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Prevention in Insurance Markets

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ABSTRACT. – This paper considers a competitive insurance market under moral hazard and adverse selection, in which preventive efforts and self-protection costs are unobservable by insurance companies. Under reasonable assumptions, the conclusions of Rothschild and Stiglitz (1976) are preserved in our context even if it involves moral hazard. The riskier agents in equilibrium, who would also be the riskier agents under perfect information, receive their moral hazard contract. For other agents, adverse selection reduces coverage, increasing likewise their preventive effort with respect to the hidden-action situation.

Aléa moral et sélection adverse sur le marché de l'assurance

RÉSUMÉ. – Nous considérons un marché concurrentiel de l'assurance avec aléa moral et sélection adverse, dans lequel l'effort de prévention et le coût d'auto-protection sont une information privée pour l'assuré. Les mécanismes en jeu sont identiques à ceux de Rothschild et Stiglitz (1976). L'agent dit à « haut risque » est celui dont la probabilité d'accident est la plus grande en information parfaite, il reçoit son contrat d'aléa moral. L'autre agent, le moins risqué à l'équilibre, voit sa couverture réduite par rapport aux risques élevés et par rapport à son contrat d'aléa moral. La sélection adverse stimule ainsi la prévention.

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1 Introduction

Consequences of adverse selection in insurance markets are well known, when the probabilities of loss are given ex ante. However, this is not always the case. Particularly, when policyholders can protect against risk via prevention, their probabilities of loss depend on the proposed contracts and become endogenous. Adverse selection thus occurs as long as unobservable characteristics affect risk, prevention being observable or not. As they are crucial points, prevention and adverse selection should be both taken into account by a relevant model of insurance. We propose in this paper a simple model of insurance markets, and focus on the consequences of adverse selection when prevention exists.

Prevention in adverse selection models has mainly been studied under the assumption that both preventive efforts and prevention costs cannot be observed. This creates a situation of double asymmetric information, in which the policyholders choose preventive effort and have private information about their risk. In this area, Stewart (1994), Chassagnon and Chiappori (1997) and De Meza and Webb (2001) are concerned with a competitive setting, while Jullien, Salanié and Salanié (2000) focus on monopoly contracts. In Stewart (1994) and Chassagnon and Chiappori (1997), agents differ in terms of probability of loss and prevention cost, whereas they differ by their risk aversion in De Meza and Webb (2001) and Jullien, Salanié and Salanié (2000).

In a competitive setting, Stewart (1994), Chassagnon and Chiappori (1997) and De Meza and Webb (2001) characterize the properties of equilibrium. Stewart adapts the equilibrium concept of Riley (1979) to a moral hazard setting, assuming that equilibrium contracts are separating and actuarial. Agents have a priori the same probability of loss but differ in their prevention cost, the reduction of probability being more expensive for high-type agents than for low-type ones. In equilibrium, only low-type agents suffer from adverse selection, which reduces their coverage but enhances their prevention.

Chassagnon and Chiappori (1997) refer to the a priori probability of loss to define types. The probability of loss is assumed to depend in a monotonic way on type, the high-risk agent being the one who has the highest probability of loss for a given effort. They show that equilibrium contracts are separating and actuarial. When the so-called "single crossing" property holds, features à la Rothschild and Stiglitz appear: one type of agent receives his moral hazard contract while the other receives the best contract compatible with incentive considerations. However, the agent who receives his moral hazard contract may be either the high-type or the low-type, depending on parameters values. Moreover, when the single-crossing property does not hold, another kind of equilibrium may emerge, in which both types suffer from adverse selection, because all incentive constraints are binding.

The two papers thus predict rather different results. In Chassagnon and Chiappori, the key point seems to be the "single-crossing". De Meza and Webb (2001) defend this point of view. When agents differ by their risk aversion, one of them being risk neutral, the agents' indifference curves cross twice, and a pooling equilibrium exists in the market. The assumption of risk neutrality is however a limit case; when agents have CARA utility functions with different risk aversion indices, this property of "multiple-crossing" no longer holds, even if pooling equilibria are feasible in a monopoly context (Jullien, Salanié and Salanié (2000)).

What determines risk when prevention exists? An agent characterized by high-risk a priori could end up being the low-risk one ex-post, if he prevents intensively. Depending on equilibrium contracts, the risk level is therefore endogenous. However, to define types, we could refer to the perfect information equilibrium (when both prevention and type are observable) in which insurers propose full insurance contracts and require efficient preventive effort. One of the two types is thus more risky ex-post, either because his prevention is less efficient (a small increase in effort would entail a low reduction of probability of loss) or because his prevention is more expensive (a small decrease of the probability of loss would imply a high additional prevention cost). This shows that optimal prevention depends both on its efficiency (measured by the marginal probability of loss) and on its difficulty (measured by the marginal effort cost). In this paper, the agent associated with the high-risk in a perfect information competitive equilibrium is called the high-type or the high-risk, and we assume that the identity of the high-risk is not reversed when the loss increases.

This assumption has strong effects. When prevention is unobservable, agents faced with a given contract behave so that the high-risk agent under perfect information is the high-risk agent ex-post. This has two consequences. On the one hand, for a given contract, insuring the low-risk agent is always more profitable than insuring the high-risk one. On the other hand, the agents' indifference curves cross only once in the indemnity-premium plane (such a property, however, requires continuity of effort; see Kambia-Chopin (2003)).

These two properties imply that the competitive process works as in Rothschild and Stiglitz (1976). When equilibrium exists, high-risk agents under perfect information are the high-risk agents ex-post, and they receive their moral hazard competitive contract. Low-risk agents suffer from adverse selection, their coverage is reduced compared to the moral hazard competitive contract. But, by reducing coverage, adverse selection enhances low-risk agents' prevention. Our paper finally generalizes the findings of Stewart (1994).

The plan of the paper is as follows. Section 2 presents the model and some benchmark results. We pay particular attention to what happens when prevention is observable. We show that some monotony properties concerning the probability of loss are essential to determine the equilibrium under adverse selection, and, in particular, to determine what type receives his perfect information contract. Section 3 is devoted to the case where prevention of agents is unobservable, and section 4 concludes. All proofs are reported in the appendix.

2 The model and preliminary results

2.1 The model

We consider the interactions of agents (insureds) and companies (insurers) in an insurance market. An agent may suffer an accident, which is represented by a monetary loss d. This agent can protect against this risk, by choosing an effort level $e \in [0,+\infty]$. Effort is unobservable and reduces the probability of accident. Let $p(e,\theta) \in [0,1]$ be the probability of having a loss *d*; we assume that $p(e,\theta)$ decreases with *e*, and is such that $p(0,\theta)=1$ and $\lim_{e\to\infty} p(e,\theta)=0$.

The reduction of risk generates a cost $c(e,\theta)$ for an agent who chooses effort *e* and whose type is[1] θ. Agents' types can be high or low, $\theta \in \{\theta_L,\theta_H\}$, and affect either their marginal probability of loss or their marginal cost of effort. Moreover, types are private information, so insurers cannot observe the value of θ.

In this model, as effort is continuous, opting for a given preventive effort is equivalent to selecting a given probability of loss, and agents' choices depend both on the efficiency of prevention (measured by the marginal probability of loss) and on the difficulty of prevention (measured by the marginal cost of effort). This can be expressed by the function $\varphi(p,\theta)$, which denotes the cost an agent incurs to obtain a probability of loss equal to *p*. Let :

(1) $$\varphi(p(e,\theta),\theta) \equiv c(e,\theta) \text{ for all } e \text{ and } \theta.$$

As will be explained below, the relevant assumptions concern the function φ rather than the functions *p* and *c*. To make the first-order approach valid, we assume that $\varphi(p,\theta)$ is decreasing, convex with respect to *p*,($\varphi_p(p,\theta)<0$ and $\varphi_{pp}(p,\theta)>0$), and twice continuously differentiable. Moreover, $\varphi_p(1,\theta)=0$ and $\lim_{p\to 0}\varphi_p(0,\theta)=-\infty$.

Agents are risk-averse, and their VNM utility function (they differ only by their type), denoted by $u(.)$, is increasing, concave with respect to wealth, and twice continuously differentiable. It is separable with respect to the cost of effort. An insurance contract, denoted by $C=(\alpha,\beta)$, plans to pay a premium $\beta \geq 0$ and to receive an indemnity $\alpha \geq 0$ in case of accident. Denoting by ω the agent's initial wealth, the agent's expected utility $V(\alpha,\beta,e,\theta)$ and the insurer's expected profit $\pi(\alpha,\beta,e,\theta)$ are thus given by:

(2) $$V(\alpha,\beta,e,\theta) \equiv (1-p(e,\theta))u(w-\beta)+p(e,\theta)u(w-d-\beta+\alpha)-c(e,\theta)$$

(3) $$\pi(\alpha,\beta,e,\theta) \equiv \beta - p(e,\theta)\alpha .$$

Insurance companies compete in contracts as described in Rothschild and Stiglitz (1976) (denoted by RS in what follows). Companies anticipate that the insurance contract chosen by agents affects their preventive efforts:

DEFINITION: *A set of contracts* $S^* = \{C_1,C_2,..,C_n\}$ *is a RS equilibrium with hidden action if:*

(1) agents opt for their best contract in S^**, then choose their optimal preventive effort,*

1. The model is similar to Chassagnon and Chiappori (1997)' one.

(2) any contract in S^* is chosen in equilibrium and no insurer makes negative (expected) profit,

(3) no contract proposed in addition to the equilibrium contracts by an entering company can be profitable.

Let us finally define two benchmark contracts. The first one is the competitive contract under perfect information (insurers observe both types and prevention), denoted by $C_K^{PI} = (\alpha_K^{PI}, \beta_K^{PI}, e_K^{PI}), K = H, L$. The second one is the competitive contract under moral hazard[2] (insurers observe types but do not observe prevention) denoted by $C_K^M = (\alpha_K^M, \beta_K^M)$, leading to an effort e_K^M, and a probability of loss $p_K^M, K = H, L$[3].

2.2 Who is the high risk?

In our setting, prevention depends on equilibrium contracts, so the probability of loss is endogenous. As a consequence, an agent could be more risky a priori but have in equilibrium the lowest probability of loss. Indeed, a given type could have a smaller marginal probability of loss but a higher marginal cost, so that increasing prevention would be both more expensive for a type and more efficient in terms of risk reduction.

Let us define the high risk (associated with the high type θ_H) as the type of the agent whose probability of loss is the highest in the perfect information competitive equilibrium. Assume moreover that the identity of the high risk does not depend on the value of the loss, that is:

(4) $$p(e_H^{PI}, \theta_H) > p(e_L^{PI}, \theta_L), \text{ for all } d > 0.$$

Recall that under perfect information, insurers offer a full insurance actuarial contract, and ask for a level of prevention which maximises the agent's expected utility $u(w - pd) - \varphi(p, \theta)$, so that:

(5) $$-du'(w - p(e^{PI}, \theta)d) - \varphi_p(p(e^{PI}, \theta), \theta) = 0.$$

(4) is thus equivalent to

(6) $$\varphi_p(p, \theta_H) < \varphi_p(p, \theta_L) \text{ , for all } p \in [0, 1[.$$

This is the key assumption of this paper: to reduce the probability of loss, the high-risk agent (θ_H) bears an increase of prevention cost higher than that of the

2. As pointed out by a referee, this contract may not be unique. In case of multiplicity, the agent's expected utility being identical, our analysis applies for any of the moral hazard contracts.
3. These two contracts maximise the agent's expected utility $V(\alpha, \beta, e, \theta)$ subject to a non-negative profit constraint $\pi(\alpha, \beta, e) \geq 0$. Concerning the moral hazard contract, an incentive constraint must be taken into account, $V_e(\alpha, \beta, e, \theta) = 0$. This last problem has been studied by Arnott and Stiglitz (1988) and Arnott (1991).

low-risk agent (θ_L); in other words, when agents are fully insured, prevention is more efficient for the low risk than for the high risk.

Our definition of the high risk can be compared to the one proposed by Chassagnon and Chiappori (1997) (CC hereafter), who assumed that the high type θ_H is more risky a priori, that is $p(e,\theta_H) > p(e,\theta_L)$ for all *e*. When the high-type agent is more risky a priori, our assumption about the prevention cost function makes our analysis more restrictive than that of CC, but we do not assume that an agent (either the high type or the low type) is more risky a priori. Moreover, note that we consider a continuous effort. The following example (where (6) holds) enlightens the differences between the two approaches.

Example: *Let* $c(e,\theta) = \theta[eh(\theta) + \exp(-eh(\theta))]$ *and* $p(e,\theta) = \exp(-eh(\theta))$ *where* $\theta_H > \theta_L > 0$ *and* $h(\theta) > 0$. *Using (1), function* φ *is given by* $\varphi(p,\theta) = \theta(p - \log p)$.

Agent θ_H has the highest probability of loss under perfect information. Note that, if $h(\theta_H) < h(\theta_L)$, the high risk a priori is θ_H but conversely when $h(\theta_H) > h(\theta_L)$, the high risk a priori is θ_L (this may occur if his marginal prevention cost is low enough).

2.3 Market equilibrium under adverse selection

In this setting, an insurance contract is described by three elements: the premium, the indemnity and the level of prevention, that is $C = (\alpha, \beta, e)$. Proposition 1 presents the competitive equilibrium. As in RS, insurers offer a menu of separating actuarial contracts, and one of the two types obtains his (full insurance) perfect information contract. As usual, the contract obtained by the other type is designed taking into account incentive considerations. What is new here is the fact that, depending on the parameters of the model, either the high risk or the low one may be insured as in the perfect information case.

Proposition 1: *i) If* $p(e_H^{PI},\theta_H) \geq p(e_H^{PI},\theta_L)$, *agent* θ_H *obtains in equilibrium (if it exists) his perfect information contract. The contract of agent* θ_L *maximises his expected utility, given that agent* θ_H *prefers his own contract and the insurer's profit is non-negative.*

ii) If $p(e_L^{PI},\theta_L) \geq p(e_L^{PI},\theta_H)$, *agent* θ_L *obtains his perfect information contract and the contract of agent* θ_H *maximises his expected utility, given that agent* θ_L *prefers his own contract and the insurer's profit is non-negative.*

Note that Proposition 1 omits some feasible configurations. Particularly, Proposition 1 does not consider the case where the perfect information probabilities of loss are such that $p(e_H^{PI},\theta_H) < p(e_H^{PI},\theta_L)$ and $p(e_L^{PI},\theta_L) < p(e_L^{PI},\theta_H)$.

When prevention can be controlled, the fact that the probability of loss is either increasing or decreasing with respect to type for all *e* is thus a key point in determining the equilibrium. Indeed, assume that the probability of loss increases with respect to type, that is $p(e,\theta_H) \geq p(e,\theta_L)$. The assumption of point *i* holds, so the high-risk agent (who has the highest risk ex-post under perfect information, that is our θ_H) obtains his perfect information contract, whereas the low type suffers from adverse selection. On the contrary, assume the probability of loss decreases with respect to type, that is $p(e,\theta_L) \geq p(e,\theta_H)$. This time, point *ii* applies, and the agent θ_L, who is the low risk under perfect information, obtains his perfect information (full insurance) contract.

The identity of the type who obtains full insurance in equilibrium may thus reverse (section 2.2 provides a numerical example of this phenomenon). The perfect information contract is proposed in equilibrium to an agent, on condition that this contract makes no loss if it is chosen by the other type of agent.

What role does the single-crossing property play in this result? A sufficient condition to ensure that separating contracts are proposed in equilibrium is the following: given a contract *C*, one can find another contract $C(\varepsilon)$, close to *C* in terms of profit, which allows to separate the types. When the contract has only two dimensions (as in RS), this property is equivalent to the single-crossing one (i.e. the indifference curves of the two types cross only once in the (α,β) plane). When, as here, the contract has three dimensions, given a contract $C = (\alpha, \beta, e)$, types can be separated if there exists $C(\varepsilon) = (\alpha + \varepsilon_1, \beta + \varepsilon_2, e + \varepsilon_3)$ (with $\varepsilon_1, \varepsilon_2$ and ε_3 tending to zero) prefered to *C* by only one type (θ_K), that is such that (where V_x^K is the derivative of agent θ_K's expected utility with respect to *x*):

$$dV(C,\theta_K) = V_\alpha^K \varepsilon_1 + V_\beta^K \varepsilon_2 + V_e^K \varepsilon_3 > 0$$

$$dV\left(C,\theta_{K'}\right) = V_\alpha^{K'} \varepsilon_1 + V_\beta^{K'} \varepsilon_2 + V_e^{K'} \varepsilon_3 < 0$$

As long as the probabilities of loss differ across types for the contract *C*, $C(\varepsilon)$ exists with $\varepsilon_3 = 0$; in other words, the single-crossing in the (α,β) plane allows to separate types. When, conversely, the two types have the same probability of loss for a contract *C*, separating types requires different preventive efforts. If the marginal probability of loss (or the marginal prevention cost) does not depend on the type in *C*, $V_e^K \neq V_e^{K'}$, the single-crossing in *C* holds in the (α,*e*) plane. However, our assumptions are compatible with some cases in which the single-crossing in other planes does not hold. The fact that the single-crossing property may not hold in some points does not prevent the competitive process from working here (see the proof of Proposition 1).

3 Equilibrium under moral hazard and adverse selection

When prevention is unobservable, agents' efforts depend on type and on contracts proposed by insurers. Taking into account the optimal effort choice, the expected utility of an agent and the expected profit of an insurer may be rewritten as functions of contracts and types:

(7) $$U(\alpha,\beta,\theta) \equiv V(\alpha,\beta,e(\alpha,\beta,\theta),\theta) \equiv \max_e V(\alpha,\beta,\theta)$$

(8) $$\Pi(\alpha,\beta,\theta) \equiv \beta - p(e(\alpha,\beta,\theta),\theta)\alpha .$$

This rewriting eliminates moral hazard, reducing our model to one with adverse selection where the objectives are now $U(.)$ and Π. In such a context, the reasoning proposed by RS applies to our market, except that utility functions $U(.)$ and $\Pi(.)$ have different properties than those used by RS. In particular, $\Pi(\alpha,\beta,\theta)$ is not linear with respect to the premium and $U(.)$ could be non-concave.[4]

However, some properties can easily be shown. First, as long as contracts offer a partial coverage $\alpha \leq d$, the first-order approach is still valid under our assumptions.[5] The preventive effort is such that the first-order optimality condition holds:

(9) $$V_e(\alpha,\beta,e,\theta) = p_e(e,\theta)\{u(w-d-\beta+\alpha) - u(w-\beta) - \varphi_p(p(e,\theta),\theta)\} = 0.$$

Denoting by $p(C,\theta)$ the probability of loss chosen by an agent θ faced with a contract $C = (\alpha,\beta)$, (9) implies:

(10) $$\varphi_p(p(C,\theta_H),\theta_H) = \varphi_p(p(C,\theta_L),\theta_L);$$

hence, taking into account (6) and the fact that φ is convex,

(11) $$p(C,\theta_H) > p(C,\theta_L).$$

(11) implies that the indifference curves of the two types, which are increasing, cross only once in (α,β) plane: agent θ_H 's indifference curve has a steeper slope than agent θ_L 's one.[6]

(11) has another consequence: for a given contract, agent θ_L behaves so that he is less risky than agent θ_H, so insurers prefer the low type to the high type.

4. Arnott and Stiglitz (1988) have shown that the insurer's iso-profit curve is not concave. Stewart (1994) used simulations to show that the insured's indifference curve is concave for high levels of coverage and convex for low ones.

5. Offering over-insurance $(\alpha \geq d)$ leads to a loss probability equal to 1, which is uninsurable, and thus not feasible in equilibrium.

6. Indeed, taking the derivative of (7) gives (denoting by u_N (resp. u_A) the utility of the agent when no (resp. one) accident occurs):

$$U_\beta(\alpha,\beta,\theta) = -(1-p(C,\theta))u'_N - p(C,\theta)u'_A < 0 \text{ and } U_\alpha(\alpha,\beta,\theta) = pu'_A > 0.$$

The slope of the indifference curve is then:

$$\frac{d\beta}{d\alpha} = -\frac{U_\alpha(\alpha,\beta,\theta)}{U_\beta(\alpha,\beta,\theta)} = \frac{p(C,\theta)u'_A}{(1-p(C,\theta))u'_N + p(C,\theta)u'_A}.$$

The insurer's profit (8) thus increases with the premium and decreases with the indemnity.[7]

Proposition 2 states that these conditions ensure that the equilibrium has the properties underlined by RS.

PROPOSITION 2: *i) In equilibrium (if it exists), insurers propose to the high type his moral hazard contract* C_H^M *. The contract designed for the low type maximises his expected utility, given that the insurer's profit is non-negative and the high risk prefers his own contract.*

ii) Moreover, agent θ_L *is the low-risk agent, that is* $p(C_L^{SM}, \theta_L) < p(C_H^M, \theta_H)$. *Both his premium and his indemnity are smaller than those of agent* θ_H.

As in RS, an equilibrium contract yields no profit for the insurer; the equilibrium is fully separating, so the agents' contractual choices reveal their types. Finally, only type θ_L suffers from adverse selection, the high type receiving his perfect information contract, whether insurers observe types or not.

The low-risk agent under perfect information is the low-risk agent under moral hazard and adverse selection. Indeed we have seen that, faced with the same contract, the high type θ_H behaves so that his probability of loss is higher than the probability of loss of the low type θ_L. We show in the appendix that any pair of incentive contracts has the same property, so that the high (respectively low) type can be associated with the high (resp. low) risk. This implies that the contract designed for agents θ_L involves a smaller indemnity than the one designed for agents θ_H. Indeed, in equilibrium, the coverage is partial and the insurance unit price (which equals the probability of loss) is actuarial, implying that agents prefer, for the same price, that the coverage increases (Mossin 1968). If, in equilibrium, agents θ_L had a higher indemnity, agents θ_H would prefer the contract designed for agents θ_L to their own, as they would benefit not only from a higher coverage but also from a smaller insurance unit price.

As in RS, the low-risk agent obtains an insurance contract which has both the lowest reimbursement and the lowest insurance premium, which once again confirms the positive correlation between ex-post risk and coverage, as in Chiappori et al (2002).

As under perfect information, the low-risk agent is the one who bears the lowest marginal cost (measured with respect to the probability $\varphi_p(p,\theta)$), given the probability of loss. It is thus possible that, in equilibrium, the low risk suffers the more important cost (if the cost function admits for instance a fixed cost as in $\varphi(p,\theta) = \theta(p - \log p) - k(\theta)$).

7. Differentiating (9) shows that the probability of loss decreases when the premium increases and increases with the indemnity:

$$\frac{\partial p(C,\theta)}{\partial \beta} = \frac{u'_N - u'_A}{\varphi_{pp}} < 0 \quad \text{and} \quad \frac{\partial p(C,\theta)}{\partial \alpha} = \frac{u'_A}{\varphi_{pp}} > 0.$$

hence $\frac{\partial \Pi(\alpha,\beta,\theta)}{\partial \beta} = 1 - \alpha \frac{\partial p(C,\theta)}{\partial \beta} > 0$ and $\frac{\partial \Pi(\alpha,\beta,\theta)}{\partial \alpha} = -p - \alpha \frac{\partial p(C,\theta)}{\partial \alpha} < 0.$

Finally, contrary to the equilibrium under adverse selection, the high-risk agent may be the agent who is the less risky a priori (as long as $p(e,\theta_L) > p(e,\theta_H)$ for all *e*), and his welfare is reduced relative to the situation of moral hazard in this case. The fact that the probability of loss is either increasing or decreasing with respect to type, which would be essential to determine the equilibrium when prevention can be controlled, plays no role when moral hazard is present.

In RS, adverse selection reduces the equilibrium premium and the equilibrium indemnity. To determine if the presence of adverse selection has the same consequence here, the contract C_L^{SM} must be compared to the moral hazard contract C_L^M. Proposition 3 studies that point.

PROPOSITION 3: *Adverse selection reduces the probability of loss, the indemnity and the premium of agent* θ_L *(* $p(C_L^{SM},\theta_L) \le p(C_L^M,\theta_L)$ *and* $\alpha_L^{SM} \le \alpha_L^M$ *).*

A priori, such a result looks like the one in RS: with exogenous probabilities of loss, when agent θ_H is insured with agent θ_L's contract, he acquires coverage α_L^{SM} for a unit price equal to the probability of loss of a low risk. As the indemnity is lower than the loss *d*, he prefers the coverage to increase, so incentive considerations reduce coverage and lead the insurer to offer under-insurance contracts to agents θ_L.

When probabilities of loss are endogenous, however, two effects matter. Indeed, for actuarial contracts, the probability of loss of agent θ_L depends on the indemnity, and, the more important is the coverage, the more important will be the probability and hence the premium. When agent θ_H chooses the contract designed for agent θ_L, his expected utility increases with coverage for a constant premium/coverage ratio but a higher coverage also makes the insurance price increase. The two effects thus play in opposite directions, and one of them does not dominate the other: in the appendix we show that expected utility increases with coverage when it is low enough, and decreases with coverage when it is high enough. However, the slope of expected utility (taken as a function of indemnity) is lower for the agent θ_L than for agent θ_H. Intuitively this means that if agent θ_H is indifferent between two levels of indemnity, agent θ_L prefers the lowest. In other words, faced with the choice between a high premium with a high indemnity or a low premium with a low indemnity, the low-risk agent is more favourable than the high-risk agent to a reduction of the premium. Even if the mechanism at play is more complex than in RS, adverse selection has the same consequence; namely, it reduces the indemnity of the low-risk agent. In a moral hazard setting, this enhances prevention.

4 Conclusion

Prevention, whether observable or not, does not change the way adverse selection works in insurance markets. Competition makes insurers propose separating and actuarial contracts, one of these being unaffected by hidden information.

Such a conclusion can be linked to those of Jullien, Salanié and Salanié (2000) and Fagart (2002), who show in two different settings of bilateral monopoly, that optimal contracts with adverse selection and moral hazard have the same properties as adverse selection contracts. Here, however, competition acts in different ways depending on whether prevention is observable or not: the low-risk agent (who has the lowest probability of loss when information is perfect and when both type and prevention are unkown) might become the high risk ex-post when only prevention is observable. This is the case even if the single-crossing property holds, and rests on the fact that, being observable, prevention is used to screen agents.

Proposition 2 confirms the positive correlation between risk and coverage in equilibrium. Empirical studies have tried to test this prediction in insurance data: this positive correlation property holds in some insurance markets but not in others, creating a debate.

Finally, Proposition 3 shows that adverse selection enhances prevention: the positive correlation property is thus reinforced by adverse selection.

Even if adverse selection improves prevention, it does not improve welfare, unless prevention creates a positive externality, which the competitive market does not consider. Indeed, assume that prevention increases the agent's wealth (for instance, a part of the agent's total loss is reimbursed to the agent by a mutual fund, like in France for health care, the remaining part of the loss being insured in the insurance market). By enhancing prevention, adverse selection makes agents θ_H better off, as their wealth is increased compared to perfect information situation. Depending on the size of the wealth effect, agents θ_L could benefit from adverse selection too. This issue remains a topic for future research.

5 Appendix

5.1 Proof of Propositions 1 and 2 *i*

We prove here Propositions 1 and 2 *i* together. The proof is written by using the notation of section 3, where the expected utility is U and the expected profit is Π. The proof of point *i* of Proposition 1 is similar, replacing U and Π by V and π. To obtain the proof of point *ii* of Proposition 1, it is necessary to reverse indices H and L.

Denote by C_H^* the contract which maximises $U(C_H,\theta_H)$ subject to $\Pi(C_H,\theta_H) \geq 0$ and C_L^{RS} the one which maximises $U(C_L,\theta_L)$ subject to $\Pi(C_L,\theta_L) \geq 0$ and the incentive constraint $U(C_L,\theta_H) \leq U(C_H^*,\theta_H)$.

Lemma 4: *If C_H is an equilibrium contract for agent θ_H, then $U(C_H,\theta_H) \geq U(C_H^*,\theta_H)$.*

Assume that in equilibrium $U(C_H,\theta_H) < U(C_H^*,\theta_H)$. Let $C(\varepsilon)$ be the same contract as C_H^* except that the premium is increased by $\varepsilon > 0$. As *U(.)* continuously decreases and Π*(.)* continuously increases with respect to the premium, there exists $\varepsilon > 0$ such that $U(C(\varepsilon),\theta_H) > U(C_H,\theta_H)$ and $\Pi(C(\varepsilon),\theta_H) > 0$. Thus an entrant can offer $C(\varepsilon)$, so he attracts all the agents θ_H and makes profits by insuring them. Moreover,

(1) in the context of Proposition 1 (point i), $C_H^* = C_H^{PI}$ and we assume that $p(e_H^{PI},\theta_H) \geq p(e_H^{PI},\theta_L)$;

(2) in the context of Proposition 2 (moral hazard and adverse selection), we know that $p(C_H^M,\theta_H) \geq p(C_H^M,\theta_L)$.

As a consequence, the entrant obtains positive profit if it insures agents θ_L with the contract $C(\varepsilon)$, hence a contradiction.

Lemma 5: *Let C_J be an equilibrium contract for the agent J, $J = H,L$, then* $\Pi(C_J,\theta_J) = 0$.

We know from the preceding step that $\Pi(C_H,\theta_H) \leq 0$. Let C_L be an equilibrium contract for agents θ_L. If $\Pi(C_L,\theta_L) > 0$, two cases are feasible:

(1) the probabilities of loss of the two types are different when they choose C_L. This always holds for Proposition 2, and occurs according to the value of parameters in Proposition 1. As the indifference curves of the two types cross only once (in C_L) in the plane (α,β), there exists $\varepsilon > 0$ and a contract $C(\varepsilon)$ such that

$$U(C(\varepsilon),\theta_L) > U(C_L,\theta_L) \text{ and } U(C_L,\theta_H) > U(C(\varepsilon),\theta_H)$$

$$\Pi(C(\varepsilon),\theta_L) \geq \Pi(C_L,\theta_L) - \varepsilon > 0.$$

If an entrant company offers $C(\varepsilon)$, it attracts all the agents θ_L but only them, indeed, all the contracts C_H chosen in equilibrium by agents θ_H are such that $U(C_H,\theta_H) \geq U(C_L,\theta_H) > U(C(\varepsilon),\theta_H)$. As a consequence, the entrant gets a positive profit, what is impossible;

(2) in the second case, the two types' probabilities of loss are identical in C_L (only feasible for Proposition 1). In that case, consider the contract $C(\varepsilon)$ identical to C_L except that the premium is reduced by $\varepsilon > 0$, with $\Pi(C_L,\theta_L) - \varepsilon > 0$. An entrant company that proposes $C(\varepsilon)$ attracts all the agents θ_L and realizes profit by insuring them. Moreover, as agents θ_H have the same probability of loss, the entrant realizes (a strictly positive) profit whatever agents he insures. Hence a contradiction.

Consequently, any contract designed for an agent θ_L generates no profit, that is $\Pi(C_L,\theta_L) \leq 0$. As the same property holds for agents θ_H we have in equilibrium $\Pi(C_H,\theta_H) = \Pi(C_L,\theta_L) = 0$.

Lemma 6: *In equilibrium $C_H = C_H^*$ and $C_L = C_L^{RS}$.*

C_H^* is the unique contract such that agents θ_H obtains an expected utility higher than $U(C_H^*,\theta_H)$ and insurers make non negative profit. Thus, the two preceding lemmas imply that $C_H = C_H^*$.

If C_L is an equilibrium contract for agents θ_L; according to the definition of C_L^{RS}, the two preceding lemmas imply that $U(C_L,\theta_L) \le U(C_L^{RS},\theta_L)$. Assume this inequality is not binding, $U(C_L,\theta_L) < U(C_L^{RS},\theta_L)$. There exists $\varepsilon > 0$ and $C(\varepsilon)$ (identical to C_L^{RS} except that the premium is increased by ε) such that $U(C_L^{RS},\theta_L) > U(C(\varepsilon),\theta_L) > U(C_L,\theta_L)$ as *U(.)* is decreasing with respect to the premium. Moreover, $\Pi(C(\varepsilon),\theta_L) > 0$ and $U(C(\varepsilon),\theta_H) < U(C_L^{RS},\theta_H) \le U(C_H^*,\theta_H)$. Consequently, an entrant who offers $C(\varepsilon)$, realizes positive gains, hence a contradiction.

5.2 **Proof of Proposition 2 *ii***

Let (α_L,β_L) and (α_H,β_H) be two contracts such that incentive constraints (12) and (13) below hold and denote by p_H (resp. p_L) the probability of loss chosen by the agent θ_H (resp. θ_L) insured by (α_H,β_H) (resp (α_L,β_L)):

$$(1-p_L)u(w-\beta_L)+p_L u(w-d-\beta_L+\alpha_L)-\varphi(p_L,\theta_L) \ge Max_p\{(1-p)u(w-\beta_H)+pu(w-d-\beta_H+\alpha_H)-\varphi(p,\theta_L)\}; \quad (12)$$

$$(1-p_H)u(w-\beta_H)+p_H u(w-d-\beta_H+\alpha_H)-\varphi(p_H,\theta_H) \ge Max_p\{(1-p)u(w-\beta_L)+pu(w-d-\beta_L+\alpha_L)-\varphi(p,\theta_H)\}. \quad (13)$$

First, taking into account the fact that (12) holds for $p = p_H$ and (13) holds for $p = p_L$, then summing these two inequalities gives $\varphi(p_L,\theta_H)-\varphi(p_L,\theta_L) \ge \varphi(p_H,\theta_H)-\varphi(p_H,\theta_L)$. Equation (6) ensures that the function $\varphi(p,\theta_H)-\varphi(p,\theta_L)$ increases with respect to *p*, as a consequence $p_H \ge p_L$.

Moreover, since (C_L^{SM},C_H^M) is such that the incentive constraints (12) and (13) hold, the agent θ_H is the high-risk agent in equilibrium.

If the agent θ_H chooses the contract of agent θ_L (indemnity α_L, premium $\beta_L = p_L\alpha_L$), he obtains an expected utility:

$$U(\alpha_L,p_L\alpha_L,\theta_H) \ge (1-p_H^M)u(w-p_L\alpha)+p_H^M u(w-d+(1-p_L)\alpha)-\varphi(p_H^M,\theta_H). \quad (14)$$

But, the right term in (14) is an increasing function of α for $\alpha \le d$, and a decreasing function of p_L. If, in equilibrium, $\alpha_L > \alpha_H^M$, the agent θ_H would prefer the contract of agents θ_L for two reasons: the probability of loss p_L is smaller than p_H^M and the indemnity α_L higher than α_H^M, hence a contradiction.

5.3 **Proof of Proposition 3**

If the incentive constraint of the program defining C_L^{RS} is not binding, the solution is C_L^M and Proposition 3 holds. We thus assume in what follows that the incentive constraint is binding.

The contract of agent θ_L is such that the insurer obtains no profit, $\beta = p_L\alpha$ and the incentive constraint (9) holds. Taking into account the value of the premium, (9) allows to define a function $p_L(\alpha)$, which is the probability chosen by agent θ_L when the indemnity is α and the premium is actuarial:

(15) $$u(w-d+(1-p_L(\alpha))\alpha)-u(w-p_L(\alpha)\alpha)-\varphi_p(p_L(\alpha),\theta_L)=0.$$

The function $p_L(\alpha)$ is defined from $[0,d]$ in $[p_L^{\min},1]$ (with $p_L(d)=1$ and $p_L^{\min}=p_L(0)$). It is continuous and a simple calculus allows to show that it increases with α, that is:

(16) $$p'_L(\alpha)=-\frac{(1-p_L)u'_A+p_L u'_N}{\alpha[u'_N-u'_A]-\varphi_{pp}(p_L(\alpha),\theta_L)}>0.$$

Let $A(\alpha,\theta)$ be the expected utility obtained by the agent θ when he chooses the actuarial contract $C(\alpha)=(\alpha,p_L(\alpha)\alpha)$. Note that when agent θ_L is the one who chooses $C(\alpha)$, he chooses the probability of loss $p_L(\alpha)$. We have:

(17) $$A(\alpha,\theta)=Max_p\{(1-p)u(w-p_L(\alpha)\alpha)+pu(w-d+(1-p_L(\alpha))\alpha)-\varphi(p,\theta)\}.$$

Given that p designs the probability of loss chosen by agents (that is $p_L(\alpha)$ for agent θ_L and $p(C_L(\alpha),\theta_H)>p_L(\alpha)$ for agent θ_H), differentiating A with respect to α gives:

(18) $$\frac{\partial A(\alpha,\theta)}{\partial\alpha}=-(1-p)p_L(\alpha)u'_N+p(1-p_L(\alpha))u'_A-p'_L(\alpha)\alpha\{(1-p)u'_N+pu'_A\}.$$

As $\frac{\partial A(0,\theta)}{\partial\alpha}>0$ and $\frac{\partial A(d,\theta)}{\partial\alpha}<0$, A is not monotonic with respect to α. On contrary, the difference between the expected utility of agent θ_L and the expected utility of θ_H decreases in α, indeed:

(19)
$$\begin{aligned}A(\alpha,\theta_L)-A(\alpha,\theta_H)&=Min_p\{[p_L(\alpha)-p](u_A-u_N)+\varphi(p,\theta_H)-\varphi(p_L(\alpha),\theta_L)\}\\&=Min_p\{[p_L(\alpha)-p]\varphi_p(p_L(\alpha),\theta_L))+\varphi(p,\theta_H)-\varphi(p_L(\alpha),\theta_L)\};\end{aligned}$$

hence $$\frac{\partial[A(\alpha,\theta_L)-A(\alpha,\theta_H)]}{\partial\alpha}=p'_L(\alpha)[p_L(\alpha)-p(C_L(\alpha),\theta_H)]\varphi_{pp}(p_L(\alpha),\theta_L))<0.$$

Moreover, as the incentive constraint is binding, we have

(20) $$A(\alpha,\theta_H)=U(C_H^M,\theta_H).$$

Let us show that this equation has several roots. First, note that agent θ_H prefers the moral hazard contract of agent θ_L, C_L^M, to his own C_H^M, so $U(C_H^M,\theta_H) < A(\alpha_L^M,\theta_H)$. Second, the contract C_H^M gives an expected utility higher than the utility given by the no-insurance contract (for which $\alpha = \beta = 0$, that is $A(0,\theta_H) < U(C_H^M,\theta_H)$). Finally, the no-insurance contract is prefered to a full insurance one ($\alpha = d$, $p_L(d) = 1$, and $\beta = d$) as $Max_p\{(1-p)u(w) + pu(w-d) - \varphi(p,\theta)\} > u(w-d) - \varphi(1,\theta)$. Thus, we obtain:

$$A(d,\theta_H) < A(0,\theta_H) < U(C_H^M,\theta_H) < A(\alpha_L^M,\theta_H).$$

The curve $A(\alpha,\theta_H)$ is thus first increasing, then decreasing, and equation (20) has at least two solutions, one of these being smaller than α_L^M. We have now to determine which solution makes agent θ_L better off.

If (20) holds, agent θ_L gets an expected utility $A(\alpha,\theta_L) = U(C_H^M,\theta_H) + A(\alpha,\theta_L) - A(\alpha,\theta_H)$. As a consequence, the best solution for agent θ_L is the solution of (20) which maximises the difference $A(\alpha,\theta_L) - A(\alpha,\theta_H)$, hence the lowest root of (20). ■

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