demand of capital for investment is the demand of capital, backed by their savings, of all agents whose inheritance is above the inheritance threshold:

$$N_t \int_{\underline{z}_t}^{z_t^{max}} \frac{\frac{\lambda\rho}{r_{t+1}}}{1-\frac{\lambda\rho}{r_{t+1}}} s\left(z, R_{t+1}\right) dG_t\left(z\right)$$

where $s\ (z, R_{t+1})$ designates the savings of entrepreneurs, and $G_t\,(z)$ with values in (0,1) is the distribution function of the inheritance z defined in R_+ between z^{min} and z^{max}. Aggregate net supply of capital is the aggregate saving of all constrained agents. That is all agents whose inheritance is below the inheritance threshold:

$$N_t \int_{z_t^{min}}^{\underline{z}_t} s\left(z, r_{t+1}\right) dG_t\left(z\right)$$

where $s(z,r_{t+1})$ designates the savings of lenders or constrained agents. The credit market equilibrium condition is thus:

$$\frac{\frac{\lambda\rho}{r_{t+1}}}{1-\frac{\lambda\rho}{r_{t+1}}}\left[1-G_t\left(\underline{z}\right)\right]\left[\gamma\left(w\left(1-\theta\right)+E_t\left[z\,\middle|\,z \geq \underline{z}_t\right]\right)-\left(1-\gamma\right)\frac{p}{R_{t+1}}\right]$$
$$= G_t\left(\underline{z}\right)\left[\gamma\left(w\left(1-\theta\right)+E\left[z\,\middle|\,z \leq \underline{z}_t\right]\right)-\left(1-\gamma\right)\frac{p}{r_{t+1}}\right] \qquad (11)$$

The following proposition establishes the existence and the uniqueness of the short-run equilibrium.

PROPOSITION 1. *At any date t given $G_t(z)$, there exists a unique equilibrium with inequality characterized by* $r_{t+1} \in (\lambda\rho,\rho)$ and $\underline{z}$

PROOF. The system equation that defines the temporary equilibrium is a function of r_{t+1} and $\underline{z}_t$ Since $\underline{z}_t$ is a function of r_{t+1}, then the equilibrium exists and is unique if, for any equilibrium interest rate $\lambda\rho < r_{t+1} < \rho$, there exists a unique $\underline{z}_t$. Let us write $\underline{z}_t$ as a function of r_{t+1} by equation (11) so that $\underline{z}_t \equiv \Psi\left(r_{t+1}\right)$. One can rewrite the credit market equilibrium condition as a function of r_{t+1}, and $\underline{z}_t$ as $\Phi\left(\underline{z}_t, r_{t+1}\right)=0$. One can show that $\Phi\left(\Psi\left(r_{t+1}\right), r_{t+1}\right)$ is monotonic and decreasing[5] with r_{t+1}. Since $\lim_{r_{t+1}\to\lambda\rho} \Phi\left(\Psi\left(r_{t+1}\right), r_{t+1}\right)=+\infty$ and $\lim_{r_{t+1}\to\rho} \Phi\left(\Psi\left(r_{t+1}\right), r_{t+1}\right)<0$, then r_{t+1} exists and is unique. Since $\psi(r_{t+1})$ is monotonic and increasing then for any equilibrium r_{t+1}, $\underline{z}_t$ is well defined and unique.

5. Calculus details and graphic illustration are available on request.

3 STEADY STATES EQUILIBRIA

Since a complete analysis of the joint dynamics of the interest rate and the wealth distribution is beyond the scope of this paper, we will focus the analysis on the steady states equilibria which may emerge in this context. The steady state is associated with the limit distribution, $G_\infty(z)$, and the limit interest rate, r_∞. It is the state which replicates itself over time, once the economy is settled in, and where all the households hold a constant wealth.

In the long run constrained agents' inheritances converge to $z_\infty^c < \underline{z}_\infty$:

(12) $$z_\infty^c = \frac{\phi r_\infty w(1-\theta)+\phi p}{(1-\phi r_\infty)}$$

Individuals who inherit more than $\underline{z}_\infty$ invest in entrepreneurial projects, as do their offspring throughout all generations. Their bequests converge to $z_\infty^u \geq \underline{z}_\infty$:

(13) $$z_\infty^u = \frac{\phi\left(\frac{(1-\lambda)\rho r_\infty}{r_\infty-\lambda\rho}\right)w(1-\theta)+\phi p}{\left(1-\phi\frac{(1-\lambda)\rho r_\infty}{r_\infty-\lambda\rho}\right)}$$

As seen in equation (13), the wealth of the unconstrained converges only when the steady-state interest rate satisfies $r_\infty > \lambda\rho/(1-\phi(1-\lambda)\rho)$. The long-run inheritance threshold is given by the steady state interest rate and the minimum investment. It is written:

(14) $$\underline{z}_\infty = \frac{\frac{(1-\gamma)p}{r_\infty}+((1-\lambda)\rho)^\gamma \frac{1}{r_\infty}(r_\infty-\lambda\rho)^{1-\gamma}\kappa}{\left(\frac{(1-\lambda)\rho}{r_\infty-\lambda\rho}\right)^\gamma-(1-\gamma)} - w(1-\theta)$$

A necessary condition of existence of $\underline{z}_\infty$ is $r_\infty < \left[\lambda\rho+(1-\lambda)\rho/(1-\gamma)^{1/\gamma}\right]$ which is verified. To summarize,

PROPOSITION 2. *In a steady state with inequality,* $r_\infty \in I \equiv \left(\frac{\lambda\rho}{1-\phi(1-\lambda)\rho},\rho\right)$ *and agent's wealth is equal to either* $z_\infty^c = \frac{\phi r_\infty w(1-\theta)+\phi p}{(1-\phi r_\infty)}$ *or* $z_\infty^u = \frac{\phi\left(\frac{(1-\lambda)\rho r_\infty}{r_\infty-\lambda\rho}\right)w(1-\theta)+\phi p}{\left(1-\phi\frac{(1-\lambda)\rho r_\infty}{r_\infty-\lambda\rho}\right)}$.

Note that the interval $r_\infty \in I \equiv \left(\frac{\lambda\rho}{1-\phi(1-\lambda)\rho},\rho\right)$ reduces to $r_\infty = \rho$ if $\lambda = 1$. Thus, the lemma also implies that $\lambda < 1$ is a necessary condition for the existence of a steady state with inequality. Denote $s_\infty^u\left(r_\infty, z_\infty^u\right)$ as the steady state unconstrained agents' savings and $s_\infty^c\left(r_\infty, z_\infty^c\right)$ as the steady state constrained agents' savings and let us define $\alpha_\infty \equiv G_\infty\left(\underline{z}_\infty\right) < 1$ as the steady state fraction of constrained households. The steady state credit market equilibrium condition yields the following relationship between α_∞ and r_∞:

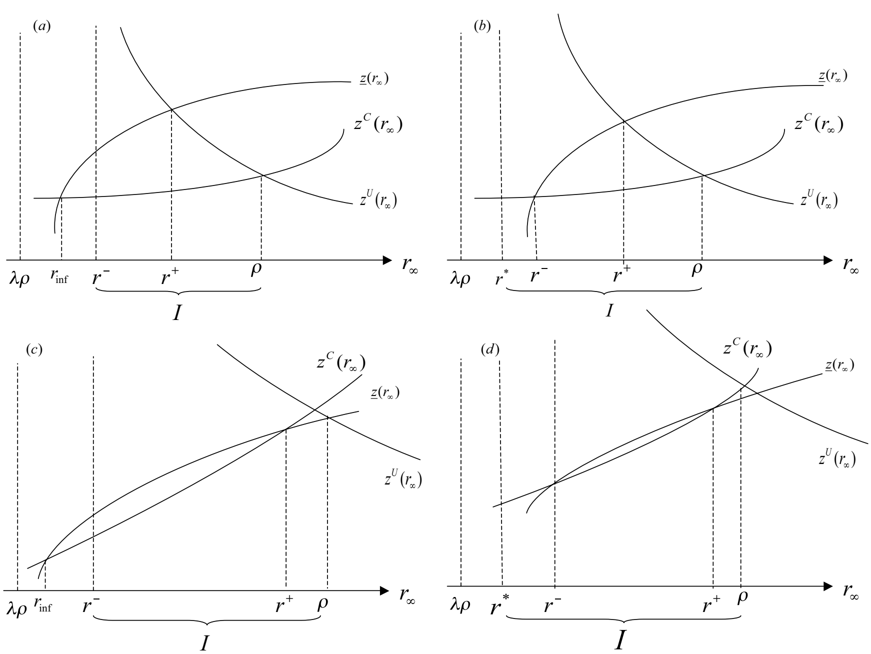
$$\alpha_\infty\left(r_\infty\right) = \frac{\lambda\rho s_\infty^u\left(r_\infty, z_\infty^u\right)}{\left[\left(r_\infty - \lambda\rho\right)s_\infty^c\left(r_\infty, z_\infty^c\right) + \lambda\rho s_\infty^u\left(r_\infty, z_\infty^u\right)\right]} \tag{15}$$

One can show that α_∞ is decreasing in $r_\infty \in I$, and is in the range $(\lambda, 1)$. This suggests that for any steady state interest rate, $r_\infty \in I$, one can always find a unique value of $\alpha_\infty \in (\lambda, 1)$ that satisfies equation (15). Thus, to demonstrate the existence of a two-point steady state equilibrium, it suffices to check the inequalities in the following condition:

$$z_\infty^c\left(r_\infty\right) < \underline{z}_\infty\left(r_\infty\right) \leq z_\infty^u\left(r_\infty\right) \tag{16}$$

FIGURE 1

Steady states configurations



To summarize the solution[6], let us distinguish between two cases. In the first one $\underline{z}_\infty(r_\infty)$ intersects with the two curves $z^c_\infty(r_\infty)$ and $z^u_\infty(r_\infty)$ in the interval $(\lambda\rho, \rho)$, and, in the second one it intersects only with $z^c_\infty(r_\infty)$ in this interval. In the first case, denote r_{inf} the solution of $z^c_\infty(r_\infty) = \underline{z}_\infty(r_\infty)$, and r^+ the solution of $\underline{z}_\infty(r_\infty) = z^u_\infty(r_\infty)$. Then there exists a continuum of steady state with $r_\infty \in (r^-, r^+)$ where $r^- \equiv \max\left\{\frac{\lambda\rho}{1-\phi(1-\lambda)\rho}, r_{\text{inf}}\right\}$ and $\alpha_\infty = \alpha_\infty(r_\infty) \in \left(\alpha(r^+), \alpha(r^-)\right)$. Figure (a) illustrates the solution $\left(\frac{\lambda\rho}{1-\phi(1-\lambda)\rho}, r^+\right)$ and figure (b) illustrates the solution (r_{inf}, r^+) of this first case. In the second case, denote r_{inf} and r^+ the two solutions of $z^c_\infty(r_\infty) = \underline{z}_\infty(r_\infty)$. Then there exists a continuum of steady state with $r_\infty \in (r^-, r^+)$ where $r^- \equiv \max\left\{\frac{\lambda\rho}{1-\phi(1-\lambda)\rho}, r_{\text{inf}}\right\}$ and $\alpha_\infty = \alpha_\infty(r_\infty) \in \left(\alpha(r^+), \alpha(r^-)\right)$. Figure (c) illustrates the solution $\left(\frac{\lambda\rho}{1-\phi(1-\lambda)\rho}, r^+\right)$ and figure (d) illustrates the solution (r_{inf}, r^+) of this second case.

In figure (*b*), $r^-=r_{\text{inf}}$ is greater than $r^* \equiv \frac{\lambda\rho}{1-\phi(1-\lambda)\rho}$ and r^+ is the solution of $\underline{z}_\infty(r_\infty) = z^u_\infty(r_\infty)$. Any $r_\infty \in (r^-, r^+)$ corresponds to a steady state with inequality characterized by a two-point distribution of wealth. The degree of inequality differs across the steady states. A high interest rate is associated with lesser inequality, both in terms of relative groups and wealth. Indeed a higher interest rate implies that $z^u_\infty(r_\infty)$ is smaller (the rich are less rich), that $z^c_\infty(r_\infty)$ is greater (the poor are richer) and that $\alpha_\infty(r_\infty)$ is smaller (the fraction of constrained agents is smaller).

4 PENSION REFORM AND WEALTH DISTRIBUTION

In this section we study how a higher contribution rate to the pension system affects equilibrium interest rate and inequality. It modifies both individual and aggregate savings, and therefore aggregate supply and demand of capital. This will in turn affect the interest rate and the inheritance threshold between constrained and unconstrained agents.

4.1 When PAYg system increases the interest rate

The pension reform in this framework is an increases of θ. That is an introduction or an increase of the size of the PAYg system. Such a reform has different

6. All calculus details in this section are available on request to author.

effects on agents' decisions at individual level and on credit market equilibrium at macroeconomic level. The effects on agents' savings is usual. Indeed, as the increases of the contribution rate to the pension system decreases their disposable wealth, their savings decreases. However the effect on the equilibrium interest rate is worth noting.

We have seen that the equilibrium interest rate is given by the credit market equilibrium condition (CMEC), equation (11) which depends on the wealth threshold given by equation (10). To analyze the effect of the pension reform at the macroeconomic level, we have to consider the joint effect on these two equations. From equation (10), one can see that $\underline{z}_t$ is a function of r_{t+1} and θ. To determine the effect of a reform of the pension system ($d\theta > 0$) on the wealth threshold, we must differentiate $\underline{z}_t\left(r_{t+1},\theta\right)$. We arrive at

$$d\underline{z}_t = \frac{\partial \underline{z}_t}{\partial \theta} d\theta + \frac{\partial \underline{z}_t}{\partial r_{t+1}} dr_{t+1} \tag{17}$$

There are two types of effects on the wealth threshold: a quantity effect given by $\frac{\partial \underline{z}_t}{\partial \theta} d\theta$ and a price effect given by $\frac{\partial \underline{z}_t}{\partial r_{t+1}} dr_{t+1}$. The quantity effect is positive. Indeed an increase of θ on agents' first period revenue reduces their disposable wealth and therefore their savings. As the minimum investment on the project is constant the wealth threshold is relatively higher. This means that the number of constrained agents increases and therefore the credit supply increases, what tends to decrease the interest rate.

The price effect depends on the variation of the equilibrium interest rate. Note that $\frac{\partial \underline{z}_t}{\partial r_{t+1}}$ is positive since that an increase of the interest rate make the down payment required for borrowing greater and therefore the wealth threshold relatively higher. The sign of the variation of the interest rate dr_{t+1} is given by the changes in the credit market equilibrium. If the interest rate decreases then the price effect on the inheritance threshold is negative and opposite to the quantity effect on the inheritance threshold. However the inheritance threshold increases because the quantity effect is a direct effect which is greater than the price effect which is indirect. But the decreasing effect of the interest rate on the wealth threshold limits the increase of this latter.

To check the variation of the interest rate implied by the reform let us rewrite the CMEC, equation (11) as $\Phi\left(\underline{z}_t, r_{t+1},\theta\right) = 0$. Differentiating this equation yields:

$$\frac{\partial \Phi\left(\underline{z}, r_{t+1},\theta\right)}{\partial \theta} d\theta + \frac{\partial \Phi\left(\underline{z}, r_{t+1},\theta\right)}{\partial \underline{z}} d\underline{z} + \frac{\partial \Phi\left(\underline{z}, r_{t+1},\theta\right)}{\partial r_{t+1}} dr_{t+1} = 0$$

Substituting equation (17) into the differentiated CMEC gives

$$\left(\underbrace{\frac{\partial \Phi\left(\underline{z}, r_{t+1},\theta\right)}{\partial \underline{z}}}_{<0} \underbrace{\frac{\partial \underline{z}_t}{\partial r_{t+1}}}_{>0} + \underbrace{\frac{\partial \Phi\left(\underline{z}, r_{t+1},\theta\right)}{\partial r_{t+1}}}_{<0}\right) dr_{t+1} = -\underbrace{\frac{\partial \Phi\left(\underline{z}, r_{t+1},\theta\right)}{\partial \theta}}_{<0} d\theta - \underbrace{\frac{\partial \Phi\left(\underline{z}, r_{t+1},\theta\right)}{\partial \underline{z}}}_{<0} \underbrace{\frac{\partial \underline{z}_t}{\partial \theta}}_{>0} d\theta$$

therefore

$$dr_{t+1} = -\frac{\left(\frac{\partial\Phi(\underline{z}_t, r_{t+1}, \theta)}{\partial\theta} + \frac{\partial\Phi(\underline{z}_t, r_{t+1}, \theta)}{\partial\underline{z}_t}\frac{\partial\underline{z}_t}{\partial\theta}\right)}{\left(\frac{\partial\Phi(\underline{z}_t, r_{t+1}, \theta)}{\partial\underline{z}_t}\frac{\partial\underline{z}_t}{\partial r_{t+1}} + \frac{\partial\Phi(\underline{z}_t, r_{t+1}, \theta)}{\partial r_{t+1}}\right)} d\theta < 0 \tag{18}$$

An increase in the contribution rate to the pension system leads to a decrease of the equilibrium interest rate. The variation of the interest rate depends on the relative variation of the credit supply and the credit demand. The pension reform causes a decrease of both the level of credit supply and demand because agents' savings fall after the reform. But, aggregate credit demand decreases more relatively to aggregate credit supply. The aggregate credit demand decreases more because some unconstrained agents become constrained after the reform. This is due to the increase of the wealth threshold generated by the reform. Recall that an increase of the wealth threshold signifies that the level of inheritance required to become unconstrained is higher. It is therefore some middle class agents who were previously unconstrained who become constrained. Therefore the interest rate decreases to balance the credit market.

The decrease of the interest rate generated by the PAYg reform is not standard in the OLG framework. Indeed, in OLG model the introduction of a PAYg system diminishes the accumulation of capital in the economy and therefore generates an increase of the marginal productivity of capital. At equilibrium the interest rate which is equal to the marginal productivity of capital increases. However, the introduction of the imperfections in the credit market, as shown by PIKETTY [1997] disconnects the equilibrium interest rate and the marginal productivity of capital in this model. Therefore at equilibrium the interest is not still equal to the marginal productivity of capital.

The variation of the interest rate has an asymmetric effect on the return of the different assets in the economy. A decrease of the interest rate increases the net return of investing in the project, while it decreases the return on lending in the credit market. As the poor agents are the lender in the credit market they lose out from the pension reform. The rich-borrowers benefit from the reform as their borrowing rate diminishes. This magnifies the conflict between constrained and unconstrained agents over a pension reform. To summarize,

PROPOSITION 3. *A pension reform, namely an increase of the size of the pension system leads to a decrease of the equilibrium interest rate and an increase of the number of constrained agents in this economy. Since the credit market interest rate is the return of poor-lender assets, its decrease disfavors them, while it increases the net return of rich-borrower assets. Therefore unconstrained-rich agents benefit from the reform at the expense of constrained-poor agents.*

4.2 **The role of wealth distribution on pension reform's effects**

The degree of inequality and dispersion in the initial wealth distribution determines the magnitude of the effect of the pension reform. Indeed increasing the contribution rate to the pension system leads to an increase of the inheritance threshold. But the magnitude of this increase depends on the degree of inequality in the wealth distribution. The number of agents who fall into the constrained categories and the part of the aggregate wealth they hold determines the importance of the variation of the interest rate. We simulate[7] the model in order to exhibit the different effects of the initial distribution on the equilibrium level and how it affects the magnitude of the impact of the pension reform.

Wealth distribution and inequality level

The table A summarizes the evolution of the number of constrained agents in the economy $\underline{Z}$ when the Gini index of the initial distribution of wealth increases. Recall that the Gini index expresses the degree of inequality and wealth concentration in the distribution. As seen in table A, when the Gini index is low 0.31, that is when there is a low level of inequality in the initial distribution of wealth, the model generates the lowest level of number of constrained agents 78.9% for a contribution rate to the pension system of 20%. As the Gini index of the initial distribution raises to 0.71 the number of constrained agents increases up to 83.1%. This increase of the number of constrained agents with the Gini index is also true when the contribution rate to the pension system is 35%. Also the interest rate is higher when the Gini index is low and it decreases with the Gini index as seen in table B. Therefore to a low level of initial inequality is associated at equilibrum a lower level of constrained agents and a higher interest rate. The intuition is that a low

TABLE A
Wealth distribution and variation of constrained agents

		$\underline{Z}$		$\Delta\underline{Z}$
		$\theta = 20\%$	$\theta = 35\%$	
I_{Gini}	0,31	78.9%	81.1%	+2.2%
	0,51	80.5%	83.5%	+3%
	0,71	83.1%	86.4%	+3.3%

TABLE B
Wealth distribution and variation of interest rate

		$\underline{r}$		$\Delta\underline{r}$
		$\theta = 20\%$	$\theta = 35\%$	
I_{Gini}	0,31	4.5%	4.46%	-0.04%
	0,51	4.46%	4.39%	-0.07%
	0,71	4.39%	4.30%	-0.09%

7. We calibrate the model to produce equilibrium with inequality. The parameters are $\lambda = 0.3$, $\gamma = 0.55$, $\sigma = 0.4$, $n = 1\%$, $\rho = 6\%$ and a level of minimum investment consistent with the distribution. The distribution is a log-normal random draw. We use Matlab to simulate the model.

Gini index means that wealth is more concentrated, that is the rich in this economy hold a lower share of total wealth. Therefore the demand of capital is relatively lower what implies a higher equilibrium interest rate to balance the credit market.

Wealth distribution and pension reform

The magnitude of the effects of the pension reform in terms of variation of the number of constrained agents and of the interest rate depends on the degree of inequality in the wealth distribution. The more the wealth distribution is inegalitarian the more the impacts of the pension reform is important. When the Gini index is 0.71 an increase of the contribution rate to the pension system from 20% to 35% leads to a variation of the number of constrained agents of + 3.3% and a variation of the interest rate of – 0.09 point, while when the Gini index is 0.31 the variations of the number of constrained agents is + 2.2% and the variation of the interest rate is – 0.04 point. The reason is that the increase of the contribution rate raises the wealth threshold but the number of agents that become constrained and the share of the total wealth they hold is very different between the two economies. When the Gini index is low the concentration[8] of agents in the bottom of the distribtion or around the wealth threshold is less important compared to a distribution with a high Gini index. Moreover the rich agents in the high Gini index economy hold a higher share of total wealth in the economy compared to the total share of the wealth hold by the rich in a low Gini index economy. Therefore more agents around the wealth threshold are concerned by the reform in an economy with a high Gini index than in an economy with a low Gini index. As the number of agents that become constrained in an economy with a high Gini index is higher and those agents hold a relatively important share of the total wealth, the interest rate varies more to balance the credit market. Therefore the impacts of the pension reform are more important when the economy is more inegalitarian.

5 Concluding remarks

There is a conventional view that in the OLG framework the PAYg system leads to less capital accumulation and a high interest rate but is more inequality-reducing than a privately funded system. The PAYg pension system may reduce inequality in at least two ways. One is that the PAYg system reduces intragenerational inequality when the benefits from the system are not exactly proportional to the contributions of agents. Another is that when individuals face risk in earnings or assets returns, the PAYg system by spreading the risk between individuals moderates the transmission of individual risk into inequality (see Deaton, Gourinchas and Paxson [2000]).

By taking into account different assets returns and unequal access to them, we find that the PAYg pension system generates a lower level of interest rate and increases wealth inequality. Moreover, the more the initial distribution is inegalitarian, the

8. Recall that the draw of the wealth distribution follows a log-normal law.

greater the negative effect of the pension reform, that is, the greater the increase in the number of constrained-poor agents and the greater the decrease of the interest rate. As the variation of the interest rate causes a variation of assets returns in the economy, it modifies agents' net wealth gains. Since the credit market interest rate is the return of poor-lender assets, its decrease disfavors them, while it increases the net return of rich-borrower assets. Therefore unconstrained-rich agents benefit from the reform at the expense of constrained-poor agents. ■

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