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Is Public Health Insurance an Appropriate Instrument for Redistribution? (*)

Dominique Henriet[1] **and Jean-Charles Rochet**[2]

ABSTRACT. – The share of the public sector in health insurance provision varies enormously from country to country. It is larger in more redistributive countries. We provide a possible theoretical explanation for these facts: a public health insurance system, financed by taxes, can be an efficient means of redistribution, complementary to income taxation. This relies on the assumption of a negative correlation between income and morbidity. We examine the empirical validity of this assumption on macro data.

La fourniture publique d'assurance maladie est-elle un instrument de redistribution adapté ?

RÉSUMÉ. – La part du secteur public en assurance maladie varie considérablement d'un pays à l'autre. Elle est plus importante dans les pays à forte tendance redistributice. Nous nous proposons d'en donner une explication théorique plausible : un système d'assurance maladie, financé par l'impôt, peut être un outil efficace de redistribution, complémentaire à la taxation du revenu. Ce résultat repose sur l'hypothèse d'une corrélation négative entre morbidité et revenu, corrélation qui semble être validée sur données macroéconomiques.

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1. D. Henriet : GREQAM, IDEP, 2 rue de la Charité, 13002 Marseille, et Ecole Centrale Marseille. dominique.henriet@egim-mrs.fr
2. J. C. Rochet : GREMAQ, IDEI, Université de Toulouse I, Manufacture des Tabacs, 21 allée de Britenne, 31000 Toulouse. rochet@cict.fr

1 Introduction

The importance of public systems in the provision of health insurance varies enormously from country to country: in some countries (like the USA), it is limited to specific categories of households (Medicare for the elderly, Medicaid for the poorer); in other countries (like most countries of the European Union) it is universal and compulsory. The generosity of coverage is also variable:

TABLE 1
Universality and generosity of public health insurance in several OECD countries (source: OECD, 1993, cited by Besley and Gouveia 1994)

	% population covered		% cost sharing	
	1981	1990	1981	1990
France	99	99.5	75	75.1
Germany	90	92.2	95	92
Italy	100	100	79.4	75.9
Sweden	100	100	95.5	94
UK	100	100	93	93
USA	42	44	61	61

The share of GNP that is dedicated to health expenses and the fraction that is publicly provided also vary a lot from country to country. For example, table 2 shows the ratios health expenses/GNP and public health expenses/GNP for several OECD countries in 2002:

TABLE 2
Health expenses in 2002 in several OECD countries (source: OECD 2003)

Country	France	Germany	Italy	Sweden	UK	USA
Health expenses (% of GNP)	9.7	10.9	8.5	9.2	7.7	14.6
Publicly provided (% of GNP)	7.4	8.6	6.4	7.9	6.4	6.6
Public share (%)	76	78	76	85	83	45

It is also interesting to look at the evolution of these figures in the recent years:

TABLE 3
Evolution of health expenses (in % of GNP) in several OECD countries (source OECD 2003)

Country	France	Germany	Italy	Sweden	UK	USA
1981	7.9	8.7	6.7	9.5	6.1	9.6
1990	8.8	8.4	8.1	8.6	6.2	12.4
1994	9.8	8.6	8.5	7.5	7.1	14.1

TABLE 4
Proportion of publicly provided health expenses (% of total health expenses) in several OECD countries (source OECD 2003)

Country	France	Germany	Italy	Sweden	UK	USA
1981	79	75	79	92	89	42
1990	75	72	77	89	84	42
1994	75	70	73	83	83	44

Even if the differences between Europe and the USA have decreased, they remain considerable.

A natural question is therefore: for a given country or government, how to determine the "optimal" amount of public provision of health insurance? Given the growing share of health expenditures in the GDPs of developed countries, this question has been (and is still to some extent) the object of an intense political debate[1]. A number of European countries, including the Netherlands, Spain, Sweden and the U.K. have reformed in depth their health care system, while others, like France are considering to do so. In the US, the Clinton administration has tried, without success, to introduce universal coverage in the early 1990s (see the symposium on Health Care Reform of the Journal of Economic Perspectives (1994) 8(3) and also Diamond (1991) for an analysis of the US case).

The theoretical literature discusses several possible reasons for public intervention in the domain of health insurance.[2] For example Poterba (1994), classifies as follows the possible justifications for such an intervention:

- the existence of market failures in the insurance sector, due for instance to adverse selection and moral hazard problems[3].

1. As McGuire *et al*. (1991) put it: "In some countries there is concern about excess spending on medical services, whilst in others under-funding is a major political issue... At one extreme it [health care] is delivered by the public sector and financed as part of general taxation. At the other it is privately delivered and charged for, much like any other good or service."
2. In some countries, like the UK, health care is directly provided by the public sector. We focus here on the public provision of health **insurance**, and consider that health care is privately provided. However our model could also apply to the British system.
3. Together with the well known agency problems identified by Arrow (1963), the most convincing example of market failure in the health insurance sector is probably the difficulty to provide insurance against long term illnesses. Cochrane (1995) suggests a system of severance payments that theoretically overcomes this difficulty.

• the existence of externalities (associated for instance to the risk of contagion of some illnesses) or even of a strategic behavior of some individuals who might take too many risks (or not enough precautions), anticipating that the government will come to their rescue expost if the risks materialize.

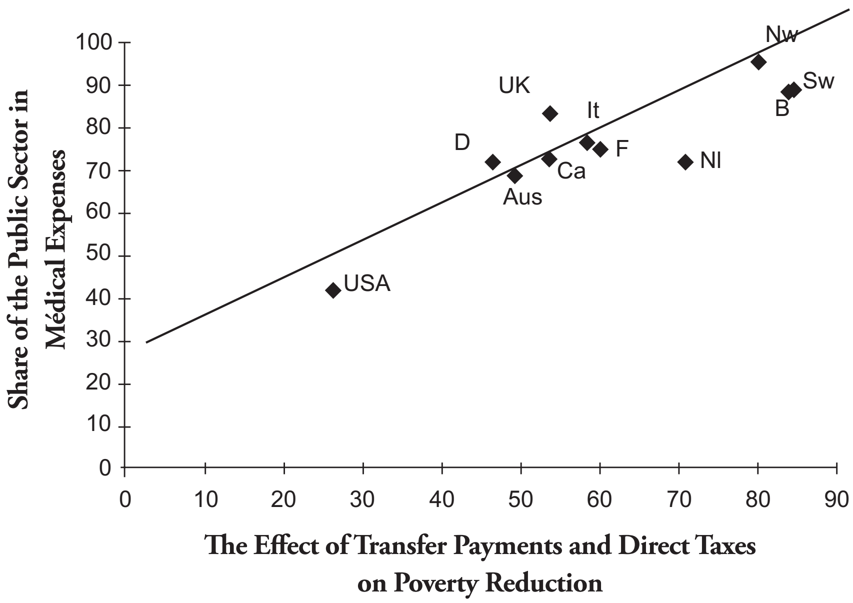
• irrationality of households: The idea is that some fraction of the households do not assess correctly the risks and consequences of illness.

• finally, equity considerations are often put forward. Some argue that basic health care must be accessible to all, but also that a public health insurance system, financed by taxes, can be used as an indirect means for redistributing income across households.

Since we want to explain cross-country differences, only the last justification seems appropriate. Indeed there is no reason to think that the magnitudes of market failures, externalities, or households' irrationality are so different from one country to another that they can explain the large differences in public coverage shown in tables 1 and 4. On the other hand, countries and governments obviously exhibit important differences in their preferences for redistribution. For example, Awad and Israeli (1997) measure the effect of transfer payments and direct taxes on poverty reduction in several OECD countries. Their results indicate a large cross-country variation in the measure of effective redistribution. Figure 5 plots these measures (on the horizontal axis) together with an indicator of the importance of public health insurance. As a matter of fact, it seems that there is a large positive correlation between preferences for redistribution and the share of the public sector in health expenses:

FIGURE 1

Public health insurance and redistribution



Source: Awad and Israeli, 1997

Although this positive statistical relation (between the importance of public health insurance and political preference for redistribution) is not really surprising, its explanation lacks a theoretical foundation. Indeed, if a government wants to redistribute income towards the more needy, why can't he use a direct method such as a negative income tax or a guaranteed income? Moreover, there are many reasons to think that, all things being equal, a government agency will be less efficient than a private insurance company for providing health insurance.[4] Also, and last but not least, there is a theoretical result by Atkinson and Stiglitz (1976) according to which, under very general assumptions, it is always inefficient to use any other instrument than income taxation for redistributive purposes.

We study in this paper a small variation on the classical income tax model of Mirrlees (1971) or Atkinson and Stiglitz (1976), in which a risk of illness is introduced. This risk varies across the population and is not observable a priori by individuals. The government has two policy instruments: the income tax schedule and public insurance coverage (which may also depend on income). There exists also a perfectly competitive market for complementary insurance. We determine under what conditions will public provision of health insurance be used as a redistributive device. We show that this will occur only if on average the probability of illness (or morbidity index) is larger for lower income groups. In that case, and not surprisingly, different governments will choose different levels of public coverage according to their preference towards redistribution.

We then examine the empirical validity of our condition on international macro data. Accordingly there seems indeed to exist a strong negative correlation between income and morbidity. Our conclusion is therefore that public health systems, financed by taxation, are in principle a good redistributive instrument, given that income and morbidity are negatively correlated.

The rest of the paper is organized as follows: section 2 discusses the related literature. Section 3 presents the theoretical analysis, while section 4 discusses empirical evidence. Section 5 concludes and indicates directions for future research. Most mathematical proofs are given is the appendix.

2 The Related Literature

The idea that public provision of health insurance can be used as a redistributive device is not new. Blomqvist and Horn (1984) study a model where individuals differ by two parameters: productivity and probability of illness (morbidity). They show that, except in special cases, public provision of health insurance (which they model as a lump sum benefit to ill people) can be a useful complement to linear taxation for redistributive puposes. Rochet (1991) extends their model to a continuum (bidimensional) distribution of types and shows that a negative cor-

4. The contrary argument is often put forward, given that public insurance systems are usually bigger than private companies and able to exploit scale economies. We conjecture that, once size is controlled for, the reverse is probably true. Indeed, incentives for cost minimization are likely to be stronger in a private system. So, in the long run, private provision of health insurance is likely to be more efficient than public provision.

relation between productivity and morbidity is a necessary and sufficient condition for full public health insurance to be optimal. Cremer and Pestieau (1996), (see also Petretto, 1999), confirm Rochet's result in a model with a discrete distribution of types. Boadway *et al.* (2004) introduce moral hazard and adverse selection in Rochet's model.

Our objective in this paper is to use this idea that public provision of health insurance can be an efficient tool for redistribution[5] in order to explain the positive correlation (across counties) between marginal tax rates (which indicate social preference to redistribution) and the extent of public provision of health insurance.

3 The Theoretical Analysis

In order to study the redistributive implications of public health insurance, we use as a starting point the classical income taxation model of Mirrlees (1971), in which individuals determine their labor supply L by maximizing a utility function $u(Y, L)$, where Y denotes net income (after tax). Individuals only differ by the marginal productivity w of their labor.

We extend this model by introducing a risk of illness: each individual has a probability π of falling ill, in which case he or she will have to incur the cost D of a treatment. To keep things as simple as possible, we consider only one type of illness (and treatment) and we exclude the possibility of adverse selection on π[6]. We assume that individuals and firms have the same information set: they observe w but not π. In this sense there is no asymmetry of information between (private) firms and individuals. Moreover, in conformity with most of the income tax literature, we assume that the private sector is perfectly competitive[7]. This means that an individual of "type" w can obtain a wage rate w on the labor market and can buy (private) health insurance at a fair premium rate $E[\pi \mid w] \overset{d}{=} \pi(w)$, equal to the conditional expectation of the probability of illness.

Since insurance markets are efficient, individuals will typically seek a complete coverage: if a compulsory coverage $q \leq D$ is provided publicly, they will buy the complementary coverage $(D-q)$ from a private insurer. This is the case for example if individuals have Von Neumann Morgenstern preferences[8].

Indeed, let K denote the demand of supplementary insurance bought from the private sector at the fair price $\pi(w)$, given that q is publicly supplied. The best choice

5. The same reasoning can be applied to justify in-kind transfers (see for instance Bergstrom and Blomqvist, 1996; Blomqvist and Christiansen, 1995 and Cremer and Gahvari, 1997).
6. In a previous paper, one of us (Rochet, 1991) has studied the case of (bidimensional) adverse selection on π as well as on individual productivities. Although the techniques are much more sophisticated, the results are similar to those we obtain in this paper. Besides, empirical studies do not seem to conclude in favor of adverse selection in the health insurance sector (see Genier, 1997 and the references therein).
7. Therefore in our model, public health insurance is not motivated by any market imperfection.
8. However, VNM preferences are not necessary for this property: It is even more true if agents have uncertainty aversion *à la* Yaari (1984) or Green and Jullien (1988). In this case they would buy complete insurance even if there is a loading factor on insurance premiums.

for the individual is to buy K that maximizes the level of expected utility (we note $d\mu(\pi/w)$ the conditional distribution of π knowing w):

$$\int\left\{\pi u\left(Y-D+q-K-\pi(w)K,L\right)+(1-\pi)u\left(Y-\pi(w)K,L\right)\right\}d\mu(\pi/w).$$

This is equal (by definition of $\pi(w)$) to:

$$\pi(w)u\left(Y-D+q-K-\pi(w)K,L\right)+\left(1-\pi(w)\right)u\left(Y-\pi(w)K,L\right),$$

which is maximum for $K=D-q$.

Therefore, the indirect utility function of individuals becomes $u\left(Y-\pi(w)(D-q),L\right)$. Except when $q=D$ (full public coverage) this utility function is now type dependent, since in general $\pi(w)$ depends on w. This will be the crucial ingredient of the redistributive role of public insurance: The market price of coverage is type dependent. In particular, when π is decreasing, that is when morbidity is greater for poor people, the private complementary coverage is less expensive for rich people than for the poor. This point is crucial for the sequel: it means that private complementary insurance implies reverse redistribution.

The government does not observe w (nor π), but observes gross income Z. In this context, public policy is composed of two instruments: $T(Z)$, the taxation function and $Q(Z)$ the public provision of health insurance function.

The indirect utility achieved by an individual w will then be:

(1) $$U(w)=\max_{Z} u\left(Z-T(Z)-\pi(w)\left(D-Q(Z)\right),\frac{Z}{w}\right)$$

It is clear that the choice of these instruments by a government depends in a complex fashion on the political system and on the characteristics of the economy. The bottom line of this article is that countries with high marginal tax rates are also more likely to exhibit high levels of public coverage. Since we don't want to enter the difficult problem of modelling the political process by which these policy instruments are chosen, we will use a revealed preference argument (at the government level).

For this argument to work we only need to assume that governments select Pareto optimal combinations of these policy instruments: therefore we assume there exists a system of positive weights[9] $\alpha(w)$ such that the States maximize a weighted sum of individual utilities and government revenue[10]:

$$W=E\left[\alpha(w)U(w)+\lambda\left(T\left(Z(w)\right)\right)-(1+\gamma)\pi(w)Q\left(Z(w)\right)\right]$$

9. These weights do not necessarily represent the actual distribution of political power among income classes. They may rather be understood as representing the social preference for redistribution. For example, Benabou and Ok (1997) have a model in which even people with income below average will not support high rates of redistribution, because of the prospect of upward mobility for themselves or their children.

10. Although the technical resolution looks similar to that of the normative model, our formulation is very flexible. For example a median voter model is compatible with the approach if we take $\alpha(w)=1$ when w equals the median productivity, and 0 elsewhere. Similarly, in probabilistic voting models *à la* Coughlin (1992), elected governments choose tax schedules that maximize welfare functions of the Nash type.

where λ stands for the opportunity cost of public funds and the parameter $\gamma \geq 0$ is here to capture the notion that public insurance is presumably less efficiently provided than private insurance.

3.1 Optimal Linear Taxation

We start with the technically simplest case where the income tax T is restricted to be linear[11] and Q to be uniform[12]:

(2) $$T(Z) = tZ - G.$$

(3) $$Q(Z) = q$$

Here $t \in]0,1[$ is the (constant) marginal tax rate and G is a guaranteed income. This case has been studied by Sheshinski (1972), Dixit and Sandmo (1977) and Stiglitz (1976). The policy instruments of the government are then t, G and q.

The optimal policy is such that (t, G, q) maximize

(4) $$W = E\left[\alpha(w)U(w) + \lambda\left(tw\ell(w) - G - (1+\gamma)\pi(w)q\right)\right]$$

where $\ell(w)$ is the labor supply.

In order to make explicit the impact of (t, G, q) on $U(w)$ and $\ell(w)$ we use the following notation:

(5) $$V(w, I) = \max_L u(wL + I, L),$$

(6) $$L(w, I) = \arg\max_L u(wL + I, L).$$

11. Actually, there even exists a technically simpler case where individual labor supply is totally inelastic. In this case, the implementation of a public insurance scheme can only have an impact on the individuals' decision to participate in the labor force. For example it has been argued (Cutler, 1994, p. 20) that in the US "empirical estimates suggest that up to one-quarter of the approximately 4 million welfare recipients would enter the labor force if health insurance were available continuously". Similarly, Newhouse (1994, p. 9) argues that "the loss of Medicaid benefits [for welfare mothers who start working] is a disincentive to work". On the other hand Newhouse (1994, p. 9) states that "by choosing to pay the employer's share of the premium for retired workers and providing an income-conditioned subsidy for the remaining amount, the Health Security Act will encourage early retirement". It is clear that these examples only concern marginal participants in the labor force (namely here welfare mothers and potential retirees). Since we want to analyze the impact of universal coverage on the incentives to work for the whole population, we need a model where individual (or rather, households) labor supply is elastic.
12. In the next subsections, we extend the analysis by allowing nonlinear income taxation and a variable level of public coverage across income classes.

We obtain an explicit expression of W as a function of (t,G,q):

$$W(t,G,q)=E\Big[\alpha(w)V\big(w(1-t),G-\pi(w)(D-q)\big) \\ +\lambda\Big(twL\big(w(1-t),G-\pi(w)(D-q)\big)-G-(1+\gamma)\pi(w)q\Big)\Big] \tag{7}$$

The question we examine now is the following: under what conditions is it the case that W is maximized for a combination $\left(t^*,\, G^*,\, q^*\right)$ of policy instruments such that full insurance is publicly provided (*i.e.* $q^* = D$)?[13] Notice that in this case the choice of t and G is exactly the same as in the classical problem solved by Sheshinski (1972):

$$W(t,G,D)=E\Big[\alpha(w)V\big(w(1-t),G\big)+\lambda\Big(twL\big(w(1-t),G\big)\Big)\Big]-\lambda\big[G+(1+\gamma)\pi_a D\big].$$

Indeed, the only additional term with respect to Sheshinski's analysis is the cost of public insurance $\lambda(1+\gamma)\pi_a D$ (where $\pi_a = E\big(\pi(w)\big)$ is the average probability of illness in the population). Since we have taken λ to be exogenous, this additional term does not alter the choice of t^* and G^*. Let us now briefly recall the results of Sheshinski (1972) and Dixit and Sandmo (1976). For this we need some notation:

$$b(w)=\frac{\alpha(w)}{\lambda}\frac{\partial V}{\partial I}\big(w(1-t),G\big)+tw\frac{\partial L}{\partial I}\big(w(1-t),G\big). \tag{8}$$

$b(w)$ is the net social marginal valuation of income.

$$\varepsilon_{LL}(w)=\frac{w(1-t)}{L\big(w(1-t),G\big)}\left(\frac{\partial L}{\partial w}\right)_U. \tag{9}$$

$\varepsilon_{LL}(w)$ is the compensated elasticity of labor supply.

PROPOSITION 1. *(adapted from Dixit and Sandmo (1976)): When $q^* = D$, the marginal tax rate t^* and guaranteed income G^* are chosen in such a way that*

$$E\big[b(w)\big]=1, \tag{10}$$

$$\frac{t^*}{1-t^*}=-\frac{\operatorname{cov}(b,Z)}{E(Z\varepsilon_{LL})}, \tag{11}$$

where $Z(w)=wL\left(w\left(1-t^*\right),G^*\right)$ denotes the gross income of an individual of type w.

13. The reason why we focus on this corner solution is technical: when $q^* = D$, our problem boils down to the classical optimal tax problem. Solving for the general case is more complex.

Given this result, full public coverage will be optimal only when the first order condition is satisfied, namely:

$$\frac{1}{\lambda}\frac{\partial W}{\partial q}\left(t^{*},G^{*},D\right)\geq 0.$$

Using formula (7) we obtain:

$$\begin{aligned}\frac{1}{\lambda}\frac{\partial W}{\partial q}&=E\left[\left(\frac{\alpha(w)}{\lambda}\frac{\partial V}{\partial I}+tw\frac{\partial L}{\partial I}\right)\pi(w)-(1+\gamma)\pi(w)\right],\\&=E\left[b(w)\pi(w)-(1+\gamma)\pi(w)\right].\end{aligned}$$

Given formula (8) this can also be written:

$$\frac{1}{\lambda}\frac{\partial W}{\partial q}\left(t^{*},G^{*},D\right)=\operatorname{cov}(b,\pi)-\gamma\pi_a. \tag{12}$$

Therefore we obtain our first result:

PROPOSITION 2 *Full public coverage will be chosen (for a given weight function $\alpha(\cdot)$) only if*

$$\operatorname{cov}(b,\pi)\geq\gamma\pi_a, \tag{13}$$

i.e. if the covariance between the net social marginal valuation of income and the probability of illness is positive and large enough.

Although proposition 2[14] does not give any explicit condition relating t^{*} (as obtained from proposition 1) and q^{*}, the similarity between formulas (11) and (13) is striking. Roughly speaking, formula (11) establishes that $\frac{t}{1-t}$ is proportional to (minus) the covariance between b (the social valuation of income) and gross income Z. Similarly, formula (13) establishes that full public coverage will be chosen as soon the covariance between b and π, the probability of illness, is positive and large enough.

When b is a non increasing[15] function of w, (13) amounts essentially to requiring that $E[\pi/w]$ be also a non increasing function of w. Although it is difficult to give a precise measure of the empirical counterpart of π, (that we will call the

14. As well known, optimal tax problems are not necessary concave. Therefore we only have necessary conditions.
15. Since Z increases with w, this implies that $\operatorname{cov}(b,Z)<0$ and the marginal tax rate defined by (10) is indeed positive.

morbidity index), we will see in our empirical section that it is reasonable indeed to postulate a negative statistical relation between π and *w* (or *Z*). To be concrete, let us assume for example that $E\left[\pi / w\right]$ is an affine function of $Z\left(w\right)$:

$$E\left[\pi / w\right] = -k_0 Z\left(w\right) + k_1, \tag{14}$$

where k_0 is a positive coefficient, and k_1 is a constant. An immediate consequence of propositions 1 and 2 is then:

PROPOSITION 3. *If the (conditional) morbidity index* $E\left[\pi / w\right]$ *is a decreasing affine function of income, then full public coverage will be chosen as soon as the marginal tax rate* t^* *is high enough. More precisely, this will be the case if it satisfies:*

$$\frac{t^*}{1-t^*} \geq \frac{\gamma \pi_a}{k_0 E\left(Z \varepsilon_{LL}\right)}, \tag{15}$$

where $\left(-k_0\right)$ *is the coefficient of the affine regression of* π *on income.*

Proof of proposition 3: From relation (14) we deduce immediately

$$\operatorname{cov}\left(b, \pi\right) = -k_0 \operatorname{cov}\left(b, Z\right).$$

Therefore, formula (11) implies that

$$\frac{t^*}{1-t^*} = \frac{\operatorname{cov}\left(b, \pi\right)}{k_0 E\left(Z \varepsilon_{LL}\right)}.$$

Thus a sufficient condition for (13) is:

$$\frac{t^*}{1-t^*} \geq \frac{\gamma \pi_a}{k_0 E\left(Z \varepsilon_{LL}\right)}$$

The intuition behind propositions 2 and 3 is simple when $b\left(\cdot\right)$ (the net social valuation of income) is a non increasing function of *w*. This means that, ideally, society would like to redistribute more income from the rich to the poor[16] but cannot do so with only two instruments, t and G. On the other hand if $\pi(w)$, the market value of health insurance, is greater for lower income classes, then the public provision of this health insurance is tantamount to a targeted redistribution of income. If public insurance is efficiently provided $(\gamma = 0)$ then this additional instrument will always improve (weighted) social welfare. If $\gamma > 0$, this is still the case when the function $b(\cdot)$ is steep enough (strong redistributional objectives) which materializes by a large marginal tax rate.

One may argue superficially that a positive relation between public coverage and taxation is purely mechanical, since public insurance is typically financed by taxation. This is not true here since we distinguish two taxation instruments: the marginal tax rate t and the guaranteed income G. Thus we can neutralize the impact of increasing q by decreasing G of an amount equal to the average cost of public insurance[17] $(1+\gamma)\pi_a$. A priori this should be totally unrelated to changes in t.

There is an obvious criticism to our interpretation of public insurance as a redistributive tool. It relies on a very general result of Atkinson and Stiglitz (1976). According to this result, under mild assumptions on households' preferences, it is inefficient to use any other instrument than (optimal) income taxation for redistributive purposes. Therefore it is important to check if our result only stems from our restriction to (suboptimal) linear taxation or if it is robust to the introduction of optimal (nonlinear) taxation. Before considering the general (but complex) model of Mirrlees (1971), we examine this question within the more intuitive discrete model of Stiglitz (1982).

3.2 **Optimal Nonlinear Taxation 1: The Discrete Case**

We now particularize our model, by considering (as in Stiglitz, 1982) that there are only two classes of individuals: the "poor" (or unskilled workers) of productivity w_1 and the "rich" of productivity $w_2 > w_1$. On the other hand, we now allow for nonlinear taxation. As in subsection 2.1, we want to see if the introduction of a public insurance scheme can improve the (weighted) social welfare criterion:

$$W = E\Big[\alpha(w)U(w)+\lambda\Big(T\big(Z(w)\big)-(1+\gamma)\pi(w)Q(w)\Big)\Big].$$

If we use the mechanism design approach, the revelation principle implies that no generality is lost by using a direct revelation mechanism:

$$w \to \big(Y(w),\ell(w),Q(w)\big),$$

(recall that Y is after tax income, ℓ is labor supply and Q is public coverage), and imposing incentive compatibility conditions. For all w and w' one must have:

16. When $b(\cdot)$ is decreasing or not monotonic (indicating social preferences towards reverse redistribution) condition (13) is likely to be violated. This is consistent with the main message of this paper: public health insurance is less developed in less redistributive countries.

17. This is because we have taken λ, the shadow cost of public funds, as exogenous.

$$(16) \qquad u\big(Y(w)-\pi(w)\big(D-Q(w)\big),\ell(w)\big) \geq u\left(Y(w')-\pi(w)\big(D-Q(w')\big),\frac{w'\ell(w')}{w}\right).$$

For notational simplicity, we will use indices $\left(Y_i,\pi_i,q_i,\ell_i \text{ and } \alpha_i\right)$ instead of functions $\left(Y(w),\pi(w),Q(w),\ell(w) \text{ and } \alpha(w)\right)$. We also introduce the "net-net" income of a type i individual (after tax and private insurance charges):

$$(17) \qquad C_i = Y_i - \pi_i\left(D-q_i\right), \quad i=1,2.$$

It is interesting to rewrite the objective function W and the incentive compatibility constraints with these new notations. We obtain:

$$(P_1)\begin{cases} \max \quad W = E\left[\alpha_i u\left(C_i,\ell_i\right)+\lambda\left(w_i\ell_i - C_i - \gamma\pi_i q_i\right)\right] \\ \forall i,j \quad u(C_i,\ell_i) \geq u\left(C_j+\left(\pi_j-\pi_i\right)\left(D-q_j\right),\frac{w_j}{w_i}\ell_j\right) \ \left(IC_{ij}\right)\end{cases}$$

Several remarks are in order at this stage:

- With the transformation of W, it is clear that, since γ is non negative (*i.e.* public insurance is at best as efficient as private insurance) W is decreasing in q_i. Therefore, the only reason why public insurance might be used is alleviation of incentive constraints.
- To see how this might work, consider the "normal" regime where the binding constraint is IC_{21} (see below for the conditions that imply this), which means that redistribution is limited by the disincentives to work of the more productive. Now if $\pi_1-\pi_2$ is positive (which means that the more productive are also less often ill) then increasing q_1 leads to decreasing the right hand side of IC_{21}, and thus to slackening the incentive constraint.
- The reason why the Atkinson-Stiglitz (1976) result does not apply to our context is that the varying morbidity rate $\pi(w)$ makes individual preferences $U\left(Y-\pi(w)\left(D-q\right),\ell\right)$ type dependent. Therefore, changing q alters the marginal rate of substitution between consumption and labor (and thus, incentives to work) in a manner that changes across productivity classes.

Before solving P_1, let us notice that if we impose full public insurance (*i.e.* $q_j \equiv D$)[18] this program is equivalent to the two-class optimal tax problem solved by Stiglitz (1982). Stiglitz identifies three regimes:

a) The "normal" regime, where redistribution is limited by work disincentives of the more productive (and thus IC_{21} is the only binding constraint). It occurs

18. As we will see, this is never optimal in a discrete model.

when the political weight of the poor is not too low:[19] $\alpha_1 \geq \alpha^*$. In this regime, the marginal tax rates are non negative.

b) the "undistorted" regime, where none of the incentive compatibility constraints are binding, and as a result marginal tax rates are zero. This corresponds to lump sum taxation, and occurs when the political weight of the poor satisfy $\alpha^{**} \leq \alpha_1 < \alpha^*$, where α^{**} is a second threshold (typically very low).

c) Finally the "regressive" regime, where the political weight of the poor is so low that transfers actually take place from the poor to the rich, and marginal tax rates are non positive.

Using again casual revealed preference arguments (and in particular the fact that, as far as we know, zero or negative marginal tax rates on income are unfrequent in practice), we will exclude **b)** and **c)** and concentrate on the "normal" regime. The following proposition characterizes the marginal tax rates[20] T_i' and public coverages q_i at the solution of P_I.

PROPOSITION 4. *In the "normal" regime, the solution of P_I is characterized by:*

(18) $$\frac{T_1'}{1-T_1'} = \frac{\mu_{21}}{\lambda} u_C^{21} \left(1 - \frac{w_1}{w_2} \frac{(u_L/u_C)\left(C_1, \frac{w_1 \ell_1}{w_2}\right)}{(u_L/u_C)(C_1, \ell_1)} \right)$$

(19) $$q_1 = D \Rightarrow \frac{\gamma \pi_1}{\pi_1 - \pi_2} \leq \frac{\mu_{21}}{\lambda} u_C^{21},$$

and

(20) $$T_2' = q_2 = 0.$$

Here, μ_{21} represents the (normalized) multiplier associated to the constraint IC_{21} and u_C^{21} denotes the marginal utility of consumption for an individual of type 2 who "mimics" an individual of type 1:

(21) $$u_C^{21} = u_C\left(C_1, \frac{w_1, \ell_1}{w_2}\right).$$

Proof of proposition 4: see the appendix.

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19. The threshold α^* is greater than $\frac{1}{2}$, which means that the utilitarian case $\alpha_1 = \alpha_2 = \frac{1}{2}$ belongs to this regime.
20. In a discrete model, there are an infinity of tax schedules that can implement the second best solution. T_i' represents the implicit marginal tax rate defined by $T_i' = 1 + w_i \frac{u_C}{u_L}(C_i, L_i)$.

The relation between public coverages q_i and marginal tax rates T_i' is again striking in proposition 4. First, both T_2' and q_2 are always zero: this is the famous result of "no distortion at the top", crucially related to the artificial assumption that the support of w is bounded above. We will relax this assumption in the next subsection. More interesting is the comparison between formulas (18) and (19), from which we deduce the following property:

COROLLARY 1. *Full public insurance will be provided to the poor only when marginal tax rates are high enough:*

$$q_1 = D \Rightarrow \frac{T_1'}{1-T_1'} \geq \frac{\gamma\pi_1}{\pi_1 - \pi_2}\left(1 - \frac{w_1}{w_2}\frac{(u_L/u_C)\left(C_1, \frac{w_1\ell_1}{w_2}\right)}{(u_L/u_C)(C_1,\ell_1)}\right). \tag{22}$$

Therefore, we confirm the result of subsection 3.1 according to which public insurance systems are more likely to be present in countries where marginal tax rates are high, which seems consistent with casual empiricism.

In order to go further, and obtain conditions that only depend on exogenous variables, let us simplify problem P_1 by assuming a constant elasticity of labor supply[21] ε:

$$u(C,\ell) = u\left[C - v(\ell)\right], \tag{23}$$

where u is concave increasing, such that $u'(+\infty) = 0$, and

$$v(\ell) = \frac{\ell^{1+1/\varepsilon}}{1+1/\varepsilon}. \tag{24}$$

With this specification, we obtain the following comparative statics properties, which confirm our intuition that public health insurance will be observed in countries where marginal tax rates are high.

PROPOSITION 5. *Under the specification (23), (24),*

1. The marginal tax rate is a decreasing function of α_2 (the political weight of the "rich"), f_1 (the proportion of "poor" people) and w_2 (the productivity of the "rich", while the ratio $\frac{w_2}{w_1}$ is kept constant).

2. Similarly, the corner solution $q_1 = D$ (full public health insurance for the poor) will be less likely when either of these parameters α_2, f_1 or w_2 increases.

21. Indeed, (22) and (23) imply that the labor supply function $L(w,I)$ equals w^{ε}.

3. Moreover, an increase in γ *(the inefficiency parameter of public health insurance) or a decrease in* $\frac{\pi_1 - \pi_2}{\pi_1}$ *(the inequality of morbidity indices) makes public insurance less likely.*

Proof: see the appendix.

These results suggest the robustness of the positive relation between the share of the public sector in health insurance and the political preference towards redistribution (materialized by high progressivity of the income tax). However they are still unsatisfactory for (at least) two reasons:

- Even in the less "redistributive" countries like the US, the poorest people have access to public health insurance (namely the Medicaid system). Therefore the real question is whether the middle class should also have access to public coverage. Clearly, a two-class model is insufficient to determine this.
- Moreover, the assumption that productivities are bounded above (or more precisely that there is a class of people who are the "most productive" in the economy) leads to the counter factual prediction of zero marginal tax rates at the top.

The only way to get around these difficulties is to examine what happens to public health insurance in a continuum income tax model *à la* Mirrlees (1971), where the support of *w* is an interval $\left[w_0, +\infty\right[$. This is what we do now.

3.3 Optimal Nonlinear Taxation 2: The Continuum Case

Following Mirrlees (1971), consider now the case where w is distributed on an interval $\left[w_0, +\infty\right[$ with a density $f(w)$. The two instruments of the government are the tax function $T(Z)$ (where $Z = w\ell$ is the gross income) and the public coverage function $Q(Z)$ which in full generality can depend also on income. Using as before the revelation principle, this is equivalent to a direct mechanism $w \to \left(C(w), \ell(w), q(w)\right)$ where $C(w)$ denotes net net income (after deduction of taxes and private insurance premiums), $\ell(w)$ is labor supply and $q(w) = Q\left(w\ell(w)\right)$ is public coverage. The same transformation as the one used in subsection 3.2 yields to maximizing the weighted social welfare function:

$$E\left[\alpha(w)U(w) + \lambda\left\{w\ell(w) - C(w) - \gamma\pi(w)q(w)\right\}\right], \tag{25}$$

under the incentive compatibility constraints, which we can write as mentioned earlier:

$$\forall w \quad U(w) = \max_Z u\left(Z - T(Z) - \pi(w)\left(D - Q(Z)\right), \frac{Z}{w}\right). \tag{1}$$

Adopting the same method as Mirrlees (1971) (*i.e.* the first order approach), we replace the constraint (1) by its first order counterpart, obtained by using the envelope theorem:

$$(26) \quad \dot{U}(w) = -\dot{\pi}(w)\big(D - q(w)\big)u_C\big(C(w), \ell(w)\big) - \frac{\ell(w)}{w}u_L\big(C(w), \ell(w)\big).$$

This allows us to transform our problem into an optimal control problem, where $U(\cdot)$ is the state variable, and (26) the state equation. The difference with the Mirrlees case is that we have now two control variables: labor supply $\ell(\cdot)$ (as in the Mirrlees model) and public coverage $q(\cdot)$. The net-net income $C(w)$ is then obtained implicitly by the identity

$$(27) \qquad U(w) = u\big(C(w), \ell(w)\big),$$

which gives C as an implicit function of U and ℓ: $C(w) = Y\big(U(w), \ell(w)\big)$.

Thus our problem writes finally:

$$P_2 \begin{cases} \max_{U,\ell,q} \quad E\Big[\alpha(w)U(w) + \lambda\big\{w\ell(w) - Y\big(U(w), \ell(w)\big) - \gamma\pi(w)q(w)\big\}\Big] \\ \qquad\qquad \text{under the constraint} \\ \dot{U}(w) = -\dot{\pi}(w)\big[D - q(w)\big]u_C\big(Y(U,\ell), \ell\big) - \frac{\ell}{w}u_L\big(Y(U,\ell), \ell\big). \end{cases}$$

Finding a closed-form solution of P_2 is impossible: this is a classical drawback of the nonlinear income tax model. However, it is easy to find a simple necessary condition for full public coverage to be provided to productivity class w (*i.e.* $q(w) = D$). As in previous models, it involves the marginal tax rates.

PROPOSITION 6. *A necessary condition for* $q(w) = D$ *in the solution of* P_2 *is:*

$$(28) \qquad \left(\frac{T'}{1-T'}\right)\big(Z(w)\big) \times \left(-\frac{w\dot{\pi}(w)}{\pi(w)}\right) \geq \gamma\left(1 + \frac{Lu_{LL}}{u_L}\right).$$

Proof: See the appendix.

As in the optimal taxation literature, the marginal tax rate is itself determined by a classical implicit equation involving, in particular a term related to the social preference for redistribution: roughly speaking, when $\alpha(\cdot)$ is more redistributive, T' is more likely to be high (see appendix).

Together with the marginal tax rate T' (computed for the gross income $Z(w)$ of type w), formula (28) involves three terms:

- the elasticity of morbidity with respect to w,
- γ, the relative efficiency parameter of private over public insurance,
- and finally a term that depends on individual preferences:

$$\varepsilon^* = 1 + \frac{Lu_{LL}}{u_L}.$$

This term is a measure of the elasticity of labor supply[22] that appears in the income tax literature (see for example Atkinson and Stiglitz (1980) p. 418). $\varepsilon^* - 1$ corresponds to the derivative of L with respect to w, holding the marginal utility of income constant. Notice that when public provision of insurance is efficient $(\gamma = 0)$ and when redistribution is "normal" $\left(T'\left(Z(w)\right) > 0\right)$ then (28) is equivalent to $\dot{\pi}(w) \leq 0$.

Assuming π is decreasing, condition (28) is hence more likely to be fulfilled when T' is large enough, that is to say when there is a rather strong preference for redistribution. Conversely, if π were increasing, public provision would never be efficient.

Hence this continuous model enforces the intuition described in the two previous sections. The main result obtained previously is robust since it clearly applies in the general case of non linear taxation.

4. Morbidity and Income: Empirical Evidence

The crucial ingredient for our analysis is the statistical relation between morbidity and income We now explore the empirical validity of this assumption, using the cross-country study of Van Doorslaer *et al.* (1997).

Although there is a lot of statistical evidence for a negative correlation between **mortality rates** and income, the relation between **morbidity** and income is less well known. One reason may be the difficulty to measure morbidity. However, Van Doorslaer *et al*. (1997) provides very detailed statistical evidence on cross country variations in income-related health inequalities. Using large samples (3.300 to 22.000 adults in each country) drawn from 9 national health surveys (Finland, East and West Germany, Netherlands, Spain, Sweden, Switzerland, UK and USA), the authors compute average health indices per income class. These indices correspond to self-assessed health scores[23] corrected for demographic variables like age and sex. From these indices, the authors construct ill-health concentration curves for each country. Roughly speaking, the Status Adjusted Health concentration function $L(x)$ measures the average morbidity index among the people with an income less or equal to the x^{th} quantile of income in the country. Formally:

$$L(x) = \frac{1}{\pi_a} \int_0^x \Pi\left[F^{-1}(s)\right] ds,$$

where $\Pi(Y)$ represents the morbidity index as a function of after tax income Y, π_a is the average of $\Pi(Y)$ in all the population, and $F(Y)$ denotes the propor-

22. However it is not the classical measure ε_{LL} (see our formula (9)).
23. These scores are computed by assuming the existence of latent self-assessed health variable with a standard lognormal distribution.

tion of people with income less or equal to Y. The values of $L(x)$ for 4 different countries are given in Table 6:

TABLE 6
Cumulative Status Adjusted Health per Cumulative Income

Income Quantile	10%	20%	30%	40%	50%	60%	70%	80%	90%
Sweden	11.5	22.5	32.2	42.2	52	63	71.8	80.7	90.2
US	14.8	28	39.5	50	59.5	69	78	86	94
Spain	12.5	24	35	45	55	65	74	84	92
Switzerland	12.2	24	35	46	55	65	74	83	92

Source: *van Doorslaer et al., 1997*

Formally, the derivative of L gives exactly the function $x \to \frac{\Pi(F^{-1}(x))}{\pi_a}$, which is the relative morbidity index of the x^{th} quantile. Approximating this derivative by the rate of increase of L we obtain the following table:

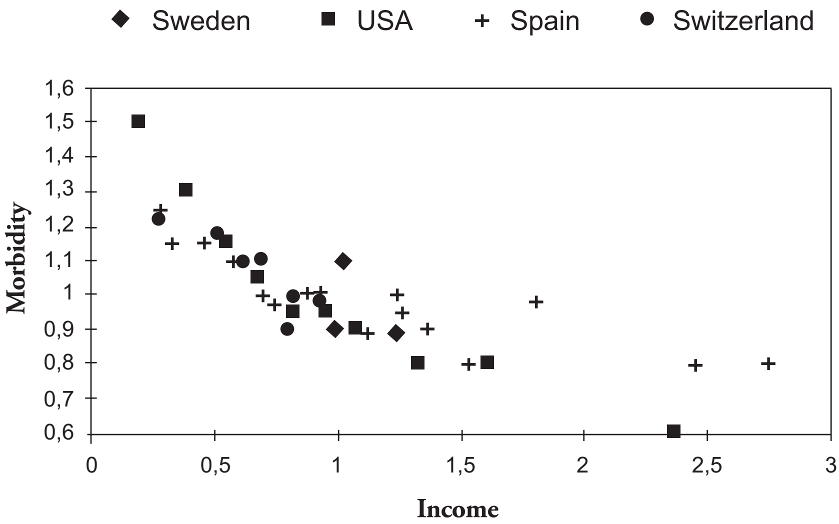
TABLE 7
Relative Morbidity Index per Income Decile

Income Quantile	10%	20%	30%	40%	50%	60%	70%	80%	90%	100%
Sweden	1.15	1.1	0.97	1	0.98	1.1	0.88	0.89	0.95	0.98
US	1.5	1.3	1.15	1.05	0.95	0.95	0.9	0.8	0.8	0.6
Spain	1.25	1.15	1.1	1	1	1	0.9	1	0.8	0.8
Switzerland	1.22	1.18	1.1	1.1	0.9	1	0.9	0.9	0.9	0.8

Table 7 clearly indicates that there is indeed a strongly negative statistical relation between income and morbidity. It is even clearer if we plot the function $Y \to \Pi(Y)$ by using the income distributions of the different countries[24]:

24. Income is disposable income normalized such that the average in each country is 1. Morbidity is the morbidity index computed by Van Doorslaer *et al*. (1997), and normalized such that the average in each country is 1. (source: Van Doorslaer *et al*. (1997), Atkinson *et al* (1994) and our own calculations).

FIGURE 2
Income and Morbidity



5 Conclusion

We have identified in this article a possible line of explanation for the cross-country variation of the public sector share in the provision of health care. If morbidity is negatively correlated with income (which seems empirically confirmed) then public provision of health insurance is theoretically an efficient instrument for income redistribution, complementary to income taxation. In particular our model predicts that the share of the public sector in health insurance should be positively correlated with marginal income tax rates. A first natural extension of our work would be an empirical test of this prediction.[25]

However introduction of ex post moral hazard would complicate the analysis: fully insured people tend to consume more medical services than they need. Moreover, poor people are less likely to buy complementary insurance and, even when insured, tend to consume less medical services than the rich. In a recent paper Boadway and al. (2004) introduce moral hazard and adverse selection in the model of redistributive health insurance of Rochet (1991). They show that the optimal coverage by the public insurance system is less than full (due to moral hazard). It would be interesting to extend our analysis to their model and check that, even with

25. We must admit that our theory does not explain the existence of the Medicare system: if public provision of health insurance is aimed at redistributing income to the poorer without altering too much incentives for work, it is hard to understand why in the less redistributive country, the USA, all retirees (for whom there is no problem of work disincentives) have access to a subsidized public system, independently on their income.

moral hazard and adverse selection, more redistributive countries provide more public health insurance.

Finally, we have the feeling that the same type of reasoning could be applied as well to the public provision of education.[26] There are also huge cross crountry differences in this sector, and the usual arguments in favor of public provision of schooling (market failures, externalities) are not appropriate for explaining these cross country differences. However, it seems natural to conjecture that the "market price" of education is greater for children coming from lower income groups. Therefore, a simple adaptation of our theoretical analysis would lead to the prediction that public education is more likely to be provided in countries where society has a strong preference towards redistribution, which materializes by high marginal income tax rates.

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Appendix

Proof of Proposition 4

Denoting by f_i the frequency of type i in the population $(i = 1,2)$ and by $\mu_{21} f_1$ the multiplier associated to the constraint IC_{21}, the Lagrangian of P_I (in the normal regime) writes:

$$\mathcal{L} = \sum_{i=1}^{2} f_i \left(\alpha_i u(C_i, \ell_i) + \lambda \left(w_i \ell_i - C_i - \gamma \pi_i q_i \right) \right)$$
$$+ \mu_{21} f_1 \left[u(C_2, \ell_2) - u\left(C_1 + (\pi_1 - \pi_2)(D - q_1), \frac{w_1 \ell_1}{w_2} \right) \right].$$

The first order conditions for maximizing $\mathcal{L}$ are:

(1) $$\frac{\partial \mathcal{L}}{\partial C_1} = f_1 \left[\alpha_1 u_C (C_1, \ell_1) - \lambda - \mu_{21} u_C^{21} \right] = 0$$

(2) $$\frac{\partial \mathcal{L}}{\partial \ell_1} = f_1 \left[\alpha_1 u_L (C_1, \ell_1) + \lambda w_1 - \mu_{21} u_L^{21} \frac{w_1}{w_2} \right] = 0$$

(3) $$\frac{\partial \mathcal{L}}{\partial C_2} = (f_2 \alpha_2 + \mu_{21} f_1) u_C (C_2, \ell_2) - \lambda f_2 = 0$$

(4) $$\frac{\partial \mathcal{L}}{\partial \ell_2} = (f_2 \alpha_2 + \mu_{21} f_1) u_L (C_2, \ell_2) + \lambda f_2 w_2 = 0$$

(5) $$\frac{\partial \mathcal{L}}{\partial q_1} = f_1 \left[-\gamma \pi_1 \lambda + \mu_{21} (\pi_1 - \pi_2) u_C^{21} \right]$$

(6) $$\frac{\partial \mathcal{L}}{\partial q_2} = -f_2 \alpha_2 \gamma \pi_2 \le 0 \ \Rightarrow \ q_2 = 0.$$

It is well known that, in such a discrete model, optimal tax schedules are piecewise linear and thus, not differentiable everywhere. However it is natural to define an "apparent" marginal tax rate T_i' by the relation:

$$\frac{T_i'}{1-T_i'} = -1 - w_i \frac{u_C}{u_L}(C_i, \ell_i).$$

Using this definition, it is immediate to deduce from (A.1) to (A.4) the desired properties:

$$(\text{A.3}) + (\text{A.4}) \Rightarrow \frac{w_2 u_C}{u_L}(C_2, \ell_2) = -1 \Rightarrow T_2' = 0.$$

$$(\text{A.1}) \Rightarrow \frac{\mu_{21}}{\lambda} u_C^{21} = -1 + \frac{\alpha_1}{\lambda} u_C (C_1, \ell_1)$$

$$(\text{A.2}) \Rightarrow \frac{\mu_{21}}{\lambda} u_L^{21} \frac{w_1}{w_2} = w_1 + \frac{\alpha_1}{\lambda} u_L (C_1, \ell_1).$$

Therefore

$$\frac{\mu_{21}}{\lambda} u_C^{21} \left[1 - \frac{w_1}{w_2} \frac{(u_L/u_C)_{21}}{(u_L/u_C)_1} \right] = \frac{\mu_{21} u_C^{21}}{\lambda} - \frac{\mu_{21}}{\lambda} \frac{w_1}{w_2} \frac{u_L^{21}}{u_L^1} u_C^1$$
$$= -1 + \frac{\alpha_1}{\lambda} u_C^1 - \frac{w_1 u_C^1}{u_L^1} - \frac{\alpha_1}{\lambda} u_C^1 = -1 - w_1 \frac{u_C^1}{u_L^1},$$

which establishes (18).

Finally, (A.5) gives that:

$$q_1 = D \quad \Leftrightarrow \quad \frac{\partial \mathcal{L}}{\partial q_1} \geq 0 \quad \Leftrightarrow \quad \frac{\gamma \pi_1}{\pi_1 - \pi_2} \leq \frac{\mu_{21} u_C^{21}}{\lambda},$$

which establishes (19).

Proof of Proposition 5

With the particular specification (21), (22), the formulas of proposition 4 become:

$$\text{(7)} \qquad \frac{T_1'}{1-T_1'} = \frac{\mu_{21}}{\lambda} u'\left(C_1 - v\left(\frac{w_1 \ell_1}{w_2} \right) \right) \left(1 - \left(\frac{w_1}{w_2} \right)^{1+1/\varepsilon} \right),$$

since

$$\frac{u_L}{u_C} = v'(\ell) = \ell^{1/\varepsilon}.$$

Let us denote by τ the quantity

$$\frac{\mu_{21} u'\left(C_1 - v\left(\frac{w_1 \ell_1}{w_2}\right)\right)}{\lambda}.$$

The first order conditions (A.1) to (A.4) give:

$$\alpha_1 u'\left(C_1 - v(\ell_1)\right) = \lambda(1+\tau),$$

$$\ell_1 = \left[\frac{w_1}{1+\tau\left(1-\left(\frac{w_1}{w_2}\right)^{1+1/\varepsilon}\right)}\right]^{\varepsilon}$$

(8) $$\left(\alpha_2 + \mu_{21}\frac{f_1}{f_2}\right) u'\left(C_2 - v(\ell_2)\right) = \lambda,$$

and

$$\ell_2 = w_2^{\varepsilon}.$$

The last condition is given by the binding constraint (IC_{21}) which implies:

(9) $$C_2 - v(\ell_2) = C_1 - v\left(\frac{w_1 \ell_1}{w_2}\right).$$

Let us denote by Δw the (positive) quantity $\left(1-\left(\frac{w_1}{w_2}\right)^{1+1/\varepsilon}\right)$.

(A.9) can also be written as:

(10) $$C_2 - v(\ell_2) = C_1 - v(\ell_1) + v(\ell_1)\Delta w,$$

whereas we can also write:

$$\tau = \frac{\mu_{21}}{\lambda} u'\left(C_2 - v(\ell_2)\right).$$

Therefore (A.8) is equivalent to:

$$\frac{\alpha_2}{\lambda} u'\left(C_2 - v(\ell_2)\right) + \tau \frac{f_1}{f_2} = 1.$$

So that we have:

$$C_2 - v(\ell_2) = u'^{-1}\left[\frac{\lambda}{\alpha_2}\left(1 - \tau \frac{f_1}{f_2}\right)\right],$$

$$C_1 - v(\ell_1) = u'^{-1}\left[\frac{\lambda(1+\tau)}{\alpha_1}\right],$$

and

$$\ell_1 = \left(\frac{w_1}{1+\tau \Delta w}\right)^{\varepsilon}.$$

We are thus left with one equation with one unknown τ, obtained by plugging the above formulas into (A.10):

(11) $$0 = u'^{-1}\left[\frac{\lambda}{\alpha_2}\left(1 - \tau \frac{f_1}{f_2}\right)\right] - u'^{-1}\left[\frac{\lambda(1+\tau)}{\alpha_1}\right] - (\Delta w)\left(\frac{w_1}{1+\tau \Delta w}\right)^{1+\varepsilon}.$$

Let us denote by θ a generic parameter of this equation, that we rewrite as

(12) $$\phi(\theta, \tau) = 0$$

where $\phi(\theta, \tau)$ is by definition the righthand side of (A.11).

Because of our assumptions on u, ϕ is an increasing function of τ such that $\phi(\theta, +\infty) = +\infty$. Therefore, (A.12) has a unique solution $\tau(\theta)$, which is positive if and only if

(13) $$\phi(\theta, 0) = u'^{-1}\left(\frac{\lambda}{\alpha_2}\right) - u'^{-1}\left(\frac{\lambda}{\alpha_1}\right) - (\Delta w) w_1^{1+\varepsilon} < 0.$$

(A.13) defines implicitly the minimum value α^* of α_1 such that we are in a normal regime (recall that $\alpha_2 = 1 - \alpha_1$, and notice that (A.13) is satisfied in particular when $\alpha_1 = \alpha_2 = 1/2$). The condition for $q_1 = D$ can now be written (see (A.5)) as:

(14) $$\tau(\theta) \geq \frac{\gamma \pi_1}{\pi_1 - \pi_2},$$

where θ represents any parameter that appears in (A.11), for example α_2, f_2 or w_1. Similarly, (A.7) can be written as

$$\frac{T_1'}{1-T_1'} = \tau(\theta)\Delta w.$$

It remains to determine the sign of $\frac{d\tau}{d\theta}$, which by the implicit function theorem (and the fact that $\frac{\partial\phi}{\partial\tau} > 0$) is the opposite of the sign of $\frac{\partial\phi}{\partial\theta}$. Immediate computations show that:

$$\frac{\partial\phi}{\partial\alpha_2} > 0, \quad \frac{\partial\phi}{\partial f_1} > 0, \quad \frac{\partial\phi}{\partial w_1} < 0,$$

which implies that

$$\frac{\partial\phi}{\partial w_2} > 0 \quad (\text{when } \Delta w \text{ is kept constant})$$

This establishes the comparative statics properties stated in proposition 5.

Proof of Proposition 6

The Hamiltonian of P_2 is:

$$H = f(w)\left[\alpha(w)U + \lambda\left(w\ell - Y(U,\ell) - \gamma\pi(w)q\right)\right]$$
$$+p\left\{\dot{\pi}(w)(D-q)u_C\left(Y(U,\ell),\ell\right) + \frac{\ell}{w}u_L\left(Y(U,\ell),\ell\right)\right\},$$

where p is the costate variable, which satisfies the differential equation:

$$\dot{p}(w) = -\frac{\partial H}{\partial U} = -f(w)\alpha(w) + \frac{\partial Y}{\partial U}\left[\lambda f(w)\right.$$
$$\left. + p(w)\left\{\dot{\pi}(w)\left(D-q(w)\right)u_{CC} + \frac{\ell(w)}{w}u_{LC}\right\}\right].$$

The optimal values of ℓ and q are given by:

$$\frac{\partial H}{\partial \ell} = \lambda f(w)\left[w - \frac{\partial Y}{\partial \ell}\right] - p(w)\frac{\partial}{\partial L}\left[\frac{Lu_L}{w} + \dot{\pi}(D-q)u_C\right]$$

$$\frac{\partial H}{\partial q} = -\lambda\gamma\pi(w)f(w) + p(w)\dot{\pi}(w)u_c.$$

Although the determination of the optimal ℓ and q are clearly very complex, it is relatively simple to find a condition under which $q(w)=D$ (*i.e.* full public coverage is provided to individuals of type w).

Indeed, in this case $\frac{\partial H}{\partial q}$ must be non negative, so that (A.16) implies that $\dot{\pi}$ is necessarily negative (this is because p is itself negative) and that

$$u_C\left[\frac{-p(w)}{\lambda w f(w)}\right]\left(-\frac{w\dot{\pi}}{\pi}\right)\geq\gamma. \tag{15}$$

Moreover, when $q(w)=D$, (A.15) can be transformed into more classical expressions (see Atkinson and Stiglitz (1980) p. 416):

$$T'=1-\frac{1}{w}\frac{\partial Y}{\partial \ell}=\left[\frac{-p(w)}{\lambda w f(w)}\right]\frac{\partial}{\partial L}\left(-\frac{Lu_L}{w}\right),$$

and

$$\frac{T'}{1-T'}=\left[-\frac{p(w)}{\lambda w f(w)}\right]\left(\frac{-wu_C}{u_L}\right)\frac{\partial}{\partial L}\left(-\frac{Lu_L}{w}\right). \tag{16}$$

Multiplying (A.16) by $\left(-\frac{w\dot{\pi}}{\pi}\right)$ and comparing with (A.15) one sees that for full public coverage to be chosen $\left(q(w)=D\right)$ it is necessary that

$$\left(\frac{T'}{1-T'}\right)\left(-\frac{w\dot{\pi}}{\pi}\right)\geq\gamma\frac{1}{u_L}\frac{\partial}{\partial L}\left(Lu_L\right)=\gamma\left(1+\frac{Lu_{LL}}{u_L}\right),$$

which was to be established.