ANNALES D'ÉCONOMIE ET DE STATISTIQUE. – N° 83/84 – 2006

Long Term Care: the State and the Family*

Pierre PESTIEAU† and Motohiro SATO‡

ABSTRACT. – In this paper we study the optimal design of a long term care policy in a setting that includes two types of care to dependent parents: financial assistance and assistance in time by children. The instruments are subsidies to aiding children, financed by a flat tax on earnings. The only source of heterogeneity is children's productivity. Parents can influence their children by leaving them gifts before they know whether or not they will need long term care, yet knowing the productivity of the children. The tax-transfer policy is shown to depend on its effect on parental gifts, on children's labor supply, on the distribution of wages and on consumption inequality between parents and children and between children having dependent parents and children having healthy parents.

L'assurance autonomie : l'Etat et la famille

RÉSUMÉ. – Nous étudions la politique optimale de financement de deux types de soins aux personnes dépendantes : soins privés financés par leurs enfants et aide en temps fournie par leurs enfants. Le planificateur social peut utiliser un certain nombre d'instruments: subventions aux enfants « aidants », financées par un impôt sur les revenus, notamment. La seule source d'hétérogénéité est la productivité des enfants. Les parents peuvent influencer leurs enfants en leur laissant des transferts avant qu'ils ne sachent leur degré d'autonomie.

* This paper was presented at the Third Journées d'Economie Publique Louis-André Gérard-Varet, Marseille, June 10-11, 2004. We are grateful to the discussant, Cecilia Garcia Penalosa for helpful comments and suggestions. This research was begun with our colleague and friend, Maurice Marchand, who passed away suddenly in July of 2003.
† P. PESTIEAU: CREPP, University of Liège, CORE, PSE and CEPR.
P. Pestieau@ulg.ac.be.
‡ M. SATO: Hitotsubashi University. satom@econ.hit-u.ac.jp.

1 Introduction

The ongoing demographic ageing process represents a major challenge for the way our economies are organized both from a social, as well as from an economic point of view. Ageing can be felt across a large array of domains touching all age groups, ranging from the very young to the oldest old. One often cited example is the provision of long-term care insurance to the oldest old, be it under the form of a private or a public system. Only a handful of countries or regions have set up such long-term care insurance systems which, incidentally, are also sometimes called dependency insurance. The relative scarcity of such systems, and the difficulties of organizing them, is linked to some conceptual problems intrinsically due to the issue at hand. First, a definition of who is a person in need of long-term care cannot always be stated objectively. However, this is not a sufficient reason to justify the lack of long-term care insurance programs around the world, since disability insurance systems are plagued by the same kind of problem but do exist. The second, and probably more fundamental reason, is that a lot of long-term care is not provided through a formal market mechanism, but rather through informal family arrangements. In this respect, the problem is similar to the child-care market, where family care is competing with market-provided care in private or public arrangements. From a social point of view, this duality of providers is an interesting one, as these two types of providers seem to function on a very different basis. While institutional care is essentially a provision of a contribution-based service by a public or private (for-profit or non-profit) provider, family care is at least partly motivated by some degree of altruism, which in turn implies that the caregiving family member also derives utility from this activity. Further, while institutional care usually implies some degree of public subsidization, and hence inter-family redistribution, this does not always hold true for family-care arrangements.

Yet the analogy between these two forms of care is limited. In contrast to the child-care market, the costs involved are much larger in long-term care insurance, as costs of medical and non-medical care are much more expensive at the end of the life-cycle than at the beginning because of the vastly different physical conditions of the people in question. Hence, the choice between family or institutional care has important budgetary implications that a government or a social planner cannot ignore. Another problem raised by cares such as child care or long term care provided out of altruism is that, depending on the opportunity cost of time, they can be provided directly by children in units of time, or obtained from the market through financial aid from children. One sees from this quick overview that the analysis of long term care is very complex, and that all aspects cannot be dealt with at the same time.

Given the complexity of the issue at hand, we have to limit the scope of our analysis and adopt a rather simple model. We look at a society consisting of a number of pairs of parent-child. In this model parents are not altruistic, while children have a specific type of altruism: in that they are ready to help their parents if these lose their autonomy. In the absence of government policy dependent parents can be helped in two ways: either children give them some financial aid that allows them to buy appropriate market services, or they provide them with assistance in time. Children have different productivities, and parents have a uniform endowment (wealth, pension). Market productivity varies, but productivity in terms of helping dependent parents is the same for all. As a consequence, one expects that children of dependent parents will be divided into two groups. The low market productivity group will help their parents with time, and the other one will provide financial assistance. Before knowing their own health status parents can give part of their endowment to their children anticipating that in case of dependence they will need their aid.

We then introduce public policy consisting of two instruments: a uniform payroll tax and a subsidy for dependent parents receiving assistance (in kind or in cash) from their children.

We are ultimately interested in the optimal policy chosen by a utilitarian government. But before doing that, we analyze the comparative statics of our model. In particular, we study the effect of policy variables and exogenous variables on the segmentation of our society into two groups.

Quite clearly such a model does not include all the aspects of long term care and it does rest on a number of assumptions. Some are pretty realistic; others are made to keep the analysis within reasonable limits. The only heterogeneity comes from differences in market productivity. The other characteristics such as altruism, initial endowment, productivity in assistance to dependent parents are equal for all.[1] The instruments are payroll tax and lump-sum subsidy. These restrictive policies are adopted for the sake of simplicity. Also, we assume away private long term care insurance, which is quite realistic. The main feature of our model is that there are two ways of taking care of dependent parents, and that this segmentation depends on the children's income.

The rest of the paper is organized as follows. The next section presents the basic model and some comparative statics results. The following section is devoted to the design of optimal tax transfer policy. A final section concludes.

2 The Basic Model

2.1 The *laisser-faire*

We consider a family consisting of a parent and his altruistic child. All families are *ex ante* identical except for the market productivity of children denoted w with density $f(w)$, distribution $F(w)$ and support (w_-, w_+). The parent is confronted with the risk of losing his autonomy and becoming dependent with probability π. This corresponds to a utility loss of D. Once he becomes dependent, the parent needs assistance from his child, that is, long term care. Assistance is given in terms of time or of financial support. To secure such an assistance in case of dependency

1. In Jousten *et al.* (2005), the optimal long term care policy is analyed when the only source of heterogeneity is children's altruism.

the parent chooses to leave an *inter vivos* gift G to his child out of his initial endowment I. He consumes d^D if dependent and d^N if autonomous.

His expected utility can be written as:

$$V = \pi\left[v\left(d^D\right) - D + H\right] + (1-\pi)v\left(d^N\right) = v(d) - \pi(D-H)$$

where $d^D = d^N = I - G$, I being his initial endowment (the same for all), and H is the help he gets from his child expressed in utility terms as well. Below we show that, H, the assistance from the child, depends on that gift G. The parent knowing this relation chooses G such that

$$-v'\left(d^D\right) + \pi\frac{\partial H}{\partial G} = 0.$$

In other words, the marginal utility of consumption is equal to the marginal increase in assistance induced by the gift with the probability of dependency as a weight.

Turning to the children, we first note that their utility is contingent as they also want to help their parents in case of dependency. Denoting their utility by $u(\cdot)$ and their consumption by $c^j\ (j = D, N)$, we have

$$U^D = u\left(c^D\right) + \beta\left(v\left(d^D\right) + H - D\right)$$

and

$$U^N = u\left(c^N\right) + \beta v\left(d^D\right)$$

where $c^D = (1-h)w + G - s, c^N = w + G$ and $\beta \leq 1$ is a factor of altruism. Market labor supply is $(1-h)$ with h being the aid in time provided to dependent parents and s, is the amount of financial aid. As we show h and s are mutually exclusive.

It is now time to define H. We assume that each child has one unit of time endowment. He can devote part of it to labor market in which case he earns w and he can devote another part of it to his parent. If he provides h to his parent given a constant productivity ω_0, this amounts to a help of $\omega_0 h$. This child will also earn $(1-h)w$ as market earnings. Instead a child may want to help his dependent parent through financial aid, s, which is used to purchase market nursing services.

By assuming perfect substituability between these two forms of assistance, namely by positing:

$$H(\omega_0 h, s) = H(\omega_0 h + s)$$

with $H' > 0$ and $H'' < 0$, we know that children with $w \leq \omega_0$ will have $h \geq 0$, $s = 0$ and those with $w > \omega_0$, $h = 0$ and $s \geq 0$. Focusing on interior solutions, we have, either $1 > h > 0$ or $s > 0$. Formally, for $w \leq \omega_0$, h^* is the solution of

$$u'\left(c^D\right)w = \beta H'\left(\omega_0 h\right)\omega_0$$

and for $w > \omega_0$, s^* is the solution of

$$u'\left(c^D\right) = \beta H'\left(s\right).$$

Each parent can decide to leave his child a certain fraction of his endowment taking into account the functional link between *h* or *s* and *G*. When making this choice, he does not know yet whether or not he will need long term care but he knows his child's productivity. There is no parental altruism. The reason for such an early gift is insurance; it is also the only way to obtain care. With $\pi = 0$, there would not be such a gift. The optimal amount of gift will depend on *w*.

2.2 **Tax-Transfer Instruments**

We now introduce two tax-transfer instruments: an income tax rate *t* and a flat subsidy σ for children providing assistance in time to their parents and for children supplying financial assistance. In other words, the subsidy is the same for the two types of assistance. We use the subscript 1 and 2 to denote the group of children helping their parents in time units or in cash respectively.

For the sake of clarity, let us explicit the sequence of decisions.

- Stage 1. The government chooses its policy instruments σ and τ.
- Stage 2. Each parent chooses *G* knowing the ability of his child, but not his future state of health.
- Stage 3. The state of health is known. The child receives *G* in any case, but only helps his parent if dependent by providing either *h* or *s*.

As usual we proceed backward and start with the child's decision. His problem can easily be stated as depending on whether $w \lessgtr \frac{\omega_0}{1-t}$. This means that compared to the *laissez-faire* less children provide financial assistance. Denoting $w\left(1-t\right)$ by ω, it consists in maximizing:

$$U_1^D = u\left(\omega\left(1-h\right)+G_1+\sigma\right)+\beta\left(H\left(\omega_0 h\right)+v\left(I-G_1\right)-D\right)$$

or

$$U_2^D = u\left(\omega+G_2+\sigma-s\right)+\beta\left(H\left(s\right)+v\left(I-G_2\right)-D\right)$$

where the superscript *D* denotes that child's utility concerns the state of nature where his parent has lost his autonomy. In the other state, the child does not have any choice. His utility is simply:

$$U_i^N = u\left(\omega+G_i\right)+\beta\, v\left(I-G_i\right).$$

Taking as given G_i, the child maximizes U_i^D, which yields the following supply functions:

$$h = h(\omega, G_1 + \sigma)$$

or

$$s = s(\omega + G_2 + \sigma).$$

Note that the subsidy and the gift have the same effect, but the subsidy is flat whereas the gift varies with *w*. We can also introduce the children's indirect utility functions (without the altruistic component):

$$u_1^D = u\Big(\omega\big(1 - h(\omega, G_1 + \sigma)\big) + G_1 + \sigma\Big) = u_1^D\,(\underset{+}{\omega}, \underset{+}{G_1}, \underset{+}{\sigma})$$

and

$$u_2^D = u\big(\omega + G_2 + \sigma - s(\omega + G_2 + \sigma)\big) = u_2^D\,(\underset{+}{\omega}, \underset{+}{G_2}, \underset{+}{\sigma}).$$

where the sign of the partial derivatives are given under each argument for well behaved utility functions.

These decisions are made by children after G_i has been determined by their parents. The choice of G_i results from maximizing either one of the following expressions:

$$V_1 = v(I - G_1) + \pi\, H\big(\omega_0 h(\omega, G_1 + \sigma)\big) - \pi D$$

or

$$V_2 = v(I - G_2) + \pi\, H\big(s(\omega + G_2 + \sigma)\big) - \pi D.$$

Again we obtain a supply function G_i and an indirect utility function V_i such that:

$$\begin{array}{lcl} G_1 = G_1(\omega, \sigma) & , & G_2 = G_2(\omega, \sigma); \\ \tilde{V}_1 = \tilde{V}_1(\underset{-}{\omega}, \underset{+}{\sigma}) & , & \tilde{V}_2 = \tilde{V}_2(\underset{+}{\omega}, \underset{+}{\sigma}). \end{array}$$

To have a better grasp at this problem, we now turn to a simple illustration, using logarithmic utility functions. The advantage of such an illustration is to give us the profile of $m(= h\omega_0 + s)$ and *G* along the *w*-axis. It also gives us unambiguous comparative statics. Finally, it provides intuitive cases where corner solutions appear.

2.3 The Log-Linear Example

As parents move first and children second, we start by looking at the problem of each child. If his parent is healthy, he does not help him and benefit from the transfer G, if any. If his parent loses his autonomy, he helps him with h or s depending on his productivity.

2.3.1 Child's Problem

Each child solves the following problem:

$$Max\ U^D = \ln\left(\omega(1-h)+G-s+\sigma\right)+\beta\ln\left(\omega_0 h+s\right)-\beta D+\beta\ln\left(I-G\right)$$

where $\omega = w(1-t)$. This yields the following solutions:

- $\omega < \omega_0$:

$$\begin{aligned} h^* &= \frac{\beta}{1+\beta}\left(1+\frac{G+\sigma}{\omega}\right) && \text{if } G+\sigma \le \omega/\beta \\ &= 1 && \text{otherwise} \end{aligned}$$

- $\omega > \omega_0$:

$$s^* = \frac{\beta}{1+\beta}\left(\omega+G+\sigma\right).$$

This implies that $h\omega_0$ is non increasing and s is increasing in w as it appears on Figure 1.

2.3.2 Parent's Problem

Ex ante before knowing his health status the parent solves the following problem:

$$Max\,V = \ln\left(I-G\right)+\pi\ln\left(\omega_0 h+s\right)-\pi D.$$

Again we have to distinguish the two regions depending on $\omega \gtreqless \omega_0$. One can easily show that

$$G_1^* = Max\left[\min\left[\frac{\pi I-\omega-\sigma}{1+\pi},\frac{\omega}{\beta}-\sigma\right],0\right]$$

When w is very small, the parent does not leave any gift as it does not has any incitative effect on his child. When $w = \dfrac{\beta\sigma}{1-t}$, then the gift becomes positive and increases with w. The child devotes all his time to his dependent parent, but this has a cost. When w reaches the value $\underline{w} = \frac{\pi\beta(I+\sigma)}{1+\pi+\beta}\frac{1}{1-t}$, both h and G decrease with w up to the threshold $\frac{\omega_0}{1-t}$.

Turning to regime 2 where $w > \frac{\omega_0}{1-t}$, we obtain:

$$G_2^* = Max\left[\frac{\pi I - \omega - \sigma}{1+\pi}, 0\right].$$

As w increases, G_2^* decreases up to the point $w = \frac{\pi I - \sigma}{1-t}$ where G_2^* is equal to 0. For higher values of w, altruistic children help their parents without any gift.

We can now represent the values of $m(=\omega_0 h \text{ or } s)$ and of G along the w-axis.

Using the same framework, we can also represent the profile of the utility of the parent on Figure 3.

We have to restrict the parameters so that $\beta\sigma < \underline{w}(1-t) < \omega_0 < \pi I - \sigma$. When $t = \sigma = 0$, this amounts to $0 < \frac{\pi\beta I}{1+\pi+\beta} < \omega_0 < \pi I$.

In the remainder of this paper, we assume that the range of wages (w_-, w_+) is included in the interval $\left(\underline{w}, \frac{\pi I - \sigma}{1-t}\right)$ such that m is first decreasing and then increasing and G is first increasing and then decreasing.

FIGURE 1
Child's assistance

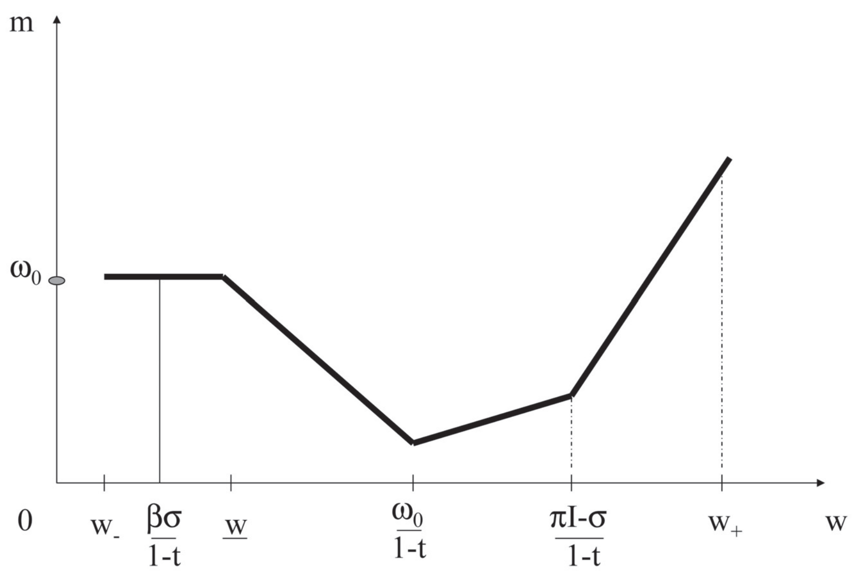


FIGURE 2
Parent's gift

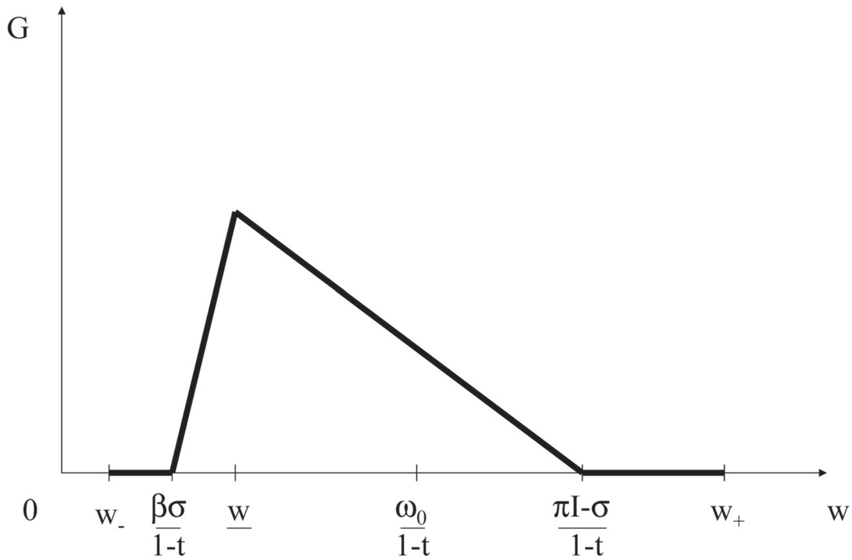
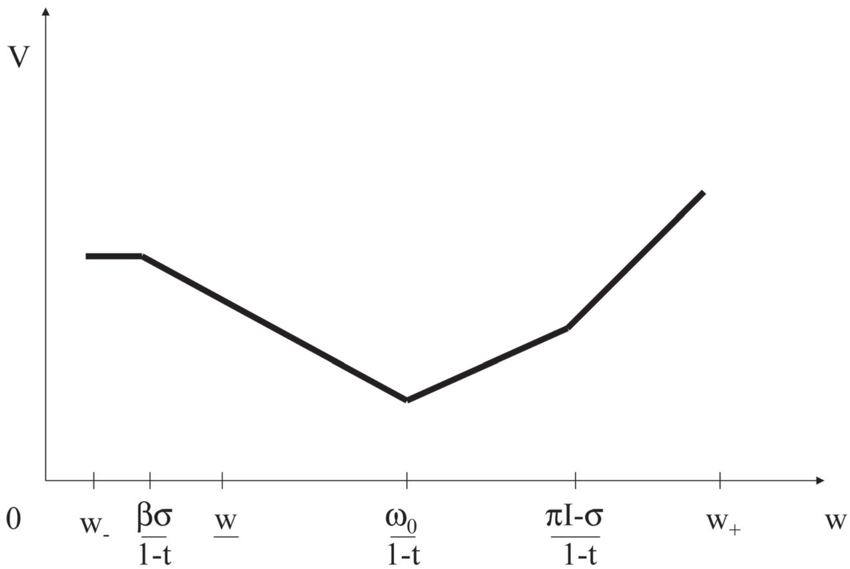


FIGURE 3
Parent's expected utility



3 Tax Transfer Policy

3.1 Unconstrained First-Best

As a benchmark we first consider the resource allocation that a social planner would implement if it had full control of the choice variables. The objective that we find appropriate is the sum of individual utilities after having removed the altruistic component. In other words, we consider as welfare function:

$$SW=\int_{w^-}^{w^+}\left\{\pi\left[u\left(c^D\right)+v\left(d^D\right)-D+H\right]+(1-\pi)\left[u\left(c^N\right)+v\left(d^N\right)\right]\right\}dF(w).$$

This view is not properly utilitarist; if we were adding individual utilities, this would amount to weight the welfare of the elderly people not by 1 but by $(1+\beta)$ which would be questionable.[2]

We can consider two resource constraints. In the first, the social planner assign all children with productivity below ω_0 to long term care not only of their own parents but also of other dependent parents. In the second, children can only work for their own dependent parents.

Let us look at the case where h can be provided to any dependent parent. Note that in this first-best the social planner controls also d^N even though in the decentralized solution children do not help their non dependent parents.

We expect that regardless of w: $u'\left(c^D\right)=u'\left(c^N\right)=v'\left(d^D\right)=v'\left(d^N\right)=H'(m)$ where $m=\omega_0 h+s$.

It is clear that h is either equal to 1 or to 0, depending on whether $w \lesseqgtr \omega_0$. The resource constraint is given by $\pi m+c+d=F(\omega_0)+\int_{\omega_0}^{w^+} w\,dF(w)+I$. If $u=v=H$, this gives:

$$c=d=m=\frac{F(\omega_0)\omega_0+\int_{\omega_0}^{w^+} w\,dF(w)+I}{2+\pi}.$$

We could also consider another first-best wherein low ability children could only help their own parents. This would restrict the total amount of resources, but the equality between marginal utilities would still hold.

2. See on this Hammond (1987) or Cremer and Pestieau (2001).

3.2 The Second-Best Problem

Within this simple setting the problem of the social planner is to maximize the utilitarian sum of utilities (without the altruistic component) subject to the revenue constraint that aggregate taxes and aggregate subsidies are equal. This is simply:

$$\pi\sigma N = t\overline{w}N - t\pi N \int_{w_-}^{\frac{\omega_0}{1-t}} wh(\omega, G_1 + \sigma)\, dF(w)$$

or

$$\pi\sigma = t\, Ey$$

with $Ey = \overline{w} - \pi \int_{w_-}^{\frac{\omega_0}{1-t}} wh\, dF(w)$ is average earnings income of children. N is the total number of children or of parents. We use the operator E instead of the integral over the range (w_-, w_+).

In the log-linear case, we have

$$Ey = \overline{w} - \frac{\pi^2}{1+\pi}\frac{\beta}{1+\beta}\frac{1}{1-t}\int_{w_-}^{\frac{\omega_0}{1-t}} [\omega + I + \sigma]\, dF(w).$$

Therefore,

$$\frac{\partial Ey}{\partial t} = -\frac{\pi^2}{1+\pi}\frac{\beta}{1+\beta}\frac{1}{(1-t)^2}\left[\omega_0 \frac{\omega_0 + I + \sigma}{1-t} + \int_{w_-}^{\frac{\omega_0}{1-t}} (w + I + \sigma)\, dF(w)\right] < 0$$

$$\frac{\partial Ey}{\partial t} = -\frac{\pi^2}{1+\pi}\frac{\beta}{1+\beta}\frac{1}{1-t}\int_{w_-}^{\frac{\omega_0}{1-t}} dF(w) < 0.$$

The problem of the social planner can be defined by the following Lagrangean:

$$\mathcal{L} = \int_{w_-}^{\frac{\omega_0}{1-t}} \left(\tilde{V}_1 + \overline{u}_1\right) dF(w) + \int_{\frac{\omega_0}{1-t}}^{w_+} \left(\tilde{V}_2 + \overline{u}_2\right) dF(w) - \mu\left[\pi\sigma - tEy\right],$$

where μ is a positive Lagrange multiplier and $\overline{u}_i = \pi u_i^D + (1-\pi) u_i^N$.

The FOC's are given in the appendix by (A.1) and (A.2). These conditions can be simplified using a reduced notation: $m = \omega_0 h + s$, $y_1^D = w(1-h)$, $y_2^D = y_1^N = y_2^N = w$.

$$\begin{aligned}(1)\qquad \frac{\partial \mathcal{L}}{\partial t} &= (1-\beta)\pi EH'(m)\frac{\partial m}{\partial t} + E\left[\pi u'\left(c^D\right)y^D + (1-\pi)u'\left(c^N\right)y^N\right] \\ &\quad - E\left[\overline{u}'(c) - v'(d)\right]\frac{\partial G}{\partial t} + \mu\left[\overline{y} + t\frac{\partial Ey}{\partial t}\right] = 0.\end{aligned}$$

$$\begin{aligned}(2)\qquad \frac{\partial \mathcal{L}}{\partial \sigma} &= (1-\beta)\pi EH'(m)\frac{\partial m}{\partial t} + E\pi u'\left(c^D\right) + E\left[\overline{u}'(c) - v'(d)\right]\frac{\partial G}{\partial \sigma} \\ &\quad - \mu\left[\pi - t\frac{\partial Ey}{\partial \sigma}\right] = 0\end{aligned}$$

where $\overline{u}'(c) = \pi u'\left(c^D\right) + (1-\pi)u'\left(c^N\right)$ is the average marginal utility of a child with productivity *w*. In the setting adopted here children consume less than their parents and hence $\overline{u}'(c) > v'(d)$. In the log-utility case, we have: $d = \frac{1}{1+\pi}(\omega + I + \sigma)$, $c^D = \frac{\pi}{1+\pi}\frac{1}{1+\beta}(\omega + I + \sigma)$ and $c^N = \frac{(\omega + I)\pi}{1+\pi} - \frac{\sigma}{1+\pi}$, with $d > c^N > c^D$ if σ is not too large.

We now use the superscript *c* to denote the compensated effect of *t* and σ and rewrite (1) and (2) as:

$$\begin{aligned}(3)\qquad \frac{\partial \mathcal{L}^c}{\partial t} &= \frac{\partial \mathcal{L}}{\partial t} + \frac{Ey}{\pi}\frac{\partial \mathcal{L}}{\partial \sigma} = (1-\beta)\pi EH'(m)\frac{\partial m^c}{\partial t} + E\left[\overline{u}'(c) - v'(d)\right]\frac{\partial G^c}{\partial t} \\ &\quad - \pi \operatorname{cov}\left(u'\left(c^D\right), y^D\right) - (1-\pi)\operatorname{cov}\left(u'\left(c^N\right), y^N\right) \\ &\quad + (1+\pi)\overline{y}^N E\left[\overline{u}'\left(c^D\right) - u'\left(c^N\right)\right] + \mu t\frac{\partial Ey^c}{\partial t} = 0.\end{aligned}$$

In (3) the effects of *t* on *m*, *G* and *Ey* are taken in compensated terms. In other words, using the revenue constraint, they include the direct effect of an increase in *t* and the indirect effect of an increase in σ, given that $\frac{d\sigma}{dt} = \frac{Ey}{\pi}$ for *Ey* being fixed.

Equation (3) can be rewritten as follows:

$$(4)\qquad \frac{\partial \mathcal{L}^c}{\partial t} = [1] + [2] - [3] + [4] + [5].$$

where

$$[1] = (1-\beta)\pi EH'(m)\frac{\partial m^c}{\partial t}$$

$$[2]=E\left[u'(c)-v'(d)\right]\frac{\partial G^{c}}{\partial t}$$

$$[3]=\pi\operatorname{cov}\left(u'\left(c^{D}\right),y^{D}\right)+(1-\pi)\operatorname{cov}\left(u'\left(c^{N}\right),y^{N}\right)$$

$$[4]=(1-\pi)\overline{y}^{N}E\left[\overline{u}'\left(c^{D}\right)-u'\left(c^{N}\right)\right]$$

$$[5]=\mu t\frac{\partial E\overline{y}^{c}}{\partial t}.$$

With this notation we can write our formula for t:

$$t=\frac{[1]+[2]-[3]+[4]}{-[5]} \qquad (5)$$

We now interpret formula (5) by considering each of its components.

[1] The first term in the numerator reflects the paternalistic action of the social planner. If $\beta=1$, namely if the social planner and children have the same view on the parents' utility, this term vanishes. For $\beta<1$, both t and σ are desirable if $\frac{\partial m^{c}}{\partial t}>0$. In other words, if the tax-transfer policy encourages assistance and if the social planner puts more weight on the parents than the children, then it should be encouraged. This will be the case if the majority of children has a productivity below $\frac{\omega_0}{1-t}$.[3] One cannot however excludes $\frac{\partial m^{c}}{\partial t}>0$ in which case, a paternalistic government will choose a lower tax transfer than if it were not paternalistic.

[2] The second term reflects the effect of the tax on gifts that expectedly narrow the difference between the marginal utilities of children and parents. That difference is known to be positive in the log case $\left(d>c^{N}>c^{D}\right)$. If the tax-transfer package induces additional gift then it will be higher the wider the consumption

3. More precisely, using the logarithmic example, we see that $\frac{\partial m^{c}}{\partial t}$ is positive for $w<Max\left[\frac{\omega_0}{1-t},\frac{Ey}{\pi}\right]$. Roughly speaking, if the majority of children have a low productivity, namely a w below $\frac{\omega_0}{1-t}$ and $\frac{Ey}{\pi}$, one can expect the tax-transfer policy to stimulate m.

gap between the parents and the child. Note that if the majority of children have a productivity below $\frac{\omega_0}{1-t}$, $\frac{\partial G^c}{\partial t} < 0$ and thus this second term is negative.[4]

[3] This covariance term expresses the concern for equity. The two covariances are negative and they increase (in absolute value) with the concavity of $u(c)$ and the inequality of w. As it appears, there is a covariance for each state of nature. This is the traditional equity term that one finds in the literature on linear income tax.

[4] The fourth term in the numerator depends on the gap between children's consumption levels in the two state of nature. To the extent that $c^D < c^N$, this term pushes for relatively higher tax-transfers.

It is interesting to observe that we have here a number of sources of inequality: wage inequality, inequality between children with and without dependent parents, inequality among parents leaving different gifts. For the first two, we have some redistribution. Not for the last one.

We now turn to the denominator of (5), that is the term [5]. Using the log linear illustration, it clearly appears that $\frac{\partial Ey^c}{\partial t}$ is negative. This is because both the tax on earnings and the subsidy tend to foster h and thus to discourage market labor supply.

Formula (5) can be viewed as an extension of the optimal linear income tax. The standard formula only includes [3] and [5], the equity and the efficiency terms. In this problem the only redistribution is among wage earners in a certainty setting. Here the tax-transfer scheme is also used for redistribution between children and parents and across states of the world. Indeed, in the extreme case where $\beta = 1$ and $\pi = 1$, we would have the standard linear income tax formula with the covariance term in the numerator and the efficiency terms in the denominator, but with an addition: the tax incidence on gifts as a way of equalizing the marginal utility of income of parents and that of children. In this particular case where there is no uncertainty, parents make gifts to their children because this is the only way to get support: private insurance is assumed away.

4 Conclusion

The purpose of this paper was to design an optimal tax transfer policy for long term care. The setting was relatively simple. Each elderly person has an altruistic child who will help him in case of loss of autonomy. Help can be of two types: time for low productivity children, cash for high productivity children. To foster help from their children parents can *ex ante* make a gift to their children. The government can subsidize children's assistance.

4. With the logarithmic utility, $\frac{\partial G^c}{\partial t} < 0$ for $w < Ey/\pi$.

Two questions can be raised in conclusion. Is it realistic? Is it not too simplistic? The two questions are naturally related. As mentioned in the introduction, the problem of long term care is very complex. It is also relatively new. There is no much evidence on the socio-economic characteristics of people suffering from loss of autonomy and on those of their close relatives. If productivity was the only distinctive factor, the pattern discussed in this paper could be quite natural. There are however other characteristics. For example, altruism is not uniform across families. Some elderly don't even have children to care about them. Introducing differential altruism along with differential productivity could complicate the model. It is for example clear that in that case public nursing homes would cater not only to parents of middle productivity children with altruism, but also to all parents with non altruistic children. In this case we are faced with a moral hazard problem if altruism cannot be observed (see Jousten *et al,* 2005). Another difficulty that we have assumed away is that loss of autonomy may not be observable. This leads to another moral hazard problem as it is tempting for healthy parents to mimick unhealthy parents. Again this would add an additional constraint to the design of an optimal tax-transfer scheme.

The model used in this paper can be extended in various directions, which represent avenues of future research. First, it would be interesting to consider the possibility of public nursing homes and/or of a private insurance market. Some parents could find desirable to go to a nursing home keeping their resources I or to purchase a long term care private insurance.

Also one could consider that parents don't have the same income and that their income be more or less correlated with that of their children.

Another extension would be to consider that all children are not necessarily altruistic, in which case parents have to resort to either public nursing homes or to private insurance.

Finally, we have considered the possibility of offering a different subsidy for assistance in time and assistance in cash. It makes the analysis quite more complex. Naturally, the subsidy on financial aid is lower than that on time spent with dependent parents; it can even be negative.

References

Cremer H. and Pestieau P. (2001). – "Non Linear Taxation of Bequests, Equal Sharing Rules and the Trade-off between Intra- and Inter-Family Inequalities", *Journal of Public Economics*, 79, pp. 35-54.

Hammond P. (1987). – "Altruism", in *The New Palgrave: a Dictionary of Economics*, J. Eatwell, M. Milgate and P. Newman (eds), Macmillan, 85-86.

Jousten A., Lipszyc B., Marchand M. and Pestieau P. (2005). – "Long Term Care Insurance and Optimal Taxation for Altruistic Children", *FinanzArchiv*, 61, pp. 1-18.

Appendix

$$
\begin{aligned}
\text{(A.1)}\quad \frac{\partial \mathcal{L}}{\partial t} = & -\int_{w_-}^{\frac{\omega_0}{1-t}} \Bigg\{ \pi H'(\omega_0 h)\omega_0 \frac{\partial h}{\partial \omega}(1-\beta) + \pi\, u'\left(c_1^D\right)\left(1-h+\left(1-\omega\frac{\partial h}{\partial G}\right)\frac{\partial G_1}{\partial \omega}\right) \\
& + (1-\pi)u'\left(c_1^N\right)\left(1+\frac{\partial G_1}{\partial \omega}\right) - v'(d_1)\frac{\partial G_1}{\partial \omega}\Bigg\} w dF(w) \\
& -\int_{\frac{\omega_0}{1-t}}^{w_+} \Bigg\{ \pi H'(s)\frac{\partial h}{\partial \omega}(1-\beta) + \pi\, u\left(c_2^D\right)\left(1+\left(1-\frac{\partial s}{\partial G}\right)\frac{\partial G_2}{\partial \omega}\right) \\
& + (1-\pi)u'\left(c_2^N\right)\left(1+\frac{\partial G_2}{\partial \omega}\right) - v'(d_2)\frac{\partial G_2}{\partial \omega}\Bigg\} w dF(w) \\
& + \Bigg\{ \mu\overline{y} - t\frac{\omega_0^2}{(1-t)^3} h(\omega_0, G_1+\sigma) \\
& - t\int_{w_-}^{\frac{\omega_0}{1-t}} w^2 \left(\frac{\partial h}{\partial \omega} + \frac{\partial h}{\partial G}\frac{\partial G_1}{\partial \omega}\right) dF(w) \Bigg\} = 0
\end{aligned}
$$

$$
\begin{aligned}
\frac{\partial \mathcal{L}}{\partial \sigma} = & -\int_{w_-}^{\frac{\omega_0}{1-t}} \Bigg\{ \pi H'(\omega_0 h)\omega_0 \frac{\partial h}{\partial \omega}(1-\beta) + \pi\, u'\left(c_1^D\right)\left(1-\omega\frac{\partial h}{\partial G}\right)\left(1+\frac{\partial G_1}{\partial \sigma}\right) \\
& + (1-\pi)u'\left(c_1^N\right)\frac{\partial G_1}{\partial \sigma} - v'(d_1)\frac{\partial G_1}{\partial \sigma}\Bigg\} dF(w) \\
\text{(A.2)}\quad & -\int_{\frac{\omega_0}{1-t}}^{w_+} \Bigg\{ \pi H'(s)\frac{\partial s}{\partial G} + \pi\, u'\left(c_2^D\right)\left(1-\frac{\partial s}{\partial G}\right)\left(1+\frac{\partial G_2}{\partial \sigma}\right) \\
& + (1-\pi)u'\left(c_2^N\right)\left(1+\frac{\partial G_2}{\partial \sigma}\right) - v'(d_2)\frac{\partial G_2}{\partial \sigma}\Bigg\} dF(w) \\
& -\mu\Bigg\{ \pi + t\int_{w_-}^{\frac{\omega_0}{1-t}} w\frac{\partial h}{\partial G}\left(1+\frac{\partial G_1}{\partial \sigma}\right) dF(w)\Bigg\} = 0.
\end{aligned}
$$