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Supplementary Insurance with *Ex-Post* Moral Hazard: Efficiency and Redistribution

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ABSTRACT. – This paper investigates the topping-up scheme in health insurance when both public and private insurers use a linear contract. It is shown that, with identical consumers, the second-best allocation is obtained. Whereas, introducing consumers' heterogeneity with respect to the wage rate when labour supply is endogenous, public coverage is uniform, and health expenditures are financed by linear taxation, it is shown that the optimal public coverage is negative and consumers are under-insured.

Assurance complémentaire avec risque moral *ex-post* : efficacité et redistribution

RÉSUMÉ. – Cet article analyse les systèmes de santé de type « topping-up » lorsque l'assureur public et l'assureur privé utilisent un contrat linéaire. Dans le modèle, les consommateurs diffèrent par rapport à leur productivité marginale, l'offre de travail est endogène, la couverture publique est uniforme et les dépenses de santé sont financées par la taxation linéaire. Le résultat principal montre que la couverture publique optimale est négative et les consommateurs ont une couverture sub-optimale.

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1 Introduction

This paper focuses on the relationship between health insurers and consumers when insurers are both private and public. In particular it analyses the effect of *ex-post* moral hazard on the demand for care when consumers are covered by a mixed insurance scheme. The mixed system explicitly considered in this paper is the topping-up scheme (as opposed to the opting-out) which is actually the most widespread system in developed countries.[1] This system is characterized by public insurance covering a part of the individuals' health expenditure (a package of essentials), and a voluntary private policy topping up the remaining services.

As it is generally known, *ex-post* moral-hazard is a consequence of health insurance: insurance coverage reduces the marginal price of care and induces additional consumption. This inefficiency is obviously increased by the presence of supplementary coverage because the latter reduces the marginal price of care even more. Since the mid-1980s developed countries have significantly and progressively increased the role of voluntary insurance in their health systems, and some authors are skeptical about the way private policies have been introduced and mixed schemes have been designed. In particular Blomqvist and Johansson (1997), Selden (1997), Pauly (2000) argue that private supplementary coverage can have adverse spillover effects, increasing the cost of public coverage. But none of these authors has analyzed in detail the inefficiency induced by mixed health insurance coverage. With this respect this paper analyses the game where the public insurer moves first and perfectly anticipates the choice of insurance firms in the private market. It is shown that, at the equilibrium, the public insurer provides zero coverage and the second-best allocation is implemented.

As empirical evidence has shown in the case of the Medicare program in the US and of French mixed coverage (Wolfe and Goddeeris, 1991; Cartwright *et al.,* 1992; Ettner, 1997; Atherly, 2002; Buchmueller *et al.,* 2004 and references therein), another important problem connected with currently available mixed systems is that supplementary insurance is purchased by wealthier consumers. In particular, in many developed countries public coverage is uniform and limited and richer people complement public insurance with private coverage. As a result, supplementary insurance can seriously affect horizontal equity.

A recent literature has analyzed mixed schemes in order to investigate whether public coverage is welfare improving when consumers can supplement in the private market. Rochet (1991), Cremer and Pestieau (1996) and Henriet and Rochet (1998) have showed that this is true if public insurance redistributes both from the rich to the poor and from the low- to the high-risk, that is if the correlation between wage rates and morbidity is negative (which seems empirically verified). Petretto (1999) and Boadway *et al.* (2001) and (2004) have extended the previous models to the case of *ex-post* moral hazard (and, for the last work, adverse selection in the insurance market). Results are however ambiguous: the public insurer may want to set either a tax or a subsidy on health care expenses. Only with quasi-linear utility

1. The countries in which the opting-out system has developed are Germany, Ireland and the Netherlands. Most other countries are characterized by the topping-up system: Finland, France, Belgium, Sweden, the UK, the US, Canada and Australia.

functions and for a sufficiently high negative correlation between wage rates and morbidity, Boadway *et al.* (2001) and (2004) have found that a positive public coverage is welfare improving.

These papers have provided interesting insights. Nevertheless, some important questions are still open. How does moral hazard affect the optimal public coverage when a private one is available? What are the consequences of mixed coverage on redistribution? These questions are particularly relevant in two situations. First, when the difference in consumers' morbidity is negligible. Second, when the negative correlation between wage rates and morbidity is not sufficiently high (so that a positive public coverage does not allow to redistribute from low-risk / high-revenue to high-risk / low-revenue people). In order to provide an answer to the previous questions, this work focuses on *ex-post* moral hazard and considers heterogenous individuals: low- and high-revenue consumers.[2] It is shown that, when moral hazard is sufficiently high, the rich buy more private coverage, and thus over-consume more than the poor. As a consequence, the optimal public coverage is negative: health care consumption must be taxed to discourage private policy purchase.

Moreover, this model shows how reverse redistribution in public health care financing can arise. Let us assume that institutional and/or political constraints on public policy exist such that the public insurer *cannot* internalize, that consumers also buy the private policy and always supplies a positive public coverage. In addition to this, let us assume that the level of ex post moral hazard is sufficiently high. The result is that the rich net contribution to health care financing can be lower than the poor one, where *net contribution* is the fiscal revenue raised from a group minus the health care subsidy paid to such a group. This could explain the empirical evidence on inequity in access to health care in countries with supplementary insurance.

Organization of the Paper

In section 2 consumers are identical. A graphical representation of moral hazard and its effect on expected consumption of the composite good is proposed in section 2.1. Section 2.2 characterizes the demand for the private coverage and describes the negative externality on the public insurer's contract. In section 2.3, the equilibrium policies are derived when the public insurer plays the role of the Stackelberg leader (sub-game perfect equilibrium). Optimal policies when the game is simultaneous are briefly discussed. In section 2.4 the consequences of moral hazard are analyzed when the public insurer does not internalize the effect of private coverage on treatment consumption. In section 3 consumers' heterogeneity is introduced: individuals differ in their wage rate and labor supply is endogenous. In section 3.3 the optimal equilibrium policies are derived. Then, section 3.4 considers reverse redistribution that can arise when the public insurer does not internalize the private policy purchase. Section 4 concludes.

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2. Here consumers only differ in their wage rate and, in this sense, the framework is less general than Petretto (1999), Boadway *et al.* (2001) and (2004). However, increased simplicity allows an accurate analysis of the consumers' demand for supplementary private coverage. In particular, revenue effects in the demand for private insurance turn out to be crucial in explaining the redistributive role of mixed coverage.

2 The Model with Identical Consumers

In the following model a part of individuals' health expenditure is covered by public insurance, a part by private insurance, and the last part is out-of-pocket.

In this section consumers are identical and have mass 1. Their utility is a function of health, the benefits of the health care received, and the income available to be spent on other goods after the cost of treatment has been deducted. In this section W is exogenous. Consumers are ill with probability p. When ill, they are subject to a negative health shock whose monetary equivalent is $\overline{h}$. Health can be recovered according to a strictly concave function $h(x)$ representing the monetary benefits from health care consumption, where x denotes the quantity of treatment.[3] h is increasing in x and ranges from 0 to $\overline{h}$. The marginal productivity of x is decreasing and the third derivative of h is positive. The lower bound of x is zero and its upper bound is set at $\overline{x}$ such that $h'(\overline{x}) = 0$.[4] The standard assumption is that $h(x) < \overline{h}$ for every possible level of treatment consumption. Nevertheless, in this paper let us assume that $h(\overline{x}) = \overline{h}$, that is the upper bound for treatment implies complete recovery. This simplifies computation.

It is assumed that technology for medical treatment is linear and subject to constant returns to scale. Marginal cost is constant and normalized at one. Consumers directly purchase on the market the chosen quantity of treatment, which implies that the physician is acting as a perfect agent for his patients.

Using a strictly concave function $U(\cdot)$ to represent the risk-averse consumers' preferences, the expected utility without any insurance is:[5]

$$EU = pU\left[W - x - \overline{h} + h(x)\right] + (1-p)U(W) \tag{1}$$

Henceforth C_I will denote the consumer's composite commodity involving all goods other than health care when ill and C_0 the composite commodity when healthy. The indifference curves represent combinations of wealth in the two states of nature which yield constant expected utility. Indifference curves have slope $\frac{dC_I}{dC_o} = -\frac{1-p}{p}\frac{U'(C_0)}{U'(C_I)}$. From the consumer's budget constraint, whose slope is $-\frac{1-p}{p}$, expected wealth is $W - p\left[x + \overline{h} - h(x)\right]$. Let us define the *net monetary loss* due to illness as X, where $X \equiv x + \overline{h} - h(x)$.

When the consumer is not insured, he chooses his treatment consumption according to the first-order condition:

3. A *state-independent* utility function is also used in all the theoretical papers mentioned in the introduction except Petretto (1999). Even if this function does not allow to capture the irreplaceable property of health, it makes demand for health care treatable since no income effects are present.
4. An upper bound on treatment can be justified by limits on care imposed either by insurers, the government, or providers. Also there may be levels of care beyond which health no longer improves (Selden, 1993).
5. The same model is used in Barigozzi (2004) to analyze secondary prevention reimbursement. In that model the health recovery function $h(\cdot)$ depends both on treatment and secondary prevention.

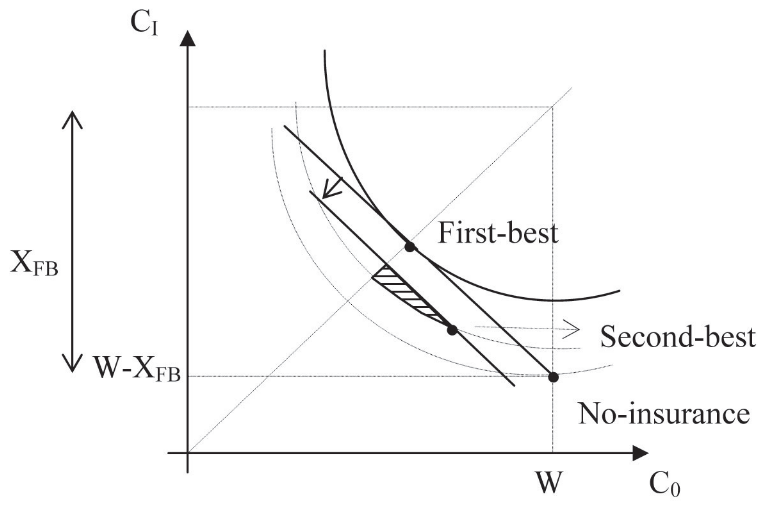
(2) $$h'(x) = 1$$

Since in the previous expression the marginal cost and the marginal benefit of treatment are equalized, from now on the amount of treatment verifying equation (2) will be referred as x_{FB}. Such an amount is the efficient one, in fact, as it will be discussed in few lines, it corresponds to the first-best consumption. Notice that there is no income effect in (2), treatment demand only depends on consumption price.

Before analyzing a standard insurance contract with co-payment, it is useful to consider the first-best allocation of this simple model. First-best insurance can be implemented when the insurance firm perfectly observes the consumer's state of health, in this case it can offer two monetary transfers contingent upon disease: a lump-sum contract. The consumer receives T_I in case of illness and T_0 when healthy, where $pT_I + (1-p)T_0 = 0$. Such a contract leads to full insurance $\left(C_I = C_0 = C^{FB}\right)$. As it is generally known, in full insurance the indifference curves are tangent to the budget constraint. With first-best insurance treatment price is not distorted and consumers choose the efficient quantity of treatment x_{FB}. This implies that, in first-best, consumption is equivalent to $C^{FB} = W - p\left[x_{FB} + \bar{h} - h(x_{FB})\right] = W - pX_{FB}$.

In figure 1 the two axes respectively indicate the composite good when the consumer is healthy (C_0) and when she is sick (C_I). In the figure the no-insurance and the first-best allocations are shown. Notice that, in the figure, the net monetary loss $X_{FB} = x_{FB} + \bar{h} - h(x_{FB})$ can be directly read on the vertical axis.

FIGURE 1
The efficient quantity of treatment and the second-best contract



Let us assume, now, that the illness status is not perfectly observable either by public or private insurers. As a consequence all insurers offer a contract where the indemnity is directly related to the health care costs. Notice that, in order to obtain reimbursement, consumers must generally show to the insurer a doctor's certification or a hospital/doctor's bill. As a consequence, it is reasonable to assume that health care consumption is *ex-post* verifiable such that non-linear contracts could be analyzed.[6] Nevertheless, for the sake of tractability, in this paper we analyze *linear* contracts.

Comments on Modelling Choices

In order to maintain the model as simple as possible we assume that the private market is competitive so that private insurance firms make zero profit. Thus, both the public and the private premiums are fair. In this setting, when the economy is characterized by a representative consumer, we assume that both the public and the private insurance contract are of the same type and defined by the premium and a cost-sharing parameter. In fact, there are no redistributive concerns for the public insurer so that a standard insurance contract suffices to illustrate the main trade-offs. On the contrary, when consumers differ in their revenue (section 3), we let the social planner use two types of policy instruments, a linear progressive income tax and social insurance (with a *uniform* cost-sharing parameter), so that concerns on redistribution can be properly addressed.[7] Indeed, considering a uniform public coverage financed by linear taxation allows to take into account both redistribution and the norm of universal and equal access to health care characterizing many European countries.

As for the timing for public and private insurers' decisions, the modeling option more appropriate for the purpose of this paper requires the public insurer moving first anticipating the reaction of private firms, *i.e.* a *sequential* timing.[8,9] A 'game theoretical' justification relies on the fact that the public insurer can credibly commit so as to ignore whatever strategy of the private one. When designing the optimal policy, the public insurer may well be constrained because of institutional and / or political reasons (see sections 2.4 and 3.4). This assumption can also be justified from an historical perspective. A common trend in the reform of health care systems in industrialized countries emerges with a clear shift from public financed systems to mixed one (mainly due to the huge pressure on government budget).

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6. Non-linear reimbursement has been investigated in Blomqvist (1997) where identical consumers purchase insurance from a unique firm and insurance coverage leads to moral hazard. Results show that, according to intuition, non-linear health insurance redistributes from those who are relatively well (and thus do not have large health expenditures) to those who are seriously ill and face large expenditures. In fact, while the formers have relatively low marginal utility of income, the latters have higher marginal utility. The optimal amount of redistribution depends on the price elasticity of demand for health care, that is on moral hazard. It is also optimal to impose a deductible for low health expenditures since moral hazard is high and benefits from insurance low.
7. The same assumption concerning the public insurance contract is in Petretto (1999) and Boadway *et al.* (2001) and (2004) where *ex-post* moral hazard is considered. Papers without moral hazard (Rochet 1991, Cremer and Pestieau 1996 and Henriet and Rochet 1998) also treat the case of a *non-linear* tax policy.
8. The sequential approach has been explicitly used in Petretto (1999), Boadway *et al.* (2001) and (2004); implicitly, it has been also used in Rochet (1991), Cremer and Pestieau (1996) and Henriet and Rochet (1998).
9. For the sake of completeness, in section 2.3 we will present a brief discussion on the *simultaneous* game between public and private insurers. A simultaneous timing has been considered in Blomqvist and Johansson (1997) and Selden (1997), where consumers are homogenous and health insurers identical.

2.1 With the Public Insurer only: the Second-Best

First of all, let us consider the case where only the public insurer offers a contract to the representative consumer. The public contract is denoted as (T,α), where T is the actuarially fair public premium $(T = p\alpha x)$ and α is a cost-sharing parameter. Hence, $(1-\alpha)$ is consumers' out-of-pocket expense when they buy one unit of treatment. With the contract (T,α), consumers' expected utility becomes:

$$EU = pU\left[W - T - (1-\alpha)x - \bar{h} + h(x)\right] + (1-p)U(W-T) \tag{3}$$

When choosing treatment quantity x^*, consumers take the premium T and the cost-sharing parameter α as given, such that:

$$x^* = x(\alpha): \qquad h'(x) = 1-\alpha \tag{4}$$

When the cost-sharing parameter α is positive, the consumption price for treatment decreases. This implies that $x^* > x_{FB}$: overconsumption of treatment arises. Here it is the problem of *ex-post* moral hazard in health insurance.[10] Moreover, by differentiating (4) it follows that $\frac{\partial x}{\partial \alpha} = -\frac{1}{h''(x)} > 0$.[11] Thus, the higher is insurance coverage, the higher is over-consumption. With a slight abuse of language, from now on $\frac{\partial x}{\partial \alpha}$ will be often referred as the *level of moral hazard* caused by the insurance coverage α.

In order to optimally choose T and α, the public insurer takes into account the choice of treatment made by consumers and solves the following program:

$$\text{(P1)} \quad \begin{cases} \max\limits_{T,\alpha} EU = pU\left[W - T - (1-\alpha)x - \bar{h} + h(x)\right] + (1-p)U(W-T) \\ \\ s.t.: T = p\alpha x \\ \qquad h'(x) = 1-\alpha \end{cases}$$

Treatment demand does not depend either on revenue or on the composite good in the illness status, therefore we can substitute the budget constraint into the public insurer's objective function. From the first order condition with respect to α, an implicit expression for the cost-sharing parameter in second-best can be found:

$$\alpha^{SB} = \frac{(1-p)x\left[U'(C_I) - U'(C_0)\right]}{\frac{\partial x}{\partial \alpha} E\left[U'(C)\right]} \tag{5}$$

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10. A number of empirical studies have analyzed the impact of cost-sharing on the consumption of health care. Since insurance is thought to increase demand for health care by reducing its marginal price, the price elasticity of demand for care is directly relevant to the moral hazard effect. To date, the most important empirical study is The Rand Health Insurance Experiment which estimated, on average, the elasticity of demand at -0.2 (Manning *et al.,* 1987).
11. See also the first lines of appendix 1.

where $E\left[U'(C)\right] \equiv pU'(C_I)+(1-p)U'(C_0)$ is expected marginal consumption of the composite good.[12]

DEFINITION 1. $\varepsilon_{x,\alpha} \equiv \frac{\partial x}{\partial \alpha}\frac{\alpha}{x} > 0$ *is the coverage elasticity of treatment demand.*

DEFINITION 2. $\pi(x) \equiv -x\frac{h'''(x)}{h''(x)} > 0$.[13]

LEMMA 1. ***(Concavity)*** *a sufficient condition for problem P1 to be concave in* α *is* $\varepsilon_{x,\alpha} > \frac{1}{p}\left(\pi(x)\right)^{-1}$.

Proof. See the appendix A.1.

Notice that the sufficient condition in lemma 1 implies that problem P1 is well-behaved when moral hazard is sufficiently high.

LEMMA 2. ***(Second-best)*** *when only one insurer provides coverage and consumers are homogeneous, the second-best allocation is obtained. The second-best coverage* α^{SB} *is positive and lower than one. Moreover, the lower is moral-hazard, and the higher is consumer's risk aversion, the higher is* α^{SB}.

Proof. Let us consider equation (5). In general no over-insurance $(\alpha > 1)$ *can arise because for* $x > \overline{x}$ *the marginal benefit from treatment becomes negative. Thus,* $C_0 \geq C_I$ *and* $U'(C_I) - U'(C_0) \geq 0$ *holds. The higher is risk aversion, the higher is the difference between the two marginal utilities. Moreover,* $\frac{\partial x}{\partial \alpha} > 0$*, such that the cost-sharing parameter* α *is positive. Finally, since coverage is inversely related to* $\frac{\partial x}{\partial \alpha}$*, the lower is the level of moral-hazard, the higher is* α^{SB}.

Now let us examine the consequence of moral hazard on the expected composite good $W - pX$.

REMARK 1. *Under ex-post moral hazard, the insurance coverage reduces expected consumption of the composite commodity.*

Proof. Under the contract (T,α) *the expected composite good becomes* $W - p\left[x^* + \overline{h} - h\left(x^*\right)\right] = W - pX^*$. *Whereas, without any insurance coverage, the expected composite good is* $W - pX_{FB}$. *The function* $h(\cdot)$ *is concave and* $x^* > x_{FB}$, *thus* $X^* = x^* + \overline{h} - h\left(x^*\right) > X_{FB} = x_{FB} + \overline{h} - h\left(x_{FB}\right)$ *and* $W - pX^* < W - pX_{FB}$.

12. Notice that, because of moral-hazard, neither $\alpha = 0$ nor $\alpha = 1$ are possible solutions for equation (5). In fact, for $\alpha = 0$, it is $C_I = W - x_{FB} - \overline{h} + h(x_{FB}) = W - X_{FB}$ and $C_0 = W$, such that $C_0 > C_I$. While, for $\alpha = 1$, $C_I = W - T - \overline{h} + h(\overline{x}) = W - p\overline{x}$ and $C_0 = W - p\overline{x}$, such that $C_I = C_0 = C_I$, where FI stands for full-insurance.

13. The function $\pi(x) \equiv -x\frac{h'''(x)}{h''(x)}$ recalls the index of relative prudence for the *utility* functions. The economic interpretation in term of the *recovery* function h(x) is difficult to establish.

Remark 1 implies that the consumers' budget constraint shifts down when insurance coverage is purchased. Figure 1 shows the new budget constraint and the consumers' allocation (the second-best) under the contract (T,α). The following equation, directly coming from (5), can be interpreted in term of the trade-off between risk-spreading and efficiency:

$$\varepsilon_{x,\alpha^*} = \frac{(1-p)\left[U'(C_I)-U'(C_0)\right]}{E\left[U'(C)\right]} \tag{6}$$

The coverage elasticity of treatment demand, *i.e.* the level of moral hazard, indicates the cost of insurance *for the representative consumer*. In fact, the coverage α leads to over-consumption: the expected composite good decreases and the budget constraint moves down; as a result, consumers' utility falls. On the other hand, the consumer, moving to the left *on* his budget constraint (see figure 1), reaches partial insurance and his utility increases. Thus, (6) shows that the optimal cost-sharing parameter α is such that the marginal cost of insurance coverage (the left hand side) is equal to the marginal benefit (the right hand side). Obviously, in second-best the consumers' utility is lower than in first-best, but it is higher than in the absence of insurance.

Some comparative statics concerning the cost-sharing parameter will be particularly useful in section 3.

LEMMA 3. ***(Insurance coverage as a normal good)*** *$U'''(\cdot)<0$ is a sufficient condition for insurance coverage to be a normal good. Whereas, if $U'''(\cdot)>0$, a necessary condition is:*

$$C.1: \quad \varepsilon_{x,\alpha} > \frac{(1-p)\left[U''(C_I)-U''(C_0)\right]}{E\left[U''(C)\right]}$$

If $U'''(\cdot)>0$ and the opposite of $C.1$ holds, than insurance coverage is an inferior good.

Proof. See the appendix A.2.

The standard assumption in Decision Theory is that $U'''(\cdot)>0$. Without moral hazard, when marginal utility is convex, the lower is consumers' revenue and the higher are risk-aversion and the demand for insurance. In fact, the lower is the revenue, the higher is marginal utility from increasing consumption of the composite good in the "bad" state of nature. For this reason insurance is generally considered an "inferior good".[14] On the contrary, concerning supplementary insurance, as mentioned in the introduction, empirical evidence shows that the rich buy more private coverage than the poor. The previous lemma can explain such empirical

14. See, as an example, Briys *et al*. (1989) who show how changes in consumers' initial wealth affect the optimal insurance coverage. In particular, the optimal coverage decreases with revenue if and only if the Arrow-Pratt index of absolute risk aversion is a non increasing function.

evidence. *When moral hazard is sufficiently high, insurance becomes a normal good.* In fact, a high level of moral hazard implies that an increase in coverage leads to a large increase in premium. The premium is paid in both states of nature and, when it is high, it brings to an important fall in expected consumption of the composite good. In such a case, the lower is the revenue, the higher is the marginal cost from decreasing consumption of the composite good in both states of nature.

On the contrary, when $U'''(\cdot) < 0$, marginal utility is concave such that the higher is consumers' revenue, the higher are risk-aversion and the demand for insurance. Thus, whatever the level of moral hazard, the rich buy more insurance coverage than the poor.

Later on the standard case $U'''(\cdot) > 0$ will be considered.

2.2 The Private Insurance Market

Let us assume that private firms offer a contracts (P, β) where P is the premium and β is the cost-sharing parameter. Let us also assume that the private market is competitive so that insurance firms make zero profit and the premium P is actuarially fair. Later on, for the sake of exposition, the representative firm in the insurance market will be referred as *the* private *insurer*.

With mixed coverage, consumers' expected utility becomes:

$$pU\left[W - T - P - (1-\alpha-\beta)x - \bar{h} + h(x)\right] + (1-p)U(W - T - P)$$

Now, the purchased quantity of treatment x^{**} is determined by:

$$x^{**} = x(\alpha+\beta): \qquad h'(x) = 1 - \alpha - \beta \tag{7}$$

With the private coverage, the marginal cost of treatment decreases, it follows that $x^{**} > x^{*}$: over-consumption increases and the consumers' budget constraint shifts down even more.

The private insurer takes both T and α as given. This is true whatever the assumption on the timing for public and private insurer's decision. In particular, the private firm takes as given the public insurer's contract both when the latter moves first and when the two insurers play simultaneously.

When the private insurance market is perfectly competitive, the private firm solves problem (P2) below.

$$\text{(P2)} \quad \begin{cases} \max\limits_{P,\beta} pU\left[W - T - P - (1-\alpha-\beta)x - \bar{h} + h(x)\right] + (1-p)U(W - T - P) \\ \\ s.t.: P = p\beta x \\ \qquad h'(x) = 1 - \alpha - \beta \end{cases}$$

Substituting the premium P in the objective function and rearranging the first-order condition with respect to β, it follows:

$$pxU'(C_I) = p\left(x + \beta\frac{\partial x}{\partial \beta}\right)E\left[U'(C)\right] \tag{8}$$

where x, C_I and C_0 depend, now, on the mixed coverage. The left-hand side of (8) represents marginal benefit and the right-hand side marginal cost from an increase in coverage. In fact, a higher coverage β leads to more treatment consumption and more recovery in the illness status (left-hand sides), while it leads to less consumption of the composite good in both states of nature because the insurance premium increases (right-hand sides).

Some considerations on the negative externality that the private coverage exerts on the public contract are valuable at this stage. Rearranging (8) it is possible to calculate the derivative of the consumer's expected utility with respect to β when $\beta = 0$:

$$\left.\frac{\partial EU}{\partial \beta}\right|_{\beta=0} = p(1-p)x\left[U'(C_I) - U'(C_0)\right] > 0. \tag{9}$$

REMARK 2. Given a *level of coverage, consumers are better off if they can buy some more coverage from another insurer.*

Considering the consumer's point of view, this shows that, once the public contract (α, T) is established, a positive private coverage is welfare-improving. A new contract, which brings the consumer into the shadow area of figure 1, increases his expected utility. However, as shown in the next section, a mixed coverage with both α and β non-negative is inefficient. Remark 2 holds because in program (P2) the public premium T is considered as given. In particular, the private insurer does not internalize that an increase in the private coverage β makes treatment demand increase which, in tour, makes the public premium T raise. Thus, consumption of the composite good decreases in both states of nature. As a result, too much coverage is offered by the private firm and a negative pecuniary externality on the public insurer's contract is produced. Notice that, if the pecuniary externality is internalized, first-order condition (8) becomes:

$$pxU'(C_I) = p\left(x + \beta\frac{\partial x}{\partial \beta} + \alpha\frac{\partial x}{\partial \beta}\right)E\left[U'(C)\right] \tag{10}$$

The right-hand side of (10) takes into account the *total* marginal cost of the private coverage. This implies that the efficient value of private coverage given by (10) is lower than the one determined from (8).[15]

15. Since $\frac{\partial x}{\partial \beta} = \frac{\partial x}{\partial \alpha}$, when we set $\beta = 0$ in (10), the latter equation becomes equivalent to the first-order condition of problem (P1). This implies that, when the externality is internalized, the expression (9) gives, as expected, $\left.\frac{\partial EU}{\partial \beta}\right|_{\beta=0} = 0$.

Before analyzing the sequential game in section 2.3, some considerations about the interaction between the private and the public coverage are useful. What follows is both interesting *per se* and necessary to characterize the equilibrium of the sequential game with heterogenous consumers in section 3.3.

The higher is total coverage $(\alpha+\beta)$, the higher is overconsumption of care. As a consequence, using the terminology of Industrial Organization, we expect that private and public coverage are *strategic substitutes*: when α increases, private coverage becomes less attractive for consumers $\left(\frac{\partial^2 EU}{\partial\alpha\partial\beta}<0\right)$.

LEMMA 4. *When the condition in lemma 1 holds,* $\varepsilon_{x,\alpha}>\frac{1}{p}$ *is a sufficient condition for public and private coverage to be strategic substitutes.*

Proof. See the appendix A.3.

In this model with no revenue effects in treatment demand, the coverage elasticity of demand $\varepsilon_{x,\alpha}$ represents the 'pure' welfare loss of price induced moral hazard. Nyman (1999) argues that an important but often neglected gain from cost-sharing insurance policy is the income transfer effect that results in additional consumption of medical care and other goods and services when ill. This income effect is related to redistribution from those who remain healthy to those who become ill and can be represented by the term $\frac{1}{p}$. The condition $\varepsilon_{x,\alpha}>\frac{1}{p}$ thus indicates that the cost of pure moral hazard is larger than the benefit from insurance and partial coverage must be used to curb overconsumption of care. As a consequence, when moral hazard is sufficiently high, increasing public coverage reduces the benefit from private insurance: public coverage leads to a (partial) crowding-out of private insurance (see also proposition 2). Finally, notice that when $\pi(x)$ is lower than one, the condition in lemma 4 implies that in lemma 1. Otherwise, the condition in lemma 1 is sufficient both for concavity and strategic substitutability between public and private coverage.[16]

2.3 The Optimal Mixed Coverage

As was discussed before, analyzing the sequential game between the public and the private insurer it is reasonable to attribute the first move to the public insurer. The game is as follows: in *stage 1* the public insurer chooses its policy (T,α) without observing either consumers' demand for treatment or private coverage. The public insurer anticipates the effect of its policies both on the insurance market and on consumers' behavior. In *stage 2* the competitive insurance industry sells contracts (P,β) to consumers. Profits are zero such that the premium P is fair. In this stage, (T,α) are considered as given and consumers' behavior is correctly anticipated. In *stage 3* consumers choose the treatment quantity given (T,P,α,β). The equilibrium is assumed to be sub-game perfect.

16. For the logaritmic function, π (x) is equal to 1 and the two conditions are equivalent.

PROPOSITION 1. *When consumers are homogeneous and insurers play sequentially, the optimal public coverage is zero. Private coverage corresponds to the second-best coverage.*

Proof. See the appendix A.4.

The intuition for the previous proposition is as follows. How it was emphasized by remark 2, the pecuniary externality arises because the private firm takes the public contract (T, α) as given. Thus, even if the public insurer correctly anticipates consumers' behavior, when a positive public coverage is offered, the externality always leads to over-insurance with respect to the second-best. The second-best allocation can be obtained only if the public coverage is zero. In other words, the public insurer anticipates that the private firm reaction function is given by (8). By setting $\alpha = 0$ the pecuniary externality is completely internalized: (8) and (10) become equivalent.

Otherwise, when the insurers play simultaneously, both offer a positive coverage and the consumer is over-insured (see the appendix 5.5). This proves that, contrary to the Stackelberg equilibrium, the Nash equilibrium is inefficient. In fact, in the simultaneous game, two pecuniary externalities arise: both insurers take the premium of the other as given. As a result the marginal cost of coverage is underestimated *by the two* insurers (this can be easily seen by comparing equations (8) and (10) for both insurers' reaction function).[17]

2.4 Mixed Coverage with Constrained Public Insurer

In this section the public insurer does not anticipate that the consumer will buy a private coverage so that it does not act strategically. Such behavior can be the consequence of political or institutional constraints which are not explicitly modelled in this analysis but are important in reality.

As shown in the previous section, when consumers are homogeneous it would be optimal to provide a zero public coverage. In alternative, when consumers are heterogenous and moral hazard is high, it would be optimal to provide a negative public coverage (see section 3.3). However, such optimal policies are not easy to implement. Even though in the last decades fiscal pressure has led many governments to reduce public coverage and boost private supplementary insurance, it may be difficult for these governments to obtain the political consensus to cease the provision of public insurance or to tax treatment.

When the public insurer is constrained to ignore consumer's purchase of supplementary insurance, it solves problem (P1) as in section 2.1, while the private firm solve problem (P2).

17. Blomqvist and Johansson (1997) do not solve the simultaneous game, anyway they claim that "mixed equilibrium leads to lower welfare than the second-best equilibrium" (page 512). Appendix A.5 provides a detailed analysis of the inefficiency caused by the mixed coverage.

REMARK 3. *When the public insurer is constrained, consumers are over-insured. Moreover, the lower is moral-hazard, and the higher is consumer's risk aversion, the higher is private coverage* β.

Proof. From lemma 2 the public insurer sets $\alpha = \alpha^{SB}$. *From remark 2 the private coverage* β *is positive and verifies condition (8). Total coverage is* $\alpha^{SB} + \beta$.

An important point concerns the externality inflicted on the public insurer by the private coverage. Here we have *two different* pecuniary externalities. The first externality is the one analyzed in section 2.2: the private insurer takes the public premium T as given. The second externality arises because the public insurer does not anticipate the effect of the private coverage on treatment choice. In particular the premium T is calculated upon x^* while, under mixed coverage, the expected consumption is $p\alpha^{SB}x^{**} > T = p\alpha^{SB}x^*$. Thus, the public premium T does not pay the public insurer for the expected cost of treatment: the public insurer faces a budget deficit equal to $p\alpha^{SB}\left(x^{**} - x^*\right)$. Notice that $C_0 = W - T - P$ and $C_I = W - T - P - \left(1 - \alpha^{SB} - \beta\right)x^{**} - \bar{h} + h\left(x^{**}\right)$ where $T = p\alpha^{SB}x^*$ and $P = p\beta x^{**}$. As a result, with mixed coverage expected consumption of the composite good is $W - pX^{**} + p\alpha^{SB}\left(x^{**} - x^*\right)$ instead of $W - pX^{**}$, and the budget constraint moves down less than it should.

REMARK 4. *When the public insurer is constrained, expected consumption of the composite good increases of the amount* $p\alpha^{SB}\left(x^{**} - x^*\right)$. *Such an amount corresponds to a public budget deficit.*[18]

The previous environment could describe some mixed insurance schemes implemented in the real world, the French system being a major example[19], together with Medicare in the US[20]. The analysis in this section shows that the problem with supplementary insurance is not just that it leads to more overconsumption of care. More important is the (second) externality which arises if the public insurer does not anticipate the effects of the private coverage on treatment choice. In particular, such externality makes consumers better off (at least in the short run), as stated by remark 4. If the public contract is not accurately modified, the introduction of supplementary insurance leads to public deficit, a problem the governments are extremely concerned about and which, paradoxically, has frequently motivated the introduction of a private policy.

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18. Given the aim of this section, we do not explicitly discuss the issue of financing this deficit. Instead we discuss the consequences of a constrained public policy that could describe the choice of some governments in the real world.
19. The "ticket modérateur" $(1 - \alpha)$ has always played a great role throughout the history of French health insurance scheme. It corresponds, on average, to the 25% of consumers' health expenditures. However, 86% of the population have private insurance that pays all or part of patients' share of the costs, thus lessening its impact. It is often observed that the French public system is facing chronic deficits.
20. More than half Medicare enrollees have private complementary insurance. In particular, in 2001, 23% of Medicare beneficiaries purchased Medigap coverage.

3 Heterogeneous Consumers

In order to investigate the redistributive implications of supplementary insurance under moral hazard, consumer's heterogeneity is here introduced with respect to wage rates. Consumers are characterized by two different productivity levels and their expected utility now is:

$$pU\left[w_i l_i - x_i - \bar{h} + h(x_i) - v(l_i)\right] + (1-p)U\left[w_i l_i - v(l_i)\right]$$

where $i = L, H$ and $w_H > w_L$ are the wage rates. The proportion of high- and low-income individuals in this economy is respectively λ_H and $\lambda_L = 1 - \lambda_H$. The function $v(\cdot)$ represents disutility from labor supply l_i and is increasing and strictly convex. Given that no adverse selection is considered, in order to simplify the notation high and low income groups are characterized by the same risk of illness *p*. Moreover, when ill, both income groups suffer the same exogenous monetary loss $\bar{h}$ and they benefit from health care consumption according to the same function $h(x_i)$.

As was discussed in section 2, in the case of heterogenous consumers it is plausible to assume that the public and the private insurers are different. On one side, public coverage is uniform while the private one is not. On the other side, public coverage is financed through linear taxation, while the private one is financed by a type-dependent premium.

As for the informational structure of the model we follow the optimal income taxation literature. Only individual labour income $w_i l_i$ is taken into account by the government when designing the taxation function used to finance public insurance. Almost all the literature on social insurance and redistribution mentioned in the introduction follows this approach. The public insurer does not observe either consumers' health status and wage rates, or individual demands for aggregated consumption, leisure or insurance. Income $w_i l_i$, preferences, and the distribution of individuals by type *i* are observable. The public insurer finances the uniform (linear) subsidy α with a linear tax on income. Thus, the public insurer's instruments are (t, G, α), where *t* is the linear tax and *G* is a lump sum transfer. The public insurer maximizes an utilitarian social welfare function and wants to redistribute from high- to low-types.

As before, the competitive insurance industry sells private contracts (P_i, β_i) to consumers. Private insurers do not observe consumers' health status.[21] Profits are zero such that private premiums are fair $(P_i = p\beta_i x_i)$.

21. In the literature on social insurance and redistribution, insurance firms are better informed than the government. This assumption ensures that private firms provide insurance efficiently, so that government intervention is not motivated by market failures. Using the same approach, this model shows that government intervention in the insurance market is welfare improving when consumers are heterogeneous and the government is concerned by redistribution. In particular, since the rich buy more private coverage and consume more treatment than the poor, a linear tax on health care consumption increases redistribution.

Given (t,G,α,β_i,P_i) consumers maximize their utility with respect to labor and treatment:

$$\max_{x_i,l_i} pU\left[(1-t)w_il_i + G - P_i - (1-\alpha-\beta_i)x_i - \bar{h} + h(x_i) - v(l_i)\right]$$
$$+(1-p)U\left[(1-t)w_il_i + G - P_i - v(L_i)\right]$$

Considering consumers' first-order conditions, labor supplies and treatment demand respectively verify:

$$l_i^I = l_i^0 = l_i^*(w_i,t): \qquad (1-t)w_i = v'(l_i) \tag{11}$$

$$x_i^{**} = x_i(\alpha+\beta_i): \qquad h'(x_i) = 1-\alpha-\beta_i \tag{12}$$

Labor supply is the same in both health status and is, obviously, negatively affected by the tax t $\left(\frac{\partial l_i}{\partial t} < 0\right)$. Moreover, more productive consumers supply more labor $(l_H > l_L)$ and have a higher post-tax revenue: $W_H \equiv (1-t)w_H l_H$. As before, there are no income effects in the demand for treatment, as a consequence, *if consumers are not insured or if they have the same private coverage, both types choose the same quantity of treatment*.[22]

Let us consider the utilitarian optimum of the model. When the public insurer observes both the consumers' type and the health status, he solves the following problem:

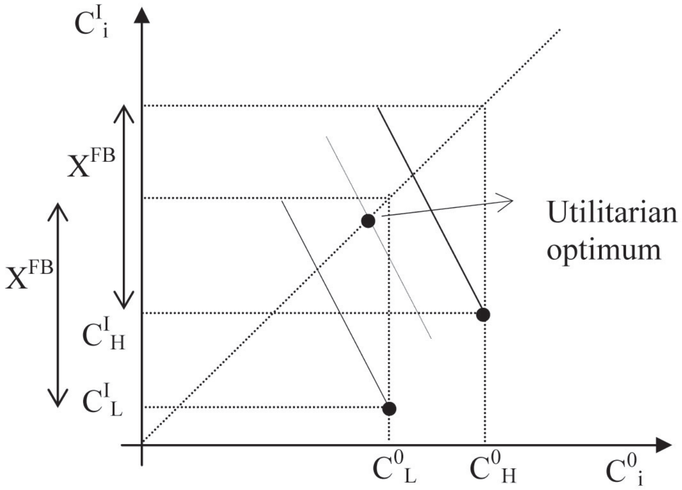
$$\begin{cases} \max\limits_{T_i^I,T_i^0,l_i,x_i} \sum_i \lambda_i \left\{ pU\left[w_il_i + T_i^I - x_i - \bar{h} + h(x_i) - v(l_i)\right]\right\} \\ \qquad\qquad +(1-p)U\left[w_il_i + T_i^0 - v(l_i)\right] \\ \\ s.t.: \qquad p\sum_i \lambda_i T_i^I + (1-p)\sum_i \lambda_i T_i^0 = 0 \end{cases}$$

Obviously there is no role for the private market because the utilitarian optimum leads to full insurance: $C_i^I = C_i^0$ for $i = L,H$. Moreover, $0 > T_L^0 > T_H^0$ and $T_L^I > T_H^I > 0$, that is, the high-type consumers pay a higher premium in good health and receive a lower transfer when ill.

22. Notice that income effects in treatment demand would reinforce the results. In fact, here the focus is on the case where coverage is a normal good: the high-income group buys more coverage and consumes more treatment.

As in the case without insurance, labor supply and treatment quantity are not distorted: l_i^{FB} is such that $w_i = v'(l_i)$ for $i = L, H$, and x^{FB} is such that $1 = h'(x)$. Note that, because $\frac{\partial}{\partial w_i}\left[w_i l_i^*(w_i) - v\left(l_i^*(w_i)\right)\right] > 0$, in the no-insurance case $C_H^0 > C_L^0$ and $C_H^I > C_L^I$, for $i = L, H$. The no-insurance allocation and the utilitarian optimum are represented in figure 2.

FIGURE 2



When the dimension of the low-income group λ_L and/or the wage difference $(w_H - w_L)$ are sufficiently high, or when the risk aversion is sufficiently low, the high-income group is better off with the no-insurance allocation than with the utilitarian optimum.

3.1 With the Public Insurer only

When only the public insurer provides coverage, it solves the following problem:

$$\text{(P3)} \quad \begin{cases} \max\limits_{t,G,\alpha} \sum_i \lambda_i \left\{ pU\left[(1-t)w_i l_i + G - (1-\alpha)x - \bar{h} + h(x) - v(l_i)\right] \right\} \\ \qquad + (1-p)U\left[(1-t)w_i l_i + G - v(L_i)\right] \\ \\ s.t.: \quad t\sum_i \lambda_i w_i l_i - G - p\alpha x = 0 \qquad (\delta) \\ \qquad\quad h'(x) = 1 - \alpha \end{cases}$$

where δ is the Lagrangian multiplier for the budget constraint. Notice that, since public coverage is uniform, both income groups consume the same treatment quantity. From first-order conditions with respect to G and α:

(13)
$$\bar{\alpha} = \frac{(1-p)x\sum_i \lambda_i \left[U'\left(C_i^I\right) - U'\left(C_i^0\right)\right]}{\frac{\partial x}{\partial \alpha}\sum_i \lambda_i E\left[U_i'(C)\right]}$$

The optimal public coverage $\bar{\alpha}$ depends on the *average* difference between marginal utilities. All the considerations on the cost-sharing parameter α made in lemma 2 also hold here. In particular $\bar{\alpha}$ is positive and lower than one. However, notice that, since consumers are heterogeneous, the public coverage $\bar{\alpha}$ corresponds here to a 'third best'. In fact, not only lump sum transfers contingent to the health status are impossible, the optimal public coverage is also derived under a uniformity constraint.

3.2 The Private Insurance Market

Given (t, G, α) and anticipating the consumers' choice, private insurers solve the following program:

(P4)
$$\begin{cases} \max_{\beta_i, P_i} pU\left[(1-t)w_i l_i + G - P_i - (1-\alpha-\beta_i)x_i - \bar{h} + h(x_i) - v(l_i)\right] \\ \qquad + (1-p)U\left[(1-t)w_i l_i + G - P_i - v(L_i)\right] \\ \\ s.t.: \quad P_i = p\beta_i x_i \\ \qquad\quad h'(x_i) = 1-\alpha-\beta_i \end{cases}$$

Remark 2 still holds: *consumers always choose a positive private coverage*. Substituting P_i in the objective function of P4 and calculating the first-order condition with respect to β_i, private coverage verifies:

(14)
$$\beta_i = \frac{(1-p)x_i\left[U'\left(C_i^I\right) - U'\left(C_i^0\right)\right]}{\frac{\partial x_i}{\partial \beta_i} E\left[U'(C_i)\right]}$$

Corollary 1 directly follows from lemma 3 and remark 2.

COROLLARY 1. *Let us assume that* $U'''(\cdot) > 0$. *When moral hazard is sufficiently high such that* $C.1$ *holds, the high-income group buys more private coverage than the low-income one* $(\beta_H > \beta_L)$. *When moral hazard is low such that* $C.1$ *is not satisfied, the opposite holds* $(\beta_L > \beta_H)$.

Empirical evidence shows that moral hazard is high in the case of ambulatory care and specialist services. Corollary 1 suggests that the rich are likely to buy more supplementary coverage for such services than the poor. This is in line with Doorslaer *et al*. (2000) who find a pro-rich bias in the use of specialist services. The previous considerations prove that the debate on general practitioners as gatekeeper in regulating access to specialist services is relevant also for equity reasons.

3.3 The Optimal Mixed Coverage

Again the public insurer moves first and correctly anticipates the second stage in which private firms offer private coverage. The following proposition can be stated.

PROPOSITION 2. *When consumers are heterogeneous, insurers play sequentially and condition C.1 holds: (i) the optimal uniform public coverage is negative, (ii) the high-income group purchases more private coverage and is better insured than the low-income one; both groups are under-insured.*

Proof. (i) See the appendix A.6. (ii) According to first-order condition (14) and corollary 1, $\beta_H > \beta_L > 0$. *As shown in the appendix A.6, public coverage leads to less than complete crowding-out of private insurance* $\left(-1 < \frac{d\beta_i}{d\alpha} < 0\right)$. *This implies that the sign of* $\frac{\partial(\alpha+\beta_i)}{\partial\alpha} = 1 + \frac{d\beta_i}{d\alpha}$ *is positive. Thus, even if a reduction in public coverage* α *makes private coverage* β_i *increase,* aggregate *coverage* $\alpha + \beta_i$ *always decreases. As a result each income group is less insured than with the optimal uniform public coverage* $\bar{\alpha}$.

The result stated by proposition 2 is not surprising. The public insurer's objective is to redistribute from the high- to the low-income group and insurance coverage is provided by the private market as well. Moreover, public coverage is constrained to be uniform whereas private coverage is type-dependent; this implies that the latter is a more efficient instrument to smooth consumption. It was assumed that moral hazard is sufficiently high to make the high-income group buy more coverage. As a result, high-revenue consumers are better insured and purchase more treatment. A uniform positive public coverage would favor the high-income group more than the low-income one. By taxing health care expenses, on the contrary, the public insurer indirectly taxes private coverage purchase and increases the level of redistribution.

3.4 Mixed Coverage with Constrained Public Insurer: Reverse Redistribution

As in subsection 2.4, let us consider the consequences of moral hazard when the public insurer does not anticipate that consumers purchase a private coverage. Public coverage $\bar{\alpha}$ verifies (13). The public insurer's budget constraint is as in program P3: $t\Sigma_i \lambda_i w_i l_i - G - p\bar{\alpha}x^* = 0$ where $x^* = x(\bar{\alpha})$.

REMARK 5. *Under condition C.1, when the public insurer is constrained, all consumers are over-insured. However the high-income group buys more coverage than the low-income one. Thus, the high-income group overconsumes treatment more than the low-income one.*

Proof. The public insurer sets $\alpha = \overline{\alpha}$. *From remark 2 and corollary 1, when condition C.1 holds the private coverage* β_i *is such that* $\beta_H > \beta_L > 0$. *Total coverage is* $\overline{\alpha} + \beta_i$ *and the rich are more insured than the poor. Since* $x_i^{**} = x(\overline{\alpha} + \beta_i)$, $x_H^{**} > x_L^{**} > x^*$*: the rich overconsume more than the poor.*

Let us consider again the externality inflicted on the public insurer. The tax t is not high enough to cover health care costs: the public insurer faces a budget deficit equal to $p\overline{\alpha}\left[\lambda_H\left(x_H^{**} - x^*\right) + \lambda_L\left(x_L^{**} - x^*\right)\right]$. Notice that *reverse redistribution* arises if:

$$tw_H l_H - p\overline{\alpha}x_H^{**} < tw_L l_L - p\overline{\alpha}x_L^{**} \tag{15}$$

where $tw_i l_i - p\overline{\alpha}x_i^{**}$ is one group's *net contribution* to health care financing, that is the fiscal revenue raised from that group minus the *effective* health care subsidy $p\overline{\alpha}x_i^{**}$ paid to such a group. Thus, reverse redistribution means that the high-income group's net contribution is lower than the low-income group's one.[23] Rearranging (15): $t\left(w_H l_H - w_L l_L\right) < p\overline{\alpha}\left(x_H^{**} - x_L^{**}\right)$. Thus, the higher is moral hazard and/or the lower is the wage rate difference and the more likely is reverse redistribution. Notice that this analysis does not consider income effects in the demand for treatment. Income effects would increase the difference $\left(x_H^{**} - x_L^{**}\right)$ and make reverse redistribution even more likely.

Some recent works present supplementary coverage as one source of the increase in rich people's medical expenses and the cause of serious inequity in the delivery of medical care (Doorslaer *et al.* 2000, Atherly 2002, Buchmueller *et al.* 2004)[24]. As stated above, this seems to be true in particular for specialist services whose demand is more elastic with respect to coverage and it is characterized by higher income effects.

As mentioned in the introduction, Petretto (1999), Boadway *et al.* (2002), (2004) showed that, when the negative correlation between wage rate and morbidity is sufficiently high, it is optimal to set a subsidy on health care expenses. On the contrary, when the negative correlation between wage rate and morbidity is not sufficiently high, the social planner should tax health expenses. In the real world, mixed health insurance schemes are characterized by a positive public coverage and we do not know whether this corresponds to an efficient coverage given a high correla-

23. Public deficit will be presumably financed in subsequent periods. If a lump sum tax will be used or if consumers live only one period, reverse redistribution is not affected by future taxation.
24. Doorslaer *et al.* (2000), page 572, write: "higher income groups may have better or quicker access to certain services because they are more likely to have supplementary private insurance cover, as in Finland, Sweden, the UK and in the US for Medicare patients".

tion between wage rate and morbidity or whether it represents an extremely inefficient policy due to a constrained public insurer. The above mentioned empirical evidence about inequity in the consumption of medical care could suggest that the latter is the case. This section offers a possible theoretical explanation of reverse redistribution.

Let us consider consumption of the composite commodity in the two states of nature: $C_i^0 = (1-t)w_i l_i + G - P_i - v(l_i)$
and $C_i^I = (1-t)w_i l_i + G - P_i - v(l_i) - (1-\bar{\alpha}-\beta)x_i^{**} - \bar{h} + h(x_i^{**})$
where $G = tE(wl) - p\bar{\alpha}x^*$ and $P_i = p\beta_i x_i^{**}$.
Expected consumption is $(1-t)w_i l_i + tE(wl) - v(l_i) - pX_i^{**} + p\bar{\alpha}(x_i^{**} - x^*)$ instead of $(1-t)w_i l_i + tE(wl) - v(l_i) - pX_i^{**}$ and, again, consumers' budget constraint moves down less than it should for both income groups. However the externality favors the high-income group more than the low-income one.

REMARK 6. *Under condition C.*1*, when the public insurer is constrained, the pecuniary externality imposed by each income group on the public insurer is:* $p\bar{\alpha}(x_i^{**} - x^*)$*, where* $p\bar{\alpha}(x_H^{**} - x^*) > p\bar{\alpha}(x_L^{**} - x^*)$*. Reverse redistribution (*$tw_H l_H - p\bar{\alpha}x_H^{**} < tw_L l_L - p\bar{\alpha}x_L^{**}$*) arises if moral hazard is sufficiently high and/or the wage rate difference is sufficiently low.*

4 Concluding Remarks

This paper investigates the topping-up scheme in health insurance when both public and private insurers use linear contracts and play sequentially. The insurance relationship is characterized by *ex-post* moral-hazard: insurance coverage leads to overconsumption of care. In the first part of the paper the optimal public coverage is derived when consumers are homogeneous and, in the second part, when they differ in their wage rate. In the case of homogenous consumers, the public insurer should not intervene in the health insurance market. In the case of heterogeneous consumers it is shown that, when *ex-post* moral hazard is sufficiently high, supplementary insurance has a negative effect on redistribution because the rich buy more coverage and overconsume more than the poor. As a consequence, the optimal public coverage is negative: health care consumption must be taxed to discourage private policy purchase. As a result, all consumers are under-insured.

Considering together the results of this model and that of the previous literature on social insurance and redistribution, the following can be argued. When the negative correlation between wage rate and morbidity is not sufficiently high, the social planner should tax health expenses *because the negative effect of supplementary insurance on redistribution prevails over the positive effect due to the correlation between wage rate and morbidity*.

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Appendix

A.1 The Concavity of Problem P1

Before calculating the second order condition of P1, let us consider the demand for treatment as defined by (4). By differentiating (4) it follows that:

$$\frac{dx}{d\alpha} = -\frac{1}{h''(x)} > 0 \tag{16}$$

Thus, $\frac{d^2x}{d\alpha^2} = \frac{d}{d\alpha}\left(-\frac{1}{h''(x)}\right) = \frac{h'''(x)}{\left[h''(x)\right]^2}\frac{dx}{d\alpha}$. Which leads to:

$$\frac{d^2x}{d\alpha^2} = -\frac{h'''(x)}{\left[h''(x)\right]^3} > 0 \tag{17}$$

The second-order condition of P1 with respect to α can be written as:

$$\begin{aligned} & pU'(C_I)\left(-2p\frac{\partial x}{\partial \alpha} + \frac{\partial x}{\partial \alpha} - p\alpha\frac{\partial^2 x}{\partial \alpha^2}\right) + pU''(C_I)\left(-px - p\alpha\frac{\partial x}{\partial \alpha} + x\right)^2 \\ & + (1-p)U'(C_0)\left(-2p\frac{\partial x}{\partial \alpha} - p\alpha\frac{\partial^2 x}{\partial \alpha^2}\right) + (1-p)U''(C_0)\left(-px - p\alpha\frac{\partial x}{\partial \alpha}\right)^2 \end{aligned} \tag{18}$$

A *sufficient condition* for (18) to be negative is: $\frac{\partial x}{\partial \alpha} - p\alpha\frac{\partial^2 x}{\partial \alpha^2} < 0$. Using (16) and (17) the previous inequality can be rewritten as:

$$\frac{\partial x}{\partial \alpha}\frac{1}{x} < p\frac{\alpha}{x}\frac{\partial x}{\partial \alpha}\frac{h'''(x)}{\left[h''(x)\right]^2}$$

or:

$$-\frac{1}{h''(x)}\frac{1}{x} < p\varepsilon_{x,\alpha}\frac{h'''(x)}{\left[h''(x)\right]^2} \tag{19}$$

From (19) the condition in lemma 1 concavity can be immediately derived.

A.2 Insurance Coverage as a Normal Good

By totally differentiating the first-order condition of problem P1 with respect to α and W it follows: $\frac{d\alpha}{dW} = -\frac{\frac{dFOC\alpha}{dW}}{\frac{dFOC\alpha}{d\alpha}}$, where the denominator is negative under the condition in lemma 1 and:

$$\frac{dFOC\alpha}{dW} = (1-p)x\left[U''(C_I)-U''(C_0)\right]-\alpha\frac{\partial x}{\partial\alpha}E\left[U''(C)\right]$$

Since $sign\left(\frac{d\alpha}{dW}\right) = sign\left(\frac{dFOC\alpha}{dW}\right)$, the sign of (20) is crucial. Rearranging (20), $\frac{dFOC\alpha}{dW}$ is positive if:

$$\varepsilon_{x,\alpha} > \frac{(1-p)\left[U''(C_I)-U''(C_0)\right]}{E\left[U''(C)\right]} \tag{20}$$

A.3 Proof of Lemma 4

The cross derivative of consumers' expected utility in program P2 is:

$$\begin{aligned}\frac{\partial^2 EU}{\partial\alpha\partial\beta} &= pU'(C_I)\left(-p\frac{\partial x}{\partial\beta}+\frac{\partial x}{\partial\beta}-p\alpha\frac{\partial^2 x}{\partial\alpha\partial\beta}\right)\\ &+pU''(C_I)\left(-px-p\alpha\frac{\partial x}{\partial\alpha}+x\right)\left(-p\alpha\frac{\partial x}{\partial\beta}+x\right)\\ &+(1-p)U'(C_0)\left(-p\frac{\partial x}{\partial\beta}-p\alpha\frac{\partial^2 x}{\partial\alpha\partial\beta}\right)\\ &+(1-p)U''(C_0)\left(-px-p\alpha\frac{\partial x}{\partial\alpha}\right)\left(-p\alpha\frac{\partial x}{\partial\beta}\right)\end{aligned} \tag{21}$$

Notice that, considering consumers' demand for treatment, $\frac{\partial x}{\partial\beta} = \frac{\partial x}{\partial\alpha} > 0$ and $\frac{\partial^2 x}{\partial\alpha\partial\beta} = \frac{\partial^2 x}{\partial\alpha^2} > 0$. The third and the fourth term in (21) are negative. The first term is negative when the condition in remark 1 is verified. The second term can be rewritten as:

$$pU''(C_I)\left(-p\alpha\frac{\partial x}{\partial\beta}+x\right)^2 - p^2xU''(C_I)\left(-p\alpha\frac{\partial x}{\partial\alpha}+x\right)$$

which is negative if $-p\alpha\frac{\partial x}{\partial\alpha}+x<0$. Rearranging the previous inequality it follows that $\varepsilon_{x,\alpha} > \frac{1}{p}$ is a sufficient condition to have $\frac{\partial^2 EU}{\partial\alpha\partial\beta} < 0$.

A.4 **Proof of Proposition 1**

The game is solved by backward induction.

In the third stage, treatment demand is given by equation (7) and $x^{**} = x(\alpha+\beta)$. The indirect utility function is $v = v(T+P, \alpha+\beta)$. Applying the envelope theorem it follows:

(22) $$\frac{\partial v}{\partial T} = \frac{\partial v}{\partial P} = -E\left[U'(C)\right], \quad \frac{\partial v}{\partial \alpha} = \frac{\partial v}{\partial \beta} = xpU'(C_I)$$

In the second stage the private insurer solves program (P2) where contract (α, T) is taken as given and consumers' behavior is correctly anticipated. Program (P2) can be written using the indirect utility function $v = v(T+P, \alpha+\beta)$:

(23) $$\max_{\beta,P} \pounds = v(T+P, \alpha+\beta) + \lambda\left[P - p\beta x(\alpha+\beta)\right]$$

The first-order conditions are:

(24) $$\begin{aligned} P: &\quad \tfrac{\partial v}{\partial P} + \lambda = 0 \\ \beta: &\quad \tfrac{\partial v}{\partial \beta} - \lambda\left(px + p\beta\tfrac{\partial x}{\partial \beta}\right) = 0 \end{aligned}$$

Since the solution to problem (23) gives $\beta(\alpha, T)$ and $P(\alpha, T)$, the maximum value function for this problem is defined as $V(\alpha, T)$. By the envelope theorem, from (22) and the first-order conditions (24), it follows the properties of $V(\alpha, T)$:

(25) $$\begin{aligned} &\tfrac{\partial V}{\partial T} = \tfrac{\partial v}{\partial T} = \tfrac{\partial v}{\partial P} = -\lambda \\ &\tfrac{\partial V}{\partial \alpha} = \tfrac{\partial v}{\partial \alpha} - \lambda p\beta\tfrac{\partial x}{\partial \alpha} = \tfrac{\partial v}{\partial \beta} - \lambda p\beta\tfrac{\partial x}{\partial \beta} = \lambda px \end{aligned}$$

Finally in the third stage the public insurer solves program:

$$\max_{\alpha,T} \pounds = V(T, \alpha) + \gamma\left[T - p\alpha x(\alpha, T)\right]$$

The first-order conditions are:

$$(26)\qquad \begin{aligned} T: &\quad \frac{\partial V}{\partial T}+\gamma\left(1-p\alpha\frac{\partial x}{\partial T}\right)=0 \\ \alpha: &\quad \frac{\partial V}{\partial \alpha}-\gamma\left(px+p\alpha\frac{\partial x}{\partial \alpha}\right)=0 \end{aligned}$$

Using (25) and rearranging (26):

$$(27)\qquad \alpha\left(\frac{\partial x}{\partial \alpha}+px\frac{\partial x}{\partial T}\right)=0$$

Notice that $\frac{\partial x}{\partial T}=-\frac{\partial x}{\partial W}$, then $\frac{\partial x}{\partial \alpha}+px\frac{\partial x}{\partial T}=\frac{\partial x}{\partial \alpha}-px\frac{\partial x}{\partial W}$ where $\frac{\partial x^C}{\partial \alpha}\equiv\frac{\partial x}{\partial \alpha}-px\frac{\partial x}{\partial W}$ corresponds to the derivative of the compensated demand for treatment. In fact, treatment demand is $x^{**}=x\left[\alpha+\beta\left(\alpha,T\right)\right]$, and an increase in α affects x both directly and indirectly through a change in β. By differentiating treatment demand it follows that $\frac{\partial x}{\partial \alpha}+px\frac{\partial x}{\partial T}=-\frac{1}{H''(x)}\left(1+\frac{\partial \beta}{\partial \alpha}-px\frac{\partial \beta}{\partial W}\right)$ which is different from zero.[27] As a consequence, from (27) $\alpha=0$.

A.5 **Simultaneous game when consumers are identical**

As previously explained, consumers and private firms' programs are not affected by the way the public and the private insurers compete. Thus, (7) always describes consumers' choice. Assuming that a large number of competitive private firms and the public insurer simultaneously choose the premium and the cost-sharing parameter, the public insurer solves the following program:

$$(\text{P5})\begin{cases} \max\limits_{T,\alpha} p\left[U\left(W-T-P-\left(1-\alpha-\beta\right)x\right)-\bar{h}+h\left(x\right)\right]+\left(1-p\right)U\left(W-T-P\right) \\ s.t.:\ T=p\alpha x \\ \qquad h'(x)=1-\alpha-\beta \end{cases}$$

Whereas private firms still solve Programs P2. According to remark 2, given one coverage, expected utility increases when another coverage is added. Thus, in the symmetric Nash equilibrium $\left(\alpha^N,\beta^N\right)$, both private and public coverage are positive. All the considerations made before regarding the optimal coverage still hold such that α^N and β^N are both lower than one. Moreover, given that the two

27. A similar expression can be found in Boadway *et al.* (2001).

coverages are strategic substitutes (see lemma 4), $\alpha^N = \beta^N < \alpha^{SB}$ holds. From the first-order conditions of programs (P2) and (P5) it follows:

$$\text{FOC}(\alpha) \quad : \quad U'(C_I)x = \left(x + \alpha \frac{\partial x}{\partial \alpha}\right) E\left[U'(C)\right] \tag{28}$$

$$\text{FOC}(\beta) \quad : \quad U'(C_I)x = \left(x + \beta \frac{\partial x}{\partial \beta}\right) E\left[U'(C)\right] \tag{29}$$

First-order conditions (28) and (29) show that here the pecuniary externality described in section section 2.2 concerns the two insurers: the public insurer does not take into account that α also affects the premium of the private contract through treatment consumption, and the private firm does not take into account that β also affects the premium of the public contract. Reasoning as in section section 2.2, we can say that the two insurers under-estimate the effect of their strategies on consumers' expected utility. Thus, aggregate coverage $\left(\alpha^N + \beta^N\right)$ is higher than the second-best coverage α^{SB}.

Summarizing the previous discussion:

Remark 7. *When consumers are homogeneous and insurers play simultaneously,* $0 < \alpha^N = \beta^N < \alpha^{SB}$ *and* $\alpha^N + \beta^N > \alpha^{SB}$ *: consumers are over-insured.*

A.6 Proof of Proposition 2

In the third stage, labor supply is given by equation (11) and treatment demand by (12). The indirect utility function is $v_i = v_i\left(t, G - P_i, \alpha + \beta_i\right)$ where $i = L, H$. Applying the envelope theorem:

$$\begin{aligned} \frac{\partial v_i}{\partial t} &= -w_i l_i E\left[U'(C_i)\right] \\ \frac{\partial v_i}{\partial G} &= -\frac{\partial v_i}{\partial P} = E\left[U'(C_i)\right] \\ \frac{\partial v_i}{\partial \alpha} &= \frac{\partial v_i}{\partial \beta_i} = x_i p U'(C_i^I) \end{aligned} \tag{30}$$

In the second stage, private insurers solve program (P4) where (t, G, α) are taken as given and consumers' behavior is correctly anticipated. Program (P4) can be written using the indirect utility function $v_i = v_i\left(t, G - P_i, \alpha + \beta_i\right)$:

$$\max_{\beta_i, P_i} L_i = v_i\left(t, G - P_i, \alpha + \beta_i\right) + \mu_i\left[P_i - p\beta_i x_i\left(\alpha + \beta_i\right)\right] \tag{31}$$

The first-order conditions are:

$$(32)\qquad \begin{aligned} P_i: &\quad \frac{\partial v_i}{\partial P_i} + \mu_i = 0 \\ \beta_i: &\quad \frac{\partial v_i}{\partial \beta_i} - \mu_i\left(px_i + p\beta_i \frac{\partial x_i}{\partial \beta_i}\right) = 0 \end{aligned}$$

Since the solution to problem (31) gives $\beta_i(t,G,\alpha)$ and $P_i(t,G,\alpha)$, the maximum value function for this problem is defined as $V_i(t,G,\alpha)$. From equations (30) and (32), using the envelope theorem it follows:

$$\begin{aligned} \frac{\partial V_i}{\partial t} &= \frac{\partial v_i}{\partial t} = -w_i l_i E\left[U'(C_i)\right] - \mu_i p\beta_i \frac{\partial x_i}{\partial t} \\ \frac{\partial V_i}{\partial G} &= \frac{\partial v_i}{\partial G} = -\frac{\partial v_i}{\partial P_i} = E\left[U'(C_i)\right] = \mu_i \\ \frac{\partial V_i}{\partial \alpha} &= \frac{\partial v_i}{\partial \alpha} - \mu_i p\beta_i \frac{\partial x_i}{\partial \alpha} = \frac{\partial v_i}{\partial \beta_i} - \mu_i p\beta_i \frac{\partial x_i}{\partial \beta_i} = \mu_i p x_i \end{aligned}$$

Finally, in the third stage the public insurer solves program:

$$(33)\quad \max_{t,G,\alpha} L = \sum_i \lambda_i V_i(t,G,\alpha) + \gamma\left[t\sum_i \lambda_i w_i l_i(t) - G - p\alpha\sum_i \lambda_i x_i(t,G,\alpha)\right]$$

The first-order conditions are:

$$(34)\qquad \begin{aligned} t: &\quad \sum_i \lambda_i \frac{\partial V_i}{\partial t} + \gamma\left[\sum_i \lambda_i w_i l_i + t\sum_i \lambda_i w_i \frac{\partial l_i}{\partial t} - p\alpha\sum_i \lambda_i \frac{\partial x_i}{\partial t}\right] = 0 \\ G: &\quad \sum_i \lambda_i \frac{\partial V_i}{\partial G} + \gamma\left[-1 - p\alpha\sum_i \lambda_i \frac{\partial x_i}{\partial G}\right] = 0 \\ \alpha: &\quad \sum_i \lambda_i \frac{\partial V_i}{\partial \alpha} + \gamma\left[-p\sum_i \lambda_i x_i - p\alpha\sum_i \lambda_i \frac{\partial x_i}{\partial \alpha}\right] = 0 \end{aligned}$$

Rearranging first-order condition with respect to *G*:

$$\sum_i \lambda_i \left[\frac{\mu_i}{\gamma} - 1 - p\alpha \frac{\partial x_i}{\partial G}\right] = 0$$

Let us define $b_i \equiv \frac{\mu_i}{\gamma} - p\alpha \frac{\partial x_i}{\partial G}$ the net marginal social utility of income for type-i consumers. From the previous equation:

$$(35)\qquad E(b) = 1$$

It is well known in the optimal taxation theory that when $b_H < b_L$ redistributing income from type-*H* to type-*L* is socially desirable. Using (35) and rearranging first-order condition with respect to α:

$$(36) \qquad E[bx] - E[x] - \alpha \sum_i \lambda_i \left(\frac{\partial x_i}{\partial \alpha} - px_i \frac{\partial x_i}{\partial G} \right) = 0$$

where, as in appendix 5.4, $\frac{\partial x_i^C}{\partial \alpha} \equiv \frac{\partial x_i}{\partial \alpha} - px_i \frac{\partial x_i}{\partial G}$ corresponds to the derivative of the compensated demand for treatment for type-*i* consumers. Equation (36) can be rewritten as:

$$(37) \qquad \alpha = \frac{cov[b,x]}{\sum_i \lambda_i \frac{\partial x_i^C}{\partial \alpha}}$$

Presumably it is $b_H < b_L$, while, under condition $C.1$, $x_H > x_L$. Thus $cov[b,x] < 0$.

Let us analyze the sign of $\frac{\partial x_i^C}{\partial \alpha}$. What follows is adapted from Boadway *et al.* (2001). From appendix 5.4 derives that:

$$\frac{\partial x_i^C}{\partial \alpha} \equiv \frac{\partial x_i}{\partial \alpha} - px_i \frac{\partial x_i}{\partial G} = -\frac{1}{h''(x)} \left(1 + \frac{d\beta_i}{d\alpha} - px_i \frac{d\beta_i}{dG} \right)$$

In order to find the sign of $\frac{\partial x_i^C}{\partial \alpha}$ it is necessary to calculate the expressions for $\frac{d\beta_i}{d\alpha}$ and $\frac{d\beta_i}{dG}$ from the second stage, that is from the maximization of expected utility by the private firm. Notice that the result concerning strategic substitutability of section 2.2 is useful here. Under the condition in remark 4, $\frac{\partial^2 EU}{\partial \alpha \partial \beta} < 0$ holds. Thus, $\frac{d\beta_i}{d\alpha} < 0$. While, according to remark 3, $\frac{d\beta_i}{dG} > 0$. From the maximization of expected utility by the private firm, first-order condition (14) can be rewritten as:

$$(38) \qquad \Delta_i = U'(C_i^I) x_i - E\left[U'(C_i)\right] \left(x_i + \beta_i \frac{\partial x_i}{\partial \beta_i} \right)$$

By totally differentiating (38) it follows:

$$\frac{\partial \Delta_i}{\partial \beta_i} d\beta_i + \frac{\partial \Delta_i}{\partial \alpha} d\alpha + \frac{\partial \Delta_i}{\partial G} dG = 0$$

such that:

$$\frac{d\beta_i}{d\alpha} = -\frac{\frac{\partial \Delta_i}{\partial \alpha}}{\frac{\partial \Delta_i}{\partial \beta_i}} < 0 \quad \text{and} \quad \frac{d\beta_i}{dG} = -\frac{\frac{\partial \Delta_i}{\partial G}}{\frac{\partial \Delta_i}{\partial \beta_i}} > 0$$

where $\frac{\partial \Delta_i}{\partial \beta_i} < 0$ when the condition in lemma 1 holds, such that $\frac{\partial \Delta_i}{\partial \alpha} < 0$ and $\frac{\partial \Delta_i}{\partial G} > 0$. Taking into account that in stage three $\frac{dx_i}{d\alpha} = \frac{dx_i}{d\beta_i}$ and $\frac{d^2 x_i}{d\alpha^2} = \frac{d^2 x_i}{d\beta_i^2} = \frac{d^2 x_i}{d\alpha d\beta_i}$ (see appendix 5.1) we can write:

(39) $$\frac{\partial \Delta_i}{\partial G} = U''(C_i^I)x_i - E\left[U''(C_i)\right]\left(x_i + \beta_i \frac{\partial x_i}{\partial \beta_i}\right) > 0$$

(40) $$\frac{\partial \Delta_i}{\partial \alpha} = p\beta_i \frac{dx_i}{d\alpha}\frac{\partial \Delta_i}{\partial G} + \frac{dx_i}{d\alpha}U'\left(C_i^I\right) + x_i U''\left(C_i^I\right)\left[x_i - p\left(x_i + \beta_i \frac{\partial x_i}{\partial \beta_i}\right)\right] \\ -E\left[U'(C_i)\right]\left(\frac{dx_i}{d\alpha} + \beta_i \frac{\partial^2 x_i}{\partial \alpha \partial \beta_i}\right) < 0$$

(41) $$\frac{\partial \Delta_i}{\partial \beta_i} = \frac{\partial \Delta_i}{\partial \alpha} - px_i \frac{\partial \Delta_i}{\partial G} - \frac{\partial x_i}{\partial \beta_i} E\left[U'(C_i)\right] < 0$$

From (41) $\frac{\partial \Delta_i}{\partial \beta_i} < \frac{\partial \Delta_i}{\partial \alpha} < 0$. Thus, $-1 < \frac{d\beta_i}{d\alpha} < 0$: public coverage leads to less than complete crowding-out of private insurance. Notice that, concerning aggregate coverage, $\frac{\partial(\alpha+\beta_i)}{\partial \alpha} = 1 + \frac{d\beta_i}{d\alpha} > 0$ holds. This means that, even if an increase in public coverage makes private coverage decrease, aggregate coverage increases as well. Finally, using the expression in (39), (40) and (41), $1 + \frac{d\beta_i}{d\alpha} - px\frac{d\beta_i}{dG}$ can be rewritten as:

$$1 - \frac{\frac{\partial \Delta_i}{\partial \alpha}}{\frac{\partial \Delta_i}{\partial \beta_i}} + px_i \frac{\frac{\partial \Delta_i}{\partial G}}{\frac{\partial \Delta_i}{\partial \beta_i}} = \frac{-\frac{\partial x_i}{\partial \beta_i} E\left[U'(C_i)\right]}{\frac{\partial \Delta_i}{\partial \beta_i}} > 0$$

As a consequence $\frac{\partial x_i^C}{\partial \alpha}$ is positive as the denominator in (37). Thus, public coverage is negative.