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Do the Hodrick-Prescott and Baxter-King Filters Provide a Good Approximation of Business Cycles?

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ABSTRACT. – The authors assess the ability of the Hodrick-Prescott filter (HP) and the band-pass filter proposed by BAXTER and KING (BK) to extract the business-cycle component of macroeconomic time series by using two different definitions of the business-cycle component. First, they define that component to be fluctuations lasting no fewer than 6 and no more than 32 quarters; this is the definition of business-cycle frequencies used by BAXTER and KING. Second, they define the business-cycle component on the basis of a decomposition of the series into permanent and transitory components. The conclusions are the same in both cases. The filters perform adequately when the spectrum of the original series has a peak at business-cycle frequencies. When the spectrum is dominated by low frequencies, the filters provide a distorted business cycle. These findings suggest that the use of the HP and BK filters with series having the typical Granger shape is highly problematic.

Le filtre Hodrick-Prescott et le filtre Baxter-King permettent-ils d'obtenir une bonne approximation des cycles économiques?

RÉSUMÉ. – Les auteurs évaluent l'efficacité avec laquelle le filtre de Hodrick-Prescott et le filtre passe-bande proposé par BAXTER et KING permettent d'isoler la composante cyclique des séries macroéconomiques. Ils trouvent que les filtres donnent une image faussée de la réalité lorsque le spectre de la série filtrée est dominé par les basses fréquences. Comme la forme spectrale de la plupart des séries macroéconomiques a cette caractéristique, les filtres réussissent mal à isoler la composante cyclique de ces séries.

The views expressed in this study are those of the authors and do not necessarily represent those of the Bank of Canada.

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1 Introduction

Applied researchers who study business cycles often need to identify the business-cycle component of macroeconomic time series. Methods for stochastic detrending have been developed since an influential paper by NELSON and PLOSSER (1982) suggested that macroeconomic time series could be better characterized by stochastic trends than by linear trends. This has led to the increasing use of mechanical filters to identify permanent and cyclical components of time series. The most popular filter-based method is probably that proposed by HODRICK and PRESCOTT (1997). Another popular filter, the purpose of which is to isolate certain frequencies in the data, is proposed by BAXTER and KING (1995).[1]

In this paper, we assess the ability of the Hodrick-Prescott (HP) and Baxter-King (BK) filters to extract the business-cycle component of macroeconomic time series. To evaluate the performance of the HP filter, previous papers have focused on specific processes and used unclear definitions of the business-cycle component. For instance, the argument by HARVEY and JAEGER (1993) and COGLEY and NASON (1995a) that the HP filter induces spurious cyclicality is based on a comparison of the cyclical component obtained by applying the filter to the level of a random walk with the business-cycle frequencies of the same series in first differences. Our objective is to obtain more general results that could be applied to a large class of time-series processes and to provide clear indications of the appropriateness of using the HP and BK filters in applied macroeconomic work.

To achieve this objective, we need to define the business-cycle component of macroeconomic time series. In the first part of this paper, we retain the definition proposed by BURNS and MITCHELL (1946) and adopted by BAXTER and KING. These authors define the business-cycle component as fluctuations lasting no fewer than 6 quarters and no more than 32 quarters. An ideal filter would extract this specific range of periodicities without altering its properties. To assess the performance of the HP and BK filters in this context, we compare the spectra of unfiltered series at business-cycle frequencies with those of their filtered counterparts for several processes. We also compare the dynamics of the cyclical component corresponding to business-cycle frequencies with the dynamics extracted with the HP and BK filters. We find that the HP and BK filters perform well in extracting the business-cycle frequencies of time series whose spectra have a peak at business-cycle frequencies. The HP and BK filters perform poorly, however, in extracting the business-cycle frequencies of time series that have most of their power at low frequencies and that have a spectrum that decreases sharply and monotonically at higher frequencies. This means that the HP and BK filters perform poorly with series that have the typical spectral shape identified by Granger (1966).[2] We also find that these filters alter the dynamics of the cyclical component that corresponds to business-cycle frequencies.

The intuition behind our results is simple. Unlike series whose spectra have a peak at business-cycle frequencies, much of the power at business-cycle frequen-

1. See BAXTER (1994), KING, STOCK and WATSON (1995), CECCHETTI and KASHYAP (1995), and STOCK and WATSON (1998) for applications of this approach.

2. In a recent paper, PEDERSON (2001) responds to this criticism, as expressed in an earlier version of our paper. In section 3 of the current paper, we examine Pederson's arguments.

cies of series that have the typical shape is concentrated in the band where the squared gain of the HP and BK filters differs from that of an ideal filter. When applied to such series, the HP and BK filters induce spurious dynamic properties and extract a cyclical component that fails to capture a significant fraction of the variance contained in business-cycle frequencies.

Macroeconomic time series are often represented as an unobserved permanent component containing a unit root and an unobserved cyclical component. It is of interest to assess the ability of the HP and BK filters to provide a good approximation of the unobserved cyclical component in this context. We use a simulation study for this purpose. The data-generating process (DGP) that we use is a structural time-series model composed of a permanent and a cyclical component. We find that the HP and BK filters perform adequately when the spectrum of the original series (including the permanent and cyclical components) has a peak at business-cycle frequencies. When the series are dominated by low frequencies, however, the HP and BK filters provide a distorted cyclical component. Series are dominated by low frequencies when the permanent component is important relative to the cyclical component, and/or when the latter has its peak at zero frequencies. This result holds for any decomposition of a macroeconomic series as an unobserved permanent component and an unobserved cyclical component. As long as the series are dominated by low frequencies, these filters provide a distorted cyclical component.

Our findings provide a specific approach that applied researchers can use in deciding whether to use the HP and BK filters. This approach consists in estimating the spectral (or pseudo-spectral) density of the series of interest and comparing it with the processes we consider in our paper. In cases where the series have the typical Granger shape, using the HP and BK filters implies that the cyclical component is distorted. Unfortunately, most macroeconomic time series in levels have the typical Granger shape.

This paper is organized as follows. In section 2, we describe the HP and BK filters and briefly review the existing literature. In section 3, we examine how well the HP and BK filters extract frequencies corresponding to fluctuations of between 6 and 32 quarters. In section 4, we present a simulation study to assess the ability of these filters to retrieve the cyclical component of a time series composed of a random walk and a transitory component. In section 5 we offer some conclusions.

2 The HP and BK Filters

2.1 The HP Filter

The HP filter decomposes a time series, y_t, into an additive cyclical component, y_t^c, and a growth component, y_t^g,

$$y_t = y_t^g + y_t^c .$$

Applying the HP filter involves minimizing the variance of the cyclical component, y_t^c, subject to a penalty for the variation in the second difference of the growth component, y_t^g,

$$\{y_t^g\}_{t=0}^{T+1} = \arg\min \sum_{t=1}^{T} [(y - y_t^g)^2 + \lambda[(y_{t+1}^g - y_t^g) - (y_t^g - y_{t-1}^g)]^2],$$

where λ, the smoothness parameter, penalizes the variability in the growth component. The larger the value of λ, the smoother the growth component. As λ approaches infinity, the growth component corresponds to a linear time trend. For quarterly data, HODRICK and PRESCOTT propose to set $\lambda = 1600$. KING and REBELO (1993) show that the HP filter can render stationary any integrated process of up to the fourth order.

A number of authors have studied the HP filter's basic properties. As HARVEY and JAEGER (1993) and KING and REBELO (1993) show, the infinite sample version of the HP filter can be rationalized as the optimal linear filter of the trend component for the following process[3]:

$$y_t = \mu_t + \varepsilon_t,$$

where ε_t is an $NID(0, \sigma_\varepsilon^2)$ irregular component and the trend component, μ_t, is defined by

$$\mu_t = \mu_{t-1} + \beta_{t-1},$$

$$\beta_t = \beta_{t-1} + \zeta_t,$$

with $\zeta_t \sim NID(0, \sigma^2)$. β_t is the slope of the process and ζ_t is independent of the irregular component. Note that this trend component is integrated of order two; i.e., it is stationary in second differences.

SINGLETON (1988) shows that the HP filter can be a good approximation of a high-pass filter when it is applied to stationary time series. Here, we need to introduce some notions of spectral analysis. A zero-mean stationary process has a Cramer representation such as:

$$y_t = \int_{-\pi}^{\pi} e^{i\omega t} dz(\omega),$$

where $dz(\omega)$ is a complex value of orthogonal increments, i is the imaginary number $\left(\sqrt{(-1)}\right)$ and ω is frequency measured in radians; i.e., $-\pi \le \omega \le \pi$ (PRIESTLEY, 1981, chapter 4). In turn, filtered time series can be expressed as

$$y_t^f = \int_{-\pi}^{\pi} \alpha(\omega) e^{i\omega t} dz(\omega),$$

with

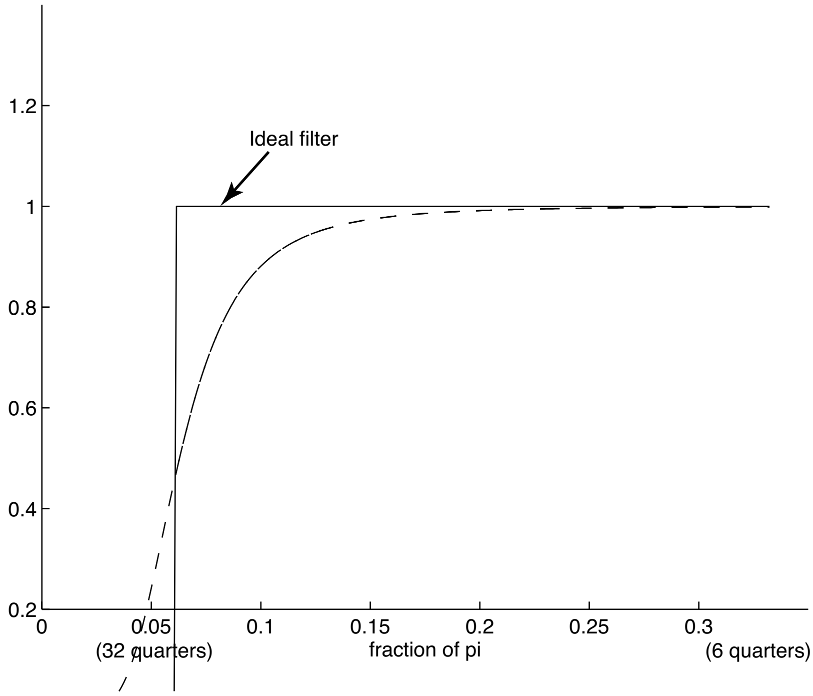
(1) $$\alpha(\omega) = \sum_{h=-k}^{k} a_h e^{-i\omega h}.$$

3. That is, the filter that minimizes the mean-square error, $MSE = (1/T) = \sum_{t=1}^{T} (\hat{y}_t^c - y_t^c)^2$, where y_t^c is the true cyclical component and $\hat{y}_t^c$ is its estimate.

Equation (1) is the frequency response (Fourier transform) of the filter. That is, $\alpha(\omega)$ indicates the extent to which y_t^f responds to y_t at frequency ω, and it can be considered as the weight attached to the periodic component $e^{i\omega t}dz(\omega)$. In the case of symmetric filters, the Fourier transform is also called the gain of the filter.

An ideal high-pass filter would remove low frequencies or long-cycle components, and allow high frequencies or short-cycle components to pass through, so that $\alpha(\omega) = 0$ for $|\omega| \leq \omega^p$, where ω^p has some predetermined value and $\alpha(\omega) = 1$ for $|\omega| > \omega^p$. Chart 1 shows the squared gain of the HP filter. We see that the squared gain is 0 at zero frequency and close to 1 from around frequency $\pi/10$ and up. Thus, the HP filter appears to be a good approximation of a high-pass filter, in that it removes low frequencies and passes through higher frequencies.

Chart 1
Squared gain of the HP filter



An important problem is that most macroeconomic time series are either integrated or highly persistent processes, so that they are better characterized in small samples as non-stationary rather than stationary processes. In their study of the implications of applying the HP filter to integrated or highly persistent time series, Cogley and Nason (1995a) argue that the HP filter is equivalent to a two-step linear filter that would initially first-difference the data to make them stationary and then smooth the differenced data with the resulting asymmetric filter. The filter tends to amplify cycles at business-cycle frequencies in the detrended data and to dampen long-run and short-run fluctuations. Cogley and Nason conclude that the filter can generate business-cycle periodicity even if none is present in the data. Harvey and

JAEGER (1993) make the same point. To better understand this result, consider the following I(1) process:

(2) $$(1-L)y_t = \varepsilon_t ,$$

where ε_t is zero-mean and stationary. KING and REBELO (1993) show that the HP cyclical filter can be rewritten as $(1-L)^4 H(L)$. We define $|HP(\omega)|^2$ as the squared gain corresponding to the HP cyclical filter, where $HP(\omega)$ is the Fourier transform of $(1-L)^4 H(L)$ at frequency ω. When the HP filter is applied to the level of the series y_t, the spectrum of the cyclical component is defined as

$$f_{y^c}(\omega) = |HP(\omega)|^2 |1-\exp(-i\omega)|^{-2} f_\varepsilon(\omega),$$

where $(1-\exp(-i\omega))$ is the Fourier transform of $(1-L)$ and $f_\varepsilon(\omega)$ is the spectrum of ε_t. The latter is well defined, since ε_t is a stationary process. Obviously, $|1-\exp(-i\omega)|^{-2}$ is not defined for $\omega = 0$. The expression $|1-\exp(-i\omega)|^{-2} f_\varepsilon(\omega)$ is often called the pseudo-spectrum of y_t (GOURIÉROUX and MONFORT, 1995).

COGLEY and NASON (1995a) and HARVEY and JAEGER (1993) calculate the squared gain of the HP cyclical component for $(1-L)y_t$. In that case, the squared gain is equal to $(1-L)^3 H(L)$, since $(1-L)^4 H(L)y_t = (1-L)^3 H(L)(1-L)y_t$. By the Fourier transform, the squared gain corresponding to the filter applied to $(1-L)y_t$ is $|HP(\omega)|^2 |1-\exp(-i\omega)|^{-2}$. The dashed line in Chart 2 represents this squared gain. These authors conclude that applying the HP filter to the level of a random walk produces detrended series that have the characteristics of a business cycle. Indeed, when this squared gain is compared with the ideal squared gain for the series in difference, we can see that the filter amplifies business-cycle frequencies and produces spurious dynamics.

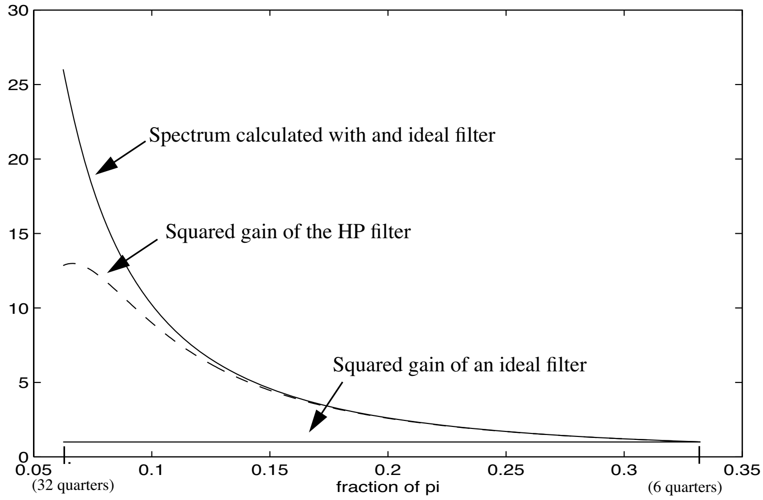
Now, suppose that ε_t in equation (2) is a white-noise process with variance equal to 2π, so that the spectrum of ε_t is equal to 1 at each frequency. We choose this example because the squared gain calculated by COGLEY and NASON corresponds to the cyclical component extracted by the HP filter in this specific case. Chart 2 shows the spectrum of the cyclical component identified with an ideal filter and the spectrum of the cyclical component identified with the HP filter at business-cycle frequencies. We can see that the effect of the HP filter is quite different depending on whether we are interested in retrieving the component corresponding to business-cycle frequencies for the level of the series y_t or for the series in difference $(1-L)y_t$.[4] Indeed, if one is to judge the performance of the HP filter by how well it extracts a specified range of periodicities, which is the first of the six objectives that BAXTER and KING (1995) try to meet in constructing their band-pass filter, the spectrum of the extracted component should be compared with the one obtained with an ideal filter. The conclusion then differs from that of COGLEY and NASON (1995a) and HARVEY and JAEGER (1993). Like them, we find that the spectrum of the cyclical component identified by the HP filter has a peak at business-cycle frequencies (30 quarters) which is absent from the spectrum of the original series. However, while they made the point that the filter generates spurious cycles when compared with the spectrum of the series in first-difference, we find that the filter

4. The fact that we are interested in extracting business-cycle frequencies from the level of integrated series may appear problematic. Note that we could also consider an AR(1) process with a coefficient of 0.95 and obtain the same result.

in fact dampens business-cycle fluctuations when compared with the spectrum of the level of the original series. Thus, the conclusion is sensitive to the definition of the business-cycle component. Moreover, the conclusion of Cogley and Nason (1995a) and Harvey and Jaeger (1993) may not hold if one is interested in the cyclical component of other processes than random walks. We come back to these points in sections 3 and 4.

Chart 2

Spectrum of y_t and of that series HP filtered (at frequencies between 6 and 32 quarters)



2.2 The BK Filter

Whereas an ideal high-pass filter removes low frequencies from the data, an ideal band-pass filter removes both low and high frequencies. Baxter and King (1995) propose a finite moving-average approximation of an ideal band-pass filter based on Burns and Mitchell's (1946) definition of a business cycle: the BK filter is designed to pass through components of time series with fluctuations between 6 and 32 quarters while removing higher and lower frequencies.

When applied to quarterly data, the band-pass filter proposed by Baxter and King takes the form of a 24-quarter moving average:

$$y_t^f = \sum_{h=-12}^{12} a_h y_{t-h} = a(L) y_t$$

where L is the lag operator. The weights a_h can be derived from the inverse Fourier transform of the frequency response function (Priestley, 1981, p. 274). Baxter and King adjust the band-pass filter with a constraint that the gain is zero at zero frequency. This constraint implies that the sum of the moving-average coefficients must be zero.

To study some time and frequency domain properties of the BK filter, assume the following DGP for y_t:

(3) $$y_t = (1-L)^{-r}\varepsilon_t ,$$

where r determines the order of integration of y_t and ε_t is a zero-mean stationary process. BAXTER and KING show that their filter can be factorized as

$$a(L) = (1-L)^2 a^*(L) ,$$

so that it is able to render stationary those time series that contain up to two unit roots.

The spectrum of the cyclical component obtained by applying the BK filter is

$$f_{y^c}(\omega) = |BK(\omega)|^2 f_y(\omega) ,$$

where $|BK(\omega)|^2$ is the squared gain of the BK filter and $f_y(\omega)$ is the spectrum of y_t. The squared gain $|BK(\omega)|^2$ is equal to $|a(\omega)|^2$, where $a(\omega)$ denotes the Fourier transform of $a(L)$ at frequency ω. The pseudo-spectrum of y_t is equal to

$$f_y(\omega) = |1-\exp(-i\omega)|^{-2r} f_\varepsilon(\omega) = 2^{-2r}(\sin^2(\omega/2))^{-r} f_\varepsilon(\omega)$$

for $\omega \neq 0$ (see PRIESTLEY,1981, p. 597), where $f_\varepsilon(\omega)$ is the spectrum of the process ε_t, which is well-defined since ε_t is stationary.

Chart 3a shows the squared gain of the BK filter and compares it with the squared gain of the ideal filter. The BK filter is designed to remove low and high frequencies from the data, and that goal is achieved. The filter passes through most components with fluctuations of between 6 and 32 quarters (respectively, $\pi/3$ and $\pi/16$), while removing components at higher and lower frequencies. However, the BK filter does not exactly correspond to the ideal band-pass filter (also shown on the graph), because it is a finite approximation of an infinite moving-average filter. In particular, at lower and higher frequencies we observe a compression effect, so that the squared gain is less than one.

As in section 2.1, we now assume that $r=1$ and that ε_t is white noise with variance equal to 2π in equation (3). The spectrum of ε_t is then equal to 1 at all frequencies and the cyclical component obtained with the BK filter corresponds exactly to the squared gain of the BK filter (the same calculation as made by COGLEY and NASON (1995a) and HARVEY and JAEGER (1993) for the HP filter):

$$|BK(\omega)|^2 |1-\exp(-i\omega)|^{-2} = |BK(\omega)|^2\, 2^{-2}(\sin^2(\omega/2))^{-1} .$$

Chart 3b shows the spectrum of the cyclical component identified with an ideal filter and that identified with the BK filter at business-cycle frequencies. The conclusion again depends on whether we are interested in retrieving the component corresponding to business-cycle frequencies for the level of the series y_t or for the series in difference $(1-L)y_t$. In the latter case, as noted by COGLEY and NASON and by HARVEY and JAEGER in the case of the HP filter, the BK filter greatly amplifies business-cycle frequencies and creates spurious cycles when compared with the ideal squared gain of the series in difference. For example, it amplifies by a factor of 10 the variance of cycles with a periodicity of around 20 quarters $(\pi/10)$. Also, as in the case of the HP filter, the business-cycle frequencies of the BK-filtered series are less important than those identified with an ideal filter, and the cyclical component identified by the BK filter has a peak corresponding to a period of 20 quarters (compared with 30 quarters in the case of the HP filter), which is absent from the spectrum obtained with an ideal filter.

CHART 3A

Squared gain of the BK filter

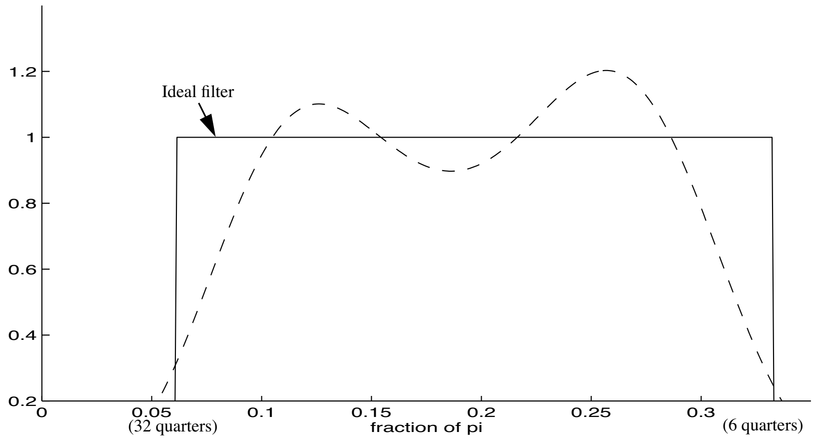
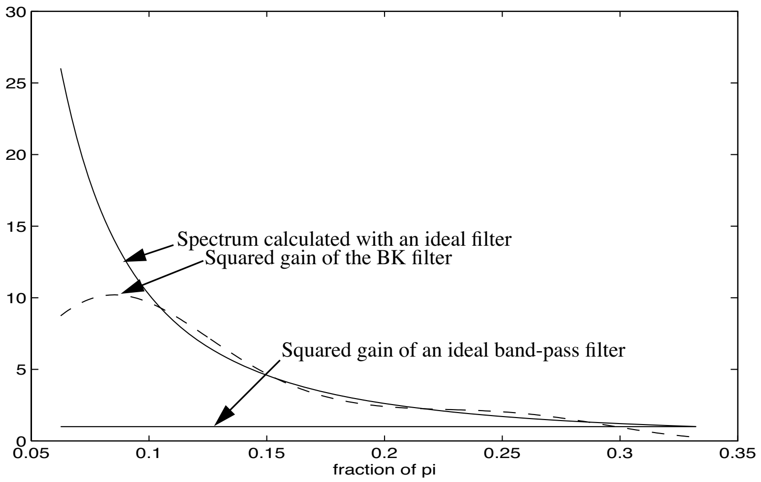


CHART 3B

Squared gain of the BK filter
(Case of a random-walk process)



3 Ability of the Filters to Extract Cyclical Periodicities

In this section, we examine how well the BK and HP filters capture the cyclical component of macroeconomic time series. BAXTER and KING's (1995) first objective is to adequately extract a specified range of periodicities without altering the properties of this extracted component. We use the same criteria to assess the performance of the HP and BK filters. We show that, when the peak of the spectral-density function of these series lies within business-cycle frequencies, these filters give a good approximation of the corresponding cyclical component. If the peak is located at zero frequency, so that the bulk of the variance is located in low frequencies, these filters cannot identify the cyclical component adequately.

To show this, we consider the following DGP,

$$y_t = \phi_1 y_{t-1} + \phi_2 y_{t-2} + \varepsilon_t \,, \tag{4}$$

where $\phi_1 + \phi_2 < 1$. A second-order autoregressive process is useful for our purposes because its spectrum may have a peak at business-cycle frequencies or at zero frequency. Although this process is stationary, by a continuity argument it provides information on the case of a non-stationary process. Indeed, in a finite sample, a non-stationary process can be approximated by a stationary process and vice versa (CAMBELL and PERRON 1991).

The spectrum of a second-order autoregressive process is equal to

$$f_y(\omega) = \frac{\sigma_\varepsilon^2}{1 + \phi_1^2 + \phi_2^2 - 2\phi_1(1-\phi_2)\cos\omega - 2\phi_2 \cos 2\omega}$$

and the location of its peak is given by

$$-\sigma_\varepsilon^{-2} f_y(\omega)^2 ((2\sin\omega)[\phi_1(1-\phi_2) + 4\phi_2 \cos\omega] \,.$$

Thus, $f_y(\omega)$ has a peak at frequencies other than zero for

$$\phi_2 < 0 \ \text{ and } \left| \frac{-\phi_1(1-\phi_2)}{4\phi_2} \right| < 1 \,. \tag{5}$$

Then $f_y(\omega)$ has its peak at $\omega = \cos^{-1}(-\phi_1(1-\phi_2)/4\phi_2)$ (PRIESTLEY, 1981). For other parameter values, the spectrum has a trough at non-zero frequencies if $\phi_2 > 0$ and $|\,\phi_1(1-\phi_2)/4\phi_2\,| < 1$.

Charts 4 and 5 show the spectrum of autoregressive processes and the spectrum of the cyclical component identified with the HP and BK filters. When the peak is located at zero frequency (i.e., most of the power of the series is located at low frequencies) the spectrum of the cyclical component resulting from the application of both filters is very different from that of the original series, especially at lower frequencies (Chart 4). In particular, the HP and BK filters induce a peak at business-cycle frequencies, even though it is absent from the original series, and they fail to capture a significant fraction of the variance contained in the business-cycle frequencies. On the other hand, when the peak is located at business-cycle frequencies, the spectrum of the cyclical component identified by HP and BK filtering matches fairly well the true spectrum at these frequencies (Chart 5). This result is robust for different sets of parameters ϕ_1 and ϕ_2 . It is interesting that the BK filter does not perform as well as the HP filter at frequencies corresponding to around 6 to 8 quarters. Indeed, the BK filter amplifies cycles of around 8 quarters

CHART 4
Series having the typical Granger shape (AR(2) coefficients: 1.26 -0.31)

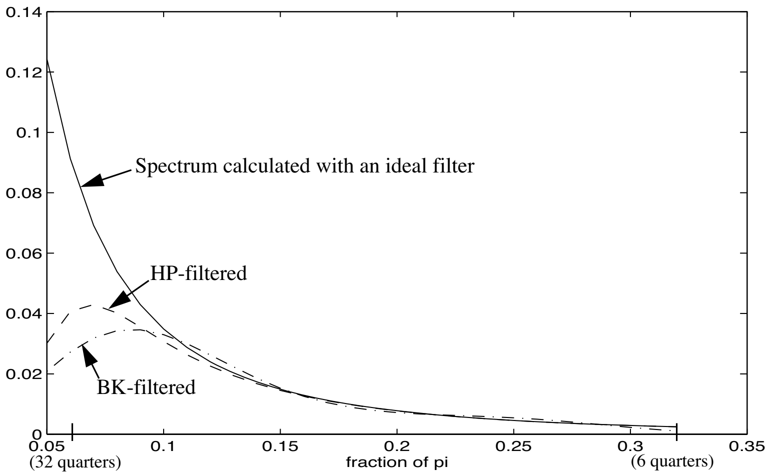
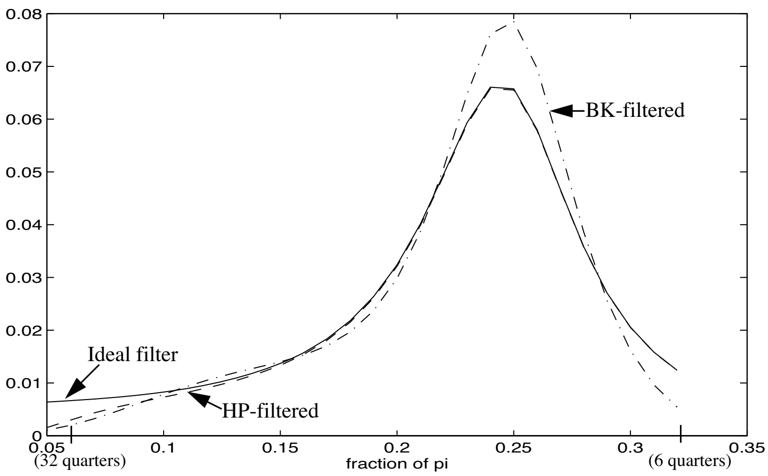


CHART 5
Series with a peak at business-cycle frequencies (AR(2) coefficients: 1.26 -0.78)



but compresses those of around 6 quarters. This results from the shape of the squared gain of the BK filter at those frequencies (Chart 3a). The absence of a peak at business-cycle frequencies does not imply that macroeconomic series do not feature business cycles; see SARGENT (1987) for a discussion. In fact, while most macroeconomic series feature the typical Granger shape, the growth rate of these series is often characterized by a peak at the business-cycle frequencies. KING and WATSON (1996) call this "the typical spectral shape of growth rates."

In response to an earlier version of this paper, PEDERSON (2001) shows that the HP filter with a smoothing parameter equal to 1600 creates fewer distortions than various others filters. Nevertheless, the impact of these distortions on the dynamics of the extracted process could be dramatic. To show this, we perform the following exercise. First, we set a DGP by a choice of $\theta = (\phi_1, \phi_2)$ for the second-order autoregressive process represented by equation (4). Second, we extract the cyclical component of the generated process with the HP or BK filters. Third, we search among second-order autoregressive processes for the parameters ϕ_1 and ϕ_2 that minimize the distance, at business-cycle frequencies, between the spectrum of the generated process at business-cycle frequencies and the spectrum of the HP- or BK-filtered processes. The problem is as follows:

$$\tilde{\theta} = \arg\min \int_{\omega 1}^{\omega 2} (S_{y^f}(\omega;\theta_0) - S_{\tilde{y}}(\omega;\theta))^2 \, d\omega ,$$

where $\omega 1 = \pi/16$, $\omega 2 = \pi/3$, $S_{y^f}(\omega;\theta_0)$ is the spectrum of the filtered DGP (where θ_0 is the vector of true values for the parameters ϕ_1 and ϕ_2), and $S_{\tilde{y}}(\omega;\theta)$ is the spectrum of the evaluated autoregressive process. Thus, if the HP and BK filters extract the range of periodicities corresponding to fluctuations of between 6 and 32 quarters perfectly, $\tilde{\theta}$ will be equal to the true vector, θ_0. Otherwise, the filter will extract a cyclical component corresponding to a second-order autoregressive process differing from the true one.

Table 1 lists our results for a DGP where the autoregressive parameter of order 1 is set at 1.20 while the parameter of order 2 is allowed to vary. Using the restrictions implied by (5), the peak of the spectrum lies within business-cycle frequencies when $\phi_2 < -0.43$. Although we report results only for the HP filter, these are almost identical to those obtained with the BK filter.[5] Results from this exercise corroborate those obtained from visual inspection. The second-order autoregressive process, which minimizes the distance between its spectrum at business-cycle frequencies and that of the business-cycle component identified by the HP and BK filters for the true process, is very different from the true second-order autoregressive process when the peak of the DGP is located at zero frequency. Even though the distance between the original and the filtered series at business-cycle frequencies does not seem to be large, the shape of their spectra differ sufficiently to induce very distinct dynamics. When the peak is located at business-cycle frequencies, the HP and BK second-order autoregressive processes are close to the true second-order autoregressive process at business-cycle frequencies.

This exercise shows that model builders need to be careful before drawing conclusions about the performance of their models based on filtered data. Using filtered data can affect these conclusions. For instance, a model-builder could be misled into concluding that his model adequately reproduces the business-cycle properties

5. These results are robust to the use of alternative values for θ_0, so that the restrictions are respected.

TABLE 1
Fitted values for the HP filter

DGP (θ_0)		HP ($\tilde{\theta}$)	
ϕ_1	ϕ_2	ϕ_1	ϕ_2
1.20	– 0.25	– 0.09	0.72
1.20	– 0.30	0.12	0.40
1.20	– 0.35	0.48	– 0.15
1.20	– 0.40	0.87	– 0.20
1.20	– 0.45	1.09	– 0.41
1.20	– 0.50	1.16	– 0.50
1.20	– 0.55	1.19	– 0.56
1.20	– 0.60	1.20	– 0.61
1.20	– 0.65	1.20	– 0.66
1.20	– 0.70	1.20	– 0.70
1.20	– 0.75	1.20	– 0.75
1.20	– 0.80	1.20	– 0.80

of the data when that conclusion depends on filter-induced distortions. Statistical tests based on dynamic properties of filtered series are thus likely to have poor properties.

The intuition behind our results is simple. Charts 1 and 3a (section 2) show that the gains of the HP and BK filters at low business-cycle frequencies are significantly smaller than that of the ideal filter. Indeed, the squared gain of the BK filter is 0.34 at frequencies corresponding to 32-quarter cycles, while that of the HP filter is 0.49. In the case of the HP filter, the squared gain does not reach 0.95 before frequency $\pi/8$ (cycles of 16 quarters). The problem is that a large fraction of the power of typical macroeconomic time series at business-cycle frequencies is concentrated in the band where the squared gains of HP and BK filters differ from that of an ideal filter. Also, the shape of the squared gain of these filters, when they are applied to typical macroeconomic time series, induces a peak in the spectrum of the cyclical component that is absent from the original series. In short, applying the HP and BK filters to series dominated by low frequencies results in the extraction of a cyclical component that does not capture an important fraction of the variance contained in business-cycle frequencies of the original series and induces spurious dynamic properties.

It is important to stress that the spectrum of the level of macroeconomic time series typically looks like that of the unfiltered series shown in chart 4. The spectrum's peak is located at zero frequency and the bulk of its variance is located in the low frequencies. This is called Granger's typical shape. Charts 6 and 7 show the estimated spectra of U.S. real GDP and consumer price inflation as well as the spectra of the filtered counterparts to those series.[6] Our results imply that the filters will not capture business-cycle frequencies well in those cases.

6. We use a parametric estimator of the spectrum. An autoregressive process was fitted and the order of that process was determined on the basis of the Akaike criteria. The same result would be obtained for a wide range of macroeconomic series in levels. Our sample is: 1963Q1-1995Q3. The data is from the U.S Bureau of Labor Statistics and the U.S. Bureau of Economic Analysis.

A macroeconomic series with the well-known Granger typical shape could be the sum of a permanent component that has a spectrum with a peak at zero frequency and a cyclical component that has a spectrum with a peak at business-cycle frequencies. The spectrum of that series could look like the one shown in Chart 6 and the spectrum of its cyclical component could look like the ones retrieved with the HP and BK filters shown in the same chart. For instance, the permanent component could be driven by a random-walk technological process with drift, while transitory monetary- or fiscal-policy shocks, among others, would generate the cyclical component with a peak in its spectrum at business-cycle frequencies. If this were true, then the HP and BK filters might be able to adequately capture the cyclical component. We examine this issue in section 4.

CHART 6
Spectrum of the logarithm of U.S. real GDP

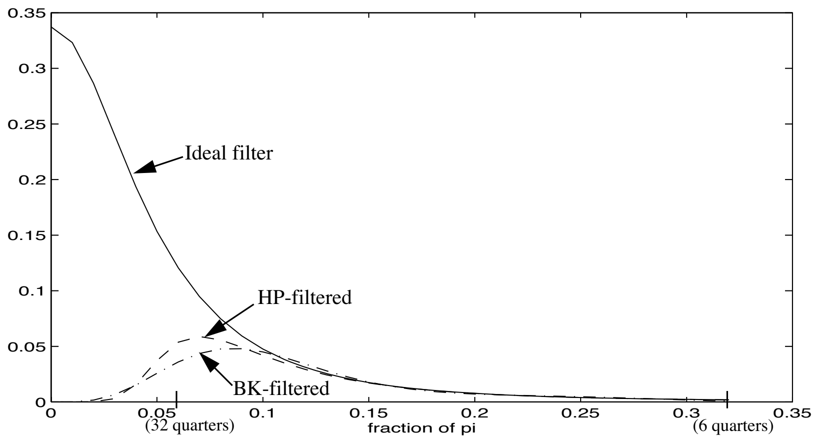
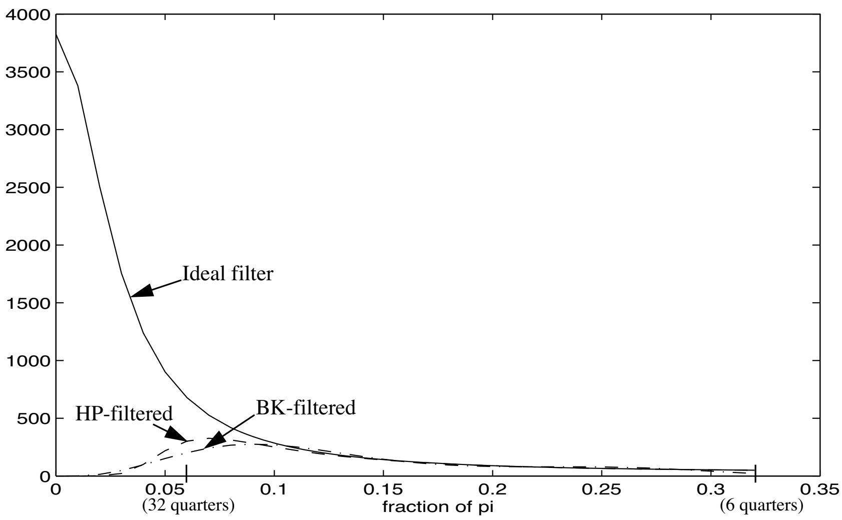


CHART 7
Spectrum of U.S. consumer price inflation



4 A Simulation Study

Consider the following DGP:

$$y_t = \mu_t + c_t \,, \tag{6}$$

where

$$\mu_t = \mu_{t-1} + \varepsilon_t$$

$$c_t = \phi_1 c_{t-1} + \phi_2 c_{t-2} + \eta_t$$

$$\varepsilon_t \sim NID(0,\sigma_\varepsilon^2)\,,\ \eta_t \sim NID(0,\sigma_\eta^2)\,.$$

Equation (6) defines y_t as the sum of a permanent component, μ_t, which in this case corresponds to a random walk, and a cyclical component, c_t.[7] The dynamics of the cyclical component is specified as a second-order autoregressive process, so that the peak of the spectrum could be at zero frequency or at business-cycle frequencies. We assume that ε_t and η_t are uncorrelated.

Data are generated from equation (6) with ϕ_1 set at 1.2 and different values for ϕ_2 to control the location of the peak in the spectrum of the cyclical component. We also vary the standard-error ratio for the disturbances $\sigma_\varepsilon / \sigma_\eta$ to change the relative importance of each component. With such DGP, we can create many types of time series. In particular, we can create series that have the following characteristics: a peak at zero frequency, most of their variance located a low frequencies, and composed of a cyclical component that has a spectrum peaking at business-cycle frequencies.[8] The aggregate spectrums of U.S. GDP and inflation and the spectrum of their cyclical component, as extracted with the HP or the BK filter shown in Charts 6 and 7, have these characteristics. Applications of the HP and BK filters to our simulated series shows whether they are able to identify cyclical components adequately.

We follow the standard practice of giving the value 1,600 to λ, the HP filter smoothness parameter. We also follow BAXTER and KING's suggestion of dropping 12 observations at the beginning and end of the sample. The resulting series contains 150 observations, a standard size for quarterly macroeconomic data. The number of replications is 500.

The performance of the HP and BK filters is assessed by comparing the autocorrelation function of the cyclical component of the true process with that obtained from the filtered data. We also calculate the correlation between the true cyclical component and the filtered cyclical component and report their relative standard deviations $(\hat{\sigma}_c / \sigma_c)$. Table 2 lists the results for the HP filter and Table 3 those for the BK filter.

7. This is WATSON's (1986) specification for U.S. real GNP. His estimates are: $\hat{\phi}_1 = 1.501$, $\hat{\phi}_2 = -0.577$, and $\hat{\sigma}_\varepsilon / \hat{\sigma}_\eta = 0.75$.
8. The model estimated by WATSON (1986) for U.S. real GNP has these characteristics.

Table 2 shows that the HP filter performs particularly poorly when there is an important permanent component. Indeed, for high $\sigma_\varepsilon / \sigma_\eta$ ratios, in most cases the correlation between the true and the filtered components is not significantly different from zero. The estimated autocorrelation function is invariant to the change in the cyclical component in these cases (the values of the true autocorrelation functions are given in brackets in the tables). When the ratio $\sigma_\varepsilon / \sigma_\eta$ is equal to 0.5 or 1 and the peak of the cyclical component is located at zero frequency $(\phi_2 > -0.43)$, the dynamic properties of the true and the filtered cyclical components are significantly different, as indicated by the estimated parameter values. In general, the HP filter adequately characterizes the series dynamics when the peak of the spectrum is at business-cycle frequencies and the ratio $\sigma_\varepsilon / \sigma_\eta$ is small. However, even when the ratio of standard deviations is equal to 0.01 (i.e., the permanent component is almost absent), the filter performs poorly when the peak of the spectrum of the cyclical component is at zero frequency. Indeed, for $\phi_2 = -0.25$, the dynamic properties of the filtered component differ significantly from those of the true cyclical component; the correlation is only equal to 0.66, and the standard deviation of the filtered cyclical component is half that of the true cyclical component.

Table 2
Simulation results for the HP filter

DGP			Estimated values				
			Autocorrelations			Corre-lation	$\hat{\sigma}_c / \sigma_c$
$\sigma_\varepsilon / \sigma_\eta$	ϕ_1	ϕ_2	1	2	3		
10	0	0	.71[0]	.46[0]	.26[0]	.08	12.96
			(.59,.80)	(.30,.60)	(.08,.43)	(– .07,.21)	(10.57,15.90)
10	1.2	– .25	.71[.96]	.47[.90]	.27[.84]	.08	4.19
			(.61,.80)	(.31,.61)	(.08,.44)	(– .11,.28)	(2.77,6.01)
10	1.2	– .40	.71[.86]	.46[.63]	.26[.41]	.13	6.34
			(.60,.80)	(.30,.60)	(.08,.44)	(– .12,.36)	(4.82,8.07)
10	1.2	– .55	.71[.77]	.46[.38]	.26[.03]	.14	6.93
			(.60,.80)	(.29,.60)	(.06,.43)	(– .08,.33)	(5.36,8.70)
10	1.2	– .75	.71[.69]	.46[.27]	.25[– .19]	.15	6.37
			(.60,.78)	(.30,.59)	(.07,.41)	(– .01,.31)	(4.79,7.95)
5	0	0	.69[0]	.45[0]	.26[0]	.15	6.50
			(.58,.78)	(.30,.58)	(.09,.41)	(.02,.27)	(5.28,7.85)
5	1.2	– .25	.71[.96]	.46[.90]	.26[.84]	.16	2.11
			(.61,.80)	(.32,.61)	(.08,.43)	(– .01,.36)	(1.43,3.04)
5	1.2	– .40	.72[.86]	.46[.63]	.25[.41]	.23	3.26
			(.61,.80)	(.31,.60)	(.08,.42)	(– .01,.45)	(2.47,4.15)
5	1.2	– .55	.71[.77]	.46[.38]	.24[.03]	.24	3.60
			(.61,.80)	(.30,.59)	(.06,.41)	(.01,.44)	(2.83,4.52)
5	1.2	– .75	.70[.69]	.43[.27]	.20[– .19]	.29	3.30
			(.61,.79)	(.26,.57)	(.00,.38)	(.11,.44)	(2.53,4.17)

TABLE 2 (concluded)

DGP			Estimated values				
			Autocorrelations				
$\sigma_\varepsilon / \sigma_\eta$	ϕ_1	ϕ_2	1	2	3	Correlation	$\hat{\sigma}_c / \sigma_c$
1	0	0	.43[0] (.27,.57)	.28[0] (.11,.42)	.20[0] (– .02,.31)	.59 (.49,.70)	1.61 (1.41,1.85)
1	1.2	– .25	.76[.96] (.67,.83)	.51[.90] (.37,.62)	.29[.84] (.11,.44)	.51 (.33,.68)	.66 (.44,.91)
1	1.2	– .40	.75[.86] (.67,.81)	.44[.63] (.28,.55)	.16[.41] (– .03,.33)	.71 (.56,.82)	1.02 (.83,1.22)
1	1.2	– .55	.72[.77] (.66,.78)	.34[.38] (.21,.47)	.01[.03] (– .17,.19)	.76 (.56,.82)	1.15 (.83,1.22)
1	1.2	– .75	.68[.69] (.63,.72)	.15[.27] (.04,.27)	– .27[– .19] (– .44,.10)	.83 (.75,.89)	1.16 (1.04,1.29)
.5	0	0	.16[0] (.01,.32)	.10[0]	.04[0] (– .10,.18)	.82 (.75,.88)	1.16 (1.07,1.27)
.5	1.2	– .25	.79[.96] (.71,.85)	.53[.90] (.38,.65)	.30[.84] (.11,.46)	.61 (.41,.79)	.55 (.37,.76)
.5	1.2	– .40	.77[.86] (.69,.81)	.43[.63] (.29,.54)	.13[.41] (– .05,.29)	.84 (.73,.92)	.87 (.74,.99)
.5	1.2	– .55	.72[.77] (.67,.78)	.28[.38] (.17,.39)	– .10[.03] (– .25,.06)	.89 (.83,.94)	.98 (.89,1.07)
.5	1.2	– .75	.67[.69] (.63,.71)	.07[.27] (– .03,.18)	– .42[– .19] (– .57,– .27)	.94 (.90,.96)	1.02 (.97,1.08)
.01	0	0	– .08[0] (– .21,.06)	– .06[0] (– .21,.06)	– .06[0] (– .19,.06)	.98 (.96,.99)	.97 (.94,.99)
.01	1.2	– .25	.80[.96] (.72,.86)	.54[.90] (.38,.67)	.30[.84] (.11,.48)	.66 (.45,.83)	.51 (.34,.69)
.01	1.2	– .40	.78[.86] (.72,.83)	.43[.63] (.30,.55)	12[.41] (– .05,.28)	.90 (.82,.96)	.81 (.71,.90)
.01	1.2	– .55	.73[.77] (.67,.77)	.26[.38] (.15,.37)	– .14[.03] (– .30,.01)	.96 (.91,.99)	.92 (.86,.96)
.01	1.2	– .75	.67[.69] (.62,.71)	.02[.27] (– .08,.13)	– .50[– .19] (– .61,– .35)	.99 (.97,1.0)	.97 (.95,.99)

The HP filter does relatively well when the ratio $\sigma_\varepsilon / \sigma_\eta$ is equal to 1, 0.5, or 0.01 and the spectrum of the original series has a peak at business-cycle frequencies (i.e., the latter frequencies contain a significant part of the variance of the series). Consequently, the conditions required to adequately identify the cyclical component with the HP filter can be expressed in the following way: the spectrum of the original series must have a peak located at business-cycle frequencies, which must account for an important part of the variance of the series. If the variance of the series is dominated by low frequencies, the HP filter does a poor job of extracting the cyclical component.

TABLE 3
Simulation results for the BK filter

DGP			Estimated values				
$\sigma_\varepsilon / \sigma_\eta$	ϕ_1	ϕ_2	Autocorrelations 1	2	3	Corre-lation	$\hat{\sigma}_c / \sigma_c$
10	0	0	.90[0] (.87,.93)	.65[0] (.52,.75)	.33[0] (.13,.51)	.03 (– .11,.16)	11.55 (9.05,14.38)
10	1.2	– .25	.90[.96] (.87,.93)	.65[.90] (.55,.74)	.34[.84] (.17,.49)	.08 (– .13,.32)	3.71 (2.34,5.45)
10	1.2	– .40	.90[.86] (.87,.93)	.64[.63] (.54,.73)	.33[.41] (.16,.48)	.11 (– .16,.36)	5.67 (4.19,7.18)
10	1.2	– .55	.90[.77] (.87,.93)	.64[.38] (.53,.73)	.33[.03] (.14,.48)	.12 (– .12,.33)	6.23 (4.71,7.93)
10	1.2	– .75	.90[.69] (.86,.92)	.63[.27] (.52,.73)	.31[– .19] (.13,.48)	.16 (– .04,.36)	5.69 (4,37,7.16)
5	0	0	.90[0] (.87,.90)	.64[0] (.53,.73)	.33[0] (.14,.49)	.05 (– .09,.20)	5.80 (4.54,7.16)
5	1.2	– .25	.90[.96] (.87,.93)	.65[.90] (.54,.73)	.34[.84] (.16,.49)	.17 (– .05,.38)	1.94 (1.25,2.74)
5	1.2	– .40	.90[.86] (.87,.93)	.64[.63] (.53,.74)	.32[.41] (.14,.49)	.23 (– .03,.47)	2.93 (2.15,3.76)
5	1.2	– .55	.89[.77] (.87,.92)	.62[.38] (.52,.72)	.30[.03] (.12,.46)	.26 (.03,.46)	3.19 (2.45,3.98)
5	1.2	– .75	.88[.69] (.85,.92)	.60[.27] (.47,.70)	.26[– .19] (.06,.44)	.28 (.09,.45)	2.97 (2.24,3.77)
1	0	0	.89[0] (.85,.92)	.61[0] (.48,.71)	.27[0] (.06,.45)	.19 (.05,.32)	1.21 (.96,1.43)
1	1.2	– .25	.90[.96] (.87,.93)	.65[.90] (.53,.74)	.34[.84] (.15,.50)	.53 (.36,.71)	.60 (.39,.84)
1	1.2	– .40	.88[.86] (.85,.91)	.58[.63] (.47,.68)	.22[.41] (.03,.39)	.70 (.55,.81)	.95 (.78,1.12)
1	1.2	– .55	.85[.77] (.81,.89)	.48[.38] (.36,.60)	.05[.03] (– .15,.24)	.73 (.61,.83)	1.06 (.89,1.23)
1	1.2	– .75	.79[.69] (.75,.83)	.27[.27] (.14,.40)	– .26[– .19] (– .45,– .06)	.79 (.69,.87)	1.08 (.96,1.20)
.5	0	0	.86[0] (.81,.89)	.50[0] (.35,.63)	.10[0] (– .12,.30)	.36 (.25,.47)	.77 (.63,.91)
.5	1.2	– .25	.90[.96] (.87,.93)	.65[.90] (.55,.74)	.34[.84] (.16,.50)	.63 (.45,.78)	.51 (.34,.71)
.5	1.2	– .40	.88[.86] (.84,.91)	.56[.63] (.43,.66)	.17[.41] (– .02,.34)	.81 (.71,.88)	.81 (.67,.93)
.5	1.2	– .55	.83[.77] (.80,.87)	.41[.38] (.30,.53)	– .06[.03] (– .24,.12)	.85 (.78,.91)	.91 (.81,1.01)
.5	1.2	– .75	.76[.69] (.72,.79)	.17[.27] (.06,.29)	– .42[– .19] (– .57,– .25)	.89 (83,.93)	.96 (.89,1.03)

TABLE 3: (concluded)

DGP			Estimated values				
			Autocorrelations				
$\sigma_\varepsilon / \sigma_\eta$	ϕ_1	ϕ_2	1	2	3	Corre-lation	$\hat{\sigma}_c / \sigma_c$
.01	0	0	.79[0] (.75,.83)	.29[0] (.16,.42)	– .22[0] (– .40,– .03)	.55 (.48,.63)	.51 (.43,.58)
.01	1.2	– .25	.91[.96] (.88,.93)	.66[.90] (.57,.74)	.35[.84] (.20,.50)	.68 (.52,.82)	.48 (.32,.64)
.01	1.2	– .40	.87[.86] (.84,.90)	.54[.63] (.44,.64)	15[.41] (– .03,.31)	.86 (.79,.92)	.76 (.65,.86)
.01	1.2	– .55	.83[.77] (.79,.86)	.39[.38] (.27,.49)	– .11[.03] (– .29,.06)	.90 (.85,.94)	.86 (.78,.92)
.01	1.2	– .75	.74[.69] (.71,.78)	.13[.27] (.03,.23)	– .48[– .19] (– .61,– .34)	.93 (.89,.96)	.92 (.86,.97)

The results for the BK filter are similar to those for the HP filter, although the dynamic properties of the BK-filtered cyclical component seem to be invariant (or almost invariant) to the true process. For example, when $\sigma_\varepsilon / \sigma_u = 0.01$, $\phi_1 = 0$, and $\phi_2 = 0$, which corresponds to the case where the cyclical component is white noise and dominates the permanent component, the BK-filtered cyclical component is a highly autocorrelated process. This difference in our results for the BK and the HP filters comes from the fact that the BK filter tries to retrieve the frequencies within 6 quarters and 32 quarters, while the HP filter tries to remove frequencies of less than 32 quarters. The autocorrelation of the cyclical component is linked to the area covered by the spectrum, which likely explains why the autocorrelations of the BK-filtered cyclical component are so high and almost invariant to the original process. Thus, the BK filter would appear to be of limited value as an instrument for identifying the cyclical dynamics of a macroeconomic time series with any confidence. As stated earlier, this result precludes the use of the BK filter to assess the internal dynamic properties of a business-cycle model, since this filter produces a series with dynamic properties that are almost invariant to the true process.

Additional simulations we performed give clear indications of the performance of the HP and BK filters when they are applied to more general decompositions between permanent and cyclical components than equation (6). For instance, the trend component can be an I(1) process with a transient dynamic (e.g., $\varepsilon_t = d(L)\zeta_t$).[9] Also, the cyclical component can be correlated with the permanent component. For example, the decomposition proposed by BEVERIDGE and NELSON (1981) implies permanent and transitory components that are perfectly correlated. To reproduce the Granger typical shape, however, any decomposition must have a permanent component that is important relative to the cyclical component, or a cyclical com-

9. LIPPI and REICHLIN (1994) argue that modeling the trend component in real GNP as a random walk is inconsistent with the standard view concerning the diffusion process of technological shocks. BLANCHARD and QUAH (1989) and KING *et al.* (1991) use a multivariate representation to obtain a trend component that has an impulse function with a short-run impact smaller than the long-run impact. Thus, the effect of the permanent shock gradually increases to its long-run impact.

ponent dominated by low frequencies. In both cases, the HP and BK filters provide a distorted cyclical component.[10]

These results should help applied researchers to decide about filtering. Although the cyclical component is not observed, the estimated spectra of an observed series can indicate whether the HP and BK filters can be used. In particular, the use of these filters is very problematic when the spectrum of an observed series has the typical Granger shape.

Our results have important implications for the evaluation of Real Business Cycle (RBC) models. Advocates of RBC models claim that these models can account for much of the business-cycle properties of U.S. output. Their claim is based on comparisons of the cyclical properties of actual HP-filtered data with series generated on the basis of models using HP-filtered data. Let us examine this procedure in the case of the two benchmark RBC models described by KING, PLOSSER and REBELO (1988a, 1988b). The deterministic growth benchmark RBC model (KING, PLOSSER and REBELO, 1988a) is characterized by a technology process following a first-order autoregressive process with a coefficient equal to 0.9. The stochastic growth model (KING, PLOSSER and REBELO, 1988b) assumes that technology follows a random walk with drift. Such technology processes are thus dominated by low frequencies. KING, PLOSSER and REBELO show that the dynamic properties of the output series resulting from the model are very close to those of the exogenous technological processes.[11] For instance, in the stochastic growth model, the output series is well-approximated by a random walk with drift. Consequently, the spectrums of the output series generated with the two benchmark models are dominated by low frequencies and are very close to the spectrum of the exogenous technology processes.

The spectrum of U.S. output data is also dominated by low frequencies but, at odds with a persistent first-order autoregressive process or a random walk, the growth rate of this series is characterized by a peak at the business-cycle frequencies.[12] The presence of a peak at business-cycle frequencies of the output growth rates corresponds to the point made by COGLEY and NASON (1995b) and ROTEMBERG and WOODFORD (1996) in the time domain.

The results from sections 3 and 4 imply that the cyclical component extracted with the HP filter for the output series resulting from the benchmark RBC models and the data are importantly distorted. In light of our results, it is very risky to draw conclusions about the adequacy of a model by comparing distorted measures of the cyclical component from the model and data series. Unless the modeller is confident that the distortions caused by the filter are innocuous for the comparison of the series, conclusions based on such criteria should be suspect. In our view, this is the main reason why RBC's advocates and subsequent studies by WATSON (1993), COGLEY and NASON (1995b) and ROTEMBERG and WOODFORD (1996), among others, draw contradictory conclusions about the adequacy of RBC models to reproduce U.S. business- cycle properties. In particular, the three aforementioned studies show that benchmark RBC models have weak propagation mechanisms, an important failure hidden from RBC's advocates by the application of the HP filter on both simulated and actual data.

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10. The results of complementary simulations with different processes are available on request. For the sake of brevity, they are not shown here.
11. See also COGLEY and NASON (1995b), ROTEMBERG and WOODFORD (1996), and WATSON (1993) on this point.
12. This is what KING and WATSON (1996) call "the typical spectral shape of growth rates."

5 Conclusions

This paper has shown that the HP and BK filters do relatively well when applied to series that have a peak in their spectrum at business-cycle frequencies. They do poorly with series whose spectrum decreases sharply and monotonically at higher frequencies; i.e., series that have the typical spectral shape identified by GRANGER (1966). These results suggest a clear strategy for applied researchers: estimate the spectral (or pseudo-spectral) density of the series of interest, so that the appropriateness of using the HP and BK filters to identify the cyclical component can be evaluated. The use of the HP and BK filters is very problematic when a series has the typical Granger shape.

Our results also allow us to understand the findings of KING and REBELO (1993) for simulated series obtained with an RBC model. It is well-known that this model has few internal propagation mechanisms.[13] Indeed, the dynamics of real output for this model correspond almost exactly to the dynamics of the exogenous shocks that affect it. KING and REBELO report persistence, volatilities, and co-movement of simulated series for the cases where the exogenous process is a first-order autoregressive process with coefficient equal to 0.9 or 1. For such processes, the spectral densities of output, consumption, and investment in levels are dominated by low frequencies. Applying the HP filter to these simulated series provides distorted cyclical properties. The same argument explains the findings of HARVEY and JAEGER (1993) and COGLEY and NASON (1995a) for a random-walk process.

What are the alternatives for a business-cycle researcher interested in measuring the cyclical properties of a macroeconomic series? In the case of the evaluation of business-cycle models, researchers are often interested only in the second moments of the cyclical component. In that case, there is no need to extract a cyclical series. KING and WATSON (1996) show how we can obtain correlations and cross-autocorrelations without filtering the observed and simulated series. The strategy consists in calculating these moments from the estimated spectral density matrix at business-cycle frequencies. An estimation of the spectral density matrix can be obtained with a parametric estimator, such as the one used by KING and WATSON, or a non-parametric estimator. The cyclical component can also be obtained in a univariate or a multivariate representation with the BEVERIDGE-NELSON (1981) decomposition. Economic theory also provides alternative methods of detrending; for instance, COCHRANE's method (1994), based on the permanent-income theory, or the Blanchard and QUAH (1989) structural decomposition.[14]

Because this paper's findings result to a large extent from the fact that the BK and HP filters differ too much from ideal filters at lower business-cycle frequencies, filters that have better properties at those frequencies need to be developed. ■

13. See Cogley and Nason (1995b), and Rotemberg and Woodford (1996) for a discussion of this point.
14. COGLEY (1996) compares the HP and BK filters with the univariate Beveridge-Nelson decomposition and Cochrane's method using an RBC model with different exogenous processes. DUPASQUIER, GUAY, and ST-AMANT (1999) discuss various methodologies.

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