ANNALES D'ÉCONOMIE ET DE STATISTIQUE. – N° 75/76 – 2004

Jeux Sans Frontières Revisited: A new Study of Tax Competition when Transportation Cost is Composite

Megan KHOSHYARAN *

ABSTRACT. – The purpose of this paper is to investigate the effects of variable population densities, composite transportation costs, and consumer preferences on tax competition. A simple formula for the number of cross border shoppers is derived which yields explicit expressions for composite reaction functions. The NASH equilibrium problem may have zero or several solutions. The major contribution of this paper is that by considering more realistic forms of the three variables, population densities, composite transportation costs, and consumer preferences, one can obtain major structural changes in the NASH Equilibrium (NE) outcome. 1) The existence of a unique NE is no longer guaranteed, 2) the size of the region is no longer significant in revenue maximization schemes, 3) the amount of cross border shopping is dependent on all three variables, 4) evolution in any or all three variables, can be PARETO-improving if, the evolution of either of these variables is correlated with the evolution of the others, 5) tax coordination is advantageous irrespective of the size of each region. The implication of these findings is that the best strategy is tax harmonization.

RÉSUMÉ. – L'objet du papier est d'étudier les effets, sur la compétition fiscale, de la variabilité de la densité de population, de la complexité des coûts de transport, et des préférences individuelles des consommateurs. Une formule simple est déduite, qui exprime le nombre de consommateurs franchissant la frontière pour effectuer leurs achats. Cette formule permet d'obtenir des expressions explicites pour des fonctions de réaction complexes. Le problème de l'équilibre de NASH peut admettre zéro ou plusieurs solutions. La principale contribution de cet article est que des changements structurels majeurs des solutions de l'Équilibre de NASH (NE) peuvent survenir lorsque l'on considère une forme plus réaliste des trois variables, densité de la population, coûts de transports complexes, préférences des usagers. 1) L'existence d'une unique solution de (NE) n'est plus garantie, 2) la taille de la région n'est plus significative pour la détermination du revenu optimal, 3) le volume des achats transfrontaliers dépend des trois variables, 4) l'évolution de l'une quelconque des trois variables peut conduire à une amélioration au sens de PARETO si l'évolution de chacune de ces variables est corrélée à l'évolution des autres, 5) la coordination des politiques de taxation est avantageuse indépendamment de la taille de chaque région. La conséquence de ces résultats est que la meilleure stratégie est l'harmonisation des politiques de taxation.

* M. KHOSHYARAN: Department of Economics, Catholic University of Leuven, Naamsestraat 69, B-3000 Leuven, Belgium, mailto:megan.khoshyaran@wanadoo.fr.

1 Introduction

Local governments have the power to levy taxes. In general fiscal decisions by one government affect the tax revenues of the others. Typically, one region's government can raise tax rates relative to those of other regions, and thus has the ability to modify the size of its tax base at the expense (or the benefit) of its neighbors. This is in evidence between the members of the European community. A new economic environment is created by gradually suppressing the trade barriers between the members while regulating the exchanges with external countries. The movements of persons and commodities are theoretically free inside the community. However, each government pursues its own fiscal and budgetary policies. In particular, each government keeps some freedom for fixing its own rates of taxation. Case in point is the commodity tax. Commodity taxes varying from country to country associated with free movement of customers induce cross-border shopping. Since cross border shopping affects government revenue, both tax competition and tax coordination have become a serious concern.

Even though the aim is to reach a deeper economic integration where coordination of broad economic policy-making objectives between governments is introduced, it is unlikely that governments would submit to general tax coordination in the near future. My view is that we are a long way away from complete tax coordination. Thus, in this paper the focus is put on tax competition and its consequences. Many researchers have looked at tax competition; for example, MINTZ and TULKENS [1985], ZODROW and MIESZKOWSKI [1986], WILDASIN [1988], BUCOVETSKY [1991], HOYT [1993], and many others. In all the models proposed fiscal equilibrium is reached using the NASH equilibrium in a non-cooperative game in which players are local governments; each adopting a strategy to maximize local taxes, while maximizing consumer utility subject to national budget constraint which relates to public expenditure levels or in other words welfare functions. In this paper attention is focused on maximizing government revenue through commodity tax competition. Few articles have been devoted to the study of commodity tax competition models: KANBUR and KEEN [1993], LOCKWOOD [1993], and OHSAWA [1994].

KANBUR and KEEN have formulated a simple model of destination-based commodity tax competition between two governments, taking cross-border shopping and the size of population into consideration. They examined the effect of population size in determining optimal taxes in a NASH equilibrium context, and analysed the existence of unique solution and its implications. The equilibrium point is the function of transportation costs bounded by the two country's reservation prices and population size when the two countries are strictly different in size. OHSAWA has proposed a model that is similar to KANBUR and KEEN's model but it varies in two aspects. First, he extends the KANBUR and KEEN's models by formulating a multi-governments model to examine how the relative spatial position of countries affects equilibrium. Second, while KANBUR and KEEN assumed that the two countries differ by population size which is uniformly distributed over two regions of equal size, OHSAWA assumes that although customers are uniformly distributed over the region, the spatial sizes of the two countries are different.

In both of these studies, the transportation variable is an integral part of the modelling effort. Let's explore this a bit further. In KANBUR and KEEN, the consumer's utility is defined by his surplus. This surplus is the function of tax differentiation, and transportation costs. Government revenue is a function of population size and consumer surplus, which is a function of transportation costs. The unique NASH equilibrium is found given structural restrictions on the transportation cost function and the consumer price acceptability. All conclusions derived from the existence of unique NASH equilibrium depend on the initial assumptions on the transport cost and the population size. Given the impact of transportation cost on the model and its implications, they give little importance to this variable. In their studies, the cost function is assumed to be a monetary value ($\delta > 0$) times the distance from the border. As such, they under-estimate the impact of transportation, specifically, the prevailing conditions of traffic at the time when the consumer decides to purchase at the border.

The same applies to OHSAWA's modelling work. In contrast, MINTZ and TULKENS [1993] do stress the importance of transportation cost function. They state that the conditions of regional market equilibrium cannot be satisfied unless the transportation cost function is an increasing and strictly convex function of commodity consumption. They also state that the uniqueness of the solution to the regional market equilibrium rests in part on the strict convexity of the transportation cost function. The statements on the form of the transportation cost function are not backed by any theoretical work that includes any functional form relating to findings in the field of traffic flow theory. It seems to the author that many statements on the transportation cost were made through logical induction and not a theoretical one. In the conclusion of their paper, MINTZ and TULKENS suggest that both the positive analysis of noncooperative NASH equilibrium and the normative equilibrium of PARETO efficient fiscal structure can be extended in several directions of which one is: to find out which characteristics of preference structure and transportation costs would induce uniqueness and stability of a noncooperative NASH equilibrium?

It is the aim of this paper to explore the effects of transportation cost on the outcome of the NASH equilibrium, its uniqueness property and the economic implications. In this paper I will conform in my assumptions with MINTZ-TULKENS by following a similar path and KANBUR-KEEN by assuming two regions of different population size. I propose to use the NASH equilibrium of a noncooperative game in which the players are local governments, their strategies are local taxes, and their payoffs are revenues. As both set of researchers have done, I consider two local jurisdictions each levying an origin-based commodity tax on a private good. Residents of each region can purchase the taxed private good either in their own region or in the other region after incurring transportation costs and paying there the local tax. Each region is assumed to choose optimally the levels of its domestic commodity tax to maximize its revenue.

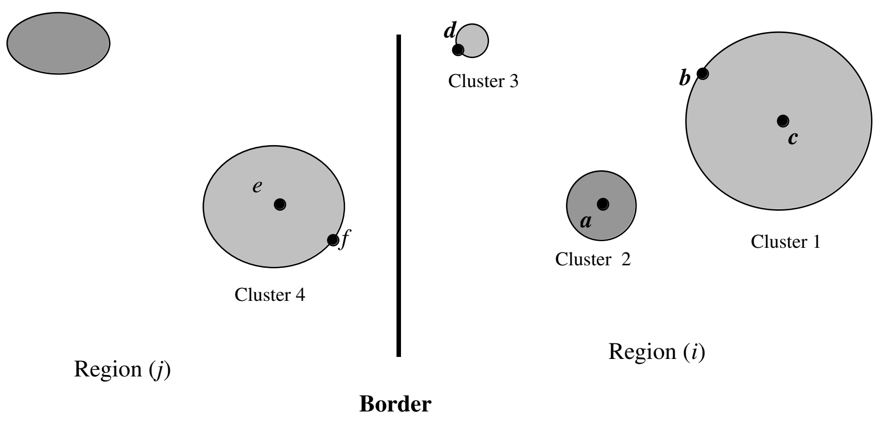
The plan of the rest of this paper is as follows: Section 2 describes the model in three sub-sections. Sub-section 2.1 models the regional population. Sub-section 2.2 models a transportation cost function that is based on population density, distance from the border, and total trip travel time. Sub-section 2.3, models the number of customers who shop in region (i). Sub-section 2.4, models consumer preference as a (Logit) probability function. In section 3, tax competition, under a non-cooperative NASH equilibrium is studied. In sub-section 3.1, general explicit expressions for the reaction functions are derived. In Sub-section 3.2, the NASH equilibrium is derived. In sub-section 3.3, two numerical examples are worked out. Section 4, explores the properties of the NASH equilibrium. Four propositions are introduced in this section.

2 The model

2.1 Modelling regional population

There are two regions and a single taxed commodity. The two regions (i) and (j) are connected by a network of roads to the border. In contrast to KANBUR and KEEN [1993], who assume different population sizes for their two countries model; here, it is the population density that is significant. Each region consists of several clusters of population. Since population density differs for each cluster within each region, it is assumed that the population is not distributed uniformly. For each cluster of population, those who live in the center of the cluster experience a higher travel time and thus travel costs if they decide to shop across the border, than those who live on the border of the cluster, consider points, c and b. For those who live in the city center, first, they experience city's dense traffic, and then other traffic that is headed to the border. Those around point b, have only to deal with the traffic that is outside of the city centers' dense traffic. The same is true for cluster a, but to a much lesser extent since the density of population is much lower. For those who live either inside or on the periphery of the cluster d, they experience little traffic

FIGURE 1

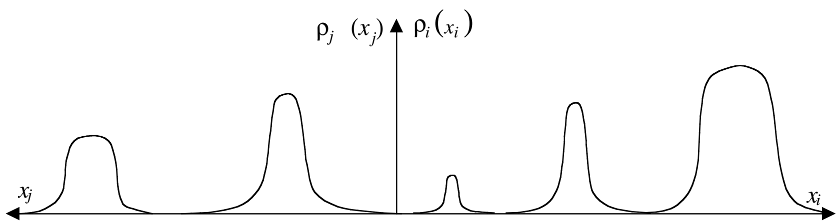


first because their own centre is small, and second they are the closest to the border. In contrast for those around points *e* and *f*, in region *j*, distance does not strictly translate into a low transportation cost, since those around the center (point *e*) still face a high density of traffic due to high density of population in spite of their proximity to the border. Those around point (*e*) might not be provoked to shop across the border if travel times stay high due to regular occurrences in traffic stream such as bands of free flows, bottlenecks, jams, synchronized moving jams, and stop and go.

My contention in this paper is that contrary to KANBUR and KEEN [1993], who assume that only those who are closest to the border will go shopping across the border if there is a tax differentiation, I assume that those on the periphery of clusters c and a might also decide to shop across the border if traffic conditions are encouraging, that is to say that if travel times are within reasonable boundaries. In other words, I assume that transportation cost τ is a function of not only distance from the border (x) but also travel-time. Figure 1 depicts the two regions with their main population clusters.

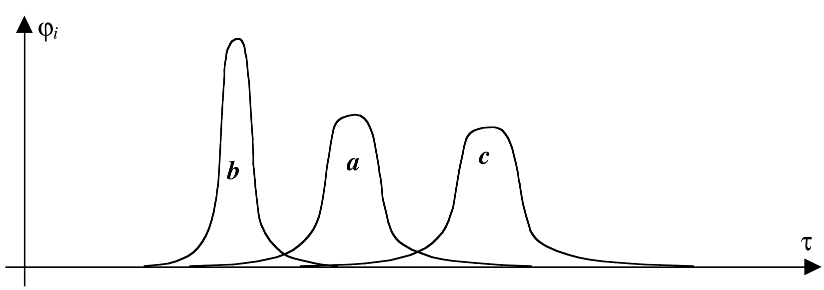
Figure (2), depicts the corresponding population densities of the two regions: $\rho_i(x_i)$ and $\rho_j(x_j)$.

FIGURE 2



The density functions φ_i of the travel costs τ at three points a, b, c are depicted in Figure 3:

FIGURE 3



Travel cost from point b to the border is less than from point a, since a lies at the centre of cluster 2 (Figure 1). Travel costs from points a and c have a wide range as it is necessary to cross wide urbanised areas in order to reach the border from these points. Therefore, in this example it is not only the customers around d who shop across the border, but also all those who are on the periphery of cluster c could also decide to shop across the border if their transportation costs stay within a reasonable range.

As is the case for many research works in this area, I assume that the model is a partial-equilibrium one. Similar to KANBUR and KEEN [1993], it is assumed that the commodity purchased in each region carries the tax that is levied in that region. Production of the commodity in both regions is assumed to take place under perfect competition and constant marginal costs. Therefore, producer prices are fixed, and are the same in both regions; thus, they are omitted in the formulation. Taxes in both regions are taken as an incentive for the consumer to shop where he/she lives or to go across the border.

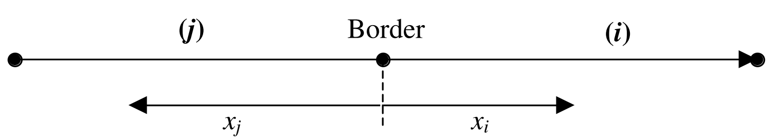
The tax on commodity in region (i) is t_i, and in region (j) is t_j. The consumer in region (i) either:

- Acquires the commodity locally in region (i) at price t_i , or
- Acquires the commodity in region (j) at a price equal to t_j, plus the cost of transportation τ_i to the border.

The market equilibrium for one region consists of its consumers' preferred choice of consumption of the produced commodity for given levels of taxes t_i, t_j. In each region, i, j the market equilibrium relative to (t_i, t_j) is defined as: the consumer in region (i) chooses to buy in region (j) if and only if

(1) $$t_j + \tau_i < t_i$$

2.2 Modelling transportation cost

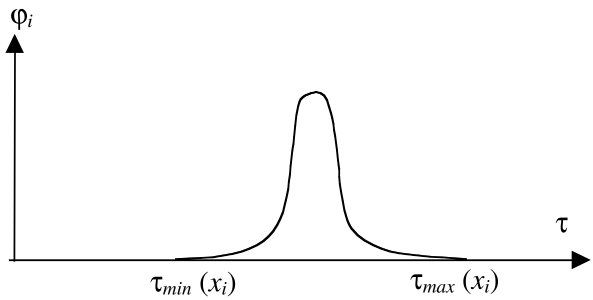


The above physical model is a linear HOTELLING [1929] like model. Consider two clusters one on each side of the border. The variables x_i and x_j represent the distance from the border. The clusters are considered bi-dimensional, since both distance and population density will be used in the forthcoming calculations. The density of population of the clusters is not assumed uniform. In each cluster, the population is assumed to have densities $\rho_i(x_i)$, and $\rho_j(x_j)$. The density accounts for the distribution of population in the cluster center, which is located at distances x_i and x_j from the border. The second assumption is the following: consumers located at distances x_i and x_j from the border do not experience the same travel times. This is due to the fact that either they do not follow the same routes to go to the border, or they do not encounter the same traffic conditions, or even do not use similar modes of transportation. Therefore, at distance say x_i, there exists a distribution of travel times and thus travel costs:

(2) $$\varphi_i(x_i, \tau_i)d\tau_i$$

At location x_i in region (i) the proportion of consumers whose travel cost is between τ_i and $\tau_i + d\varphi_i$ is equal to $\varphi_i(x_i, \tau_i)d\tau_i$, as is shown in Figure 4.

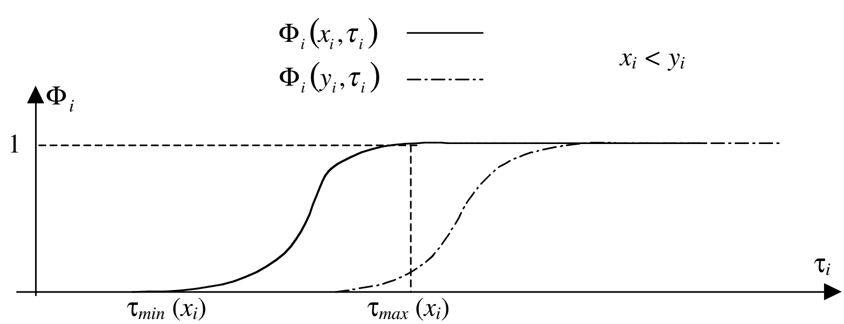
FIGURE 4



A symmetric description applies to travel costs in region (*j*). From the distribution of travel times one can find the distribution of the number of customers who shop in region (i) or in region (*j*). Thus the proportion of consumers at location x_i whose travel cost is less than τ_i is:

(3) $$\Phi_i(x_i, \tau_i) \stackrel{\text{def}}{=} \int_0^{\tau_i} \varphi_i(x_i, t)dt$$

FIGURE 5

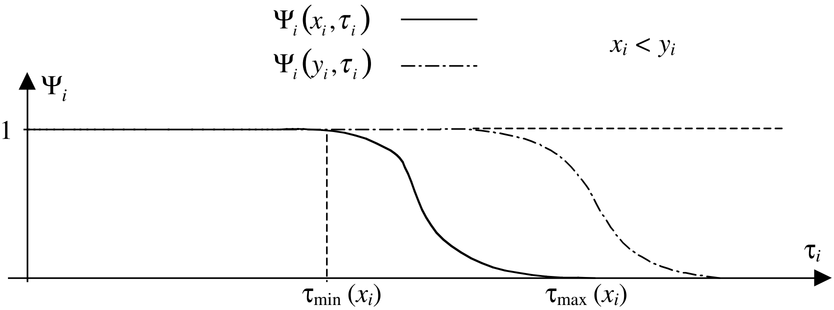


and the proportion $\Psi_i(x_i, \tau_i)$ of consumers at location x_i whose travel cost is greater than τ_i is:

(4) $$\Psi_i(x_i, \tau_i)^{\text{def}} = 1 - \Phi_i(x_i, \tau_i) = \int_{\tau_i}^{+\infty} \varphi_i(x_i, t)dt$$

The distribution functions $\Phi_i(x_i, \tau_i)$ are increasing functions in τ_i; while they are decreasing in x_i. Though, indeed, travel costs must increase with distance x_i, given the conditions of traffic, it is possible to have different travel costs traversing the same distances.

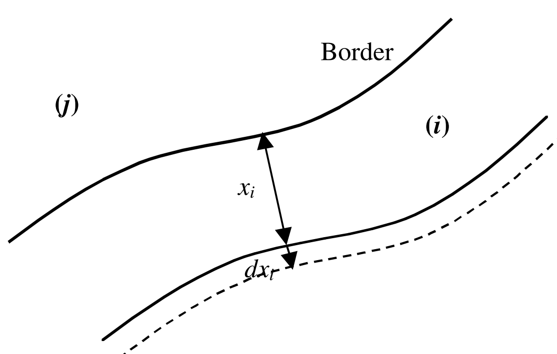
FIGURE 6



By postulating these travel cost distributions, we assume that traffic and transportation conditions are given in both regions, and that the transportation networks are at equilibrium, *i.e.* all roads in the network are used in both regions and similar roads have similar traffic patterns. Further, by postulating that travel cost distributions are independent of tax levels, we assume that the transportation equilibrium in both regions is not affected by changes in the tax levels and by the induced changes in the transportation demand.

2.3 How travel costs are used to estimate the number of customers in region (i)

FIGURE 7



Let's consider a sector (Figure 7), in region (i), between x_i and $x_i + dx_i$. The number of customers in this sector is denoted by $\rho_i(x_i)dx_i$. In the same sector, the proportion of customers whose travel cost to the border is in the interval $[\tau_i, \tau_i + d\tau_i]$ is equal to:

$$\varphi_i(x_i, \tau_i)d\tau_i$$

Thus, in region (i), the total number of customers whose travel cost is within interval $[\tau_i, \tau_i + d\tau_i]$ is equal to the integral over the whole region $(i) \stackrel{\text{def}}{=} [0, +\infty)$ of the population density $\rho_i(x_i)$ times the travel time density $\varphi_i(x_i, \tau_i)$ in sector x_i :

(5) $$P_i(\tau_i)d\tau_i \stackrel{\text{def}}{=} d\tau_I \int_0^{+\infty} \rho_i(x_i)\varphi_i(x_i, \tau_i)dx_i$$

where:

(6) $$P_i(\tau_i) = \int_{(i)} \rho_i(x_i)\varphi_i(x_i, \tau_i)dx_i$$

is the *density of population with respect to travel-costs*.

Customers of region (i) who shop in region (j) satisfy equation (1):

$$\tau_i < t_i - t_j$$

Thus the number of customers of region (i) who shop in region (j), is equal to:

(7) $$\int_{-\infty}^{t_i - t_j} P_i(\tau_i)d\tau_i$$

or

(8) $$\int_0^{+\infty} \rho_i(x_i)\Phi_i(x_i, t_i - t_j)dx_i$$

since

$$\begin{aligned}\int_0^{t_i - t_j} P_i(\tau_i)d\tau_i &= \int_{-\infty}^{t_i - t_j} d\tau_i \int_{(i)} \rho_i(x_i)\varphi_i(x_i, \tau_i)dx_i \\ &= \int_{(i)} dx_i \rho_i(x_i) \int_{-\infty}^{t_i - t_j} d\tau_i \varphi_i(x_i, \tau_i)\end{aligned}$$

Let us recall that $\Phi(x_i, \tau_i)$ is the travel time distribution in region (i). $\Phi(x_i, \tau_i)$ is the proportion of customers at location x_i, whose travel cost is less than τ_i.

The number of customers in region (i) who buy locally is given by:

(9) $$\int_0^{+\infty} \rho_i(x_i)\Psi_i(x_i, t_i - t_j)dx_i$$

$\Psi_i(x_i, \tau_i) = 1 - \Phi_i(x_i, \tau_i)$ is the proportion of customers at location x_i, whose travel cost is greater than τ_i and who do not satisfy equation (1).

Alternatively, Equation (9) can be expressed as follows:

$$(10) \qquad \int_{t_i - t_j}^{\infty} P_i(\tau_i) d\tau_i$$

Consequently the number of customers of region (j) who buy the commodity in region (i) is given by:

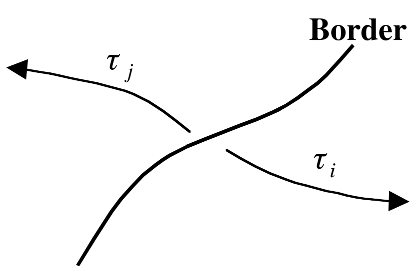
$$(11) \qquad \int_{-\infty}^{t_i - t_j} P_j(\tau_j) d\tau_j$$

Thus the total number of customers in region (i) (who pay tax t_i) is given by:

$$(12) \qquad N_i(t_i, t_j) = \int_{-\infty}^{t_j - t_i} P_j(\tau_j) d\tau_j + \int_{t_i - t_j}^{\infty} P_i(\tau_i) d\tau_i$$

Note that here τ_i and τ_j are defined as transportation costs of traveling to the border and are therefore associated with opposite directions. In principle both are positive variables. The negative values of τ_i and τ_j, imply that crossing the border for shopping may be an activity with positive utility for some customers, MOKHTARIAN [2001]. This type of customers might actually cross the border even if the commodity is more expensive on the other side of the border.

FIGURE 8



2.4 Modelling Consumer preferences

In this section the model is further developed taking into account consumer preferences. It should be noted that incorporating consumer preference does not change the structure of the model. It only enriches the model.

The customer's choice to consume locally or across the border can be described by a utility function:

$$-\theta_i \text{ cost}_i + \varepsilon_i$$

θ_i is a parameter, ε_i is an stochastic variable, and cost_i is the cost associated with alternatives of buying locally or buying across the border.

The stochastic variable ε_i represents,

– the variability due to travel costs,
– and/or the variability due to user's perception of travel costs,
– and/or the variability due to user's reaction to regional price differentiation.

The following assignments are made to distinguish costs:

$$\text{cost}_i = t_i \qquad \text{consumer buys locally}$$

$$\text{cost}_i = t_j + \tau_i \qquad \text{consumer buys across the border}$$

The probability of consumers of region (i) with travel cost τ_i buying commodity in region (j) is given by

$$F_i(\tau_i; t_i - t_j) \stackrel{\text{def}}{=} G_i(t_i - t_j - \tau_i)$$

$F_i(\tau_i; t_i - t_j)$ is the distribution function of the choice model.

The Logit distribution function is taken as an example to define $F_i(\tau_i; t_i - t_j)$:

(13)
$$\begin{aligned} F_i(\tau_i; t_i - t_j) = G_i(t_i - t_j - \tau_i) &\stackrel{\text{def}}{=} \frac{e^{-\theta_i(t_j+\tau_i)}}{e^{-\theta_i(t_j+\tau_i)} + e^{-\theta_, t_i}} \\ &= \frac{1}{1 + e^{-\theta_i(t_i - t_j - \tau_i)}} \end{aligned}$$

Other distribution functions can be used as well.

The number of customers of region (i) who shop in region (j) is given by:

(14)
$$\int P_i(\tau_i) F_i(\tau_i; t_i - t_j) d\tau_i$$

The interpretation is that there are $P_i(\tau_i)d\tau_i$ customers in region (i) with travel cost τ_i of which the fraction $F_i(\tau_i; t_i - t_j)$ of these customers shop in region (j).

Define $f_i(\tau_i; t) \stackrel{\text{def}}{=} \frac{\partial F_i}{\partial t}(\tau_i; t)$ as the density function associated with the distribution F_i.

The logit model version of $f_i(\tau_i; t)$ is given by:

(15)
$$f_i(\tau_i; t) = \theta_i \frac{e^{-\theta_i(t-\tau_i)}}{\left(1 + e^{-\theta_i(t-\tau_i)}\right)^2}$$

The number of customers of region (i) who shop in region (j) is given by Equation (14); it can also be evaluated by the following equation:

$$\begin{aligned}\int P_i(\tau_i)F_i(\tau_i; t_i - t_j)d\tau_i &= \int d\tau_i P_i(\tau_i) \int_{-\infty}^{t_i - t_j} f_i(\tau_i; t)dt \\ &= \int_{-\infty}^{t_i - t_j} dt \int P_i(\tau_i) f_i(\tau_i; t)d\tau_i\end{aligned}$$

Let us define:

$$\Sigma_i(t) \stackrel{\text{def}}{=} \int P_i(\tau_i) f_i(\tau_i; t)d\tau_i \tag{16}$$

Given the above transformation, the number of customers in region (i) who buy commodity in region (j), Equation (14), is re-written as:

$$\int_{-\infty}^{t_i - t_j} \Sigma_i(s)ds\,. \tag{17}$$

There exists a striking analogy between Equations (6), and (7), and Equations (16) and (17) .

Equation (16) is actually a convolution product which can be rewritten as:

$$\Sigma_i(t) \stackrel{\text{def}}{=} \int P_i(\tau_i) G_i'(t - \tau_i)d\tau_i$$

Contrary to transformation (6), which transforms distance to the border densities into travel cost to the border densities, transformation (16) does not yield densities relative to obvious physical variables. Still, the argument (t) in transformation (16) can be interpreted as the travel cost to the border corrected by consumer preference.

Equation (12), which gives the total number of consumers in region (i) (who pay tax t_i), can now be re-written as follows:

$$N_i(t_i, t_j) = \int_{-\infty}^{t_j - t_i} \Sigma_j(\zeta_j)d\zeta_j + \int_{t_i - t_j}^{\infty} \Sigma_i(\zeta_i)d\zeta_i \tag{18}$$

The arguments ζ_i and ζ_j must be considered positive, and should be interpreted as travel costs to the border corrected by consumer preference. In order to simplify expression (18) and to obtain arguments that span both negative and positive values, with 0 representing the border, the following transformation is introduced:

$$\begin{cases} \sigma_i(s) \stackrel{\text{def}}{=} \Sigma_i(s) \\ \sigma_j(s) \stackrel{\text{def}}{=} \Sigma_j(-s) \\ \sigma(s) \stackrel{\text{def}}{=} \sigma_i(s) + \sigma_j(s) \end{cases} \tag{19}$$

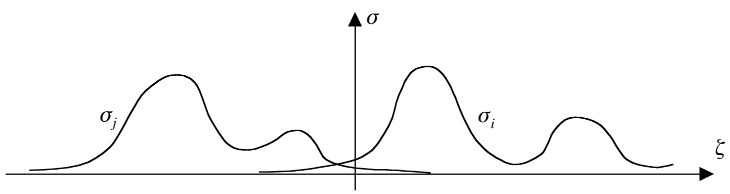
Now Equation (18) can be simplified as:

$$N_i(t_i, t_j) = \int_{t_i - t_j}^{\infty} \sigma(\zeta) d\zeta \tag{20}$$

A symmetric formula expresses the total number of customers in region (j) (who pay tax t_j):

$$N_i(t_i, t_j) = \int_{\infty}^{t_i - t_j} \sigma(\zeta) d\zeta \tag{21}$$

FIGURE 9



The summary of the modeling process is as follows:

1. Population density $\rho_i(x_i)dx_i$ is a function of distance to the border x_i.
2. Density of travel costs $\varphi_i(x_i, \tau_i)d\tau_i$ at distance x_i to the border is used to calculate the population density $P_i(\tau_i)d\tau_i$ with respect to the travel cost τ_i to the border (Equation (6)).
3. Density $f_i(\tau_i; t)dt$ of users whose travel cost to the border is τ_i takes into account individual preferences with respect to the variable t. The population density, $\Sigma_i(t)dt$, is calculated using equation (16). The variable t can be interpreted as the travel cost modified by the consumer preference.

It must be noted that densities ρ_i result from simple demographic data. Densities P_i result from transform (6). The required data is the travel cost distribution on the network, obtained by surveys, direct measurements and/or application of transportation planning models. Densities Σ_i are estimated by the convolution product (16). Parameters θ_i should be estimated before the convolution product (16) can be used. Transform (19) is purely formal and serves one purpose which is to simplify expressions (20) and (21). The resulting density $\sigma = \sigma_i + \sigma_j$ gives the population density that is corrected by effective travel costs in the network, and is smoothened by the consumer preference, Figure 9.

3 Modelling tax competition

3.1 Reaction functions

3.1.1 Introduction

In this section the aim is to find the outcome of tax competition between two regional governments. The borders are open, and tax competition is unrestricted. The approach adopted is to consider each government as a player in a game. The objective of the game for each player is to maximise its tax revenue. The strategies are local taxes, t_i, and t_j. The payoff is the local tax revenue, $R_i(t_i, t_j)$. The game is treated as a non-cooperative one of the NASH equilibrium, where each region is at its tax revenue optimum relative to the choice made by the other region.

Assume definitions (20) and (21) hold. Few hypotheses are made concerning density $\sigma = \sigma_i + \sigma_j$, except that it is positive, and has finite sum, which is the total population N of the two regions:

$$N = \int_{-\infty}^{\infty} \sigma(\zeta) d\zeta \tag{22}$$

Denote:

$$n(t) \stackrel{\text{def}}{=} \int_{-\infty}^{t} \sigma(\zeta) d\zeta \tag{23}$$

Expression (23) is positive, and increasing from 0 to N. N_i and N_j can be written using n:

$$\begin{cases} N_i(t_i, t_j) = N - n(t_i - t_j) \\ N_j(t_i, t_j) = n(t_i - t_j) \end{cases} \tag{24}$$

Tax revenue in region (i) is given by

$$R_i(t_i, t_j) = t_i \, N_i(t_i, t_j)$$

In the NASH equilibrium, the revenue of each region is maximized relative to the tax of that region. Thus the first order conditions for the equilibrium are given by:

$$\begin{cases} \dfrac{\partial}{\partial t_i} R_i(t_i, t_j) = 0 \quad i.e. \quad \dfrac{\partial}{\partial t_i}(t_i N_i) = 0 \\ \dfrac{\partial}{\partial t_j} R_j(t_i, t_j) = 0 \quad i.e. \quad \dfrac{\partial}{\partial t_j}(t_j N_j) = 0 \end{cases} \tag{25}$$

which yield the first order necessary conditions for the maximization of the revenue $R_i(t_i,t_j) = t_i\, N_i(t_i,t_j)$, of region (i). The first order conditions are not sufficient; therefore, it is necessary to add second order conditions, which will be done later. The first order condition, the system of equations (25) can be re-written as:

(26)
$$N_i(t_i,t_j) + t_i \frac{\partial N_i}{\partial t_i}(t_i,t_j) = 0$$
$$N_j(t_i,t_j) + t_j \frac{\partial N_j}{\partial t_j}(t_i,t_j) = 0$$

or

(27)
$$N - n(t_i - t_j) - t_i n'(t_i - t_j) = 0$$
$$n(t_i - t_j) - t_j n'(t_i - t_j) = 0$$

(28)
$$n(t) + t_i n'(t) = N$$
$$n(t) - t_j n'(t) = 0$$
$$t \stackrel{\text{def}}{=} t_i - t_j$$

The solution of this system (NASH equilibrium) is given later. The next section addresses the problem of determining *reaction curves*.

3.1.2 Defining reaction curves for region (i)

The system of equations (28) can be solved for t_i, where t_i is expressed as a function of parameter $t = t_i - t_j$:

$$t_i = \frac{N - n(t)}{\sigma(t)} \qquad (\text{since} \quad n'(t) = \sigma(t))$$

At this level of generality (no hypothesis is made on the form of (σ)), the reaction function cannot be expressed explicitly. Instead, it is possible to define a *reaction curve* for region (i).

t_j. can also be expressed as a function of parameter $t = t_i - t_j$:

$$t_j = t_i - t = \frac{N - n(t)}{\sigma(t)} - t$$

The expression of the *reaction curve* for region (i), $r_i\ (t)$, follows:

(29)
$$r_i(t) = \left\{ \begin{array}{ll} t_i & = \dfrac{N - n(t)}{\sigma(t)} \\ t_j & = \dfrac{-t\sigma(t) - n(t) + N}{\sigma(t)} \end{array} \right\}$$

Since the reaction curve is a parametric curve, it is necessary to determine the domain D_i of r_i. The constraints are:

$$t_i \geqslant 0\,; \quad t_j \geqslant 0\,; \left(\frac{\partial}{\partial t_i}\right)^2 R_i < 0$$

The last constraint indicates that the revenue R_i must be at its maximum, that is, it should be locally concave.

The condition $t_i \geqslant 0$ is trivial. The local concavity condition of the revenue R_i is shown below:

$$\begin{aligned} -\left(\frac{\partial}{\partial t_i}\right)^2 (t_i N_i) &= -\frac{\partial}{\partial t_i}(N - n(t_i - t_j) - t_i n'(t_i - t_j)) \\ &= 2n'(t_i - t_j) + t_i n''(t_i - t_j) \\ &= 2n'(t) + \frac{N - n(t)}{n'(t)} n''(t) \geqslant 0 \end{aligned}$$

The domain of r_i is:

$$D_i = \left\{ \begin{array}{l} t \mid -tn'(t) - n(t) + N \geqslant 0 \\ \text{and} \\ 2n'(t)^2 + (N - n(t))n''(t) \geqslant 0 \\ \text{remember that} \\ n'(t) = \sigma(t) \end{array} \right\} \tag{30}$$

It is possible (see subsection 3.3 for examples) for R_i not to be defined on some intervals, notably if t is large and $n' = \sigma(t) > 0$, in which case ($t_j < 0$) or if $n''(t) << 0$ (in which case the concavity condition is not satisfied). There is a very small chance for a closed expression relating to t_i and t_j to exist, that would be equivalent to equations (29) and (30): the expression $t_j = -t + \dfrac{N - t\sigma(t) - n'(t)}{\sigma(t)}$ is unlikely to be invertible, especially with domain (30).

Let's interpret conditions (30) that define domain D_i. The first condition means that t_j is positive. The derivative with respect to t_j of t in expression (29) is given by:

$$\frac{dt_j}{dt} = -\frac{2n'(t)^2 + n''(t)(N - n(t))}{n'(t)^2}.$$

The second condition means that t_j is a decreasing function of t. *Thus (30) means that t_j is a positive decreasing function of t.*

This condition can be visually checked by looking at the graph of the reaction curve (see examples in subsection 3.3).

3.1.3 **Defining reaction curves for region** (j)

The reaction curve for region (j) can be calculated the same way as for region (i). The first order necessary condition is derived for region (j) as follows:

$$\frac{\partial}{\partial t_j}(t_j N_j(t_i - t_j)) = 0$$

$$n(t_i - t_j) - t_j n'(t_i - t_j) = 0$$

$$t_j = \frac{n(t)}{\sigma(t)} \quad \text{and} \quad \left\{ \begin{array}{l} t = t_i - t_j \\ \sigma(t) = n'(t) \end{array} \right\}$$

and $\left(t_i = t + t_j \, ; \, t_i = t + \dfrac{n(t)}{\sigma(t)} \right)$. The reaction function t_j as function of t_i is expressed as a parameterized curve r_j:

(31)
$$r_j(t) = \left\{ \begin{array}{lcl} t_i & = & \dfrac{t\sigma(t) + n(t)}{\sigma(t)} \\ t_j & = & \dfrac{n(t)}{\sigma(t)} \end{array} \right\}$$

To determine the domain of r_j, the conditions to be satisfied are:

$$t_i \geqslant 0 \; ; \; t_j \geqslant 0 \; ; \; \left(\frac{\partial}{\partial t_j} \right)^2 (t_j - t_j N_j) < 0$$

The last expression is the second order sufficient optimality condition. The condition $t_j \geqslant 0$ is trivial. The condition $t_i \geqslant 0$ is equivalent to $n(t) + t\sigma(t) \geqslant 0$, which may be violated for $t << 0$ and $\sigma(t) > 0$. The condition $t_i \geqslant 0$ is only satisfied, if the transportation cost, *i.e.*, the value of (t), or $n(t)$ is small, and population density modified by consumer preference, and transport costs, $\sigma(t)$ is large. The second order optimality condition for $t_j N_j$ can be expressed as:

$$\begin{aligned} -\left(\frac{\partial}{\partial t_j} \right)^2 (t_j N_j) &= -\frac{\partial}{\partial t_j}(n(t_i - t_j) - t_j n'(t_i - t_j)) \\ &= 2n'(t_i - t_j) - t_j n''(t_i - t_j) \\ &= 2n'(t) - \frac{n(t)}{n'(t)} n''(t) \\ &= \frac{2n'(t)^2 - n(t)n''(t)}{n'(t)} \geqslant 0 \;\; i.e. \; 2n'(t)^2 - n(t)n''(t) \geqslant 0 \end{aligned}$$

This second order optimality condition may not be satisfied for large (n), if t increases, $n''(t) >> 0$, and the density $\sigma(t)$ increases. Finally, *the domain of* r_j, D_j , is given by:

$$D_j = \left\{ \begin{array}{ll} t| \quad n(t) - t\sigma(t) & \geqslant 0 \\ \text{and} & \\ 2n'(t)^2 - n(t)n''(t) & \geqslant 0 \end{array} \right\} \tag{32}$$

Let's interpret conditions (32) that define domain D_j. The first condition means that t_i is positive. The derivative of t_i with respect of t in (31) is given by:

$$\frac{dt_i}{dt} = \frac{2n'(t)^2 - n''(t)n(t)}{n'(t)^2}.$$

The second condition means that t_i is an increasing function of t. *Thus* equation (32) *shows that t_i is a positive increasing function of t.*

This condition can be visually verified on the graph of the reaction curve (see section 3.3).

3.2 NASH Equilibrium

In the NASH equilibrium system of equations (28):

$$\begin{cases} n(t) + t_i n'(t) = N \\ n(t) - t_j n'(t) = 0 \\ t \overset{\text{def}}{=} t_i - t_j \end{cases}$$

by adding the first two equations in the system above, and using $n' = \sigma$, it follows that:

$$N = 2n(t) + t\sigma(t) \tag{33}$$

This is the NASH equilibrium equation yielding the optimal value of t, (t^*); if and only if (t^*) is in $D_i \cap D_j$, *i.e.*

$$N - n(t^*) \geqslant t^*\sigma(t^*) \geqslant -n(t^*)$$

$$\text{and}$$

$$\frac{-2\sigma(t^*)^2}{N - n(t^*)} \leqslant n''(t^*) \leqslant \frac{2\sigma(t^*)^2}{n(t^*)}$$

Thus, not all solutions of (33) are acceptable. The acceptability of the NASH equilibrium solution depends on whether the equilibrium solution is stable or not. The stability of the NASH equilibrium solution is not explored in this paper. Taking into account that σ is a density with finite sum N, *i.e.* $n(t) \overset{\text{def}}{=} \int_{-\infty}^{t} \sigma(s)ds$ it follows that:

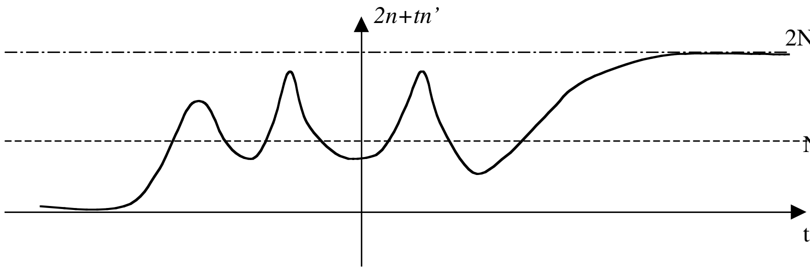
- $n(t)$ is continuous and differentiable,
- $n(t)$ is increasing,
- $n(-\infty) = 0$,
- $n(\infty) = N$.

Consider the right hand side of function (33):

$$2n(t) + tn'(t)$$

This function is continuous, and is equal to 0 if $t = -\infty$, and is equal to $2N$ if $t = +\infty$. Figure 10 depicts the form of the function above.

FIGURE 10

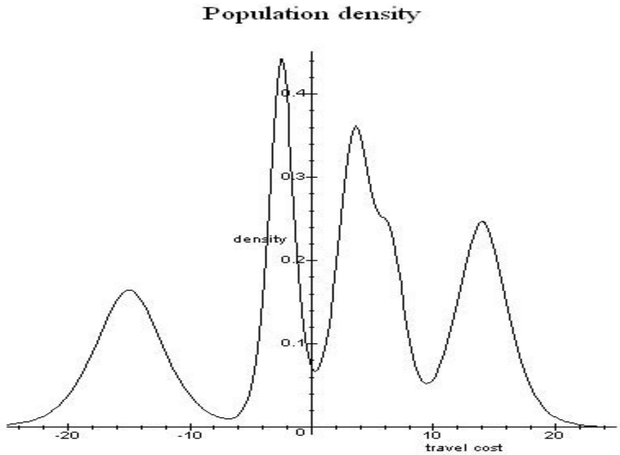


There is at least one intersection with the axis N; there may be many intersection points as is shown above. The implication is that there are possible multi-NASH equilibrium points; given that the equilibrium points fall in $D_i \cap D_j$. The oscillations of $2n(t) + tn'(t)$ are related to the density of population clusters. If travel costs are highly related to the distance from the border, as is the case in most population centers, tn' has a large value either positive or negative.

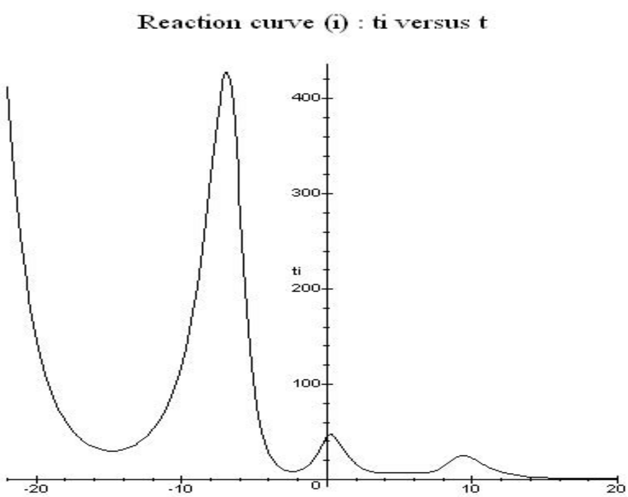
It is shown that in the general case where the transportation cost is a function of travel time and consumer preferences are incorporated, several possibilities arise: 1) either there is a unique NASH equilibrium point, or 2) there are multi-NASH equilibrium points, or 3) there is no NASH equilibrium point. The multiplicity of equilibrium points creates a problem for the players. The problem is that both players will choose the more profitable policy, and split the market both receiving lower payoffs. Therefore, the problem is which equilibrium is the right one to choose. To solve this problem it is necessary to coordinate the choices of strategies and avoid the danger of mutually inferior outcomes. The solution as Thomas SCHELLING (1970) proposed is any piece of information that the players in a coordination game would have that enables them to focus on the same equilibrium. The economic implications of this are explored in section 4.

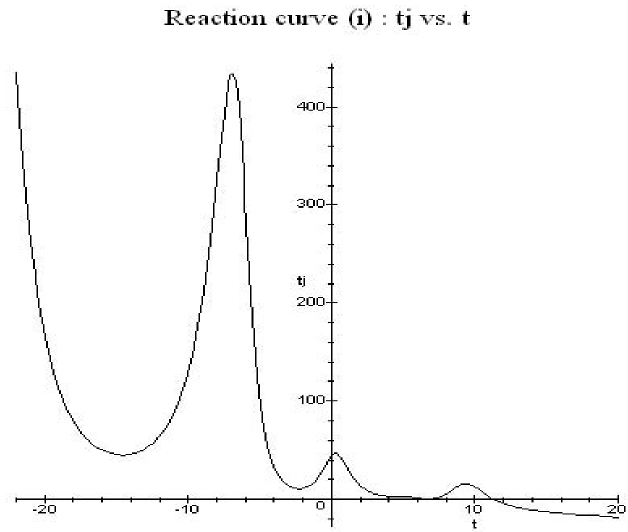
3.3 Numerical Examples

The following numerical examples are calculated using Maple. First, the following population density is considered: Both regions have two major population clusters with two density distributions.

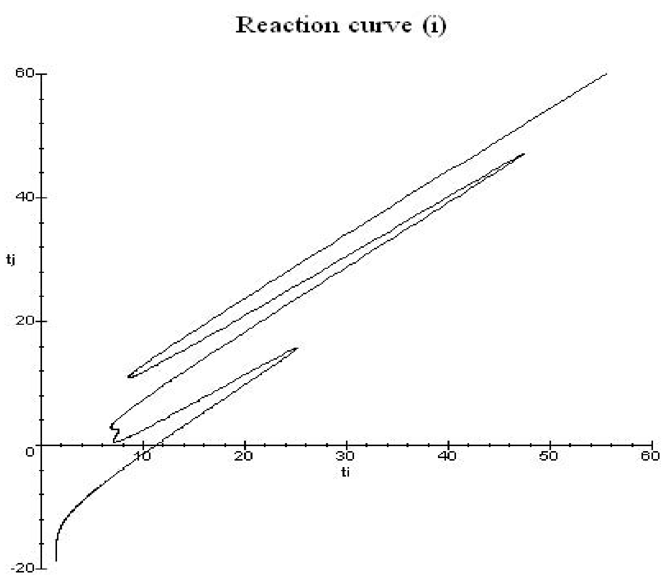


The resulting reaction function for region (i) (on the right hand side) is described by it's two coordinates t *versus* t_i, and t *versus* t_j, which are depicted below:



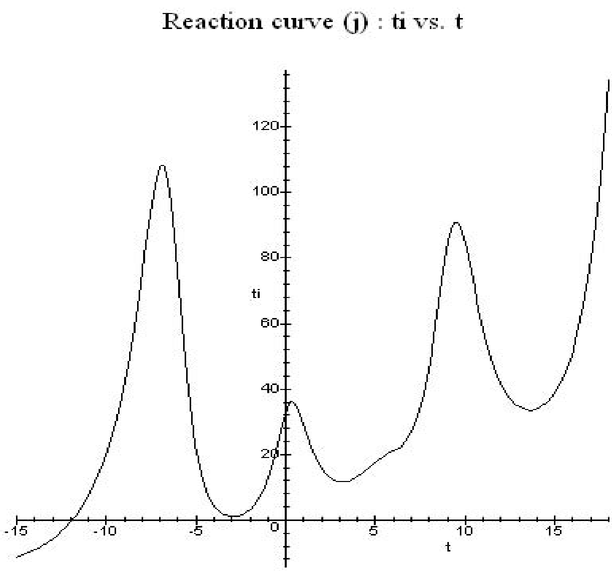


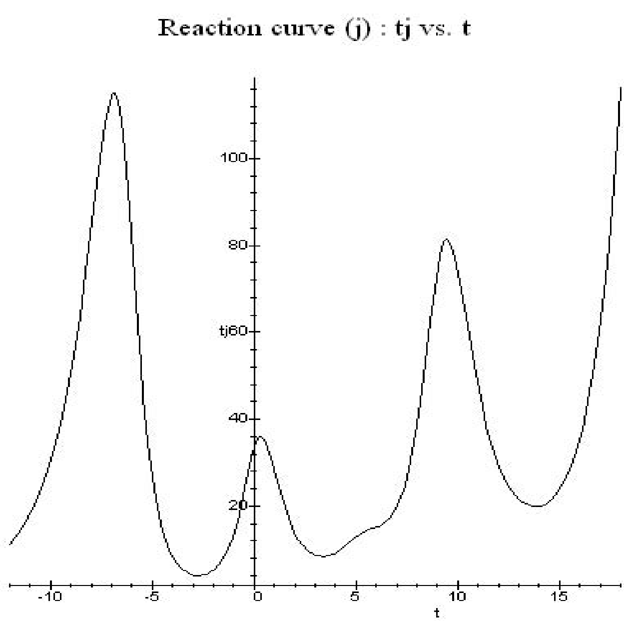
The resulting reaction function is multi-valued and is described below:



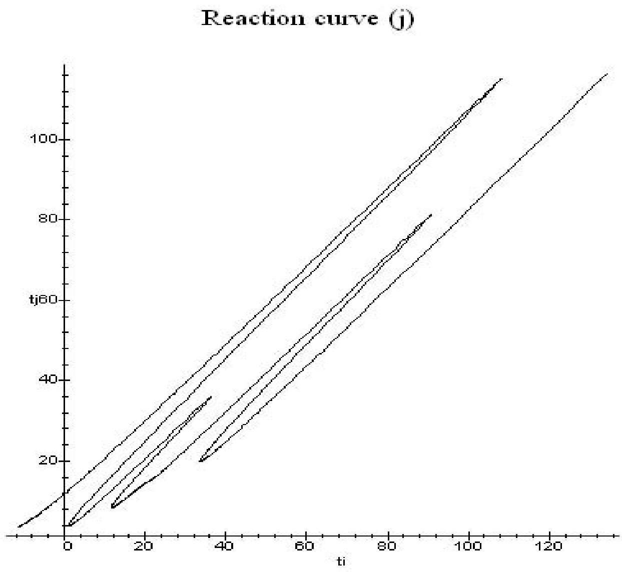
Only those values of t_i and t_j, which fall in the interval [0,20] are relevant, still it is interesting to have a global view of the reaction function.

The reaction function for region (i) (on the right hand side) is described by it's two coordinates t_i and t_j, which are functions of t, and are depicted hereafter:





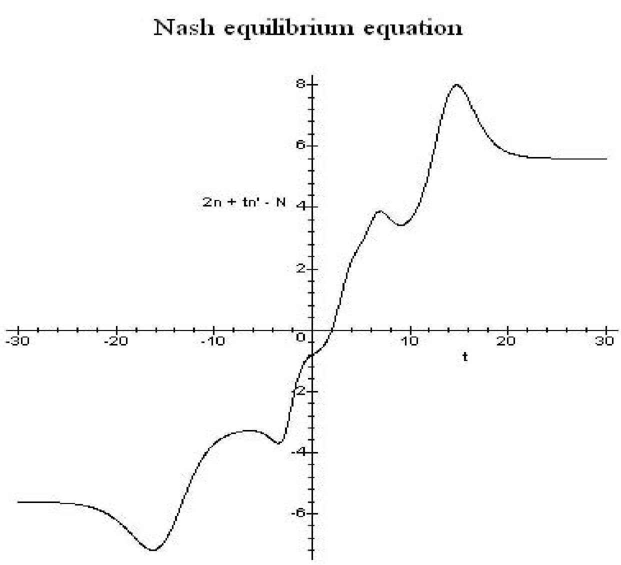
In the resulting reaction function for each value t_i there are up to three values of t_j. The multi-valued reaction function is shown below:



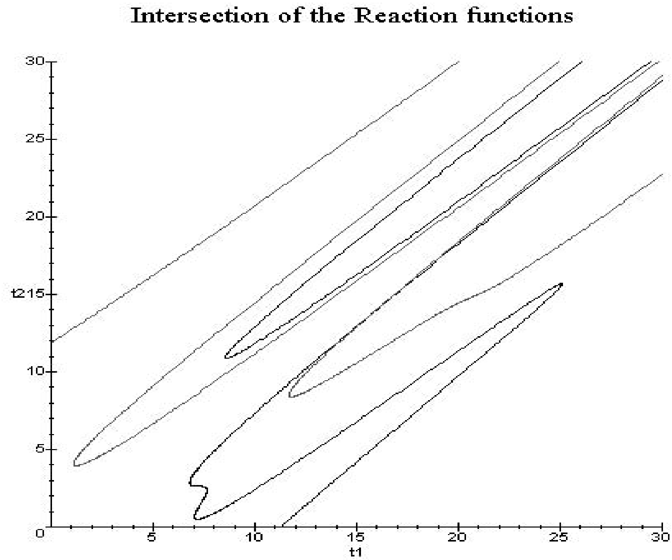
The equation yielding the NASH equilibrium points can be written as:

$$2n(t) + tn'(t) - N = 0$$

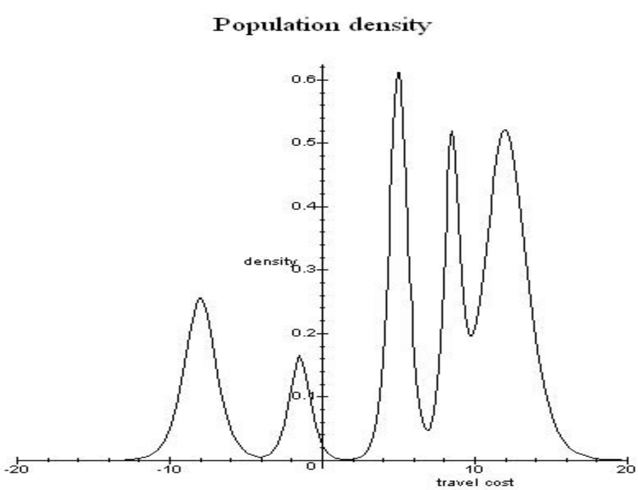
the left hand side is plotted below against t :



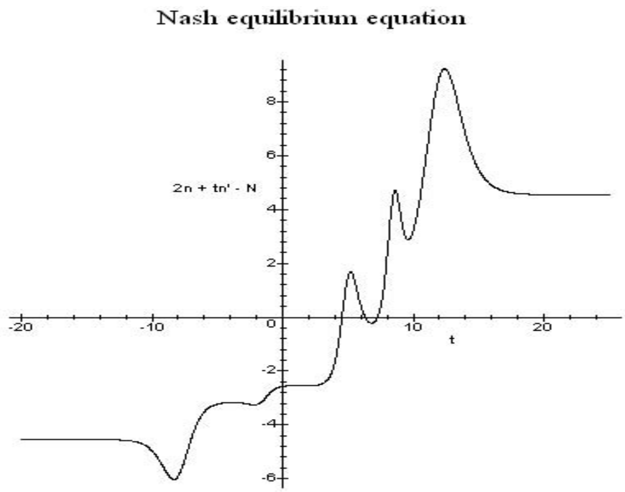
From the graph above it is clear that there is only one intersection point with the t axis. This unique solution does not correspond to the NASH equilibrium, since condition (D_j) is not satisfied.



The same intersection point is depicted above as the intersection of two reaction functions of t_i and t_j. The solution of the equation that yields NASH equilibrium points can admit several solutions. Consider the density below:



The left hand side of the equation yielding NASH equilibrium points, $2n(t) + tn'(t) - N = 0$, is plotted below against t. There are three intersection points, none of which correspond to the NASH equilibrium. Still, one of the intersection points nearly satisfies conditions (D_i) and (D_j), and would be a good candidate for a cooperative solution.

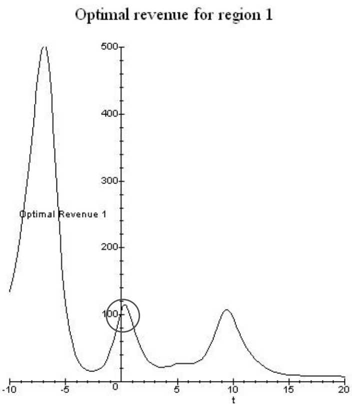


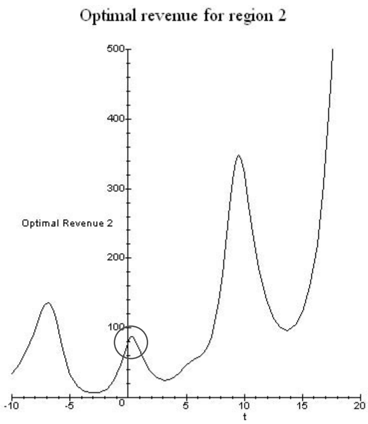
4 Properties of the NASH equilibrium

PROPOSITION 1: *The existence of multiple* NASH *equilibrium points suggests that the best strategy is tax harmonization.*

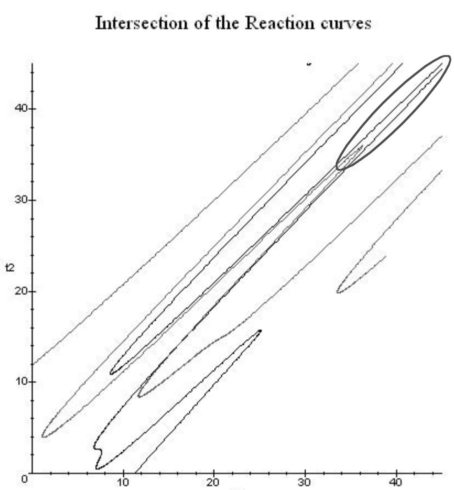
It has been shown that the NASH equilibrium points only exist where the domains, D_j of R_i , and D_j, of R_j, intersect. The characteristic of these regions depends on the slope of the population densities of clusters, transportation costs, and the degree to which consumer preferences modify multiple effects of density and travel cost. In other words, it requires a special configuration of population density, and transportation cost to make sure that NASH equilibrium points exist. The best way to assure the local concavity of the revenue function is to set coordinated tax rates that are in the definition domains D_i and D_j. To avoid the danger of choosing a mutually inferior outcome, the most common method would be to choose a PARETO efficient fiscal choice. That is to choose a pair of regional feasible fiscal choices $[((t_i^*, R_i(t_i^*)), (t_j^*, R_j(t_j^*))]$ such that there exists no alternative such pair for which the respective welfare functions of regions (i) and (j) can be improved.

As an illustration, let's consider the first example of subsection 3.3. If the optimal revenue of each region is plotted against t, one would notice that both optimal revenues admit a local maximum close to $t = 0$ (red circles).



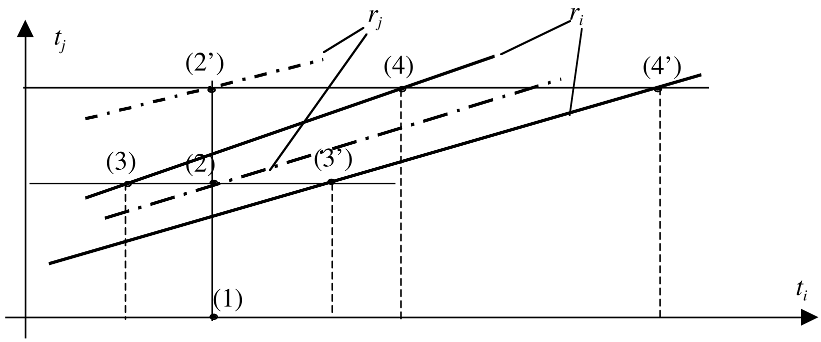


In the figure below, two regions reaction curves or functions are plotted. It can be verified that values of the two regions taxes, t_1 and t_2 are close to 40, and they correspond to an area in which the reaction curves are very close (red ellipse). These points could be potential tax coordination points.



PROPOSITION 2: *The existence of multiple valued reaction functions suggests the possibility of no* NASH *equilibrium points; thus the best strategy is tax harmonization.*

Consider the graph below:

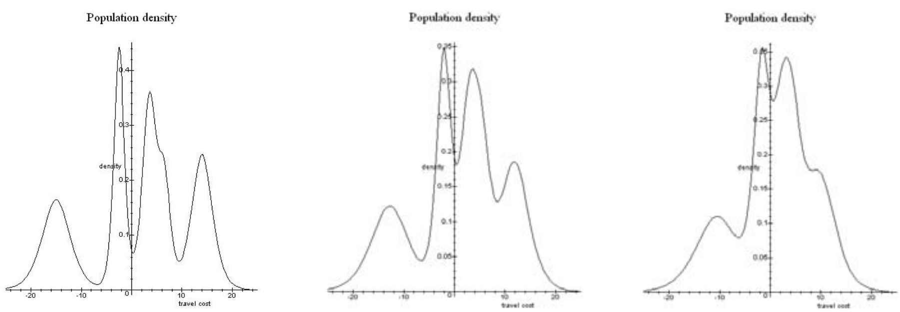


There are two branches for each of the reaction functions, (r_i, r_j). Assume that player (i) sets the tax at level 1 (point (1) on the graph). Player (j) has two choices of points: (2) and (2'). Given the reaction of player (j), player (i) will choose between {(3), (3')} or {(4), (4')}. This interaction will continue indefinitely, which implies the players will never reach equilibrium even if the reaction functions intersect. Therefore an arbitrary equilibrium point has to be set, which leads to the tax harmonization strategy.

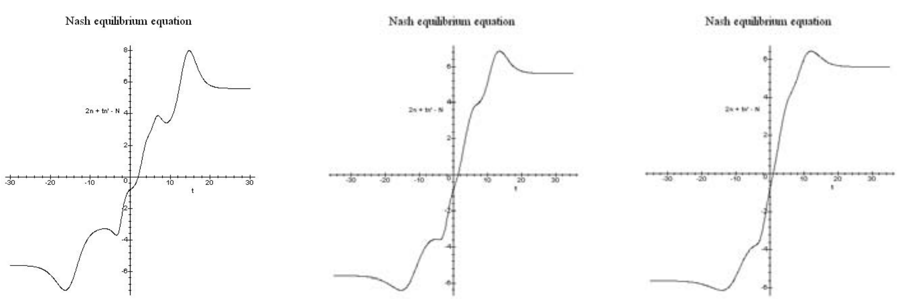
PROPOSITION 3: *In the long run the definition domains D_i, and D_j improve as a positive correlation between the population density of the clusters and the transportation costs of traveling to the border is established. Thus the stability and the superiority of the* NASH *Equilibrium points improve accordingly.*

In the short run the region that has not achieved equilibrium between population clusters and transportation costs, experiences high transportation costs. This reduces the magnitude of the (Logit) probability density, $f_i(\tau_i; t)$, of shopping across the border for the consumers in region (j) due to a high transportation cost in the region (i), and the consumers in region (i) have no incentive to go across the border due to the same high transportation costs. This implies that both regions can entirely ignore each other in setting their tax rates: there can be no tax induced border shopping, and the assumption of a constant producer price precludes any indirect interaction through terms-of-trade effects. The nature of the closed border optimum is immediate: each region will extract all the surplus of its own citizens by setting its tax at the level of their reservation price; *i.e.* $(t_c^* = v, t_c^* = V)$, asterisks indicating optimality and subscript c denoting closed border, KANBUR-KEEN [1993]. This is in a direct violation of tax coordination. It excludes any set of stable and superior coordinated strategies. Since closed border tax rates will be superior to the coordinated tax rates, any coordination will not be PARETO optimal. Therefore, in the short run any coordinated tax rate will be an unstable and inferior NASH equilibrium point. Whereas, in the long run, the expectations on the improvement of disparity between population density and transportation costs will force convergence to one of the equilibrium points, BISCHI-KOPEL [2000].

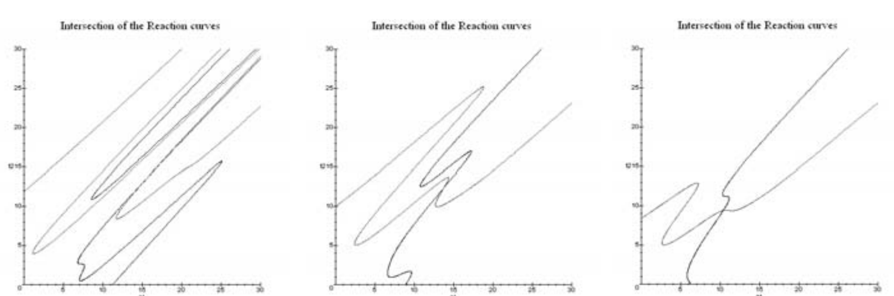
As an illustration, take the example of section 3.3. Consider three situations in which the population density ρ_i and ρ_j stay unchanged while travel costs decrease in two steps. Travel costs decrease from left to right. Decrease in travel costs affects densities Σ_i, Σ_j, and σ. The density σ is plotted for each level of travel costs. As can be seen from the first set of graphs, travel costs have a considerable impact on the density σ.



In the second set of graphs, the NASH equilibrium converges to a stable point as travel costs decrease, as is shown below:



The intersection of the reaction curves shows definite improvement, seen below. The right-most graph (lowest travel costs) the intersection point is the NASH equilibrium. The trend points to the stabilizing effect of lower costs.



PROPOSITION 4: *In equilibrium tax revenues depend on the density of population clusters, and on the travel costs. Therefore, both countries stand to gain in a non-cooperative tax competition if and only if they both have similar population cluster densities irrespective of their sizes and the travel costs to the border are at a reasonable level irrespective of distances.*

The intuition is straightforward. The revenue maximising comes from setting commodity tax rate equal to the reciprocal of the elasticity of demand. In this model, the elasticity of demand does not only come from cross – border shopping. As can be seen from the general form of the optimal NASH equilibrium solutions, it is the consumer preference that determines the probability of shopping across the border. This probability depends on the consumer utility; which is a function of the cost of travel to the border. This cost depends on travel time. Therefore, the conditions of traffic have a significant impact on the magnitude of the Logit probability of crossing the border. Starting then from a position in which $t_i = t_j$, the increase in demand that each government tries to induce by cutting its tax rate depends on the density of its population, and on the conditions of traffic on the roads. Thus, it is the country with better transportation facilities that undercuts the other country. Two countries can either have the same cluster density distribution or different ones. What propels the consumer to shop across the border is not just the price differentiation, but also easier accessibility to the border. Those regions with high cluster densities and inferior transportation facilities necessarily experience a higher transport cost and therefore, derive much lower utility in driving to the border for shopping. Both regions can benefit from opening borders, even in a competition if the transportation cost is kept at a reasonable level in both of the regions.

5 Conclusion

In this paper three major ideas were explored: cluster population density, composite transportation costs, and consumer preference. The introduction of the three elements is an extension of the idea of two regions with different population sizes introduced by KANBUR-KEEN [1993]. The aim was to explore the impacts of population density, a more complicated transportation cost function, and the impact of consumer preference through the introduction of a stochastic (Logit) choice model, on the likeliness of making cross border shopping trips, on the outcome of the NASH equilibrium, its existence and its uniqueness. Theoretical results show that in a case of this type the uniqueness of the NASH equilibrium is no longer guaranteed. There may be one or multiple or zero NASH equilibrium points. The conditions of acceptability depend on whether NASH equilibrium points fall in the intersection of the domains of the reaction functions.

Two numerical examples were worked out to illustrate theoretical ideas that were introduced. The interesting point in each example is that the reaction functions do not show a routine form which usually contains jumps, but rather, the reaction functions are multi-branch curves. This is significant in that it implies that for each choice of one player, the other player can have several choices. The only possible NASH equilibrium is the one or ones that are reached through a coordinated strategy.

The economic implications of the findings are that given the existence of multiple NASH equilibrium points, the strategy in the tax competition situation is to reach a tax coordination that coincides with one of the possible NASH equilibrium points in the set of possible points. How this coordinated tax rate should be chosen depends on the PARETO objectives of revenue maximization, and the stability of the NASH equilibrium points. It is concluded that only those NASH equilibrium points are stable that occur in regions where any evolution in the population density is followed by an evolution in the transportation system, that is, improved transportation costs. It is also concluded that if such a link does not exist, in the short run, no superior coordinated tax rate can be obtained. It is only in the long run that due to expectations from improving the link between population density and transportation costs that a convergence to a stable NASH equilibrium point can be expected.

In this paper the problem of the stability of multiple NASH equilibrium points is not explored. To solve this problem the frame of research should be changed to reflect the time dependent behavior of the two regions with respect to the development of population clusters, and transportation facilities that provide accessibility and connectibility from the clusters to the border. The better transportation network translates into better accessibility and therefore, lowers transportation costs, since fewer traffic snags can be expected.

Acknowledgements

The author would like to thank Professor Stef PROOST and Professor Bruno DE BORGER of the Catholic university of Leuven for their helpful comments. The author would also like to extend her thanks to Professor Jean-Patrick LEBACQUE of the ENPC (École Nationale des Ponts et Chausses) for his constructive and helpful comments and advice. ▼

• References

BISCHI G., KOPEL M. (2001). – « Equilibrium selection in a nonlinear duopoly game with adaptive expectations », *Journal of Economic Behavior & Organization 46*, p. 73-100.

BUCOVETSKY B. (1991). – « Asymmetric tax competition », *Journal of Urban Economics 30*, p. 167-181.

HOTELLING H. (1929). – « Stability in competition », *Economic journal 39*, p. 40-57.

HOYT W.H. (1993). – « Tax competition, NASH equilibria and residential mobility », *Journal of Urban Economics 34*, p. 358-379.

KANBUR R., KEEN M. (1993). – « Tax competition and tax coordination when countries differ in size », *American economic review 83*, p. 877-892.

LOCKWOOD B. (1993). – « Commodity tax competition under destination and origin principles », *Journal of Public Economics 52*, p. 141-162.

MINTZ J., TULKENS H. (1986). – « Commodity tax competition between member states of a federation: equilibrium and efficiency », *Journal of Public Economics 29*, p. 133-172.

MOKHTARIAN P., SALMON I. (2001). – « How derived is the demand for travel? Some conceptual and measurement considerations », *Transportation Research Part A 3*5, p. 695-719.

OHSAWA Y. (1994). – « Spatial competition models among local governments », *Regional Science and Urban Economics 29*, p. 33-51.

SCHELLING T.C. (1970). – « The strategy of conflict », *Harvard university press*.

WILDASIN D.E. (1988). – « NASH equilibria in models of fiscal competition », *Journal of Public Economics 35*, p. 229-240.

ZODROW G.R., MIESZKOWSKI P. (1986). – « Pigou, Tiebout property taxation and underprovision of local public goods », *Journal of Urban Economics 19*, p. 356-370.