

# Product Choice, Taxation and Switching Costs

G. D. MYLES\*, A. UYDURANOGLU\*\*

**ABSTRACT.** – Switching costs can prevent products that would be socially beneficial from being produced. We construct a model in which the switching cost takes the form of an investment required to manufacture a new form of an existing product. Given the cost, it is privately rational for producers not to invest. We analyze whether tax policy can change the incentives sufficiently to ensure the product is introduced and determine the welfare implications of policy. The general message of the paper is that tax policy can be employed to prevent switching costs from trapping an economy in a socially-inefficient position.

---

## Choix de produit, taxation et coûts de changement technique

**RÉSUMÉ.** – Les coûts de changement de technique peuvent rendre impossible la fabrication de produits qui pourraient pourtant être socialement utiles. Nous construisons un modèle dans lequel ces coûts prennent la forme d'un investissement. Étant donné ces coûts, il est rationnel pour les producteurs de ne pas investir. Nous analysons l'impact que la politique fiscale peut avoir sur les incitations à investir dans le changement de technologie et nous déterminons les effets de bien-être. La conclusion générale que nous en tirons est qu'effectivement la politique fiscale peut permettre de sortir d'une situation productive socialement inefficace.

---

\* G. D. MYLES, University of Exeter and Institute for Fiscal Studies, Department of Economics, Exeter EX4 4RJ, UK, [gdmyles@ex.ac.uk](mailto:gdmyles@ex.ac.uk).

\*\* A. UYDURANOGLU, Istanbul Bilgi University, Department of Economics, Istanbul, Turkey, [ayseu@bilgi.edu.tr](mailto:ayseu@bilgi.edu.tr).

# 1 Introduction

---

History can often set the economy down paths from which it can prove difficult to deviate. The adoption of the QWERTY keyboard, VHS video and Windows-based personal computers are all examples of product selection where arguably better alternatives were available at the time of selection or later became available. Although it may be socially-rational to switch to the alternative product, switching costs can ensure that it is not a privately-rational choice.

Switching costs can take several forms. Consumers can face direct costs in switching as would occur, for example, in purchasing new software after moving from Windows to Linux. Indirect costs can also arise in having to undertake training to learn to use the new system. Network costs can also hinder switching; they have certainly slowed the adoption of the new Third Generation mobile phones which can send some transmissions only to other phones from the same provider. Network costs were also responsible for demise of Betamax video after the ascendancy of VHS.

The example that originally motivated this paper presents another form of switching cost. Consider the fact that almost all vehicles available run on internal combustion engines<sup>1</sup> (either petrol, diesel or, in a very small minority, liquid petroleum gas). This is despite the fact that these vehicles are a significant source of pollution. An alternative exists in the form of electric cars which have virtually no emissions when in use. Although current electric vehicles suffer from a limited range, they would still be acceptable to many commuters if made widely available. In this case the cost that holds back their introduction is the investment required of producers to begin large-scale production of electric cars.

Our analysis chooses to focus on a model of these switching costs in production. The perspective we take is that there is an established product for which any fixed costs of production have already been sunk. There is also an alternative form of the product for which an investment must be made before production can begin. There is a demand for the alternative, but this will only be met by the manufacturer if the additional profit it generates exceeds the investment cost. The assumptions we make are general enough to permit the alternative to be viewed as either a horizontally or vertically differentiated form of the product.

Since there exist switching costs, it is necessary for the introduction of the alternative that there is an interval of time during which revenue exceeds variable costs. This requirement places a focus upon situations of imperfect competition. Given this, we study the potential role of tax policy in offsetting the switching cost and guiding the economy to a socially-preferable outcome. The analysis begins by considering a monopoly producer with the *status quo* a situation in which the monopolist does not find it profitable to undertake the investment. We show that there exist balanced-budget tax policies that induce

---

1. Further discussion of these issues can be found in OECD [1993].

the monopolist to introduce the alternative product and then examine whether such policies are welfare improving. This is followed by an analysis of the same issues in a duopoly context. The analysis becomes considerably more complex in this instance since, for any pair of tax rates, there are several potential equilibrium regimes depending on the product ranges of the two firms. In addition, as tax rates change there are discrete switches from one regime to another and consequent discontinuous changes in revenue and welfare. Despite the discontinuities, we succeed in fully determining the equilibrium and in characterizing the optimal policy.

Section 2 introduces the issues we consider and analyses tax policy with monopoly. The model is extended to duopoly in Section 3. Section 4 presents the conclusions.

## 2 Monopoly

---

In this section we assume that there is a monopolist who can choose between producing just the existing form of a product (a position we view as the *status quo*) or introducing a new form to supply alongside the original. To produce the alternative form requires an investment in new technology – the switching cost – which we assume the manufacturer cannot justify from the additional profits that would flow from producing both forms. Given this position, we investigate whether fiscal intervention can make it a profitable strategy to undertake the investment and, if it can, the welfare consequences of so doing.

### 2.1 Demand and Profit

Demand functions are defined for when only the existing form is available and for when both are available. These demand functions cannot be entirely distinct constructs since the non-availability of one form is economically identical to it being available at a prohibitive price. Conditions are therefore imposed which link the two demand functions.

To ease the discussion, we choose to refer to the existing form of the product as good 1 and the alternative as good 2. This nomenclature is not meant to detract from the basic hypothesis that these are two forms of the same basic product. When only good 1 is produced, the demand function facing the firm is

$$(1) \quad X^1 = X^1(q_1),$$

where  $X^1$  denotes quantity and  $q_1$  price. This function satisfies  $X^1_1 \equiv \frac{\partial X^1}{\partial q_1} < 0$ . When good 2 is introduced onto the market, the demand function for good 1 becomes

$$(2) \quad \hat{X}^1 = \hat{X}^1(q_1, q_2),$$

and that for good 2 is

$$(3) \quad \hat{X}^2 = \hat{X}^2(q_1, q_2),$$

where  $q_2$  denotes the price of good 2. These functions satisfy  $\hat{X}_i^i < 0$ ,  $i = 1, 2$ .

The following assumption is placed on the demand functions.

ASSUMPTION 1:

$$(i) \quad \hat{X}_2^1 > 0 \text{ and } \hat{X}_1^2 > 0.$$

$$(ii) \quad \lim_{q_2 \rightarrow \infty} \hat{X}^1(q_1, q_2) = X^1(q_1).$$

$$(iii) \quad \text{If } q_1 = q_2 = q, \text{ then } \hat{X}^1(q, q) < X^1(q) < \hat{X}^1(q, q) + \hat{X}^2(q, q).$$

Assumption (1i) is that the two forms of the product are substitutes. (1ii) links the two demand functions: after the introduction of the new form onto the market, when its price approaches infinity the demand for the original form is equal to that when only it was available. Finally (1iii) ensures that when the second form is produced, total market size increases. As already noted, these assumptions encompass the interpretations of both horizontal product differentiation or vertical differentiation.

The profit function of the firm with only good 1 in production is

$$(4) \quad \Pi^1(q_1) \equiv X^1(q_1)[q_1 - c_1],$$

where  $c_1$  is the constant marginal cost. The profit-maximizing price,  $q_1^*$ , satisfies

$$(5) \quad X^1(q_1^*) + \frac{\partial X^1}{\partial q_1}[q_1^* - c_1] = 0.$$

If good 2 were introduced onto the market, the profit function of the firm, gross of the investment cost, would become

$$(6) \quad \Pi^{1,2}(q_1, q_2) \equiv \hat{X}^1(q_1, q_2)[q_1 - c_1] + \hat{X}^2(q_1, q_2)[q_2 - c_2],$$

where  $c_2$  is the marginal cost of production for good 2. Using (6), the optimal prices  $q_1^{**}$  and  $q_2^{**}$  satisfy

$$(7) \quad \frac{\partial \hat{X}^1}{\partial q_1}[q_1^{**} - c_1] + \hat{X}^1 + \frac{\partial \hat{X}^2}{\partial q_1}[q_2^{**} - c_2] = 0,$$

and

$$(8) \quad \frac{\partial \hat{X}^1}{\partial q_2} [q_1^{**} - c_1] + \frac{\partial \hat{X}^2}{\partial q_2} [q_2^{**} - c_2] + \hat{X}^2 = 0.$$

Restrictions on the profit functions are adopted in the next assumption.

ASSUMPTION 2:

- (i) *There exists  $\hat{q}_1 < \infty$  such that  $\Pi^1(\hat{q}_1) > 0$ .*
- (ii) *There exist  $\tilde{q}_1 < \infty, \tilde{q}_2 < \infty$  such that  $\Pi^{1,2}(\tilde{q}_1, \tilde{q}_2) > 0$ .*
- (iii)  *$q_1^*, q_1^{**}$  and  $q_2^{**}$  are finite.*

The first two parts of the assumption require that there are some prices at which the firm can be profitable. Part (2iii) requires that in both cases the firm never finds it optimal to charge an infinitely high price. These restrictions could be phrased in terms of more primitive assumptions on the demand functions, but it is easier to place them directly on the profit function.

We now show that under Assumptions 1 and 2 the (gross) profit obtained from selling both goods is higher than that obtained from selling only good 1.

LEMMA 1:  $\Pi^1(q_1^*) < \Pi^{1,2}(q_1^{**}, q_2^{**})$ .

PROOF. From Assumption 1(ii), it follows that

$$(9) \quad \begin{aligned} \Pi^1(q_1) &\equiv X^1(q_1)[q_1 - c_1] \\ &= \lim_{q_2 \rightarrow \infty} \left[ \hat{X}^1(q_1, q_2)[q_1 - c_1] + \hat{X}^2(q_1, q_2)[q_2 - c_2] \right]. \end{aligned}$$

Employing Assumption 2(iii) then gives

$$(10) \quad \Pi^1(q_1^*) = \lim_{q_2 \rightarrow \infty} \Pi^{1,2}(q_1^*, q_2) < \Pi^{1,2}(q_1^{**}, q_2^{**}),$$

since  $q_2^{**} < \infty$ . ■

In the absence of the investment cost required to produce good 2, Lemma 1 shows that the firm would prefer to produce both goods. If it does not do so, it must be because the investment cost is too high relative to the increase in profit. It is also possible that the firm could produce just good 2. However, applying the analogue of Assumptions 1 and 2 to the demand for good 2 would lead to a result with the profit from just good 2 being less than the profit from both.

When the cost of investment is considered, the additional profit must exceed this in order to justify moving to the production of both goods. Denote the cost of the necessary technology for producing good 2 by  $F > 0$ . As already noted, we want to model the scenario in which without intervention the firm chooses to produce only good 1. Assumption 3 is therefore adopted.

ASSUMPTION 3: *The cost of technology,  $F$ , satisfies*

$$\Pi^{1,2}(q_1^{**}, q_2^{**}) - F < \Pi^1(q_1^*).$$

These assumptions and results place us in a position where producing both goods would be the most profitable option for the firm in the absence of the investment cost. However, the firm chooses to produce only good 1 as the increased profit from introducing good 2 does not cover the cost of switching to production of both goods.

## 2.2 Tax Policy

The intention of the analysis is to discover if tax policy can induce the firm to introduce the second form of the product alongside the first and, if it can, whether it is socially-efficient to do so. This section looks at the first of these issues and concludes that there is always a balanced-budget tax policy that ensures the second form is produced.

The tax policy we consider is a tax upon good 1 with a balanced-budget subsidy to good 2. This reduces the profit earned when only good 1 is produced while raising the contribution to profit of good 2. It remains to be shown that it can affect profit sufficiently to ensure that it becomes privately-rational for the firm to introduce good 2. It may be felt that it would be more natural to provide a direct subsidy to investment costs since this is the source of the divergence between private and social benefits. Several comments can be offered to support our choice of an indirect policy. Firstly, in the context of petrol versus electric cars which initially motivated the model, taxes are in practice differentiated between types of vehicle.<sup>2</sup> Secondly, subsidies to investment costs may fall foul of competition policy – this is especially pertinent in the European Union. From a formal perspective, there may be implementation problems with subsidizing investment costs which may be non-observable (at the very least, the true cost of investment may be private information) whereas the tax policy we consider is based on observable transactions. In addition, the tax/subsidy policy affects the monopolist's pricing decision but a subsidy to investment costs does not. This can lead to circumstances in which the indirect policy produces a socially-preferred outcome. Having said this, the analysis can easily be repeated to study the direct policy.

Denoting the tax on good 1 by  $t$ , the profit from producing just this good is given by

$$(11) \quad \Pi^1(t) \equiv \max_{\{q_1\}} \left\{ X^1(q_1) [q_1 - c_1 - t] \right\}.$$

Similarly, placing a subsidy,  $s$ , on the production of good 2 changes the profit from producing both to

---

2. For example, the annual road tax on vehicles in the UK is dependent on engine size and age. Taxes on the beneficiaries of vehicles provided as part of an employment package are differentiated according to pollution characteristics.

(12)

$$\Pi^{1,2}(t,s) \equiv \max_{\{q_1, q_2\}} \left\{ \hat{X}^1(q_1, q_2) [q_1 - c_1 - t] + \hat{X}^2(q_1, q_2) [q_2 - c_2 + s] \right\}.$$

With tax policy  $\{t, s\}$ , we define

$$(13) \quad \Theta(t,s) \equiv \Pi^{1,2}(t,s) - \Pi^1(t),$$

where  $\Theta(t,s)$  is the increase in profit from producing both forms of the product rather than just one. If the government does not intervene, then Assumption 3 guarantees that  $\Theta$  will be lower than  $F$ , so

$$(14) \quad \Theta(0,0) < F.$$

The tax policy needs to increase  $\Theta$  in order to induce the firm to produce good 2. The critical value that needs to be achieved is defined by

$$(15) \quad \Theta(t,s) = F,$$

since both forms will be produced if a policy  $\{\tilde{t}, \tilde{s}\}$  can be found with  $\Theta(\tilde{t}, \tilde{s}) \geq F$ .

To analyze this possibility, it is necessary to study the effects of the tax policy upon  $\Theta(t,s)$ . An additional assumption is imposed to aid with this.

ASSUMPTION 4:

*The demand functions satisfy:*

$$(i) \quad \hat{X}_{11}^1 \leq 0, \quad \hat{X}_{22}^2 \leq 0;$$

$$(ii) \quad \hat{X}_{12}^1 \leq 0 \text{ and } \hat{X}_{21}^2 \leq 0.$$

This assumption requires the demand functions to be concave and to become steeper in own-price as the other price increases. The first is a necessary condition for the profit function to be concave.

The effects of the tax and subsidy upon  $\Theta(t,s)$  are now given.

$$\text{LEMMA 2: (i) } \frac{\partial \Theta}{\partial s} > 0, \quad \text{(ii) } \frac{\partial \Theta}{\partial t} > 0.$$

PROOF: (i) Using the Envelope Theorem it follows directly from (11)-(13) that

$$(16) \quad \frac{\partial \Theta}{\partial s} = \frac{\partial \Pi^{1,2}}{\partial s} = \hat{X}^2 > 0.$$

(ii) Using the Envelope Theorem again,

$$(17) \quad \frac{\partial \Theta}{\partial t} = \frac{\partial \Pi^{1,2}}{\partial t} - \frac{\partial \Pi^1}{\partial t} = -\hat{X}^1 + X^1.$$

To evaluate (17) it is necessary to contrast  $\hat{X}^1(q_1^{**}, q_2^{**})$  and  $X^1(q_1^*)$ . This can be done by defining the function  $\hat{q}_1(q_2)$  by

$$(18) \quad \hat{q}_1(q_2) \equiv \arg \max_{\{q_1\}} \left\{ \hat{X}^1(q_1, q_2)[q_1 - c_1] + \hat{X}^2(q_1, q_2)[q_2 - c_2] \right\}.$$

Clearly,  $\hat{q}_1(q_2^{**}) = q_1^{**}$  and  $\lim_{q_2 \rightarrow \infty} \hat{q}_1(q_2) = q_1^*$ . Since  $q_2^{**}$  is finite, the result is proved if it can be shown that  $\hat{X}^1(\hat{q}_1(q_2), q_2)$  is decreasing in  $q_2$ . From the first-order condition for (18)

$$(19) \quad \frac{d\hat{q}_1}{dq_2} = -\frac{\hat{X}_2^1 + [q_1 - c_1]\hat{X}_{12}^1 + \hat{X}_1^2 + [q_2 - c_2]\hat{X}_{12}^2}{2\hat{X}_1^1 + [q_1 - c_1]\hat{X}_{11}^1 + [q_2 - c_2]\hat{X}_{11}^2},$$

where the denominator must be negative as the second-order condition for profit maximization. Substituting (19) into  $\frac{\partial}{\partial q_2} \hat{X}^1(\hat{q}_1(q_2), q_2)$  proves the result under the conditions of Assumption 4. ■

Having characterized the dependence of  $\Theta(\cdot)$  upon its arguments, it is now possible to determine whether any values of the tax rates exist for which  $\Theta(t, s) \geq F$  and the government budget is balanced. Insisting upon budget balance requires that

$$(20) \quad R(t, s) \equiv t\hat{X}_1 - s\hat{X}_2 = 0.$$

The production of good 2 is therefore achievable with a balanced budget if there exists a tax/subsidy pair  $(\hat{t}, \hat{s})$  such that:

- i.  $R(\hat{t}, \hat{s}) = 0$  ;
- ii.  $\Theta(\hat{t}, \hat{s}) \geq F$ .

To investigate whether these can be satisfied simultaneously, define

$$(21) \quad \Pi^2(s) \equiv \lim_{t \rightarrow \infty} \left\{ \max_{\{q_2, q_1\}} \hat{X}^1[q_1 - c_1 - t] + \hat{X}^2[q_2 - c_2 + s] \right\}.$$

Here  $\Pi^2(s)$  is the maximum profit attainable as the tax rate on good 1 tends to infinity for a given subsidy,  $s$ , to good 2. It is clear that as  $t \rightarrow \infty$  the profit maximizing solution must have  $q_1 \rightarrow \infty$  and hence  $\hat{X}^1(q_1, q_2) \rightarrow 0$ . So effectively,  $\Pi^2(s)$  is the profit made from good 2 alone.

THEOREM 3: If  $\Pi^2(0) > F$ , then there exists a pair  $\widehat{t} > 0, \widehat{s} > 0$  satisfying (i) and (ii).

PROOF: For any finite  $t$ , it is clear that  $\lim_{s \rightarrow \infty} \Theta(t, s) = \infty$ , since the firm can always choose  $q_1 \rightarrow \infty$  and hence  $\widehat{X}^1(q_1, q_2) \rightarrow 0$ , but earn infinite profit for any  $\widehat{X}^2(q_1, q_2) > 0$ . Hence the solution to  $\Theta(t, s) = F$  is finite in  $s$  for any finite value of  $t$ . In particular it is finite for  $t = 0$ .

From Lemma 2,  $\Theta(t, s)$  is monotonic in  $t$ . Therefore, for given  $s$ , it tends to its maximal value as  $t \rightarrow \infty$ . As  $\lim_{t \rightarrow \infty} \Pi^P(t) = 0$ , it follows that for finite  $s$ ,  $\lim_{t \rightarrow \infty} \Theta(t, s) = \Pi^2(s)$ . Therefore if  $\Pi^2(0) > F$ , the solution in  $t$  to  $\Theta(t, 0) = F$  must be finite.

Denote the solutions to  $\Theta(0, s) = F$  and  $\Theta(t, 0) = F$  by  $\bar{s}$  and  $\bar{t}$  respectively. Both are finite.

By definition,  $R(0, 0) = 0$ . Next note that  $R(\bar{t}, 0) \geq 0$ . Consequently, there must be some value of  $s$ , say  $\tilde{s}$ , with  $\tilde{s} \geq 0$  such that  $R(\bar{t}, \tilde{s}) = 0$ . Furthermore, since  $\Theta(t, s)$  is monotonically increasing in  $s$ ,  $\Theta(\bar{t}, \tilde{s}) \geq F$ . This observation establishes that the locus of  $R(t, s) = 0$  is below the locus of  $\Theta(t, s) = F$  at  $(0, 0)$  and at least equal to it at  $(\bar{t}, \tilde{s})$ . Since  $R(t, s)$  and  $\Theta(t, s)$  are continuous, they must intersect. This intersection defines the pair  $\widehat{t}, \widehat{s}$ . Since  $R(0, \bar{s}) < 0$ , it follows that  $\widehat{t} > 0, \widehat{s} > 0$ . ■

This theorem has proved that there is a balanced budget policy that can induce the firm to produce both forms of the product. It remains now to show whether it is in the interests of the society for the government to do this.

### 2.3 Welfare

It has been established that there is a balanced-budget tax policy that leads to the production of both forms of the product. The issue of the welfare consequences of this policy is now addressed by contrasting social welfare with this policy to social welfare in the absence of taxation.

To do this, define social welfare without policy intervention,  $W(0, 0)$ , by

$$(22) \quad W(0, 0) = U(0, 0) + \Pi^1(0),$$

where  $U(0, 0)$  is the level of utility achieved. Similarly,  $W(\widehat{t}, \widehat{s})$  is defined by

$$(23) \quad W(\widehat{t}, \widehat{s}) = U(\widehat{t}, \widehat{s}) + \Pi^{1,2}(\widehat{t}, \widehat{s}) - F.$$

The policy intervention is welfare-improving if  $W(\widehat{t}, \widehat{s})$  is greater than  $W(0, 0)$ .

Now observe that

$$(24) \quad \begin{aligned} W(\widehat{t}, \widehat{s}) - W(0, 0) &= [U(\widehat{t}, \widehat{s}) - U(0, 0)] + [\Pi^{1,2}(\widehat{t}, \widehat{s}) - F - \Pi^1(0)] \\ &= [U(\widehat{t}, \widehat{s}) - U(0, 0)] - [\Pi^1(0) - \Pi^1(\widehat{t})], \end{aligned}$$

where the second equality follows from the fact that  $\Pi^{1,2}(\widehat{t}, \widehat{s}) - F - \Pi^1(\widehat{t}) = 0$  since  $\Theta(\widehat{t}, \widehat{s}) = F$ . As  $\widehat{t} > 0$ ,  $\Pi^1(0) > \Pi^1(\widehat{t})$ . Therefore, the comparison of welfare levels depends upon (i) the increase in utility due to the availability of good 2 and (ii) the loss in profit caused by the tax on the output of good 1.

Although this representation of the problem identifies the factors at work, there is an important issue that arises before it can be taken any further: the two utility levels appearing in (24) are defined over different product ranges. The value of  $U(0, 0)$  is determined over a product range without good 2 whereas  $U(\widehat{t}, \widehat{s})$  is defined when good 2 is available.<sup>3</sup> LANCASTER [1966] provided a comprehensive discussion of the issues surrounding such expansion of the product space, and concluded that further structure (for instance the use of a characteristics model) was necessary for precise conclusions to be derived. Consequently, what we choose to do here is to work with the demand functions which have been the basis of the analysis so far and conduct welfare comparisons using consumer surplus arguments.

To allow the calculation of consumer surplus, linear demand functions are now adopted. When only good 1 is produced demand is therefore given by

$$(25) \quad X^1 = f(q_1) = a - bq_1,$$

and with both goods it is

$$(26) \quad \hat{X}^i = \frac{a}{2} - \beta q_i + \delta q_j, \quad i, j = 1, 2, \quad i \neq j,$$

where  $\beta > 0$  and  $\delta > 0$ . It should be noted that these demands do not completely satisfy Assumption 1, in particular  $\lim_{q_2 \rightarrow \infty} \hat{X}_1 \neq X_1$ . Their interpretation must therefore be of local approximations to the true demands which are valid in the region of (finite) equilibrium prices.

Welfare with only good 1 is

$$(27) \quad W^1 = \Pi^1 + Cs,$$

where  $W^1$  denotes welfare and  $Cs$  consumer surplus. Consumer surplus is defined by

---

3. To illustrate the difficulty, assume utility is logarithmic. If zero units of good 2 are consumed, then utility is  $-\infty$  regardless of the consumption of other commodities. The welfare gain of introducing good 2 is therefore infinite.

$$(28) \quad Cs = \int_0^{X^1} q_1(X) dX - X^1 q_1.$$

Similarly, when both goods are produced welfare becomes

$$(29) \quad W^{1,2} = \Pi^{1,2} + Cs^1 + Cs^2 - F,$$

where  $Cs^1$  and  $Cs^2$  are consumer surplus generated by good 1 and good 2 respectively. The value of consumer surplus for good  $i$  is

$$(30) \quad Cs^i = \int_0^{\hat{X}^i} \hat{q}_i(\hat{X}^j, \hat{X}) d\hat{X} - q_i \hat{X}^i, \quad i, j = 1, 2, \quad i \neq j.$$

Since the demand functions are linear, the consumer surplus integrals are exact measures of welfare and are path-independent.

The equilibrium is simulated by adopting the parameter values  $a = 100$ ,  $b = 1.5$ ,  $\beta = 1.2$ ,  $\delta = 0.9$ ,  $c_1 = c_2 = 2$  and  $F = 2600$ . For these values, the initial equilibrium in the absence of tax policy is summarized in Table 1. The first row gives the equilibrium values when only good 1 is produced; the second row for when both goods are produced. It can be seen that the introduction of the second form of the product raises both profit and consumer surplus. Welfare is also higher. However, the increase in profit is not sufficient to overcome the investment costs, so the firm will choose only to produce good 1. In relation to the role of a subsidy to investment costs, note that the increase in welfare is significantly less than the investment cost, so it would not pay society to directly pay the investment cost. The payment of the investment cost can only be justified alongside a policy that modifies the monopolist's pricing policy.

TABLE 1

***The Initial Equilibrium***

	$q_1$	$X^1$	$q_2$	$X^2$	$\Pi^1$	$\Pi^{1,2}$	$Cs^1$	$Cs^2$	$W$
Good 1	34	49	–	–	1568	–	800	–	2368
1 and 2	84	25	84	25	–	4067	586	586	2629

Using the results in Table 1, it follows that  $\Theta(0,0)$  is given by

$$(31) \quad \Theta(0,0) = \Pi^{1,2}(0,0) - \Pi^1(0,0) = 2499.$$

The chosen value of  $F = 2600$  ensures that  $\Theta(0,0) < F$ . The level of welfare demonstrates that it is socially beneficial if both goods could be produced but the level of profit guarantees that it is not privately efficient.

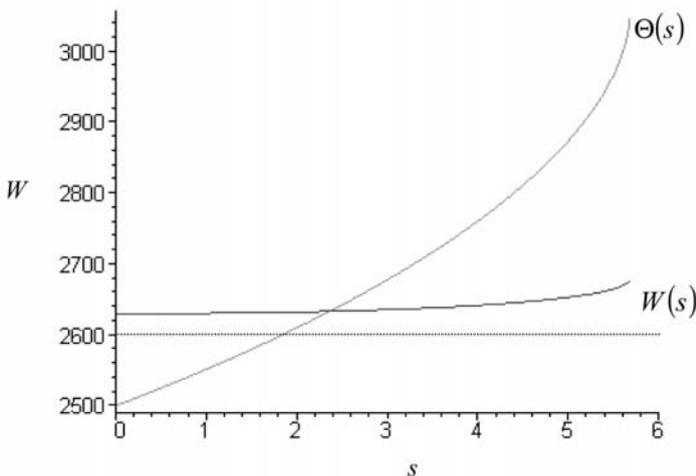
Calculating the taxes that solve  $R(\hat{t}, \hat{s}) = 0$  and  $\Theta(\hat{t}, \hat{s}) = F$  provides the results reported in Table 2. These show that the policy succeeds in inducing the firm to produce both goods and in raising welfare. Profit net of investment cost is less than with no policy intervention. Consumer surplus from good 1 is reduced from the level when there is no policy intervention but to this is added the surplus from good 2. Consequently the policy induces a transfer from the firm to consumers. The fact that there is monopoly allows some of the tax on good 1 to be absorbed by the firm (there is undershifting of the tax).<sup>4</sup> So, while being effective in modifying the firm's behavior, it is not too damaging for consumers of good 1. On the other hand, the subsidy results in a reduction in  $q_2$  compared to the outcome in Table 1, and an increase in consumption.

TABLE 2  
*Policy Intervention*

$s$	$t$	$q_1$	$X^1$	$q_2$	$X^2$	$\Pi^{1,2}$	$Cs^1$	$Cs^2$	$W$
1.9	2.2	85	22	83	27	4062	484	684	2631

It is possible to move beyond this analysis and to determine the optimal policy. By solving the budget constraint  $R(t, s) = 0$  for  $t$  as a function of  $s$ , the tax rate can be eliminated from  $\Theta(t, s)$  and  $W(t, s)$  to write  $\Theta = \Theta(s)$  and  $W = W(s)$  respectively. These functions are plotted in Figure 1. The dashed line is the critical value of 2600, so values of  $s$  where  $\Theta(s)$  is above the line will ensure that the firm produces both goods. This holds for values

FIGURE 1  
*Optimal Tax Policy*



4. See MYLES [1995] for a discussion of tax undershifting.

of  $s \geq 1.9$ . Welfare increases with  $s$  and is maximized at  $s = 5.8$ . This is the highest value of  $s$  for which a balanced-budget can be sustained. The values of the variables corresponding to the optimum outcome are given in Table 3. For this example of linear demand the optimal policy is to use the maximum possible subsidy.

TABLE 3  
*Values at Optimum*

	$s$	$t$	$q_1$	$X^1$	$q_2$	$X^2$	$\Pi^{1,2}$	$Cs^1$	$Cs^2$	$W$
Optimal Value	5.68	15.72	92	13	81	35	3943	154	1179	2676

This analysis has demonstrated that tax policy can ensure that both forms of the product are produced. An example has also been given in which the policy raises social welfare. The fact that the optimum has the subsidy set at the highest value possible is almost certainly a consequence of the linear structure of demand and is not a feature that would generalize.

### 3 Duopoly

---

Making the extension to duopoly (or oligopoly in general) raises difficulties not present in the case of monopoly. With monopoly, the firm either produced the initial form of the product or it produced both forms. Having additional firms introduces the possibility of asymmetric equilibria in which the competing firms have different product ranges. In fact, each firm has three choices of product range: it can produce only good 1, only good 2 or both goods. The choice of good 2 alone could be ruled out with monopoly because adding the production of good 1 was always marginally profitable. This argument cannot be used for duopoly since an equilibrium could well involve each firm having a monopoly in one product.

If one firm changes its product range, there will be a discrete change in the equilibrium. These discrete changes lead to discontinuities in profit, tax revenue and welfare at the points at which firms switch product ranges. The discontinuities in profit complicate the derivation of equilibrium whereas those in tax revenue and welfare add additional complexity to the welfare analysis. Furthermore, if both firms produce good 2, two sets of investment costs will have to be paid, with consequent implications for welfare. Finally, since each firm has a choice of three product ranges there are nine possible equilibrium configurations possible in a duopolistic market.

The next subsection will describe the general version of the model with duopoly. This is followed by the explicit calculation of equilibrium for an example with linear demand. Given the numerous discontinuities, this is probably as general as the analysis can be. The calculations fully determine the equilibrium regimes and isolate the optimal tax policy.

### 3.1 Equilibria

When only good 1 is produced, the inverse demand function is given by

$$(32) \quad q_1 = q_1 \left( X_1^1 + X_2^1 \right),$$

where  $X_j^i$  is the output of good  $i$  by firm  $j$ . The inverse demand functions when both goods are produced are

$$(33) \quad \hat{q}_i = \hat{q}_i \left( X_1^1 + X_2^1, X_1^2 + X_2^2 \right), \quad i = 1, 2.$$

The *status quo* is again taken to be an equilibrium in which only good 1 is produced. This equilibrium is defined by two output levels that solve

$$(34) \quad \max_{\{X_i^1\}} X_i^1 \left[ q_1 \left( X_1^1 + X_2^1 \right) - c_1 - t \right], \quad i = 1, 2.$$

When at least one firm produces variety 2, the output levels solve

$$(35) \quad \begin{aligned} & \max_{\{X_i^1, X_i^2\}} X_i^1 \left[ \hat{q}_1 \left( X_1^1 + X_2^1, X_1^2 + X_2^2 \right) - c_1 - t \right] \\ & + X_i^2 \left[ \hat{q}_2 \left( X_1^1 + X_2^1, X_1^2 + X_2^2 \right) - c_2 + s \right], \quad i = 1, 2. \end{aligned}$$

The notational convention is now adopted that the pair  $\{k, \ell\}$ , where  $k \in \{1, 2, b\}$  and  $\ell \in \{1, 2, b\}$ , denotes a regime in which firm 1 chooses product range  $k$  and firm 2 product range  $\ell$ . For firm  $j$ , product range 1 implies  $(X_j^1 > 0, X_j^2 = 0)$ , range 2 implies  $(X_j^1 = 0, X_j^2 > 0)$  and range  $b$  denotes production of both goods so  $(X_j^1 > 0, X_j^2 > 0)$ . The number of distinct equilibria can be reduced using the fact that the firms are identical so, for example, equilibrium  $\{1, 2\}$  is economically equivalent to  $\{2, 1\}$ . This leaves six potential equilibrium regimes:

- (i) 1, 1:  $X_1^1 > 0, X_2^1 > 0, X_1^2 = X_2^2 = 0$ ;
- (ii) 1, 2:  $X_1^1 > 0, X_2^1 = 0, X_1^2 = 0, X_2^2 > 0$ ;
- (iii) 1,  $b$ :  $X_1^1 > 0, X_2^1 > 0, X_1^2 = 0, X_2^2 > 0$ ;
- (iv)  $b$ , 2:  $X_1^1 > 0, X_2^1 = 0, X_1^2 > 0, X_2^2 > 0$ ;
- (v)  $b$ ,  $b$ :  $X_1^1 > 0, X_2^1 > 0, X_1^2 > 0, X_2^2 > 0$ ;
- (vi) 2, 2:  $X_1^1 = 0, X_2^1 = 0, X_1^2 > 0, X_2^2 > 0$ .

The profit levels in these equilibria are denoted by  $\Pi_j^{k, \ell}(t, s)$  where  $k, \ell = 1, 2, b$  and  $j = 1, 2$ . In defining these equilibria, it should be recalled that whenever  $X_j^2 > 0$ , the investment cost,  $F$ , must be paid and the firm is then concerned with profit net of  $F$ .

The next result provides a sufficient condition to rule out regime {2,2} as an equilibrium.

LEMMA 4: *If  $t < \min_{\{X^2\}} \widehat{q}_1(0, X^2) - c_1$ , then {2,2} cannot be an equilibrium.*

PROOF: There is no investment cost involved in producing good 1. Under the condition of the lemma, if firm  $j$  chooses  $X_j^1 = 0$  (as it does in regime {2,2}), then  $\lim_{X_{j'}^1 \rightarrow 0} \widehat{q}_1(0, X^2) - c_1 - t > 0$ . The continuity of  $\widehat{q}_1(0, X^2)$  then guarantees that  $X_{j'}^1, [\widehat{q}_1(0, X^2) - c_1 - t]$  is positive for a range of  $X_{j'}^1 > 0$ . Hence firm  $j'$  can increase profit by choosing  $X_{j'}^1 > 0$  and deviating from {2,2}. ■

As a consequence of this result, we will rule out consideration of {2,2} and hence only look at tax rates satisfying the condition of Lemma 4. This gives five potential equilibrium regimes to concentrate upon.

To proceed further, return to the fact that the *status quo* is taken to be the production of good 1 only. To sustain this, the size of the investment cost,  $F$  must satisfy

$$(36) \quad \Pi_2^{1,1}(0,0) > \Pi_2^{1,2}(0,0) - F, \quad \Pi_2^{1,1}(0,0) > \Pi_2^{1,b}(0,0) - F,$$

where the superscripts denote regimes. The conditions on  $F$  in (36) are sufficient to guarantee that firm 2 will not switch into producing just good 2 (the first inequality) or both goods (the second inequality) when there is no tax or subsidy. Since the firms are identical, these conditions on  $F$  also guarantee that firm 1 will also not switch.

## 3.2 Equilibrium and Welfare

The difficulty in analyzing this model is that there are discontinuous changes in the equilibrium as the firms switch between product ranges. As such, it bears analytical similarities to other models with discrete choices for firms such as location models in trade theory.<sup>5</sup>

In order to determine the equilibrium product ranges for any pair of tax rates  $\{s, t\}$ , it is necessary to compare the profit levels for all possible product ranges given the choice of the other firm. As far as optimality is concerned, welfare and tax revenue will also be discontinuous in the tax variables as the switch is made from one equilibrium to the next. The optimum cannot be found using standard maximization techniques but must be explicitly constructed.

---

5. For example, MOTTA and THISSE [1994].

The model with duopoly is investigated using the linear demand system in (25). The technique is to calculate the level of profit for both firms for all of the potential equilibrium regimes identified above. This is undertaken using a grid for  $s$  and  $t$ . So, for each tax policy five profit levels are found for each firm. The equilibrium is then found for each point in the grid by using standard Nash equilibrium arguments on the resulting profit levels. This assigns an equilibrium regime to each point of the grid in  $\{s,t\}$  space. These grid points then show where the switches between regimes occur and which regimes are adjacent. With this knowledge, it is possible to compute the precise boundaries between the regimes. This is possible since it is known which firm switches between regimes as each boundary is crossed and that their profit level before and after the switch must be equal on the boundary (for the other firm, there will be a discontinuous change in profit).

Doing this provides the information summarized in Figure 2. Starting from the point  $s = 0, t = 0$  where regime 1,1 holds by construction, the first change in regime occurs when firm 2 switches from product range 1 to product range  $b$ . The boundary between these two equilibrium regimes is found by plotting the locus of  $\{s,t\}$  that solve

$$(37) \quad \Pi_2^{1,1}(s,t) = \Pi_2^{1,b}(s,t) - F.$$

The next switch is from regime 1, $b$  to 1,2. The second boundary is defined by

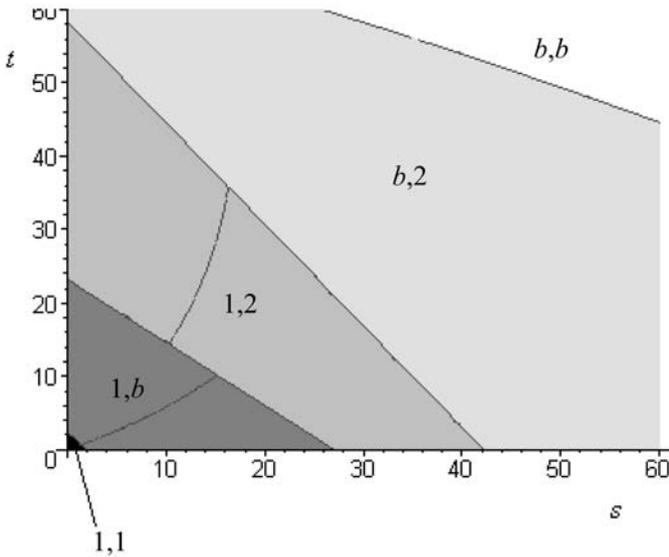
$$(38) \quad \Pi_2^{1,b}(s,t) = \Pi_2^{1,2}(s,t).$$

Continuing in this way completes the characterization of equilibrium regimes for given pairs of tax rates. The resulting regions are separated by the curves sloping from north-west to south-east in Figure 2. Only for high values of the subsidy do both firms produce both forms of the product.

The next step in the construction involves computing the points of budget-balance within each regime. These points of balanced budget are shown by the curves sloping from south-west to north-east in Figure 2. Three regimes have no points of balanced budget. For regime 1,1 this is because there is no production of good 2 to be subsidized. In contrast, for  $b,2$  and  $b,b$ , it is because the set of  $\{s,t\}$  that are budget-balanced for these regimes does not intersect with the set of  $\{s,t\}$  for which these regimes are equilibria. In fact, these regimes can only exist in combination with a budget deficit. In summary, only equilibria in regimes 1, $b$  and 1,2 can co-exist with a balanced budget.

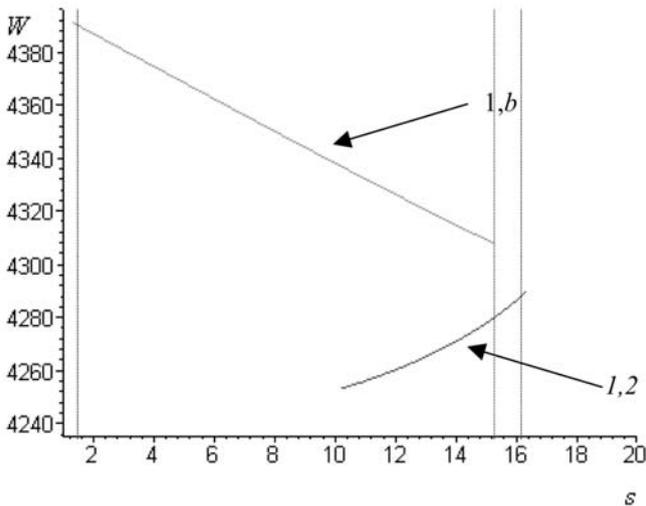
The final step in the analysis is to calculate the welfare levels along the points of balanced budget. This is done by solving the budget constraint for  $t$ , substituting into the welfare function and then plotting welfare as a function of  $s$ . This gives the curves shown in Figure 3. The curve labelled 1, $b$  shows the welfare level when firm 1 produces good 1 while firm 2 produces both. On the other curve, firm 2 produces only good 2. It can be seen that there are some values of the subsidy for which there are two welfare levels. This is just a reflection of the fact, apparent in Figure 2, that for this range of  $s$  there are two different balanced-budget equilibrium regimes depending on the accompanying value of  $t$ .

FIGURE 2  
*Equilibrium Regimes*



The maximal welfare level is attained at the lowest subsidy that supports an equilibrium in regime  $1,b$ . This is above the level with no intervention. Therefore, the government should intervene just sufficiently to ensure that one of the firms produces good 2. What is happening in this equilibrium is that competition in good 1 is sustained, whereas one firm has a monopoly in good 2 to compensate it for the payment of the investment cost. This policy configuration can secure an increase in welfare.

FIGURE 3  
*Welfare Levels*



This concludes the analysis of the duopoly case. It has been noted that there are numerous discontinuities and several potential equilibrium regimes. For the linear case, these have been resolved by the numerical analysis. This has determined that the optimal policy is to use the lowest tax and subsidy that just ensure one of the firms introduces good 2. The discontinuities are a general feature of a problem of this form. The solution that a minimal intervention should be used is likely to be a consequence of the linear structure.

## 4 Conclusions

---

The issue motivating the paper was that the deviation between private and social incentives may lead the economy to a position where the range of goods on offer is not socially efficient. We focussed on a situation where a second form of a good could only be produced if firms undertook investment. The existence of this switching cost creates the possibility that the economy may get trapped into a socially inefficient (but privately rational) equilibrium. With monopoly, the implementation of the tax policy increases the difference between the profit from producing just good 1 and that from producing both goods. It was proved that there was a balanced-budget policy that induces the monopolist to produce both forms of product. Whether this policy increases welfare is dependent upon the trade-off between the potential increase in utility and the definite loss of profit. Calculations for linear demand show that it is possible for policy to increase welfare. Therefore the tax policy can overcome the switching cost and place the economy in a socially-preferred position.

The analysis of duopoly is considerably more complex than that of oligopoly. The reason for this is the possibility of asymmetric equilibria in which the firms have different product ranges. This implies that for any pair of tax rates there are six possible equilibria. Furthermore, both tax revenue and welfare are discontinuous as firms switch between product ranges. Despite these difficulties, it is still possible to solve for the equilibrium with linear demand. It was shown that the optimal policy is that which is just sufficient to induce one firm to supply good 2 and to have both supplying good 1. This policy retains competition in the sale of good 1 but leaves a monopoly in good 2 in order to overcome the switching costs. As with monopoly, the tax policy leads to an increase in welfare.

From a policy perspective, the results of the paper suggest a positive role for tax policy in the encouragement of additional products. By tilting the balance in favor of good 2 the tax/subsidy package can provide the incentive that manufacturers require to undertake the investment needed to produce good 2. It is not certain that such a policy will raise welfare, but the examples given here suggest there are a broad range of circumstances in which it will. The general economic message of the paper is that tax policy can rescue an economy from a socially-inefficient equilibrium brought about through switching costs.

## • References

- LANCASTER K.J. (1966). – « Change and innovation in the technology of consumption », *American Economic Review*, **56**, p. 14-23.
- MOTTA M., THISSE J.-F. (1994). – « Does environmental dumping lead to delocation? », *European Economic Review*, **38**, p. 563-576.
- MYLES G.D. (1995). – *Public Economics* (Cambridge: Cambridge University Press).
- OECD (1993). – *Choosing an Alternative Transportation Fuel, Air Pollution and Greenhouse Gas Effects*, (Paris: OECD).