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Is fiscal cooperation always sustainable when regions differ in size? Lessons for the EMU

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ABSTRACT. – This paper investigates the sustainability of fiscal co-ordination between two regions according to three criteria of co-ordination: tax harmonization, fiscal cooperation and bargaining. We show that in a two-region model in which regions differ in size, bargaining is the only sustainable co-ordination policy when trigger types strategies are implemented. The sustainability of cooperative fiscal policy or tax harmonization depends on the degree of difference in regions' size. When this difference is large, such co-ordinated policies are not sustainable.

La coopération fiscale est-elle toujours soutenable quand les régions diffèrent par leur taille ? Leçons pour L'UEM

RÉSUMÉ. – Cet article est consacré à la soutenabilité de la coordination fiscale entre deux régions selon trois critères de coordination : l'harmonisation fiscale, la coopération fiscale et la négociation. Nous montrons que dans un modèle à deux régions différant par leur taille, la négociation est la seule politique de coordination stable quand des stratégies de menace sont mises en place. La soutenabilité des deux autres formes de coordination dépend du degré d'asymétrie dans la taille des régions : elle nécessite une différence de taille relativement faible.

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1 Introduction

High degree of integration combined with increasing capital mobility between countries imply an elevated level of fiscal competition between governments. The diversity of tax systems and more particularly inside EMU allows households to take advantages of the localization of their investments. In order to limit tax evasion, the EMU ECOFIN council has reached in 2000 an agreement concerning capital taxation but decisions are still limited. The agreement concerns the exchange of informations as well as a level of a minimum tax rate but does not include all of the EMU countries. Luxembourg, Belgium and Austria require more guarantees, especially from the USA, to sign the agreement. Why coordination does not still seem the best response to the tax evasion problem? This paper tries to explain why capital tax coordination is not applied and to rank different cooperative strategies in response to the problems raised by opening borders.

The literature about fiscal competition is rather important and can be divided into two types of models. The first, in the tradition of RAZIN and SADKA [1991], analyzes a two-country economy when both countries are perfectly symmetric. This literature focuses on the ability for governments to opt for non zero tax rates. The introduction of capital mobility constraint (GORDON [1992]), imperfect information sharing (BACCHETTA & ALI [1995])… are among the arguments developed in this literature. Without any particular assumption, tax harmonization is always optimal when compared to the non cooperative equilibrium since it enables governments to set non zero capital tax and improves households' welfare.

The second type of literature was initiated by ZODROW and MIESZKOWSKI [1986]. They wonder if tax competition is always sub-optimal when compared to the tax harmonization solution. In this literature, three different ideas have been developed: the choice of a tax system, the influence of multiple periods and the asymmetry between regions.

BUCOVETSKY [1991] and WILSON [1991] wonder if every proportional tax implies the same negative consequences from distortionnary taxation effect. When regions are perfectly symmetric, the reallocation between capital tax rates and labor tax rates can yield a second best equilibrium in the non-cooperative case while the social planner can not use the labor tax rate.

The dynamic dimension introduced by JENSEN and THOMAS [1991], LEE [1997] and COATES [1993] enables them to analyze whether the introduction of two periods improves or reduces fiscal competition. This two periods scheme brings the opportunity for governments to borrow (JENSEN and THOMAS [1991]), to reduce capital mobility with adjustment costs (LEE [1997]) and to take account of the previous tax rate in the current decision (COATES [1991]). JENSEN and THOMAS show that fiscal competition is then enforced when one of the regions finances its public expenditures by borrowing if there exist strategic substituabilities between regions. The opposite result emerges when there exist strategic complementarities. LEE [1997] studies the introduction of dynamics when regions are similar and COATES [1991] analyses the impact of infinite repetitions of fiscal competition. When governments take account of the previous tax rates to determine the current tax rate, optimal solution is to subside capital. This result is biased by the existence of lump sum taxation on the immobile factor.

While ZODROW and MIESZKOWSKI [1986] analyzed a multiple-region economy, BUCOVETSKY [1991] and WILSON [1991] both studied a two-region model. Regions can differ in size but capital per capita is identical in each different region. When regions differ in size, one government can benefit from manipulating the tax rate, the capital reporting between regions and by consequence the citizens' welfare. More recently, HWANG and CHOE [1995] consider two regions with asymmetric populations and asymmetric capital endowments. The choice of optimal taxation depends on the elasticity of marginal capital productivity and capital patrimony that population of each region gets. A rich small region chooses a lower tax rate than the poor one whereas a poor small region wants a tax rate higher or lower than the other one. The equilibrium tax rate depends on two factors (elasticity and endowments).

This article associates two of the above mentioned ideas: asymmetry and dynamics. The purpose of the paper is double. Taking the closed borders equilibrium as the benchmark case, the first task is to evaluate different political strategies in response to the tax evasion problem raised by opening borders. The second angle is to cope with a **sustainable** solution to tax evasion problem i) when only current revenues are considered ii) when trigger strategies are introduced. Three types of co-ordination are explored: tax harmonization, fiscal cooperation and bargaining.[1] Tax harmonization is the most appropriated co-ordinated type to analyze the EMU ECOFIN agreement presented above. Unfortunately, we will show that such a co-ordinated policy is not sustainable. This leads to wonder which kind of coordination would then succeed. That is the reason why the cooperative solution is analyzed in a second step. The results show that the sustainability of a cooperative policy is limited. Finally, the bargaining solution is studied. It corresponds to the particular solution of the cooperative problem which always allows sustainability of policy co-ordination. Nevertheless, a numerical exercise over EMU shows that bargaining is difficult to apply in Europe.

The remainder of the paper is organized as follows. Section 2 describes the model. Section 3 presents the Nash equilibrium. The tax harmonization case is analyzed in section 4, cooperation in section 5 and bargaining in section 6. Each co-ordination type is studied through a one shot and a repeated game. Section 7 presents a numerical exercise and section 8 concludes.

2 The Model

The economy is described by a partial-equilibrium model with two regions indexed by subscript i and j. Within each region, population is uniformly

1. A common tax rate fixed by a central planner refers to tax harmonization. Fiscal cooperation consists in maximizing a weighted joint objective. Bargaining refers to the Nash solution.

distributed but both populations may differ in size. There are h_i households in region i and h_j in region j. $\theta_t = \frac{h_i}{h_j}$ captures the relative size between population i and population j. Without loss of generality, we assume for the rest of the article that $\theta < 1$: region i is then less populated than region j. The return of storage technology is identical in both regions and equal to r. Taxes are levied according to the source principle in the sense that region i resident is taxed by government i on her domestic investments while she is taxed by government j on her foreign investments. Each household is endowed by one unit of capital that she invests either in her home region or abroad.

If household i invests in her region, the net return from investment is:

$$r\,(1 - \tau_{it})$$

where τ_{it} represents the tax rate levied by region i at time t. Investing abroad induces an information cost δ per unit of information.[2] We assume that $r > \delta$ the cost per unit of information is smaller than the interest rate per unit of invested capital. However households differ in the exogenous level of information (s_i) they need to invest abroad. s_i is uniformly distributed over $[0,1]$. If the required information is s, the gain from investing abroad is:

$$r\left(1 - \tau_{jt}\right) - \delta s$$

If the necessary information to be able to invest abroad is s, household i invests abroad if and only if the two following conditions are verified:

$$r\left(1 - \tau_{jt}\right) - \delta s > r\,(1 - \tau_{it})$$

or equivalently

$$\frac{r\left(\tau_{it} - \tau_{jt}\right)}{\delta} > s$$

and

$$r\left(1 - \tau_{jt}\right) - \delta s > 0$$

We assume each government to be Leviathan. Their objective is to maximize their intertemporal tax revenue $\sum_{t=0}^{\infty} \gamma^t W_{it}$ where γ represents the intertemporal discount factor and W_{it} the current tax revenue.

Current region i government revenue is then:

$$(1) \qquad W_{it}\left(\tau_{it}, \tau_{jt}\right) = \begin{cases} r\tau_{it}h_{it}\left(1 - \frac{r}{\delta}\left(\tau_{it} - \tau_{jt}\right)\right) & \text{if} \quad \tau_{it} \geqslant \tau_{jt} \\ r\left(\tau_{it}h_{it} + \tau_{it}h_{jt}\frac{r}{\delta}\left(\tau_{jt} - \tau_{it}\right)\right) & \text{if} \quad \tau_{it} \leqslant \tau_{jt} \end{cases}$$

2. See PERSSON & TABELLINI [1992] for an interpretation of the sunk cost.

Every region i resident getting less than an amount $\frac{r\left(\tau_{it}-\tau_{jt}\right)}{\delta}$ of information invests in region i while other residents getting more than $\frac{r\left(\tau_{it}-\tau_{jt}\right)}{\delta}$ of information invest abroad. Note that if both regions fix an identical level of tax rate, there are no capital flows at the equilibrium since the cost of information diminishes the net return rate of capital when citizens invest abroad.

When borders between regions are closed, both governments can entirely ignore each other setting their tax rates. The nature of the "closed borders optimum" is then immediate: governments extract the capital income of their own citizens by setting their tax rates to 1. Revenues are then $W_i^{CB}=rh_i$ and $W_j^{CB}=rh_j$. This result is now considered as the benchmark case to evaluate different political strategies (competition, harmonization, cooperation,…) when borders are open.

3 The Nash Equilibrium

This section examines the outcome when borders are open and tax competition is unrestricted. More precisely, each region chooses its tax rate and takes the other government policy as given. This leads to a Nash equilibrium. To avoid any inconsistency problem, each government is supposed to be committed at time $t+1$ to policies announced at time t.

For $\tau_{it}>\tau_{jt}$, maximization of region i revenue (1) gives:

$$\tau_{it}=\frac{1}{2}\left(\frac{\delta}{r}+\tau_{jt}\right)$$

and $\tau_{it}>\tau_{jt}$ requires $\tau_{jt}<\frac{\delta}{r}$, while $\tau_{it}<1$ requires $\tau_{jt}<2-\frac{\delta}{r}$. This previous condition is always verified since by assumption $r>\delta$ that yields $2-\frac{\delta}{r}>1$.

For $\tau_{it}<\tau_{jt}$, maximization of region i revenue gives:

$$\tau_{it}=\frac{1}{2}\left(\frac{\delta\theta}{r}+\tau_{jt}\right)>0$$

and $\tau_{it}<\tau_{jt}$ requires $\tau_{jt}>\frac{\delta\theta}{r}$

$\theta<1$ yields $\frac{\delta\theta}{r}<\frac{\delta}{r}$.

Region i best responses for $\frac{\delta}{r} > \tau_j > \frac{\delta\theta}{r}$ are:

$$\tau_{it} = \begin{cases} \frac{1}{2}\left(\frac{\delta}{r} + \tau_{jt}\right) & \text{if} \quad \tau_{jt} < \frac{\delta}{r}\sqrt{\theta} \\ \frac{1}{2}\left(\frac{\delta\theta}{r} + \tau_{jt}\right) & \text{if} \quad \tau_{jt} > \frac{\delta}{r}\sqrt{\theta} \end{cases} \tag{2}$$

and region j best responses are:

$$\tau_{jt} = \begin{cases} \frac{1}{2}\left(\frac{\delta}{r} + \tau_{it}\right) & \text{if} \quad \tau_{it} < \frac{\delta}{r} \\ \tau_{it} & \text{if} \quad \frac{\delta}{r} < \tau_{it} < \frac{\delta}{r\theta} \\ \frac{1}{2}\left(\frac{\delta\theta}{r} + \tau_{it}\right) & \text{if} \quad \tau_{it} > \frac{\delta}{r\theta} \end{cases} \tag{3}$$

There exists thus a fundamental asymmetry between the responses of the small and the large region. Region i best responses are given by (2). When τ_{jt} is sufficiently low $\left(\tau_{jt} < \frac{\delta}{r}\sqrt{\theta}\right)$, region i can set a higher tax rate than region j. Since τ_{jt} increases, it is at first optimal for region i to increase its tax rate as well. When τ_{jt} becomes sufficiently high $\left(\tau_{jt} > \frac{\delta}{r}\sqrt{\theta}\right)$, it is optimal for region i to set its tax rate below the region j tax rate by a discontinuous reduction which increases its revenue.[3] We now turn to determine which case corresponds to the equilibrium.

Combining the best responses of both regions implies a discontinuity which could make the existence of the NASH equilibrium potentially problematic. Resolving the program we obtain:

PROPOSITION 1: *Assuming $\theta < 1$ there exists a unique NASH equilibrium. The NASH equilibrium tax rates are:*

$$\tau_i^N = \frac{\delta}{r}\left(\frac{2\theta + 1}{3}\right) \tag{4}$$

$$\tau_j^N = \frac{\delta}{r}\left(\frac{\theta + 2}{3}\right) \tag{5}$$

PROOF: see appendix 1

3. See KANBUR and KEEN [1993] for more details and appropriate figures.

Note that tax rates peak when countries are perfectly symmetric. When asymmetry between regions diminishes, the difference between tax rates diminishes also. The asymmetry stemming from the difference in size requires comments about the NASH equilibrium. The small region levies a tax rate lower than the large one. The explanation is rather simple and lies in capital elasticity. Starting at $\tau_i = \tau_j$, the increase of capital demand when region i reduces its tax rate depends on the size of the other region. The small region perceives an elasticity higher than the large region and fixes a lower tax rate. The larger the difference between elasticities is, the larger the difference between tax rates is.

Revenues at Nash equilibrium when compared to the autarchic solution yield the following proposition:

PROPOSITION 2: *The large region is worse off at Nash equilibrium while the small region is better off (worse off) if difference in size is sufficiently large (small).*

PROOF: Replacing (4) and (5) in (1) gives:

$$W_i^N = h_j \delta \left(\frac{2\theta + 1}{3} \right)^2$$

and

$$W_j^N = h_j \delta \left(\frac{\theta + 2}{3} \right)^2$$

That yields

$$W_j^N - W_j^{CB} = h_j \left[\delta \left(\frac{\theta + 2}{3} \right)^2 - r \right] < 0$$

since $r > \delta$ and $\left(\frac{\theta + 2}{3} \right)^2 < 1$.

$$W_i^N - W_i^{CB} = h_j \left[\delta \left(\frac{2\theta + 1}{3} \right)^2 - \theta r \right] > 0$$

$$\Longleftrightarrow g(\theta) = 4\delta\theta^2 - 9r\theta + 4\delta\theta + \delta > 0$$

Since

$$g(0) = \delta > 0$$

$$g(1) = 9(\delta - r) < 0$$

There exists a $\bar{\theta} < 1$ such that $g(\theta) > 0$ for $0 \leqslant \theta \leqslant \bar{\theta}$. $\bar{\theta}$ is the smaller root of the equation $g(\theta) = 0$.

The comparison with the closed borders solution shows that region j is worse off at the NASH equilibrium since its citizens invest abroad while foreigners who invest in region j pay a tax rate smaller than 1. For region i the comparison shows that for $\theta < \overline{\theta}$ (large difference in size), region i benefits from opening its borders since the gains from the investments of foreign citizens are greater than the losses of fiscal revenue from the home citizens. All these results are similar to those of KANBUR and KEEN and the interpretation adapted to an international capital tax problem is also similar. The difference between the two models arises with the tax co-ordination analysis.

4 Tax harmonization: The EMU recommendation[4]

4.1 The one shot game

Suppose that both regions gather in federation. The central planner of the federation sets a common tax rate τ_t and maximizes the revenues of both regions. Afterwards, the revenue of each region is given back to each government. Tax harmonization eliminates capital flows and current governments revenues at time t are then:

$$W_{it} = r\tau_t h_{it}$$

and

$$W_{jt} = r\tau_t h_{jt}$$

Since current revenue in each region is strictly increasing in τ_t, the intertemporal revenue is also increasing in τ_t and the common tax rate from tax harmonization is then equivalent to the closed borders solution $(\tau_{it} = \tau_{jt} = 1)$.

At each time t, governments revenues issued from tax harmonization are then:

$$W_i^H = h_j r\theta = W_i^{CB}$$

and

$$W_j^H = h_j r = W_j^{CB}$$

When borders are open, the constitution of a federation leads to an equilibrium similar to the closed borders equilibrium. There is no fiscal competition

4. The terminology "tax harmonization" and "sustainability" refers to the wording used in CARDARELLI & *alii* (2002).

and tax rate is decided at the federation level which eliminates competition between regions.

The tax harmonization equilibrium when compared to the NASH equilibrium leads to the following proposition:

> PROPOSITION 3: *Harmonization to any common tax rate $\tau = 1$ improves the current revenue of the large region while worse off (improves) the current revenue of the small region if difference in size is sufficiently large (small).*

PROOF: directly from proposition 2.

On the one hand, tax harmonization is always optimal for region j at time t since harmonization avoids capital flows and enables governments to choose the maximum tax rate. On the other hand, when difference in size is sufficiently large, region i benefits from tax competition since the gain issued from revenue of foreign citizens' taxation exceeds the loss of revenue from applying a lower tax rate. When borders are open either non cooperative equilibrium or tax harmonization equilibrium may constitute a strategic response to the openness. The choice of the optimal response depends on the level of asymmetry between regions.

4.2 The repeated interactions case

In order to determine which policy (harmonization or competition) each government would decide to choose in an intertemporal scheme, trigger strategies are introduced. They are defined by:

$$\tau_{it+1} = \begin{cases} 1 & \text{if } \tau_{jt} = 1 \\ \tau_i^N & \text{otherwise} \end{cases}$$

At each time $t + 1$ government i chooses the harmonized tax rate if government j has chosen the harmonized tax rate at the previous period. If government j has deviated from tax harmonization at time t, government i plays NASH equilibrium at time $t + 1$. This strategy prevails then for the rest of the periods. The choice of government j is perfectly symmetric.

> PROPOSITION 4: *If difference in size is sufficiently large, tax harmonization is not sustainable. When difference in size is sufficiently small, tax harmonization is sustainable if both governments are sufficiently patient.*

PROOF: see appendix 2.

For region j, the result is akin to the folk theorem established in the theory of repeated game. Tax harmonization is optimal for region j whenever the government is sufficiently patient. An increase in the discount factor decreases the current value of the short run gain from deviation and increases the current value of the loss from reverting to fiscal competition. If region i is sufficiently large (small difference in size between regions), region i benefits from tax harmonization if its government is sufficiently patient. In this case, tax harmonization is sustainable when governments are both sufficiently patient. If one of them is not sufficiently patient, tax harmonization is not sustainable.

When difference in size between regions is large, tax harmonization is never sustainable. The interpretation is similar to proposition 2.

The EMU solution which consists in setting a minimum tax rate in every country and then corresponds to the harmonization case (every country chooses to set the same tax rate) is not a sustainable solution to avoid tax competition when trigger strategies are implemented. Which kind of coordination can then avoid deviation from the coordinated equilibrium? The next section studies the cooperative solution.

5 Cooperation

5.1 The one shot game

This section explores the equilibrium from cooperation between both regions. The cooperative solution solves the following program:

$$\max_{\tau_i,\tau_j}\left[\alpha W_i + (1-\alpha)\, W_j\right]$$

where α and $(1-\alpha)$ represent the weights of region i and region j in the objective. The revenue of each region corresponds to their own fiscal income.

PROPOSITION 5: *The cooperative equilibrium* $\left(\tau_i^C,\tau_j^C\right)$ *is characterized by:*

- For $\tau_i \leqslant \tau_j$

$$\left(\tau_i^C,\tau_j^C\right) = \begin{cases} \left(\frac{1}{2}\left(\frac{r+\delta\theta\alpha}{\alpha r}\right),1\right) & \text{if} \quad \alpha > \frac{r}{2r-\theta\delta} = \bar{\alpha} \\ (1,1) & \text{if} \quad \alpha < \frac{r}{2r-\theta\delta} = \bar{\alpha} \end{cases}$$

- For $\tau_i \geqslant \tau_j$

$$\left(\tau_i^C,\tau_j^C\right) = \begin{cases} \left(1,\frac{1}{2}\left(\frac{r\theta+(1-\alpha)\,\delta}{(1-\alpha)\, r\theta}\right)\right) & \text{if}\, \alpha < \frac{r\theta-\delta}{2r\theta-\delta} = \hat{\alpha} \text{ and } \theta r > \delta \\ (1,1) & \text{otherwise} \end{cases}$$

PROOF: see appendix 3.

The cooperative equilibrium does not necessarily lead to the closed borders equilibrium. In particular cases depending on the weight given to each region in the objective, the cooperative equilibrium implies that one of the tax rates is not at its maximum. The region which chooses the lower tax rate thanks to an elevated weight in the objective function, benefits from tax cooperation. Its revenue is higher than in the closed borders case since the government is able to manipulate the tax rate in order to attract foreign capital. The gain from attracting capital exceeds the loss induced by the fall in domestic revenues. The region which chooses the higher tax rate is worse off by an opposite mechanism. Note that when $\tau_i \leqslant \tau_j$, the higher is θ, and the higher is $\overline{\alpha}$. This means that when difference in size between regions is small, region i weight has to be heavy to enable region i to select a non maximum tax rate. When $\tau_i \geqslant \tau_j$ the lower is θ and the lower is $\widehat{\alpha}$. When difference in size is small, region j weight $(1-\alpha)$ has to be high to enable region j to select a non maximum tax rate.

In comparison with the NASH equilibrium, the proposition 6 follows :

PROPOSITION 6:

• *For $\tau_i < \tau_j$, if $\alpha > \overline{\alpha}$ (with $\overline{\alpha}$ defined in proposition 5) and $r\theta > \delta$, then the revenue of region i at the cooperative equilibrium is higher than its revenue at the Nash equilibrium. In addition, there exists $\overline{\overline{\alpha}} > \overline{\alpha}$ such that for any $\alpha \in \left]\overline{\alpha},\overline{\overline{\alpha}}\right[$ the revenue of region j at cooperative equilibrium is higher than its revenue at the NASH equilibrium.*

• *For $\tau_i > \tau_j$ if $\alpha < \widehat{\alpha}$ (with $\widehat{\alpha}$ defined in proposition 5) and $r\theta > \delta$, then the cooperative equilibrium is always dominated by the NASH equilibrium for the small region.*

• *In the others cases, $\left(\tau_i, \tau_j\right) = (1,1)$ and then the large region is better off at cooperative equilibrium while the small region is worse off (worse off) if difference in size is sufficiently large (small).*

PROOF: see appendix 4.

The second part of the proposition stipulates that the cooperative equilibrium when $\tau_i > \tau_j$ is suboptimal. The benefit from taxing at the maximum tax rate for the small region is dominated by he loss of revenue from the flows of capital from the small region to the large one which applies a lower tax rate. The first part of the proposition means that region i as well as region j benefit from tax cooperation when weights imply $\tau_i^C < \tau_j^C$. This equilibrium enables both regions to be richer: region i thanks to capital flows and region j thanks to a maximum tax rate. When $\tau_i = \tau_j = 1$, proposition 2 applies.

5.2 The repeated interactions case

Following the previous repeated interactions framework, trigger strategies are defined by:

$$\tau_{it+1} = \begin{cases} \tau_i^C & \text{if} \quad \tau_j^C = \tau_i^C \\ \tau_i^N & \text{otherwise} \end{cases}$$

PROPOSITION 7: *When* $\tau_i^C < \tau_j^C$ *cooperative equilibrium is sustainable if governments are sufficiently patient and if* $\alpha = \min\left[\widetilde{\alpha}_i, \widetilde{\alpha}_j\right]$.

When $\tau_i^C > \tau_j^C$ *cooperative equilibrium is never sustainable.*

When $\tau_i^C = \tau_j^C = 1$ *cooperation is not sustainable if difference in size is sufficiently large. When difference in size is sufficiently small, cooperation is sustainable if both governments are sufficiently patient.*

PROOF: directly from proposition 6 and appendix 2.

When $\tau_i^C = \tau_j^C = 1$, the result is similar to the harmonization case. When $\tau_i^C > \tau_j^C$, government i prefers deviating from the cooperative equilibrium. This result occurs for a small difference in regions size $\left(\theta > \frac{\delta}{r}\right)$ and a low weight of region i in the cooperative objective. Then the gain from taxing at the maximum tax rate at the cooperative equilibrium is dominated by the gain from the current deviation and the capital flows from region j to region i when applying the NASH equilibrium since $\tau_i^N < \tau_j^N$. The loss of revenue from applying a lower tax rate $\left(\tau_i^N < 1\right)$ is small since the difference in size is small as well. When $\tau_i^C < \tau_j^C$, the result is akin to the folk theorem for $\alpha = \min\left[\widetilde{\alpha}_i, \widetilde{\alpha}_j\right]$. If region i's weight in the cooperative objective is sufficiently high, it enables region i and j to be richer at cooperative equilibrium when compared to the Nash equilibrium. If both regions are sufficiently patient, the loss from future punishment (Nash equilibrium) dominates the current gain from deviation and each region prefers to cooperate. When governments are not sufficiently patient the opposite case occurs and cooperation is not still sustainable.

The cooperative solution can constitute a sustainable solution to avoid tax competition but the feasibility is quite limited when trigger strategies are implemented. The next section studies the bargaining solution.

6 Bargaining

6.1 The one shot game

6.1.1 Bargaining

In this section the bargaining solution is studied. The objective of each government is now to maximize the following program:

$$\max_{\tau_i,\tau_j} \left(W_i - W_i^N\right)\left(W_j - W_j^N\right)$$

The NASH bargaining solution is a particular solution of the cooperative problem (see PETIT [1989]) where α represents the bargaining power of the player. It can be rewritten as:

$$\max_{\alpha} W^B = \left(W_i^C - W_i^N\right)\left(W_j^C - W_j^N\right)$$

The determination of α is obtained by numerical simulations. The results are presented in tables 1 to 4. The impact of the relative size between regions is discussed through numerical computations.

The numerical results show that when difference in size between regions is large (small value of θ) there exists a level of bargaining power (α) such that the bargaining equilibrium corresponds to a cooperative equilibrium which differs from $(1,1)$. When difference in size is small (large value of θ), the bargaining solution corresponds to the closed borders solution. The explanation is as follows. Firstly, it is straightforward that the bargaining equilibrium is either $\left(\frac{1}{2}\left(\frac{r+\delta\theta\alpha}{\alpha r}\right),1\right)$ or $(1,1)$ since we have showed that $\left(1,\frac{1}{2}\left(\frac{r\theta+(1-\alpha)\delta}{(1-\alpha)\theta r}\right)\right)$ is always dominated by the NASH equilibrium for region i. Secondly, we know that the higher is θ, the higher is $\overline{\alpha}$. When difference in size is small, the bargaining power that maximizes the bargaining problem is very high and brings a value of τ_i higher than one. This solution is not possible and the bargaining solution is then reduced to the closed borders solution which is optimal since θ is high enough.

When θ is small, there exists a bargaining power $\alpha < 1$ which maximizes the bargaining program. When difference in size is large, region i bargaining power does not have to be very high to obtain an asymmetric cooperative equilibrium since capital flows are elevated from region i point of view and hence the gains of revenues are substantial.

For a low value of information cost $(\delta = 0.001)$, the optimal bargaining power is around $\frac{1}{2}$ for both countries even if the difference in size is very large $(\theta = 0.1)$.

6.1.2 **Bargaining with transfers**

In this section, another policy tool is introduced: μ is a revenue transfer from one region to the other. Whereas in the previous case each region received its own fiscal income, in this section we introduce a redistributive transfer which implies that the revenue of a region does not necessarily correspond to its domestic fiscal income. Region i revenue is then:

$$W_i + \mu$$

with $\mu \gtrless 0$; while region j revenue is:

$$W_i - \mu$$

Governments of both regions are supposed to bargain in order to determine the optimal tax rates as well as the optimal transfer of revenue between regions following two stages: in the first stage, they bargain on the tax rates, in the second stage, they bargain on the optimal transfer. The resolution is backward.

First stage: the program of the regions is to maximize the following objective with respect to μ

$$\left(W_i^C + \mu - W_i^N\right)\left(W_j^C - \mu - W_j^N\right) \tag{6}$$

The maximization gives:

$$\mu = \frac{\left(W_j^C - W_j^N\right) - \left(W_i^C - W_i^N\right)}{2}$$

The optimal transfer is then:

$$\mu = \frac{h_j\left((\theta - 1)\left(r - \frac{\delta}{3}(\theta + 1)\right)\right)}{2} > 0 \Longleftrightarrow \theta > \frac{3r - \delta}{\delta}$$

Replacing the optimal transfer in the objective function leads to the following maximization program:

$$\max_{\tau_i, \tau_j} W^B = \frac{1}{4}\left(W_i^C - W_i^N + W_j^C - W_j^N\right)^2$$

which can be rewritten as:

$$\max_{\tau_i, \tau_j} W^B = \left(\frac{1}{2}\left(W_i^C + W_j^C\right) - \frac{1}{2}\left(W_i^N + W_j^N\right)\right)^2$$

PROPOSITION 8: *The bargaining equilibrium with transfer is equivalent to the closed borders equilibrium.*

PROOF: The maximization program is a particular case of the cooperative program for $\alpha = \frac{1}{2}$. From appendix 5 it is then straightforward that the bargaining equilibrium with transfer is $(1,1)$.

When a transfer is introduced, the bargaining solution corresponds to the closed borders solution. Indeed, the transfer smooths the effect of bargaining power. When regions choose their tax rates, they internalize that if they can fix a tax rate lower than the other region in order to attract capital and to obtain more revenue, this additional revenue will be redistributed between both regions. The optimum converges towards the closed borders equilibrium which induces that each region has identical bargaining power and sets the maximum tax rate as if there are no capital flows.

6.2 The repeated interactions case

The repeated game is defined by :

$$\tau_{it+1} \begin{cases} \tau_i^B & \text{if} \quad \tau_{jt} = \tau_j^B \\ \tau_i^N & \text{otherwise} \end{cases}$$

According to the bargaining solution, the following proposition is immediate:

PROPOSITION 9: *The bargaining equilibrium is always sustainable when governments are sufficiently patient.*

PROOF: directly from the definition of the bargaining solution.

By definition, each economy is better off at the Bargaining equilibrium than at the Nash equilibrium. The best response to the tax evasion problem raising by opening borders is then the bargaining equilibrium. This equilibrium is indeed sustainable since none region has interest to deviate from this equilibrium.

7 Numerical exercise

7.1 Tables

For numerical computations, a value of $r = 0{,}05$ is chosen. This corresponds to an interest rate of 5%. Each table represents the bargaining solution for different values of θ and a given δ. The alternative values of δ yield the different tables. The characters in bold emphasize the effective welfare at the NASH solution.

TABLE 1
For $\delta = 0.045$

θ	α	W_i^B	(τ_i, τ_j)
0.1	0.6936	$\mathbf{9849.10}^{-8}$	(0.7659, 1)
0.3	0.6932	$\mathbf{10711.10}^{-8}$	(0.8563, 1)
0.5	0.6991	$\mathbf{102264.10}^{-8}$	(0.9402, 1)
0.7	0.7125	$\mathbf{8410.10}^{-8}$	(1, 1)
0.9	0.7363	$\mathbf{4611.10}^{-8}$	(1, 1)

TABLE 2
For $\delta = 0.03$

θ	α	W_i^B	(τ_i, τ_j)
0.1	0.6939	$\mathbf{2355.10}^{-7}$	(0.8187, 1)
0.3	0.6301	$\mathbf{3012.10}^{-7}$	(0.8835, 1)
0.5	0.6283	$\mathbf{3594.10}^{-7}$	(0.9458, 1)
0.7	0.6281	$\mathbf{4061.10}^{-7}$	(1, 1)
0.9	0.6296	$\mathbf{4144.10}^{-7}$	(1, 1)

TABLE 3
For $\delta = 0.015$

θ	α	W_i^B	(τ_i, τ_j)
0.1	0.5711	$\mathbf{4466.10}^{-7}$	(0.8904, 1)
0.3	0.5695	$\mathbf{6073.10}^{-7}$	(0.9229, 1)
0.5	0.5682	$\mathbf{7820.10}^{-7}$	(0.9549, 1)
0.7	0.5672	$\mathbf{9664.10}^{-7}$	(0.9864, 1)
0.9	0.5666	$\mathbf{11491.10}^{-7}$	(1, 1)

TABLE 4
For $\delta = 0.001$

θ	α	W_i^B	(τ_i, τ_j)
0.1	0.50498	$\mathbf{7318.10}^{-7}$	(0.9911, 1)
0.3	0.50497	$\mathbf{10209.10}^{-7}$	(0.9931, 1)
0.5	0.504969	$\mathbf{13571.10}^{-7}$	(0.9951, 1)
0.7	0.504961	$\mathbf{17400.10}^{-7}$	(0.9971, 1)
0.9	0.50495	$\mathbf{21693.10}^{-7}$	(0,9991, 1)

7.2 Interpretation to the EMU example

In this section we try to interpret our results with regard to the EMU example. We choose to analyze four couples of countries: Germany/France, France/United Kingdom, France/Denmark and France/Neertheland. For these couples of countries, the relative sizes are:[5]

Couple	θ
Germany/France	$0,75$
France/United Kingdom	$0,9$
France/Denmark	$0,3$
France/Neertheland	$0,1$

We also assume that the bargaining power of each region is the number of votes that each government gets at the Council of European Union. α represents the bargaining power of the small region and $(1-\alpha)$ the one of the large region. The following table gives the values of α for each couple:

Couple	votes	α
France/Germany	$10/10$	0,5
France/United Kingdom	$10/10$	0,5
France/Denmark	$10/3$	0,25
France/Netherland	$10/5$	0,33

According to tables 1 to 4, we can stipulate that bargaining is not feasible between Denmark and France and Netherland and France. Bargaining is feasible between Germany and France as well as between France and United Kingdom if the cost of information δ is very low (about 2% of the interest rate).

These results show that bargaining should not be based on voting but on other criteria that should be defined later.

8 Conclusion

In this paper two ideas have been investigated i) the evaluation of different political strategies when regions are integrated regarding the closed economy as the benchmark case and ii) the study of the sustainability of fiscal co-ordination following three criteria of co-ordination: tax harmonization, fiscal cooperation and bargaining. We show that tax harmonization and bargaining with transfer are equivalent to the closed borders solution. Cooperation and

5. The second country cited in the couple is the smaller. In order to fit on our theorical analysis we suppose that $\theta < 1$ is the ratio of the smaller population on the larger.

bargaining without transfer may differ from the closed borders equilibrium when the small region has a bargaining power or a weight in the cooperative objective sufficiently high with regard to the large region one.

If difference in size between regions is sufficiently large, the small region prefers to deviate from the coordinated solution in order to attract and tax capital from the other region. The gain from a higher level of capital dominates the loss from lowering tax rate.

The results of this article show that bargaining is the only sustainable co-ordinated policy when trigger types strategies are implemented. Nevertheless this type of co-ordination is very difficult to achieve. The recent recommendations of the ECOFIN Council show that European governments prefer discussing about tax harmonization or fiscal cooperation. The problem is that sustainability of cooperative fiscal policy or tax harmonization depends on the degree of difference in regions' size: tax harmonization or tax cooperation are not sustainable when difference in regions size is large. According to these results, Greece or Netherland would not have interest to support any harmonized or cooperative solution. The EMU ECOFIN council should then consider a bargaining solution rather than a tax harmonization solution to avoid tax competition. Moreover, bargaining power should not be based on voting but on other criteria that should be defined later but the European Union. ▼

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Appendix

Appendix 1 : proof of proposition 1

We set $\theta < 1$ and explore the different cases following the relation between τ_i and τ_j.

If $\tau_i > \tau_j$

Best response functions are then:

$$\tau_i = \frac{1}{2}\left(\frac{\delta}{r} + \tau_{jt}\right) \text{ if } \tau_j < \frac{\delta}{r}\sqrt{\theta} \tag{7}$$

$$\tau_j = \frac{1}{2}\left(\frac{\delta}{\theta r} + \tau_{it}\right) \text{ if } \tau_i > \frac{\delta}{r\theta} \tag{8}$$

Compiling (7) and (8) gives

$$\tau_i = \frac{\delta}{r\theta}\left(\frac{2\theta + 1}{3}\right) < \frac{\delta}{\theta r}$$

which is impossible.

If $\tau_i < \tau_j$

Best response functions are then:

$$\tau_i = \frac{1}{2}\left(\frac{\delta\theta}{r} + \tau_{jt}\right) \text{ if } \tau_j > \frac{\delta}{r}\sqrt{\theta} \tag{9}$$

$$\tau_j = \frac{1}{2}\left(\frac{\delta}{r} + \tau_{it}\right) \text{ if } \tau_i < \frac{\delta}{r} \tag{10}$$

Compiling (9) and (10) gives

$$\tau_i = \frac{\delta}{r}\left(\frac{2\theta + 1}{3}\right) < \frac{\delta}{r}$$

$$\tau_j = \frac{\delta}{r}\left(\frac{\theta + 2}{3}\right) > \frac{\delta}{r}\sqrt{\theta}$$

Note that $\tau_i = \tau_j$ is possible if $\theta = 1$. The tax rate is then $\tau_i = \tau_j = \frac{\delta}{r}$.

Appendix 2 : Proof of proposition 4

For region i

We define τ_i^D as the tax rate of the optimal deviation. τ_i^D is given solving the program:

$$\max_{\tau_i}\left[h_j r\left(\tau_i\theta+\tau_i\frac{r}{\delta}\left(1-\tau_{it}\right)\right)\right]$$

that yields

$$\tau_i^D=\frac{\theta\delta}{2r}+\frac{1}{2}$$

The gain issued from deviation is:

$$V_i^D=W_i^D\left(\tau_i^D,1\right)+\frac{\gamma}{1-\gamma}W_i^N\left(\tau_i^N,\tau_j^N\right)$$

where $W_i^D=\frac{h_j}{\delta}\left(\frac{\theta\delta+r}{2}\right)^2$ and $W_i^N=\delta h_j\left(\frac{2\theta+1}{3}\right)^2$ since $\tau_i^N<\tau_j^N$

The gain from tax harmonization is:

$$V_i^H=\frac{1}{1-\gamma}W_i^H\left(1,1\right)=h_j\frac{\theta r}{1-\gamma}$$

Let us define

$$\begin{aligned}\Psi\left(\gamma\right)&=\left(1-\gamma\right)\left(V_i^H-V_i^D\right)\\&=h_j r\theta-\left(1-\gamma\right)W_i^D\left(\tau_i^D,1\right)-\gamma W_i^N\left(\tau_i^N,\tau_i^N\right)\end{aligned}$$

Tax harmonization is sustainable if there exists $\overline{\gamma}_i\in[0,1[$ such as $\Psi\left(\gamma\right)>0$ for $\gamma>\overline{\gamma}_i$.

The study of the function gives:

$$\begin{aligned}\Psi'\left(\gamma\right)&=W_i^D-W_i^N\\&=h_j\left(\frac{1}{\delta}\left(\frac{\theta\delta+r}{2}\right)^2-\delta\left(\frac{2\theta+1}{3}\right)^2\right)\\&=\frac{\delta h_j}{36}\left(4\left(\frac{r^2}{\delta^2}-1\right)+5\left(\frac{r^2}{\delta^2}-\theta^2\right)\right.\\&\quad\left.+16\theta\left(\frac{r}{\delta}-1\right)+2\theta\left(\frac{r}{\delta}-\theta\right)\right)>0\end{aligned}$$

since $r>\delta$ and $\theta<1$, and:

$$\begin{aligned}\Psi\left(0\right)&=W_i^H-W_i^D\\&=-\frac{h_j}{4\delta}\left(\theta\delta-r\right)^2<0\end{aligned}$$

and

$$\begin{aligned}\Psi\left(1\right)&=W_i^H-W_i^N\\&=\frac{h_j}{9}\left(9r\theta-4\delta\theta-4\theta^2\delta-\delta\right)\end{aligned}$$

Then

$$\Psi(1) > 0 \iff f(\theta) = 9r\theta - 4\delta\theta - 4\theta^2\delta - \delta > 0$$

and note that $f(\theta) = -g(\theta)$ (proof of proposition 2). Hence for $\theta > \overline{\theta}$ then $\Psi(1) > 0$ and tax harmonization is sustainable if government i is sufficiently patient. For $\theta < \overline{\theta}$ (large difference in size between regions) then $\Psi(1) < 0$ and tax harmonization is not sustainable for region i.

For region j

We define τ_j^D as the tax rate of the optimal deviation. τ_j^D is given solving the program:

$$\max_{\tau_j} \left[h_j r \left(\tau_j + \tau_j \frac{r}{\delta} \theta \left(1 - \tau_{jt}\right)\right)\right]$$

that yields

$$\tau_j^D = \frac{\delta}{2\theta r} + \frac{1}{2}$$

The gain issued from deviation is:

$$V_j^D = W_j^D\left(\tau_j^D, 1\right) + \frac{\gamma}{1-\gamma} W_j^N\left(\tau_j^N, \tau_i^N\right)$$

where $W_j^D = h_j \frac{1}{\delta\theta}\left(\frac{\theta r + \delta}{2}\right)^2$ and $W_j^N = h_j \delta \left(\frac{\theta + 2}{3}\right)^2$

The gain from tax harmonization is:

$$V_j^H = \frac{1}{1-\gamma} W_j^H(1,1) = \frac{r h_j}{1-\gamma}$$

Let us define

$$\begin{aligned}\Psi(\gamma) &= (1-\gamma)\left(V_j^H - V_j^D\right)\\ &= r h_j - (1-\gamma) W_j^D\left(\tau_j^D, 1\right) - \gamma W_j^N\left(\tau_j^N, \tau_j^N\right)\end{aligned}$$

Tax harmonization is sustainable if there exists $\overline{\gamma}_j \in [0,1[$ such as $\Psi(\gamma) > 0$ for $\gamma > \overline{\gamma}_j$.

The study of the function gives:

$$\begin{aligned}\Psi'(\gamma) &= W_j^D - W_j^N\\ &= \frac{h_j}{\delta\theta}\left(\frac{\theta r + \delta}{2}\right)^2 - h_j\delta\left(\frac{\theta+2}{3}\right)^2\\ &= \frac{\delta\theta}{36} h_j \Bigg(9\left(\frac{1}{\theta^2} - 1\right) + 7\left(\frac{r^2}{\delta^2} - 1\right)\\ &\quad + 16\frac{1}{\theta}\left(\frac{r}{\delta} - 1\right) + 2\left(\frac{r^2}{\delta^2} - \theta\right) + 2\left(\frac{1}{\theta}\frac{r}{\delta} - \theta\right)\Bigg) > 0\end{aligned}$$

since $r > \delta$ and $\theta < 1$ and :

$$\Psi(0) = W_j^H - W_j^D$$
$$= -\frac{h_j}{4\delta\theta}(\theta\delta - r)^2 < 0$$

and

$$\Psi(1) = W_j^H - W_j^N$$
$$= \frac{h_j}{9}\left(9r - 4\delta\theta - \theta^2\delta - 4\delta\right) > 0$$

Appendix 3 : Proof of proposition 5

The objective is to maximize

$$W^C = \alpha W_i + (1-\alpha) W_j$$

Note that each couple $\left(\tau_i, \tau_j\right)$ with $\tau_i < 1$ and $\tau_j < 1$ is PARETO dominated by a couple $\left(\tau_i + \varepsilon, \tau_j + \varepsilon\right)$ with $\varepsilon = \min\left[1 - \tau_i, 1 - \tau_j\right]$ which implies identical capital flows but higher revenues for each government. This property shows that at least one of the tax rate is equal to one.

• For $\tau_i \leqslant \tau_j$, the objective can be rewritten as:

$$W^C = r\left(\alpha\tau_i h_i + (1-\alpha)\tau_j h_j + \frac{r}{\delta}\left(\tau_j - \tau_i\right) h_j \left(\alpha\tau_i - (1-\alpha)\tau_j\right)\right)$$

and at least $\tau_j = 1$.

If $\alpha < \frac{1}{2}$ then $\left(\alpha\tau_i - (1-\alpha)\tau_j\right) < 0$ since $\tau_i \leqslant \tau_j$ and W^C is maximal when the negative effect from $\left(\alpha\tau_i - (1-\alpha)\tau_j\right)$ is minimal that is for $\tau_i = \tau_j = 1$.

If $\alpha > \frac{1}{2}$, the first order conditions with respect to τ_i and τ_j are:

$$\frac{\partial W^C}{\partial \tau_i} = 0 \text{ implies } \alpha\theta + \frac{r}{\delta}\left(\tau_j - \tau_i\right)\alpha - \frac{r}{\delta}\left(\alpha\tau_i - (1-\alpha)\tau_j\right) = 0$$

The best response of τ_i for $\tau_j = 1$ gives:

$$\tau_i = \frac{1}{2}\left(\frac{r + \alpha\theta\delta}{\alpha r}\right)$$

and $\tau_i < 1 \Longleftrightarrow \alpha > \dfrac{r}{2r - \theta\delta} > \dfrac{1}{2}$.

Then for $\alpha > \frac{r}{2r-\theta\delta}$ the cooperative solution is $\left(\frac{1}{2}\left(\frac{r+\alpha\theta\delta}{\alpha r}\right),1\right)$ and for $\alpha \leqslant \frac{r}{2r-\theta\delta}$ the cooperative solution is $(1,1)$.

- For $\tau_j \leqslant \tau_i$ the analysis is similar. The objective function can be rewritten as:

$$W^C = r\left(\alpha\tau_i h_i + (1-\alpha)\tau_j h_j + \frac{r}{\delta}\left(\tau_i - \tau_j\right) h_i \left((1-\alpha)\tau_j - \alpha\tau_i\right)\right)$$

and we know that at least $\tau_i = 1$.

If $\alpha > \frac{1}{2}$ then $\left((1-\alpha)\tau_j - \alpha\tau_i\right) < 0$ since $\tau_j \leqslant \tau_i$ and W^C is maximal when the negative effect from $\left((1-\alpha)\tau_j - \alpha\tau_i\right)$ is minimal that is for $\tau_i = \tau_j = 1$.

If $\alpha < \frac{1}{2}$, the derivatives of W^C with respect to τ_i and τ_j give:

$$\frac{\partial W^C}{\partial \tau_j} = 0 \text{ implies } (1-\alpha) + \theta\frac{r}{\delta}\left(\tau_i - \tau_j\right)(1-\alpha) - \theta\frac{r}{\delta}\left((1-\alpha)\tau_j - \alpha\tau_i\right) = 0$$

The best response of τ_j for $\tau_i = 1$ gives:

$$\tau_j = \frac{1}{2}\left(\frac{r\theta + (1-\alpha)\delta}{(1-\alpha)\theta r}\right)$$

the condition $\tau_j < 1$ implies that $\alpha < \frac{r\theta - \delta}{2r\theta - \delta}$ and we check easily that $\frac{r\theta-\delta}{2r\theta-\delta} < \frac{1}{2}$

$$\tau_j = \begin{cases} \frac{1}{2}\left(\frac{r\theta + (1-\alpha)\delta}{(1-\alpha)\theta r}\right) \iff \alpha < \frac{r\theta-\delta}{2r\theta-\delta} \text{ for } \theta r > \delta \\ 1 \text{ otherwise} \end{cases}$$

Then:

When $r\theta < \delta$ the cooperative solution is $(1,1)$.

When $r\theta > \delta$, if $\alpha \geqslant \frac{r\theta-\delta}{2r\theta-\delta}$ the cooperative solution is $(1,1)$ and if $\alpha < \frac{r\theta-\delta}{2r\theta-\delta}$ the cooperative solution is $\left(1,\frac{1}{2}\left(\frac{r\theta+(1-\alpha)\delta}{(1-\alpha)\theta r}\right)\right)$.

Appendix 4 : Cooperative versus NASH *equilibrium*

For $\tau_i < \tau_j$ (When $\alpha > \overline{\alpha}$) we get:

For region i:

$$\begin{aligned} W_i^C &= W_i\left(\tau_i^C, \tau_j^C\right) \\ &= W_i\left(\frac{r+\alpha\theta\delta}{2\alpha r}, 1\right) = \frac{h_j}{4\alpha^2\delta}\left(r+\alpha\theta\delta\right)\left[\alpha\theta\delta + 2r\alpha - r\right] \\ W_i^C - W_i^N &= h_j\left(\left(\theta r\alpha + r\right)\left(\frac{1}{4\alpha^2\delta}\right)\left[\alpha\theta\delta + 2r\alpha - r\right] - \delta\left(\frac{2\theta+1}{3}\right)^2\right) \\ &= \Psi\left(\alpha\right) \end{aligned}$$

and

$$\begin{aligned} \Psi\left(1\right) = \frac{h_j}{36\delta}\Big[16\theta\delta\left(1-\delta\right) + 2\theta\delta\left(1-\theta\delta\right) + \delta\left(r^2-\delta^2\right) \\ +5\left(r^2-\theta^2\delta^2\right)\Big] > 0 \end{aligned}$$

is sufficient to affirm that there exists at least one value of $\alpha > \overline{\alpha}$ (since $\overline{\alpha} < 1$) such that $W_i^C > W_i^N$

For region j

$$W_j^C = W_j\left(\tau_j^C, \tau_i^C\right) = W_j\left(1, \frac{r+\alpha\theta\delta}{2\alpha r}\right) = rh_j\frac{2\alpha\left(\delta-r\right) + r + \alpha\delta\theta}{2\alpha\delta}$$

$$W_j^C - W_j^N = h_j\left(\frac{2\alpha\left(\delta-r\right) + r + \alpha\delta\theta}{2\alpha\delta}r - \delta\left(\frac{\theta+2}{3}\right)^2\right) = \Phi\left(\alpha\right)$$

and the condition

$$\Phi\left(\alpha\right) = 0 \Longleftrightarrow \alpha = \frac{r}{2\left(r-\delta\right) - \delta\theta + \dfrac{2\delta^2}{r}\left(\dfrac{\theta+2}{3}\right)^2}$$

is sufficient to assure that there exists at least one value of $\alpha > \overline{\alpha}$ such that $W_j^C > W_j^N$.

Indeed, we can easily check that $\alpha = \dfrac{r}{2(r-\delta) - \delta\theta + \dfrac{2\delta^2}{r}\left(\dfrac{\theta+2}{3}\right)^2}$ $> \bar{\alpha}$ since $\dfrac{\delta}{r}\left(\dfrac{\theta+2}{3}\right)^2 < 1$ because $\delta < r$ and $\left(\dfrac{\theta+2}{3}\right)^2 < 1$.

- For $\tau_i > \tau_j$ $\left(\text{When } \alpha < \dfrac{r\theta - \delta}{2r\theta - \delta} = \widehat{\alpha} \text{ if } \theta r > \delta\right)$ then:

For region i:

$$\begin{aligned} W_i^C &= W_i\left(\tau_i^C, \tau_j^C\right) = W_i\left(1, \frac{r\theta + (1-\alpha)\delta}{2(1-\alpha)r\theta}\right) \\ &= \frac{rh_j}{2(1-\alpha)\delta}\left[(1-\alpha)\delta(2\theta+1) + r\theta(2\alpha-1)\right] \end{aligned}$$

and

$$\begin{aligned} & W_j^C - W_j^N \\ &= h_j\left[\frac{r}{2(1-\alpha)\delta}\left[(1-\alpha)\delta(2\theta+1) + r(2\alpha-1)\right] - \delta\theta\left(\frac{2\theta+1}{3}\right)^2\right] \\ &= \Delta(\alpha) \end{aligned}$$

with

$$\begin{aligned} \Delta'(\alpha) &= \frac{h_j r^2\theta}{2\delta(1-\alpha)^2} > 0 \\ & \Delta\left(\frac{r\theta-\delta}{2r\theta-\delta}\right) = -\frac{h_j}{18\delta}\left((r\theta-\delta) + 4\theta(r-\delta) + 4\theta(r-\delta\theta)\right) \\ & < 0 \text{ since } r\theta > \delta \end{aligned}$$

Region i is then better of at the NASH equilibrium for any value of $\alpha < \dfrac{r\theta-\delta}{2r\theta-\delta} = \widehat{\alpha}$ if $\theta r > \delta$.

- In the others cases the cooperative equilibrium is given by the couple of tax rates (1,1) which is equivalent to the closed borders equilibrium and proposition 2 applies.