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Testing Seasonality in the Context of Fractionally Integrated Processes

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ABSTRACT. – We propose in this article the use of a version of the tests of ROBINSON [1994] for testing seasonally fractionally integrated processes. The tests have standard null and local limit distributions and allow us to test unit and fractional seasonal roots even with different amplitudes at different frequencies. A Monte Carlo experiment is conducted to check the power of the tests against different types of fractional alternatives and, an empirical application, using quarterly data for the U.S. total expenditure of several monetary aggregates, is also carried out at the end of the article.

Caractère saisonnier d'essai dans le contexte des processus intégrés partiels

RÉSUMÉ. – Nous proposons en cet article l'utilisation d'une version particulière des essais de ROBINSON [1994] pour examiner de façon saisonnière des processus intégrés partiels. Les essais ont des distributions nulles et locales standard de limite et nous permettent au banc d'essai et aux racines saisonnières partielles même avec différentes amplitudes à différentes fréquences. Une expérience de Montecarlo est entreprise pour vérifier la puissance des essais contre différents types de solutions de rechange partielles et, une application empirique, employant des données trimestrielles pour toute la dépense des USA de plusieurs agrégats monétaires est également effectuée à la fin de l'article.

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1 Introduction

Many macroeconomic time series contain important seasonal components and it is a common belief that modellers need to pay specific attention to the nature of seasonality rather than essentially to ignore it. As an alternative to the deterministic approaches based on seasonal dummies, stochastic models based on seasonal differencing were proposed. These models implicitly assume that the seasonal component substantially drifts over time and thus, a process observed 's' times per year would be transformed to its 's'-period difference on the assumption that the process contains an integrated seasonal component. Thus, for example, for quarterly data, $s = 4$,

$$(1) \qquad (1 - L^4)\, x_t \;=\; u_t, \qquad t \;=\; 1, 2, ...$$

where 'x_t' is the time series we observe and 'u_t' is an $I(0)$ process, defined as a covariance stationary process, with spectral density function that is positive and finite at any frequency on the spectrum. Note that the operator '$(1 - L^4)$' can be factored as '$(1 - L)(1 + L)(1 + L^2)$' and thus, x_t in (1) contains four roots of modulus unity: one at zero frequency; one at two cycles per year, corresponding to frequency π; and two complex pairs at one cycle per year, corresponding to frequencies $\pi / 2$ and $3\,\pi / 2$ (of a cycle $2\,\pi$).

A good deal of empirical work has followed this approach: HYLLEBERG et al. [1990] found evidence for seasonal unit roots in quarterly U.K. consumption and income, using a procedure that allows tests for unit roots at some seasonal frequencies, without maintaining their presence at all such frequencies. Further evidence in favour of seasonal unit roots was found in OTTO and WIRJANTO [1990] and LEE and SIKLOS [1991] for Canadian economic time series; in HYLLEBERG et al. [1993] for Japanese consumption and income; and also in LINDEN [1994] for the Finish economy. All these works are based on the HYLLEBERG et al.'s [1990] procedure. Other seasonal unit root tests are GHYSELS et al. [1994], CANOVA and HANSEN [1995], and more recently, TAM and REIMSEL [1997], the latter proposing a test for a unit root in the seasonal MA operator.

However, seasonal unit roots are only an extremely specialised form for describing the nonstationary nature of seasonality. Consider, for instance, the process,

$$(2) \qquad (1 - L^4)^d\, x_t \;=\; u_t, \qquad t \;=\; 1, 2, ...$$

with $d > 0$ and $I(0)$ u_t. Clearly, x_t has four roots of modulus unity, all with the same integration order d. In such a case, the polynomial $(1 - L^4)^d$ can be expressed in terms of its Binomial expansion, such that, for all real d,

$$(1 - L^4)^d \;=\; \sum_{j=0}^{\infty} \binom{d}{j} (-1)^j L^{4j} \;=\; 1 \;-\; d\,L^4 \;+\; \frac{d\,(d-1)}{2} L^8 \quad ... \,.$$

Clearly, if $d = 0$ in (2), $x_t = u_t$, and a weak autocorrelation structure is allowed for. However, if $d > 0$, x_t is said to be a seasonal long memory process, so-named because of the strong association between observations widely separated in time. Thus, the higher d is, the higher will be the degree of seasonal dependence between the observations. If $d \in (0, 0.5)$, x_t is still covariance stationary and mean reverting, with the effect of the shocks disappearing in the long run; if $d \in [0.5, 1)$, x_t is no longer covariance stationary but it is still mean reverting, having autocovariances, which decay much more slowly than those of a seasonal ARMA process, in fact, so slowly as to be non-summable; finally, $d \geq 1$ implies nonstationary and non-mean reverting behaviour.

Most of the literature on fractional integration has concentrated on the long run or zero frequency (i.e., using a polynomial of form: $(1 - L)^d$), and it has been identified for several macroeconomic time series in many papers. This finding is often explained using Robinson [1978] and Granger [1980] aggregation results: cross section aggregation of a large number of AR(1) processes with heterogeneous AR coefficients may create long memory at the zero frequency. Parke [1999] uses a closely related discrete time error duration model, while Diebold and Inoue [2001] relate fractional integration with regime switching models. In a recent paper, Lildholdt [2002] provides both theoretical and Monte Carlo evidence that the three types of explanations may also generate seasonal fractional integration of form as in (2).[1]

Few empirical applications have been carried out in relation to seasonal fractional models. The notion of fractional Gaussian noise with seasonality was initially suggested by Abrahams and Dempster [1979] and Jonas [1981], and extended in a Bayesian framework by Carlin et al. [1985] and Carlin and Dempster [1989]. Porter-Hudak [1990] applied a seasonally fractionally integrated model to quarterly U.S. monetary aggregate with the conclusion that a fractional model could be more appropriate than standard ARIMAs. Advantages of seasonally fractionally integrated models for forecasting are illustrated in Ray [1993] and Sutcliffe [1994], and other recent empirical applications can be found in Arteche and Robinson [2000] and Gil-Alana and Robinson [2001].

The model in (2) can also be extended to allow different integration orders at each of the seasonal frequencies, for example,

$$(1 - L^2)^{d_1} (1 + L^2)^{d_2} x_t = u_t, \qquad t = 1,2,... \tag{3}$$

or more generally,

$$(1 - L)^{d_1} (1 + L)^{d_2} (1 + L^2)^{d_3} x_t = u_t, \qquad t = 1,2,..., \tag{4}$$

for given real values d_1, d_2 and d_3. In case of (3), d_1 corresponds to the order of integration at the real roots, while d_2 refers to the complex roots. (4) is even more general, and d_1 refers now to the order of integration at the long run or zero frequency; d_2 refers to the real frequency π; and d_3 to the complex ones corresponding to the annual frequencies $\pi / 2$ and $3 \pi / 2$. Then, x_t will be stationary if all integration orders are smaller than 0.5, and we say that x_t has seasonal long memory at a given

1. Lildholdt [2002] shows that fractional integration at the seasonal frequencies may be created by: a) cross-sectional aggregation of seasonal data; b) aggregation of seasonal duration models, and c) regime-switching if the underlying Markov process possesses seasonal dependencies.

frequency if the integration order at that frequency is greater than zero. A more detailed discussion of the stochastic properties of models (2) – (4) can be found for example in HASSLER [1994], WOODWARD, CHENG and GRAY [1998] and ARTECHE and ROBINSON [2000]. Note that the formulation in (3) is much more general than (1) or (2) and thus, it permits us to examine separately the degree of dependence between the observations at each of the frequencies, i.e., the long run or zero frequency, and the annual and the biannual frequencies.

In the following section we describe several versions of the tests of ROBINSON [1994] that permit us to test seasonally fractionally integrated processes like (2), (3) and (4). Section 3 contains several Monte Carlo experiments to check the power of the tests against different fractional alternatives. The tests of ROBINSON [1994] are applied in Section 4 to several U.S. monetary aggregates, while Section 5 contains some concluding comments.

2 The Tests of Robinson (1994)

ROBINSON [1994] proposes a method for testing unit roots and other fractionally integrated hypotheses when the roots are located at any frequency on the interval [0, π]. He considers the model:

(5) $$y_t = \beta' z_t + x_t, \qquad t = 1,2,...$$

where y_t is a raw time series; β is a (kx1) vector of unknown parameters; z_t is a (kx1) vector of deterministic regressors that may include, for example, a linear time trend or seasonal dummies, and x_t in (5) satisfies:

(6) $$\rho(L;\theta) x_t = u_t, \qquad t = 1,2,...$$

where ρ (L; θ) is a prescribed function of L and the (px1) parameter vector θ, that will adopt different forms depending on the model tested. Thus, for example,

(7) $$\rho(L;\theta) = (1 - L^4)^{d+\theta}$$

when testing (2) for a given real value d; Similarly,

(8) $$\rho(L;\theta) = (1 - L^2)^{d_1+\theta_1} (1 + L^2)^{d_2+\theta_2}$$

when testing (3); or more generally,

(9) $$\rho(L;\theta) = (1 - L)^{d_1+\theta_1} (1 + L)^{d_2+\theta_2} (1 + L^2)^{d_3+\theta_3}$$

in case of testing (4) for given real values d_1, d_2 and d_3. Also, u_t in (6) must be an $I(0)$ process, with spectral density:

$$f(\lambda;\tau) = \frac{\sigma^2}{2\pi}\, g(\lambda;\tau) \qquad -\pi < \lambda \leq \pi,$$

where the positive scalar σ^2 and the (qx1) vector τ are unknown, but g is of known form. Thus, for example, if u_t is white noise, $f = \sigma^2 / 2\pi$, and $g \equiv 1$, and if u_t is AR(1) of the form: $u_t = \tau\, u_{t-1} + \varepsilon_t$, $g(\lambda_j;\tau) = \left| 1 - \tau e^{i\lambda_j} \right|^{-2}$, with $\sigma^2 = V(\varepsilon_t)$, so that the AR coefficients correspond to the parameter vector τ.

Under the null hypothesis, defined by:

$$(10) \qquad H_o:\ \theta = 0,$$

the residuals in (5) and (6) are:

$$\hat{u}_t = \rho(L)\, y_t - \hat{\beta}' w_t, \qquad t = 1, 2, \ldots$$

where $\rho(L) = \rho(L;\theta = 0)$ and $\hat{\beta} = \left(\sum_{t=1}^{T} w_t\, w_t' \right)^{-1} \sum_{t=1}^{T} w_t\, \rho(L)\, y_t$; $w_t = \rho(L)\, z_t$.

The test statistic, which is derived from the Lagrange Multiplier (LM) principle, is

$$(11) \qquad \hat{R} = \frac{T}{\hat{\sigma}^4}\, \hat{a}' \hat{A}^{-1} \hat{a}$$

where T is the sample size, and

$$\hat{a} = \frac{-2\pi}{T} \sum_j^{*} \psi(\lambda_j)\, g(\lambda_j;\hat{\tau})^{-1} I(\lambda_j);\ \hat{\sigma}^2 = \sigma^2(\hat{\tau}) = \frac{2\pi}{T} \sum_{j=1}^{T-1} g(\lambda_j;\hat{\tau})^{-1} I(\lambda_j)$$

$$\hat{A} = \frac{2}{T} \left(\sum_j^{*} \psi(\lambda_j)\psi(\lambda_j)' - \sum_j^{*} \psi(\lambda_j)\hat{\varepsilon}(\lambda_j)' \left(\sum_j^{*} \hat{\varepsilon}(\lambda_j)\hat{\varepsilon}(\lambda_j)' \right)^{-1} \sum_j^{*} \hat{\varepsilon}(\lambda_j)\psi(\lambda_j)' \right)$$

$$\psi(\lambda_j) = \mathrm{Re}\left(\frac{\partial}{\partial\theta} \log \rho(e^{i\lambda_j};0) \right);\ \hat{\varepsilon}(\lambda_j) = \frac{\partial}{\partial\tau} \log g(\lambda_j;\hat{\tau});$$

$I(\lambda_j)$ is the periodogram of $\hat{u}_t$ and $\hat{\tau} = \arg\min_{\tau\in H} \sigma^2(\tau)$, with H as a suitable subset of the R^q Euclidean space. The sum on * is over $\lambda_j = 2\pi j / T$, such that $-\pi < \lambda_j < \pi$. $\lambda_j \notin (\rho_l - \lambda_1, \rho_l + \lambda_1)$, $l = 1, 2, \ldots, s$ such that ρ_l, $l = 1, 2, \ldots, s < \infty$ are the distinct poles of $\rho(L)$. Note that $\hat{R}$ is a function of the hypothesized differenced series, which has short memory under (10), and thus, we must specify the frequencies and integration orders of any seasonal roots.

ROBINSON [1994] established that under certain regularity conditions:[2]

$$\hat{R} \rightarrow_d \chi_p^2, \quad as\ T \rightarrow \infty, \tag{12}$$

where $p = \dim(\theta)$, and also that the tests are efficient in the Pitman sense, i.e., that against local alternatives of form: $H_a: \theta = \delta T^{-1/2}$, $\hat{R}$ has a $\chi_p^2(\nu)$ distribution, with a non-centrality parameter ν that cannot (when u_t is Gaussian) be exceeded by that of any rival regular statistic. Because $\hat{R}$ involves a ratio of quadratic forms, its exact null distribution can be calculated under Gaussianity via Imhof's algorithm. However, a simple test is approximately valid under much wider distributional assumptions. Thus, a test of (10) against the alternative, $H_a: \theta \neq 0$ will reject H_o if $\hat{R} > \chi_{p,\alpha}^2$, where $P(\chi^2 > \chi_p^2) = \alpha$. OOMS [1995] proposes Wald tests based on ROBINSON'S [1994] set-up in the model given by (5) and (6) but they require efficient estimates of the fractional differencing parameters. He uses a modified periodogram regression estimation procedure due to HASSLER [1994]. Also, HOSOYA [1997] establishes the limit theory for long memory processes with the singularities not restricted at the zero frequency and proposes a set of quasi log-likelihood statistics to be applied in raw time series. Unlike these methods, ROBINSON'S [1994] tests do not require estimation of the long memory parameters since the differenced series have short memory under the null. Empirical applications of the tests of ROBINSON [1994] with $\rho(L) = (1 - L)^d$, i.e., imposing the root exclusively at the zero frequency, (but not at the seasonal ones), are GIL-ALANA and ROBINSON [1997] and GIL-ALANA [2000], and given the recent extensive theoretical literature based on seasonal fractional integration, a further study of ROBINSON'S [1994] tests in this context seems overdue.

3 A Monte Carlo Experiment

In this section we examine the finite-sample behaviour of versions of the above tests by means of Monte Carlo simulations. In ROBINSON [1994] a finite-sample experiment was also conducted, looking at the rejection frequencies of the tests when the true model was a random walk, (i.e., $(1 - L)\, x_t = \varepsilon_t$), and the alternatives were either fractional, (i.e., $(1 - L)^{1+\theta} x_t = \varepsilon_t$), or autoregressive, (i.e., $(1 - (1 + \theta L))\, x_t = \varepsilon_t$), for different values of θ. A similar study was also carried out by GIL-ALANA [1999] for monthly data.

2. These conditions are very mild, regarding technical assumptions to be satisfied by $\psi(\lambda)$.

We investigate here the power of the tests of ROBINSON [1994] when the true model contains four unit roots, that is, (1) with white noise u_t, and the alternatives are seasonally fractionally integrated, first, with the same integration order at all frequencies, i.e., (2), and then allowing different orders of integration at each of the seasonal frequencies, i.e., (3) and (4), for different real values d, d_1, d_2 and d_3. We could also have extended the analysis to the reverse situation, i.e., assuming that the true model is given by (3) or (4), and testing H_o (10) in (7) for different values of d. However, given the vast amount of empirical work based on seasonal unit root models, we have considered more convenient to present in more detail the results of the former experiments rather than the latter ones.

Across Tables 1-3 we look at the rejection frequencies of ROBINSON'S [1994] tests in a model given by (5) and (6) with $\beta = 0$ in (5), (i.e., $y_t = x_t$), and ρ in (6) given by (7) with $d = 1$ and $\theta = 0$. The alternatives will be in all cases fractional, with $\rho\ (L;\ \theta)$ given by (7), (8) and (9) with d, d_1, d_2 and d_3, equal to 1, and values of θ, θ_1, θ_2 and θ_3 equal to –1, -0.75, ..., (0.25), ..., 0.75 and 1. Thus, the rejection frequencies corresponding to $\theta = 0$ will indicate the sizes of the tests. We use Gaussian series generated by the routines GASDEV and RAN3 of PRESS, FLANNERY, TEUKOLSKY and VETTERLING [1986], with 10,000 replications of each case. The sample sizes are $T = 120$, 240 and 360 observations and in all cases the nominal size is 5%.

Each table shows in the upper part the rejection frequencies when $\rho\ (L;\ \theta) = (1 - L^4)^{1+\theta}$, that is, imposing the same integration order at each frequency. Then, we take $\rho(L;\theta) = (1-L^2)^{1+\theta_1}(1+L^2)^{1+\theta_2}$, i.e., allowing different orders of integration for the real and the complex roots, and finally, in the lower part of the tables, we take $\rho(L;\theta) = (1-L)^{1+\theta_1}(1+L)^{1+\theta_2}(1+L^2)^{1+\theta_3}$, i.e., allowing different integration orders at each frequency.

Starting with $\rho\ (L;\ \theta) = (1 - L^4)^{1+\theta}$, we observe that the size of $\hat{R}$ is too large in all cases though it approximates to the nominal value with T. Thus, it is 17.8% when $T = 120$; it becomes 10.1% when $T = 240$, and reduces to 8.0% with $T = 360$. We also observe that the test statistic is slightly biased toward positive values of θ, obtaining higher rejection frequencies for $\theta > 0$ than for $\theta < 0$, and this is observed even when $T = 360$, (Table 3), though in this case, the rejection frequencies are relatively high even for $\theta = -1$, (0.831).

Taking $\rho(L;\theta) = (1-L^2)^{1+\theta_1}(1+L^2)^{1+\theta_2}$, the sizes are now 6.9% with $T = 120$; 6.2% with $T = 240$, and 5.3% with $T = 360$. The rejection frequencies are relatively high in all cases, though a bias in favour of positive values of θ_1 and θ_2 is again observed. We see that the lowest value (apart from that corresponding to the true model) is obtained in all cases when $\theta_1 = \theta_2 = -0.25$, in which case the rejection frequencies are 0.160 with $T = 120$; 0.502 with $T = 240$, and 0.793 with $T = 360$.

The rejection frequencies of the tests of ROBINSON [1994] with $\rho(L;\theta) = (1-L)^{1+\theta_1}(1+L)^{1+\theta_2}(1+L^2)^{1+\theta_3}$ are given in the lower part of Tables 1-3. In general, we observe a dramatic reduction in the size when we move from $\rho\ (\mathrm{L};\ \theta) = (1 - L)^{1+\theta}$ to the more elaborated versions $(1-L)^{1+\theta_1}(1+L)^{1+\theta_2}$ or $(1 - L)^{1+\theta_1}(1+L)^{1+\theta_2}(1+L^2)^{1+\theta_3}$. This may appear surprising, especially, if we take into account that the test employed is a LM test, implying that is evaluated under the null $\theta = 0$ in the first case, and $\theta_1 = \theta_2 = 0$ ($\theta_1 = \theta_2 = \theta_3 = 0$) in the second (and third) cases, imposing thus, the same model under the null. This is explained by the functional form of the test statistic. In the first case, the func-

TABLE 1

Rejection Frequencies of $\hat{R}$ in (11) Against Fractional Alternatives. True Model: $(1 - L^4)\, y_t = \varepsilon_t$, and $T = 120$.

$$\rho(L;\theta) = (1-L^4)^{1+\theta}$$

θ	-1.00	-0.75	-0.50	-0.25	0	0.25	0.50	0.75	1.00
	0.604	0.566	0.447	0.153	**0.178**	0.763	0.989	1.000	1.000

$$\rho(L;\theta) = (1-L^2)^{1+\theta_1}(1+L^2)^{1+\theta_2}$$

θ_1 / θ_2	-1.00	-0.75	-0.50	-0.25	0	0.25	0.50	0.75	1.00
-1.00	0.790	0.959	0.841	0.833	0.881	0.915	0.939	0.954	0.963
-0.75	0.973	0.695	0.803	0.811	0.921	0.962	0.981	0.988	0.993
-0.50	1.000	0.981	0.493	0.648	0.913	0.979	0.995	0.998	0.999
-0.25	1.000	1.000	0.974	0.160	0.709	0.969	0.996	0.999	0.999
0.00	1.000	1.000	0.999	0.951	**0.069**	0.795	0.988	0.998	0.999
0.25	1.000	1.000	1.000	0.999	0.909	0.402	0.845	0.993	0.999
0.50	1.000	1.000	1.000	1.000	0.999	0.890	0.888	0.877	0.995
0.75	1.000	1.000	1.000	1.000	1.000	0.998	0.898	0.992	0.899
1.00	1.000	1.000	1.000	1.000	1.000	1.000	0.997	0.909	0.999

$$\rho(L;\theta) = (1-L)^{1+\theta_1}(1+L)^{1+\theta_2}(1+L^2)^{1+\theta_3}$$

(θ_1, θ_2) / θ_3	-1.00	-0.75	-0.50	-0.25	0	0.25	0.50	0.75	1.00
(-1.00, -1.00)	0.848	0.930	0.999	1.000	1.000	1.000	1.000	1.000	1.000
(-1.00, -0.50)	0.989	0.988	0.999	1.000	1.000	1.000	1.000	1.000	1.000
(-1.00, 0.00)	0.997	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
(-1.00, 0.50)	0.998	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
(-1.00, 1.00)	0.999	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
(-0.50, -1.00)	1.000	1.000	0.999	1.000	1.000	1.000	1.000	1.000	1.000
(-0.50, -0.50)	1.000	0.962	0.580	0.834	0.998	1.000	1.000	1.000	1.000
(-0.50, 0.00)	1.000	0.998	0.968	0.990	1.000	1.000	1.000	1.000	1.000
(-0.50, 0.50)	1.000	0.999	1.000	1.000	1.000	1.000	1.000	1.000	1.000
(-0.50, 1.00)	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
(0.00, -1.00)	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
(0.00, -0.50)	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
(0.00, 0.00)	1.000	1.000	0.998	0.582	**0.073**	0.756	0.999	1.000	1.000
(0.00, 0.50)	1.000	1.000	1.000	0.931	0.733	0.971	1.000	1.000	1.000
(0.00, 1.00)	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
(0.50, -1.00)	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
(0.50, -0.50)	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
(0.50, 0.00)	1.000	1.000	1.000	1.000	0.998	0.998	1.000	1.000	1.000
(0.50, 0.50)	1.000	1.000	1.000	1.000	0.799	0.497	0.921	1.000	1.000
(0.50, 1.00)	1.000	1.000	1.000	1.000	0.918	0.642	0.947	1.000	1.000
(1.00, -1.00)	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
(1.00, -0.50)	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
(1.00, 0.00)	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
(1.00, 0.50)	1.000	1.000	1.000	1.000	1.000	0.999	1.000	1.000	1.000
(1.00, 1.00)	1.000	1.000	1.000	1.000	1.000	0.985	0.983	1.000	1.000

10,000 replications were used for each case. Sizes are in bold and the nominal size was 5%.

TABLE 2

Rejection Frequencies of $\hat{R}$ in (11) Against Fractional Alternatives. True Model: $(1 - L^4)\, y_t = \varepsilon_t$, and $T = 240$.

$\rho(L;\theta) = (1-L^4)^{1+\theta}$

θ	-1.00	-0.75	-0.50	-0.25	0	0.25	0.50	0.75	1.00
	0.750	0.781	0.814	0.554	**0.101**	0.958	1.000	1.000	1.000

$\rho(L;\theta) = (1-L^2)^{1+\theta_1}(1+L^2)^{1+\theta_2}$

θ_1 / θ_2	-1.00	-0.75	-0.50	-0.25	0	0.25	0.50	0.75	1.00
-1.00	0.910	0.993	0.922	0.950	0.969	0.982	0.989	0.993	0.995
-0.75	0.998	0.891	0.919	0.972	0.992	0.998	0.999	0.999	0.999
-0.50	1.000	1.000	0.846	0.961	0.998	1.000	1.000	1.000	1.000
-0.25	1.000	1.000	0.999	0.502	0.990	0.999	1.000	1.000	1.000
0.00	1.000	1.000	1.000	0.999	**0.062**	0.995	1.000	1.000	1.000
0.25	1.000	1.000	1.000	1.000	0.999	0.762	0.997	1.000	1.000
0.50	1.000	1.000	1.000	1.000	1.000	0.997	0.999	0.998	1.000
0.75	1.000	1.000	1.000	1.000	1.000	1.000	0.997	1.000	0.997
1.00	1.000	1.000	1.000	1.000	1.000	1.000	1.000	0.997	1.000

$\rho(L;\theta) = (1-L)^{1+\theta_1}(1+L)^{1+\theta_2}(1+L^2)^{1+\theta_3}$

(θ_1, θ_2) / θ_3	-1.00	-0.75	-0.50	-0.25	0	0.25	0.50	0.75	1.00
(-1.00, -1.00)	0.963	0.993	1.000	1.000	1.000	1.000	1.000	1.000	1.000
(-1.00, -0.50)	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
(-1.00, 0.00)	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
(-1.00, 0.50)	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
(-1.00, 1.00)	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
(-0.50, -1.00)	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
(-0.50, -0.50)	1.000	0.998	0.905	0.988	1.000	1.000	1.000	1.000	1.000
(-0.50, 0.00)	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
(-0.50, 0.50)	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
(-0.50, 1.00)	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
(0.00, -1.00)	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
(0.00, -0.50)	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
(0.00, 0.00)	1.000	1.000	1.000	0.926	**0.066**	0.972	1.000	1.000	1.000
(0.00, 0.50)	1.000	1.000	1.000	1.000	0.999	1.000	1.000	1.000	1.000
(0.00, 1.00)	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
(0.50, -1.00)	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
(0.50, -0.50)	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
(0.50, 0.00)	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
(0.50, 0.50)	1.000	1.000	1.000	1.000	1.000	0.965	1.000	1.000	1.000
(0.50, 1.00)	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
(1.00, -1.00)	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
(1.00, -0.50)	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
(1.00, 0.00)	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
(1.00, 0.50)	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
(1.00, 1.00)	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000

10,000 replications were used for each case. Sizes are in bold and the nominal size was 5%.

TABLE 3

Rejection Frequencies of $\hat{R}$ in (11) Against Fractional Alternatives. True Model: $(1 - L^4)\ y_t = \varepsilon_t$, and $T = 360$.

$$\rho(L;\theta) = (1-L^4)^{1+\theta}$$

θ	-1.00	-0.75	-0.50	-0.25	0	0.25	0.50	0.75	1.00
	0.831	0.861	0.927	0.847	**0.080**	0.993	1.000	1.000	1.000

$$\rho(L;\theta) = (1-L^2)^{1+\theta_1}\ (1+L^2)^{1+\theta_2}$$

θ_1 / θ_2	-1.00	-0.75	-0.50	-0.25	0	0.25	0.50	0.75	1.00
-1.00	0.944	0.994	0.958	0.977	0.988	0.993	0.995	0.996	0.998
-0.75	1.000	0.937	0.957	0.991	0.998	0.999	1.000	1.000	1.000
-0.50	1.000	1.000	0.946	0.994	0.999	1.000	1.000	1.000	1.000
-0.25	1.000	1.000	1.000	0.793	0.999	1.000	1.000	1.000	1.000
0.00	1.000	1.000	1.000	1.000	**0.053**	0.999	1.000	1.000	1.000
0.25	1.000	1.000	1.000	1.000	1.000	0.936	1.000	1.000	1.000
0.50	1.000	1.000	1.000	1.000	1.000	0.936	1.000	1.000	1.000
0.75	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
1.00	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000

$$\rho(L;\theta) = (1-L)^{1+\theta_1}\ (1+L)^{1+\theta_2}\ (1+L^2)^{1+\theta_3}$$

(θ_1, θ_2) / θ_3	-1.00	-0.75	-0.50	-0.25	0	0.25	0.50	0.75	1.00
(-1.00, -1.00)	0.979	0.995	1.000	1.000	1.000	1.000	1.000	1.000	1.000
(-1.00, -0.50)	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
(-1.00, 0.00)	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
(-1.00, 0.50)	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
(-1.00, 1.00)	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
(-0.50, -1.00)	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
(-0.50, -0.50)	1.000	1.000	0.971	0.999	1.000	1.000	1.000	1.000	1.000
(-0.50, 0.00)	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
(-0.50, 0.50)	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
(-0.50, 1.00)	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
(0.00, -1.00)	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
(0.00, -0.50)	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
(0.00, 0.00)	1.000	1.000	1.000	0.996	**0.056**	1.000	1.000	1.000	1.000
(0.00, 0.50)	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
(0.00, 1.00)	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
(0.50, -1.00)	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
(0.50, -0.50)	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
(0.50, 0.00)	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
(0.50, 0.50)	1.000	1.000	1.000	1.000	1.000	0.999	1.000	1.000	1.000
(0.50, 1.00)	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
(1.00, -1.00)	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
(1.00, -0.50)	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
(1.00, 0.00)	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
(1.00, 0.50)	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
(1.00, 1.00)	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000

10,000 replications were used for each case. Sizes are in bold and the nominal size was 5%.

tion $\psi(\lambda_j) = \log\left|\sin\frac{\lambda_j}{2}\right| + \log\left(2\cos\frac{\lambda_j}{2}\right) + \log\left|2\cos\lambda_j\right|$; in the second case, it is a (2x1) vector of form: $\left[\log\left|\sin\frac{\lambda_j}{2}\right| + \log\left(2\cos\frac{\lambda_j}{2}\right), \log\left|2\cos\lambda_j\right|\right]^T$; while in the third case, $\psi(\lambda_j)$ is a (3x1) vector formed by each of these elements. (See, ROBINSON [1994]). Thus, though asymptotically they should all of them lead to the same value in the determinant of $\hat{A} = \frac{2}{T}\sum_j^* \psi(\lambda_j)\psi(\lambda_j)'$, in finite samples, the values are very different.[3] A bias also appears in this context of three different orders of integration, though it tends to disappear with T. Thus, for example, if $\theta_1 = \theta_2 = \theta_3 = -0.50$, the rejection frequency with $T = 120$ is 0.580, while the value corresponding to the alternative $\theta_1 = \theta_2 = \theta_3 = 0.50$ for the same T is 0.921. Moreover, increasing the sample size, these values are 0.905 and 1 with $T = 240$, and 0.971 and 1 with $T = 360$. Finally, imposing different θ_i's, we also see that the rejection frequencies increase with T, obtaining values higher than 0.97 in all cases when $T = 360$.

The same experiment was also conducted imposing the true model to be (2) with $d = 0.50$, 0.75, 1.25 and 1.50, obtaining results, in terms of size and power, similar to those reported across Tables 1-3. Therefore, we can summarise the results in these tables by saying that the tests of ROBINSON [1994] analysed in this article seem to perform quite well when testing the null hypothesis of four seasonal unit or fractional roots.

Another case of interest is to examine the power of the tests when the true model possesses different orders of integration at each of the frequencies. In particular, we examine a model of form as in (4) with $d_1 = 1$, $d_2 = 0.75$ and $d_3 = 0.50$, and look at the power properties of the different versions of ROBINSON's [1994] tests. First, we look at tests with $\rho(L;\theta) = (1 - L)^{1+\theta}$, and $\theta = -1, (0.25), 1$. In such a case, since we impose the same degree of integration at all frequencies, the rejection probabilities should be close to 1 in all cases. Next, we examine the case of $\rho(L;\theta) = (1-L^2)^{1+\theta_1}(1+L^2)^{1+\theta_2}$, with θ_1 and θ_2 again equal to -1, (0.25), 1. Finally, we also consider the case of $\rho(L;\theta) = (1-L)^{1+\theta_1}(1+L)^{1+\theta_2}(1+L^2)^{1+\theta_3}$, and we look here at alternatives with $\theta_1 = -1, (0.50), 1$; $\theta_2 = -1, (0.25), 1$; and $\theta_3 = -0.50$ (0,25), 0.50. Thus, the rejection frequencies corresponding to $\theta_1 = 0$; $\theta_2 = -0.25$; and $\theta_3 = -0.50$ will indicate the size of the tests. Similarly to the previous cases, we use 10,000 replications for a nominal size of 5%. We only report the results for a sample size of 120 observations (Table 4). As expected, the results considerably improved with $T = 240$ and 360 observations.

Starting with $\rho(L;\theta) = (1 - L)^{1+\theta}$, we see that the rejection probabilities are very high for the extreme alternatives $\theta = 1$ and -1. However, as we approach to $\theta = 0$, the values reduce, and it is 0.369 for $\theta = 0$. This may be explained by the influence of the zero frequency over the others, especially if the sample size is small. In fact, increasing T to 240 or 360, the values become respectively 0.544 and 0.799, stressing once more the asymptotic validity of the tests.

3. Note that we are assuming here white noise disturbances. Thus, $g \equiv 1$, and $\varepsilon(\lambda_j) = 0$.

TABLE 4

Rejection Frequencies of $\hat{R}$ in (11) Against Fractional Alternatives. True Model: $(1-L)(1+L)^{0.75}(1+L^2)^{0.50}y_t = \varepsilon_t$, and T = 120.

$\rho(L;\theta) = (1-L^4)^{1+\theta}$

θ	-1.00	-0.75	-0.50	-0.25	0	0.25	0.50	0.75	1.00
	0.855	0.796	0.517	0.438	0.349	0.587	0.818	0.972	0.996

$\rho(L;\theta) = (1-L^2)^{1+\theta_1}(1+L^2)^{1+\theta_2}$

θ_1 / θ_2	-1.00	-0.75	-0.50	-0.25	0	0.25	0.50	0.75	1.00
-1.00	0.990	1.000	1.000	0.991	0.971	0.946	0.919	0.904	0.891
-0.75	0.858	0.992	0.999	0.989	0.939	0.875	0.838	0.813	0.796
-0.50	0.746	0.714	0.986	0.988	0.895	0.746	0.687	0.676	0.698
-0.25	0.937	0.678	0.503	0.978	0.899	0.584	0.391	0.392	0.464
0.00	1.000	0.888	0.428	0.433	0.988	0.717	0.183	0.104	0.211
0.25	1.000	0.995	0.773	0.506	0.708	0.997	0.632	0.089	0.063
0.50	1.000	1.000	0.996	0.668	0.564	0.956	1.000	0.972	0.232
0.75	1.000	1.000	1.000	0.970	0.656	0.969	0.997	1.000	0.938
1.00	1.000	1.000	1.000	0.999	0.944	0.878	0.856	1.000	1.000

$\rho(L;\theta) = (1-L)^{1+\theta_1}(1+L)^{1+\theta_2}(1+L^2)^{1+\theta_3}$

(θ_1, θ_2) / θ_3	-1.00	-0.75	-0.50	-0.25	0	0.25	0.50	0.75	1.00
(-1.00, -0.50)	0.773	0.995	1.000	1.000	1.000	1.000	1.000	1.000	1.000
(-1.00, -0.25)	0.887	0.899	0.999	1.000	1.000	1.000	1.000	1.000	1.000
(-1.00, 0.00)	0.704	0.937	1.000	1.000	1.000	1.000	1.000	1.000	1.000
(-1.00, 0.25)	0.933	0.996	1.000	1.000	1.000	1.000	1.000	1.000	1.000
(-1.00, 0.50)	0.997	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
(-0.50, -0.50)	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
(-0.50, -0.25)	0.998	0.998	1.000	1.000	1.000	1.000	1.000	1.000	1.000
(-0.50, 0.00)	0.862	0.816	0.987	1.000	1.000	1.000	1.000	1.000	1.000
(-0.50, 0.25)	0.796	0.337	0.829	0.996	1.000	1.000	1.000	1.000	1.000
(-0.50, 0.50)	0.815	0.249	0.792	0.992	1.000	1.000	1.000	1.000	1.000
(0.00, -0.50)	0.988	0.934	0.899	0.993	0.999	1.000	1.000	1.000	1.000
(0.00, -0.25)	0.994	0.945	**0.112**	0.789	0.998	1.000	1.000	1.000	1.000
(0.00, 0.00)	0.995	0.893	0.873	0.881	1.000	1.000	1.000	1.000	1.000
(0.00, 0.25)	1.000	0.999	1.000	1.000	1.000	1.000	1.000	1.000	1.000
(0.00, 0.50)	1.000	0.995	0.996	1.000	1.000	1.000	1.000	1.000	1.000
(0.50, -0.50)	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
(0.50, -0.25)	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
(0.50, 0.00)	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
(0.50, 0.25)	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
(0.50, 0.50)	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
(1.00, -0.50)	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
(1.00, -0.25)	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
(1.00, 0.00)	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
(1.00, 0.25)	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
(1.00, 0.50)	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000

10,000 replications were used for each case. Sizes are in bold and the nominal size was 5%.

If $\rho(L;\theta)$ is given by $(1-L^2)^{1+\theta_1}(1+L^2)^{1+\theta_2}$, the rejection probabilities are relatively high in all cases, the lowest value being obtained at $\theta_1 = 0$ and $\theta_2 = -0.50$, which is the closest alternative to the true model. Finally, if $\rho(L;\theta) = (1-L)^{1+\theta_1}(1+L)^{1+\theta_2}(1+L^2)^{1+\theta_3}$, we observe that the size of the test is 0.112, which is higher than the 5% level corresponding to the nominal value. However, though not reported in the table, if $T = 240$, the value becomes 0.091 and, if $T = 360$, it is 0.059.

Clearly, when testing the null hypothesis (10) in a model given by (7) the tests have greater power than when directed against the models in (8) or (9), though the latter forms will have power against a wider range of alternatives. Finally, the performance of the tests was also evaluated against alternatives of form: $(1-\rho L^4)\, x_t = \varepsilon_t$, with different values of ρ, comparing the results of ROBINSON's [1994] tests with those based on standard seasonal unit root tests (i.e., DICKEY, HASZA and FULLER, DHF, [1984]). As we should expect, DHF [1984] perform better against these AR alternatives, but worse against the fractional alternatives used in this article.

In conclusion, given the larger flexibility permitted in the case of different orders of integration, it might be more interesting to start the analysis of the seasonal structure by performing the tests using $\rho(L;\theta)$ with the form as in (9) and, if we find cases where H_o cannot be rejected for the same values of some of the d's, perform then other versions of the tests according to this. However, given the easy in the computation of the three forms of the tests, it might also be convenient to have the whole picture of the results for all statistics and examine the whole situation, drawing then the adequate conclusions.

4 An Empirical Application

In this section the tests of ROBINSON [1994] are applied to several U.S. monetary aggregates. The series are the (seasonally unadjusted) total expenditure for $M1$, $M2$, $M3$ and L in the U.S. from 1960.1 to 1998.4, quarterly, obtained from the Federal Reserve Bank of St. Louis database (http:/research.stlouisfed.org/fred2). The definitions of these monetary aggregates are given in the Appendix.

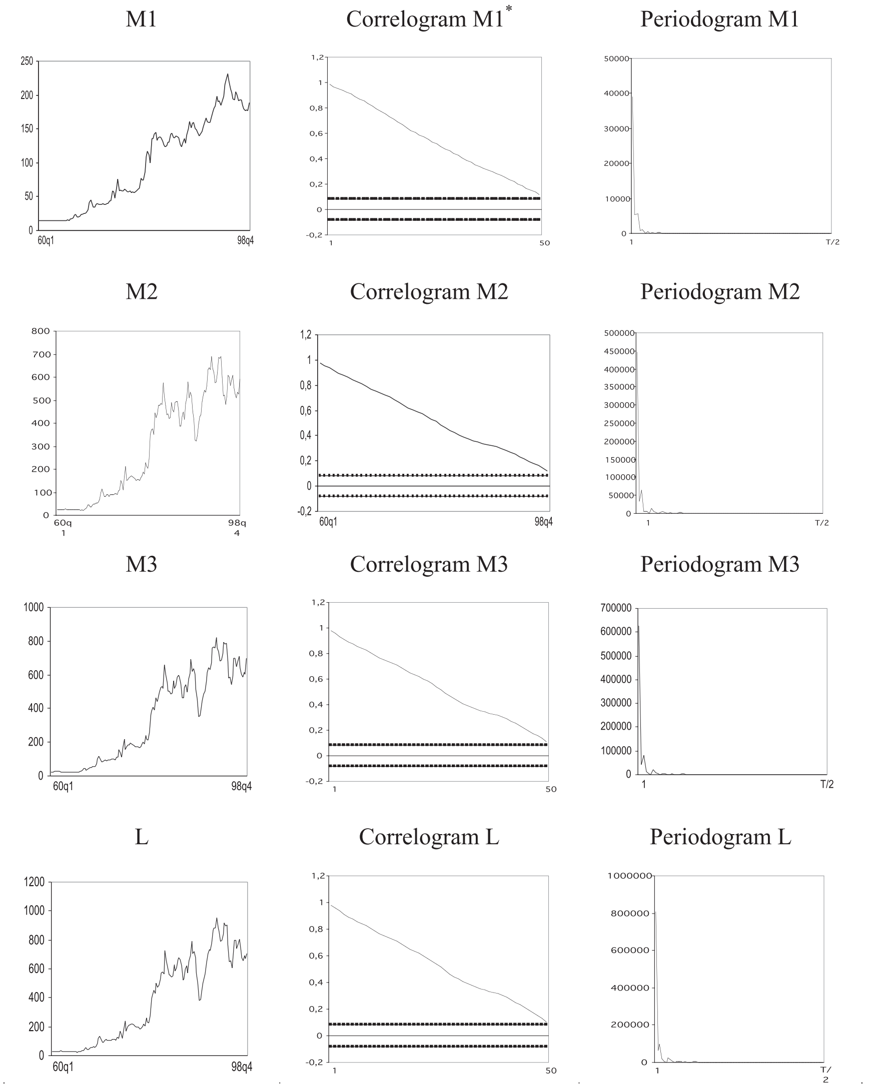
Figure 1 displays plots of the individual series along with their corresponding correlograms and periodograms. The series are clearly nonstationary, with the values in the correlograms decaying very slowly, and a large peak in the periodograms at the smallest frequency.[4]

Taking first differences, we do not display the plots, though it was observed that seasonality plays an important role, with significant values in the correlograms at the seasonal lags, and peaks in the periodograms at the seasonal frequencies. Thus, any type of analysis based on classical unit root tests (e.g. DICKEY and FULLER, [1979]; PHILLIPS and PERRON, [1988]; etc.) or even fractional models

4. Note that the periodogram is an estimate of the spectral density function. In an $I(d)$ process with $d > 0$, the spectrum is unbounded at the origin, and the periodogram should reproduce that behavior.

FIGURE 1

Original Time Series, with Their Correlograms and Periodograms



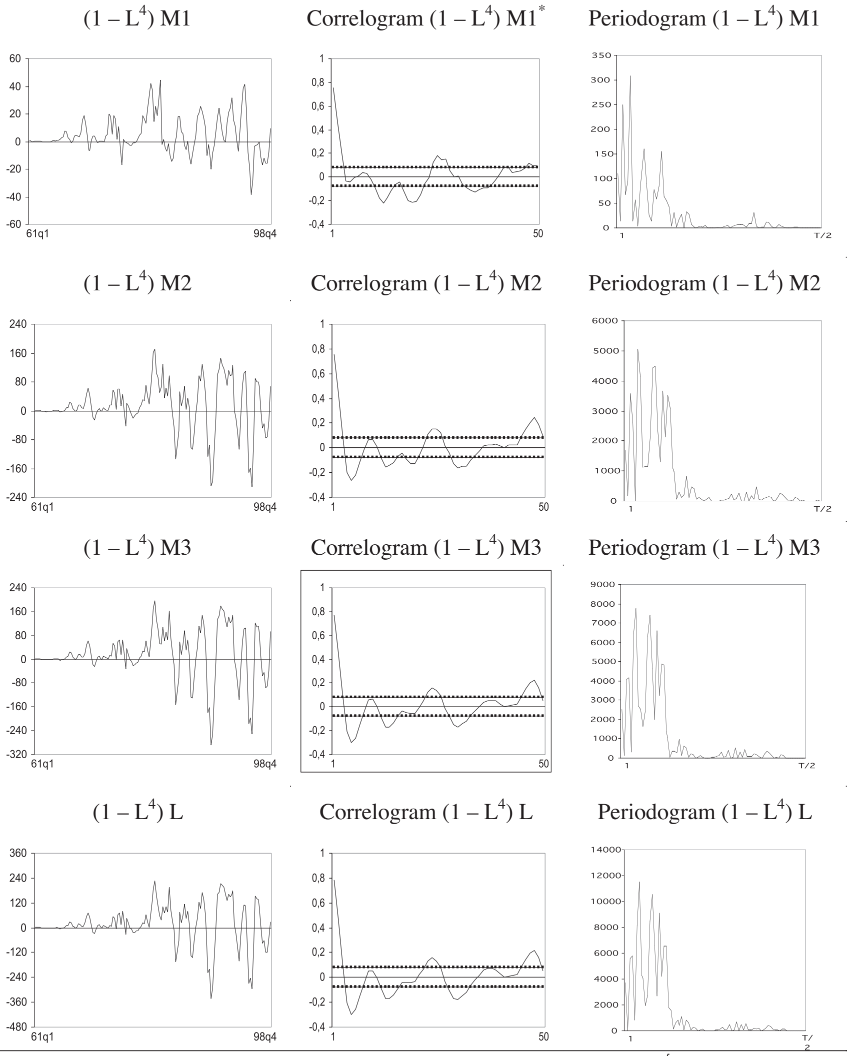
** The large sample standard error under the null hypothesis of no autocorrelation is 1/√T or roughly 0.080 for series of length considered here.*

(SOWELL, [1992]) might be biased because of the presence of seasonality. Several papers conducted by MONTANARI, ROSSO and TAQQU [1995, 1996, 1997] in a hydrological context, showed that the presence of periodicities strongly influence the reliability of the estimators at the zero frequency.[5] Figure 2 contains the plots based on the first seasonal differences. Here, the series have an appearance of stationary,

5. In another recent paper, MONTANARI, TAQQU and TEVEROWSKY [1999] performed an extensive Monte Carlo investigation in order to find how reliable the estimators for long memory are in the presence of periodicities and they concluded that the best results were those obtained using likelihood-type methods.

FIGURE 2

First Seasonal Differenced Time Series, with Their Correlograms and Periodograms



* *The large sample standard error under the null hypothesis of no autocorrelation is 1/√T or roughly 0.080 for series of length considered here.*

(though it is observed a substantial increase in the variances from the second part of the sample). However, a deeper visual inspection at the periodograms shows that the series may now be overdifferenced with respect to the seasonal frequencies.

Denoting each of the series by y_t, we initially employ throughout the null model:

$$y_t = \alpha + \beta_0 t + \sum_{j=1}^{3} \beta_j S_{jt} + x_t, \qquad t = 1, 2, ... \tag{13}$$

$$\rho(L) x_t = u_t, \qquad t = 1, 2, ... \tag{14}$$

where S_{1t}, S_{2t} and S_{3t} are seasonal (quarterly) dummy variables. These dummies were included in order to know if deterministic components play a role in describing the seasonality.[6] We take $\rho(L) = (1 - L^4)^d$; $(1-L^2)^{d_1}(1+L^2)^{d_2}$; and $(1-L)^{d_1}(1+L)^{d_2}(1+L^2)^{d_3}$, and d, d_1, d_2, d_3 equal to 0, 0.25, ..., (0.25),..., 1.75 and 2. We model the $I(0)$ disturbances u_t as white noise, and treat separately the cases of all coefficients in (13) equal to 0 a priori, (i.e., including no regressors in the undifferenced regression); α unknown and $\beta_i)_{i=0,\ldots,3} = 0$ a priori, (i.e., including an intercept); α and β_0 unknown and $\beta_i)_{i=1,\ldots,3} = 0$ (i.e., with a linear trend); α and $\beta_i)_{i=1,\ldots,3}$ unknown and $\beta_0 = 0$, (i.e., with an intercept and the seasonal dummies); and with all the coefficients unknown. However, the results were very similar across all these cases, and thus, we only report here those corresponding to the case of $\alpha = \beta_i)_{i=0,\ldots,3} = 0$ a priori, i.e., with $y_t = x_t$. Moreover, the coefficients corresponding to these deterministic components were found to be insignificantly different from zero in practically all cases where the null could not be rejected. Note that the tests are based on the null differenced model, which is short memory, and thus, standard t-tests apply.

Table 5 shows the results of the statistic $\hat{R}$ in (11), firstly with $\rho(L) = (1 - L^4)^d$, i.e., imposing the same integration order at each frequency. We see that the results are very similar for the different monetary aggregates, and the non-rejection values occur in all cases when $d = 1$, 1.25 and 1.50. We also observe that when $d = 2$, the null is less strongly rejected than for example when $d = 0$, but on the whole, these extreme values are always rejected, suggesting that the optimal local power properties of ROBINSON's [1994] tests may be supported by reasonable performance against non-local alternatives. The lowest statistics across the different values of d are obtained when $d = 1.25$ for $M1$, and when $d = 1$ for the remaining aggregates.

In view of the similarities observed across the different monetary aggregates, we concentrate only on $M2$ as the series of interest, and look at $\hat{R}$ in (11) with $\rho(L) = (1-L^2)^{d_1}(1+L^2)^{d_2}$ and $(1-L)^{d_1}(1+L)^{d_2}(1+L^2)^{d_3}$. Allowing a different integration order at the real and the complex roots, (i.e., with $\rho(L) = (1-L^2)^{d_1}(1+L^2)^{d_2}$), we observe very few non-rejection values, all of them occurring when d_1 ranges between 1 and 2, and when d_2 ranges between 0 and 0.75. Thus, we observe higher integration orders at the 0 and π frequencies than at the complex ones $\pi / 2$ and $3\pi / 2$. In fact, H_o (10) is now decisively rejected when $d_1 = d_2 = 1$, 1.25 and 1.50, contradicting the results obtained just above, and the lowest statistic is obtained when $d_1 = 1.50$ and $d_2 = 0.25$, suggesting the importance of the real roots over the complex ones. These contradictory results may be related with the different sizes and rejection frequencies observed in Table 1. If the true model were given by $(1 - L^4)\, x_t = u_t$, performing the tests of ROBINSON [1994] with (3) and $d_1 = 1.50$ and $d_2 = 0.25$, we saw in Table 1 that the rejection frequency was exactly 1. On the contrary (and though it is not reported here), performing the opposite experiment, i.e., testing (2) with $d = 1$ when the true model is given by $(1 - L^2)^{1.50}(1 + L^2)^{0.25}x_t = u_t$, the rejection frequency was 0.144, suggesting both experiments that a model with different orders of integration might be more appropriate for this series. Extending the model, and thus allowing a different integration order at each frequency, we only observe two non-rejection values, correspond-

6. Note that, in many cases, deterministic sesonality can coexist with stochastic sesonality. The same happens, for example, in non-seasonal contexts, with deterministic trends and unit roots at the zero frequency.

TABLE 5
Testing (10) in (6) with White Noise u_t

$$\rho(L;\theta) = (1-L^4)^d$$

Series / d	0.00	0.25	0.50	0.75	1.00	1.25	1.50	1.75	2.00
*M*1	370.7	340.7	162.6	24.30	**2.34’**	**.0009’**	**1.22’**	4.06’	5.94’
*M*2	368.5	270.1	76.81	7.54	**0.09’**	**0.85’**	**2.89’**	4.96’	6.72
*M*3	369.9	265.4	71.54	6.36	**0.03’**	**0.90’**	**2.78’**	4.66’	6.27
L	372.5	265.2	70.22	6.51	**0.06’**	**0.81’**	**2.66’**	4.54’	6.46

SERIES: M2 $\rho(L;\theta) = (1-L^2)^{d_1}(1+L^2)^{d_2}$

d_1 / d_2	0.00	0.25	0.50	0.75	1.00	1.25	1.50	1.75	2.00
0.00	652.2	656.8	657.8	657.0	655.5	653.7	651.8	649.9	648.1
0.25	474.1	505.8	517.1	518.2	514.6	508.9	502.5	495.9	489.5
0.50	130.2	17.84	202.4	209.3	207.1	200.7	192.9	184.9	177.3
0.75	17.53	41.25	59.51	67.35	67.69	64.33	59.66	54.86	50.40
1.00	**1.40’**	11.78	26.15	35.30	37.80	36.23	32.99	29.41	26.05
1.25	**0.71’**	**2.27’**	12.51	22.95	28.01	28.28	26.12	23.17	20.22
1.50	**4.57’**	**0.01’**	**4.41’**	14.51	22.24	24.92	24.16	21.89	19.26
1.75	10.50	**2.72’**	**0.33’**	7.14’	16.58	22.19	23.46	22.25	20.09
2.00	17.57	9.53	**1.27’**	**1.50’**	9.89	18.17	22.18	22.66	21.34

SERIES: M2 $\rho(L;\theta) = (1-L)^{d_1}(1+L)^{d_2}(1+L^2)^{d_3}$

(d_1, d_2) / d_3	0.00	0.25	0.50	0.75	1.00	1.25	1.50	1.75	2.00
(0.00, 0.00)	1004.5	1012.9	1018.3	1022.0	1024.7	1026.7	1028.0	1028.8	1029.3
(0.00, 0.50)	1013.4	1019.9	1023.9	1026.7	1028.6	1029.8	1030.7	1031.1	1031.3
(0.00, 1.00)	1017.8	1023.1	1026.4	1028.2	1029.6	1030.4	1030.9	1031.0	1031.0
(0.00, 1.50)	1020.6	1024.9	1027.3	1028.8	1029.7	1030.2	1030.4	1030.4	1030.2
(0.00, 2.00)	1022.5	1025.9	1027.8	1028.9	1029.5	1029.8	1029.8	1029.6	1029.3
(0.50, 0.00)	165.25	224.7	270.11	306.8	338.0	365.2	389.3	410.9	430.6
(0.50, 0.50)	240.9	306.6	354.7	392.8	424.7	452.4	476.8	498.6	518.3
(0.50, 1.00)	287.4	348.5	392.3	426.8	455.8	481.8	503.5	523.7	542.1
(0.50, 1.50)	321.9	376.8	416.1	447.3	473.7	496.9	517.7	536.6	553.9
(0.50, 2.00)	349.9	398.9	434.4	462.8	487.1	508.7	528.3	546.2	562.7
(1.00, 0.00)	**1.82’**	9.43	19.48	27.02	32.49	36.70	40.10	42.92	45.32
(1.00, 0.50)	11.93	25.24	41.65	55.52	67.13	77.24	86.29	94.52	102.0
(1.00, 1.00)	23.08	43.94	64.56	81.40	95.45	107.7	118.7	128.7	138.0
(1.00, 1.50)	34.06	58.86	80.11	96.78	110.4	122.3	133.0	142.7	151.8
(1.00, 2.00)	45.09	71.09	91.48	107.1	119.8	130.8	140.8	150.0	158.6
(1.50, 0.00)	13.05	22.02	37.02	49.67	59.49	67.35	73.84	79.35	84.13
(1.50, 0.50)	13.44	**6.36’**	12.19	19.14	24.62	28.82	32.11	34.72	36.81
(1.50, 1.00)	15.53	11.10	19.73	29.37	37.49	44.30	50.20	55.39	60.03
(1.50, 1.50)	15.46	17.81	30.20	42.06	51.82	60.05	67.25	73.70	79.57
(1.50, 2.00)	16.17	24.75	39.21	51.59	61.42	69.56	76.65	83.01	88.83
(2.00, 0.00)	25.69	43.36	65.23	82.72	95.77	105.7	113.7	120.3	125.9
(2.00, 0.50)	25.17	13.30	18.83	28.03	36.32	43.21	48.99	53.89	58.10
(2.00, 1.00)	31.93	11.15	10.28	15.39	20.67	25.05	28.61	31.49	33.80
(2.00, 1.50)	31.17	12.13	13.38	20.35	27.29	33.23	38.30	42.68	46.49
(2.00, 2.00)	26.64	13.53	18.82	28.04	36.36	43.32	49.25	54.42	59.02

In bold the non-rejection values of the null hypothesis (10) at the 95% significance level. : Non-rejection values at the 95% significance level with the finite sample critical values obtained in Table 5.

ing to $d_1 = 1$ and $d_2 = d_3 = 0$, (i.e., a random walk), and $d_1 = 1.50$; $d_2 = 0.50$ and $d_3 = 0.25$. These two possibilities were not allowed with the previous specifications for $\rho(L)$. The results here emphasize once more the importance of the real roots, in particular, the one at the zero frequency.

The non-rejection values obtained across Table 5 are all based on the asymptotic critical values given by the chi-squared distributions. However, since the sample size in this empirical application is not very large, we also calculated finite sample critical values of the tests by means of Monte Carlo simulations. We computed the values for the three functional forms of $\rho(L)$ with $T = 120$ and though not reported in the paper, the critical values were slightly greater than those given by the chi-squared distributions. Thus, some of the values of $\hat{R}$ in Table 5 where H_o (10) was not rejected when using the asymptotic critical values might now be rejected with the finite sample ones.

We also performed a similar experiment as in Table 1, studying the rejection frequencies of ROBINSON'S [1994] tests when $T = 120$, for the three functional forms of $\rho(L)$, but using the new finite sample critical values, and the conclusions can be summarized as follows: taking $\rho(L) = (1 - L^4)^{1+\theta}$, the size of the test reduces considerably (from 17.8% in Table 1 to 4.6%). This small size is also associated with some inferior rejection frequencies compared with Table 1, being particularly worrisome the low value obtained with $\theta = -0.25$ (0.077). Imposing $\rho(L) = (1-L^2)^{d_1}(1+L^2)^{d_2}$ and $(1-L)^{d_1}(1+L)^{d_2}(1+L^2)^{d_3}$, the sizes also reduce, getting closer to the nominal value of 5%, (4.8% and 4.9% respectively), but the rejection frequencies keep relatively high in practically all cases. (A detailed exposition of these results can be obtained in GIL-ALANA, [2003]).

Looking again at the results in Table 5 when using the finite sample critical values, the proportion of non-rejection values was higher. Thus, if $\rho(L) = (1 - L^4)^d$, H_o (10) is not rejected if $d = 1$, 1.25, 1.50 and 1.75 (and if $d = 2$ for $M1$). This is something to be expected in view of the lower rejection frequencies obtained with the finite-sample values. If $\rho(L) = (1-L^2)^{d_1}(1+L^2)^{d_2}$, we observed just one extra non-rejection value corresponding to $d_1 = 1.75$ and $d_2 = 0.75$. Finally, if $\rho(L) = (1-L)^{d_1}(1+L)^{d_2}$ $(1+L^2)^{d_3}$, the non-rejection values were exactly the same as when using the asymptotic critical values, i.e. $d_1 = 1$; $d_2 = d_3 = 0$, and $d_1 = 1.50$; $d_2 = 0.50$; $d_3 = 0.25$. Thus, we may conclude by saying that there are not large differences in the results whether we use the asymptotic or the finite sample critical values.

The results reported in this article indicate that the orders of integration may differ across the frequencies in spite of the non-rejection values obtained when testing when the same integration order at each frequency is imposed. As a final remark, we are concerned with the possibility that the true model has a different integration order at each frequency, (as it may be the case in this empirical application), but we test imposing the same order of integration at all frequencies. We see, in the lower part of Table 5, that there are two non-rejected models for $M2$, one, which is a random walk, i.e.,

(15) $$(1-L)x_t = u_t, \qquad t = 1,2,\ldots$$

and the other,

(16) $$(1-L)^{1.50}(1+L)^{0.50}(1+L^2)^{0.25}x_t = u_t, \qquad t = 1,2,\ldots,$$

both with white noise u_t. It was shown in Gil-Alana [1999] that if the true model is given by (15) and we apply Robinson's [1994] tests with $\rho(L) = (1 - L^4)$, the rejection frequency with $T = 120$ was 0.074. Similarly, we have performed the same experiment with the true model given by (16), obtaining a rejection frequency of 0.099. These extremely low values may be the reason why H_o (10) is not rejected in some cases when testing with $\rho(L) = (1 - L^4)^d$ in the upper part in Table 5. Extending the analysis, and testing (2) when the true model has different orders of integration at each frequency, the rejection frequencies were very low in all cases, suggesting the low power of the tests when we impose the same order of integration at all frequencies.

We can therefore conclude the analysis of the seasonal structure of the US monetary aggregate by saying that the root at the long run or zero frequency seems to play a much more important role than the seasonal roots and, the seasonal real one also seems to be more important than the pair of complex ones. However, we should also mention that these results have to be taken with cautions. First, when fractionally integrated hypotheses are entertained, there are an infinite number of possibilities to be examined because of the continuity over the real line. In that respect, the use of estimation rather than testing procedures might be more appropriate. Moreover, the results reported here are all based on the assumption of white noise for the underlying disturbances, not taking into account any potential weak dependence structure. However, we have to say here that we also employed some other forms for u_t. Using pure AR processes, the results did not substantially differ from those reported in the paper, suggesting that there is not weak autocorrelation at least at the zero frequency; using seasonal AR specifications for u_t, the main differences occurred in the case of $\rho(L) = (1 - L^4)^d$, obtaining smaller orders of integration in practically all cases. This may perhaps be explained because of the competition between the AR parameters and the orders of integration in describing the nonstationary seasonal component of the series.

5 Concluding Comments

We have presented in this article different versions of the tests of Robinson [1994] for testing seasonally fractionally integrated processes. The tests have several distinguishing features which make them particularly useful in the applied work: they have standard null and local limit distributions, and this limit behaviour holds across the different hypothesised values of d; Also, they allow us to test different orders of integration at different frequencies and, unlike other procedures, do not require estimation of the fractional differencing parameters.

A Monte Carlo experiment was conducted to check the power of the tests against different fractional alternatives. The results suggest that the tests of Robinson [1994] perform quite well for testing seasonal unit or fractional roots when the same order of integration is imposed at all frequencies. However, if the true model contains different integration orders for the different frequencies, the tests may have very low power, especially if the sample size is not very large.

The tests were also applied to the total expenditure of several monetary aggregates of the U.S. with quarterly data, obtaining results, which emphasize the importance of the root at the zero frequency over the others. Thus, even if the tests cannot reject a null of four unit roots, the results may be hiding the importance of some of the roots over the others, in particular, the one corresponding to the zero frequency. This is important since most of empirical works based on seasonal unadjusted data assume that seasonal unit roots are present in the data and thus, impose the same degree of integration (i.e., $d = 1$) at zero and all the seasonal components. In this article, however, we have shown that at least for the US monetary aggregate, the long run frequency plays a much more important role than the others and thus, a more careful study of the seasonal frequencies should be taken into account when seasonal unadjusted data are employed.

The article can be extended in several directions. First, we could study the consequences of imposing deterministic seasonality when the true process is stochastic (either integrated or fractionally integrated). Similarly, it may be of interest to consider the effect of misspecification in the opposite direction, i.e., if the true model is purely deterministic and we consider stochastic alternatives. Also, the Monte Carlo simulations carried out across the paper can be extended to study the power of the tests when the disturbances are weakly parametrically autocorrelated, especially in those cases where the roots are close to unit circle. Note that this is a well known problem in non-seasonal contexts (e.g. DIEBOLD and RUDEBUSCH, [1994]; HASSLER and WOLTERS, [1994]) and we should expect the same type of behaviour in seasonal models. In that respect, semiparametric methods (like those proposed by ARTECHE and ROBINSON, [2000], and ARTECHE, [2002]) can be also employed. Work in all these directions is now under progress. ■

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Appendix

The data for the monetary aggregates have been obtained from the US Federal Reserve Bank of St. Louis database. They are seasonally unadjusted, and cover the period 1960q1-1998q4. We use the following definitions of the monetary aggregates:

$M1$ = currency + travellers checks + demand deposits + checkable deposits.

$M2 = M1$ + small time deposits + money market demand deposits accounts + retail money market accounts + overnight repurchase agreements + overnight Eurodollars.

$M3 = M2$ + large time deposits + term-repurchase agreements + term Eurodollars + institutional money market funds.

$L = M3$ + other liquid assets + Treasury Bills + commercial paper + saving bounds.