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Information, Discrimation and the Long Run

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ABSTRACT. – This note studies the long-run behaviour of wages for two groups of workers in a stochastic matching model. The quality of the match is modeled as a Brownian motion with drift. Groups differ in the speed at which information relative to the quality of the match is revealed. The note shows that the expected cross-group wage differential for workers having never quit their job is everywhere at the favoured group's advantage.

Information, discrimination et long-terme

RÉSUMÉ. – Cette note étudie le comportement des salaires de deux groupes de travailleurs dans un modèle avec appariement stochastique. La qualité de l'appariement suit un mouvement Brownien avec tendance. La vitesse à laquelle la qualité de l'appariement est révélée diffère entre les deux groupes de travailleurs. Cette différence implique que l'espérance mathématique du salaire d'un travailleur n'ayant jamais changer d'emploi est à n'importe quel instant plus élevée pour les travailleurs du groupe favorisé.

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1 Introduction

Theories of discrimination in economics in general and in labour markets in particular go a long way back, to the works of EDGEWORTH [1922] and, more importantly, BECKER [1957]. Since then a large body of literature has been developing. This body is surveyed in CAIN [1986] and ALTONJI and BLANK [1999]. Within this body, one can identify many different explanations for discrimination in labour markets. We will focus on what we believe are three of the most important ones.

The first explanation, usually labelled *pure discrimination*, relies on agents' tastes, or preferences, to explain discrimination. Thus, differences in agents' preferences, be they consumers, employers or employees, lead to theories that can explain discrimination both in terms of wage differentials and segregation, that is, job segmentation within labour markets. Seminal contributions here include BECKER [1957] and ARROW [1973a].

A second strand of the literature focuses on the consequences of informational problems and asymmetries to explain the differential treatment of workers. Following PHELPS [1972] and ARROW [1973b], *statistical discrimination* is due to either ex-ante beliefs about differences in productivity across different groups of workers that are confirmed ex-post, or to problems linked to the measurability of workers' differing performances across different groups, because of differences in the noisiness of signals or tests. More recent contributions that, for example, highlight the role of human capital are LUNDBERG and STARTZ [1983], LUNDBERG [1991] and COATE and LOURY [1993].

Finally, a third and more recent body of the literature has turned to language problems to explain discrimination. The seminal papers in this area are LANG [1986] and [1993].[1] As vividly explained by LANG in his [1986] seminal contribution, language-based discrimination is mainly due to differential communication costs between agents. Indeed, Lang cites many sociological studies where it was shown that, in the USA, interactions between white and black americans who are both native speakers in English are more costly than those between all whites or all blacks, simply because of a different usage of certain words or expressions that may lead to misunderstandings.

The crucial aspect of language-based discrimination we wish to emphasize here is the fact that discrimination occurs even though agents are *identical*, *i.e.*, the productivity of the match is the same. This is one feature this paper will stick to throughout.[2] Thus, the key difference between statistical discrimination *à la* PHELPS or ARROW and language-based discrimination *à la* LANG is that, *whereas in the former case there are productivity differences across groups,*[3] this is not the case under language-based discrimination.

1. LAZEAR [1999] and SAINT-PAUL [2001] deal with language too but do not address the issue of discrimination.
2. And on which we will be more precise below (see section 3).
3. Because, for example, of differences in human capital, as in LUNDBERG and STARTZ [1983].

Turning to dynamics, the effects of market forces has been explicitely looked at in the pure discrimination literature (see *e.g.* ARROW (1973), CAIN [1986], BORJAS and BRONARS [1989] and BLACK [1995]), the main conclusion being that, even though market forces should be strong enough to eliminate discriminatory behaviour in the long run, once we allow for frictions in the markets, discrimination will typically *not* disappear (see *e.g.* BLACK [1995, p. 310]).

In the statistical discrimination literature, the persistence of discriminatory behaviour in the long run has not been tackled explicitely, even though, as CAIN [1986, p. 728] points out, informational problems leading to statistical discrimination should disappear in the long-run, as the uncertainty surrounding the workers' productivity is gradually eroded by on-the-job performance.

Does this hold true for language-based discrimination too? To the best of our knowledge, the issue has not been examined yet.

This note focuses on a slightly broader issue, namely the effects of differential cross-group speeds of information revelation and proposes a simple formalisation building on the stochastic job matching model of JOVANOVIC [1978] and [1979]. We consider a pool of workers and show that, once we partition them into two subsets characterized by different speeds of information revelation as regards the quality of their match, cross-group equilibrium wages are very unlikely to converge, if not in the very long run. This model can thus be seen as rationalizing, at least partially,[4] the propositions of the language-based discrimination literature.

The rest of the paper is structured as follows. Section 2 presents the basic model. Section 3 presents the extension to the case of two subgroups of workers facing different employer-employee communication problems and derives the main results. The last section concludes.

2 A Simple Model (JOVANOVIC [1979])

2.1 Preliminaries

Assume each firm is randomly matched to a worker and that the (true) productivity of the match is drawn from a *known*, non-degenerate Normal distribution, but that the match-specific realisation of this parameter is *unknown both to the worker and the firm*. Assume each firm's production function has the worker's labour as its only input and exhibits constant returns to scale. Firms maximise expected profits and can contract with workers individually. Suppose that the *observed* product of the match is also subject to a *known stochastic disturbance*, which is modelled here as a driftless Brownian motion.

4. We say partially because some of the implications of the model seem to contrast, *prima facie*, with the effects one would intuitively expect from language-based discrimination. We wish to thank the anonymous referees for pointing this out.

Denoting with $X(t)$ the worker's contribution to the firm's output over a period of length t, we then have:

$$X(t) = \mu t + \sigma_x Z(t), t > 0 \tag{1}$$

where $Z(t)$ is a standard Brownian motion, $\sigma_x > 0$ is the instant variance of the Brownian motion and is supposed to be a constant that is *known* and μ, the true value of the quality of the employer-employee match, is, as mentioned above, an unobserved draw from a known Normal distribution with mean $\overline{\mu}$ and variance σ_μ^2.

μ can thus be best described as the quality of a match, and *not as a measure of a worker's ability*. Indeed, observe that for any match, μ, its quality, is a draw from the same Normal distribution, *regardless* of the characteristics of the worker. Thus, the *true* quality of a match is *independent* of any groupwise difference between workers that will be introduced below.

That this is so should also become clearer later on when we describe (one of) the equilibrium wage schedules firms will offer workers. We will see that this wage schedule is a *driftless Brownian motion* whose instant variance tends to zero as time tends to infinity implying that, in the long run, 'the worker's true productivity becomes known and his wage is in the limit [constant and] *equal* to his productivity' [μ] (see JOVANOVIC [1978] p. 52).

In this model, workers constantly receive offers from competing firms and they are free to break their current match at any time and then instantly go for the best alternative offer, whose producitivity is a *newly and independently* drawn realisation μ' from the same Normal distribution. Thus there is *no unemployment* in this model and the worker's prior on-the-job history has *no importance* for the productivity of any new match he might form.

As time goes by, the employer and the employee observe the product of their match and update their belief about the true value of its quality. The productivity of the relationship is therefore modelled as a pure *experience* good, that is, a good whose quality is totally unknown at the start of the game and that will be gradually revealed through time. There is no role for screening in this model.[5]

Given the productivity's prior distribution, the employer and the employee alike update their beliefs on the value of the productivity of their match as they observe output $x = X(t)$ and time according to (see CHERNOFF [1968]):

$$\begin{cases} E(\mu|x,t) \equiv E_{x,t}(\mu) = S(t)\left(\dfrac{\overline{\mu}}{\sigma_\mu^2} + \dfrac{x}{\sigma_x^2}\right) \\ S(t) = \left(\dfrac{1}{\sigma_\mu^2} + \dfrac{t}{\sigma_x^2}\right)^{-1} \end{cases} \tag{2}$$

where $E(\mu|x,t)$ is the conditional expectation of μ given t and $X(t) = x$.

In addition, it is also possible to show (CHERNOFF [1968]) that the conditio-

5. See JOVANOVIC [1984] for a related model with screening (and unemployment).

nal expectation of μ given x and t is also normal:

$$(3) \qquad E(\mu|x,t) \sim \mathcal{N}\left(\overline{\mu}, \sigma_\mu^2 - S(t)\right), E(\mu|0,0) \equiv \overline{\mu}$$

That is, $E(\mu|x,t)$ is a driftless Brownian motion in the $p(t)$ space: $E(\mu|x,t) = Z[p(t)]$, where $p(t) = \sigma_\mu^2 - S(t)$. It is worth remarking here that $p(t)$ converges from below to σ_μ^2 as t tends to infinity.

Even more importantly, (1) and (2) imply that as time goes to infinity $E(\mu|x,t)$ tends to μ (in probability) and therefore the true quality of the match is fully revealed.

Suppose now that firms and workers have an infinite horizon, that they discount the future at the same constant discount rate r and that workers are risk-neutral.[6] Suppose too that firms offer workers life-time contracts that will always be honoured ex-post.[7] Then JOVANOVIC [1979] shows that, under these assumptions, one of the competitive equilibrium wage contracts is:

$$(4) \qquad w(t) = E(\mu|x,t)$$

This in turn implies, using (2), that the wage function satisfies the following stochastic differential equation:

$$(5) \qquad dw(t) = \frac{S(t)}{\sigma_x} dZ(t),\ w(0) = \overline{\mu}$$

that is, the wage is a driftless stochastic process with mean $\overline{\mu}$ and variance $\sigma_\mu^2 - S(t)$ and is given by the following Brownian motion:

$$(6) \qquad w(t) = Z[p(t)] + E(\mu|0,0) = \int_0^t \frac{S(\tau)}{\sigma_x} dZ(\tau) + \overline{\mu}$$

Then the sufficient statistics for our problem are (x,t), or (w,t). Notice that the wage process is defined *regardless* of the worker's (optimal) decision and that, given that $E(\mu|0,0) \equiv \overline{\mu}$, all matches start with the same initial wage offer, $\overline{\mu}$.

As was already said above, from equation (6) it should be clear by now that,

6. We are actually only assuming that workers' lifetimes are exponentially distributed random variables. This implies, as explained by JOVANOVIC [1978], pp. 18-19, that, if the model can be solved as if workers had an infinite horizon, workers will nevertheless eventually die, at least probabilistically. If workers were to live forever, given that turnover is negatively related to tenure, we would eventually end up in this model with no turnover (*i.e.* no quits), and this would violate the results we derive below, especially in sections 3.2 and 3.3. Notice too that under exponentially distributed lifetimes the remaining life expectancy is a constant, thus vindicating the absence of the worker's age in the wage function we will make use of below. See JOVANOVIC [1978] for more details.

7. There is an inherent incentive problem in this wage schedule. Indeed, the firm has always an incentive to reneg on its promised wage schedule and offer the worker a wage *lower* than $E(\mu|x,t)$ but *above* the worker's reservation wage. For a discussion of this assumption and a suggestion that reputation effects could allow for this incentive problem to be resolved, see *e.g.* JOVANOVIC [1984].

as time goes to infinity, the wage approaches a constant, given that $S(t)$ tends to 0. This constant is nothing but the true productivity of the match. Thus, at infinity at least, all will be revealed and every worker will be paid exactly according to their true productivity *in that match*.

2.2 The Worker's Optimal Decisions

Once a firm's offer accepted, the worker has always the possibility of quiting that job and accepting the best available competing offer. But, given the game's assumptions – that is, the infinite horizon, a constant and equal discount rate for both workers and firms, and the independence of successive draws for μ – this implies that the present discounted value of quiting a job and (immediately) pursuing the best alternative offer is a constant, Q, of time.

Call $V(w,t)$ the present discounted value of the current match for a worker having tenure t and a wage $w = w(t)$. Then $V(w,t)$ must satisfy:

$$V(w,t) = w\triangle t + e^{-r\triangle t} E_{w,t} V(w(t+\triangle t),t) + o(\triangle t) \tag{7}$$

where $E_{w,t}$ is the conditional expectation operator, conditional on $w = w(t)$ and $o(\triangle t)$ represents terms tending to zero faster than $\triangle t$ does.

This can be rewritten, applying Itô's lemma and after some algebra, as:

$$w - rV(w,t) + \frac{S^2(t)}{2\sigma_x^2}\frac{\partial^2 V}{\partial w^2}(w,t) + \frac{\partial V}{\partial t}(w,t) = 0 \tag{8}$$

where $\frac{\partial^2 V}{\partial w^2}$ stands for the second order derivative of V with respect to w and $\frac{\partial V}{\partial t}$ stands for the first order derivative of V with respect to t.

The worker's problem is therefore that of choosing his optimal quitting time for his current match. These optimal stopping problems[8] allow us to subdivide the (w,t) space into two sub-spaces, one of continuation of the relationship and one of quitting. We can also define a reservation wage $\theta(t)$ below which the worker always (optimally) quits. These problems are characterized by two properties that the solution must satisfy, and that are usually labelled the *value matching* and the *smooth pasting* condition. The first one says that, along the boundary between the continuation and the stopping region, the worker must be indifferent between stopping and continuing, that is, we must have $V(\theta(t),t) = Q$. The second condition tells us that along the boundary, we must have that $\frac{\partial V}{\partial t}(\cdot) = 0$. Applying these conditions to our problem we have that our reservation wage is such that:

$$\theta(t) = rQ - \frac{S^2(t)}{2\sigma_x^2}\frac{\partial^2 V}{\partial w^2}(\theta(t),t) \tag{9}$$

8. See for example DIXIT [2001] or, for a more formal treatment, SHIRYAEV [1978].

As shown by JOVANOVIC [1979], $\frac{\partial^2 V}{\partial w^2}$ is strictly positive along the boundary of the continuation region and the reservation wage is monotonically increasing in time. Thus, $\theta(t)$ is always below rQ, the flow value of quitting and, in the limit, we have that $\lim_{t\to\infty} \theta(t) = rQ$.

3 Two Sub-Groups of Workers

Suppose now we sub-divide the workers' population into two sub-populations, labelled A and B. We maintain the assumption that the true productivity of each match is still an (unknown) draw from the same (known) Normal distribution $\mathcal{N}\left(\overline{\mu},\sigma_{\mu}^{2}\right)$, *regardless* of the group the worker belongs to, but we assume that the instant variance of the Brownian motion is different across the two groups. Specifically, assume that $\sigma_{x,B} > \sigma_{x,A}$, implying that, in loose terms, if we were to take two workers that *have the same draw* μ *for their match*, the observed output of a worker belonging to goup B would nevertheless be much more irregular than the output observed for a worker belonging to group A.

Why not assume that $\sigma_{\mu,A}^2 \neq \sigma_{\mu,B}^2$ Instead? There are two reasons for this: 1) we want to maintain the assumption of identity of the generating process for the realisation of the match-specific quality for both groups of workers, that is, we want the same Normal distribution generating μ for both groups of workers, in order to be able to claim that there are no cross-group differences in the expected quality of the matches; and 2) having the same normal distribution for both groups of workers allows us to have that, as we will see below, the initial and the terminal values of the wage-tenure profiles are the same across groups, implying that market forces, if left to their own devices, will impose convergence of the wage-tenure profiles, even if only in the long run.[9]

Let us finally note here that imposing that the population is sub-divided into two groups with different instant variances in the function defining the output of the match amounts to imposing that the true quality of the match will be revealed at different speeds depending on whether the worker belongs to group A or B. But remember that the presence of different instant variances does not imply different cross-group productivity (distributions). What the different σ_x's imply is only that *the information regarding* μ *will be revealed at different speeds*.

9. That this is so was already implicitly stated in our comments above realtive to the behaviour of the equilibrium wage, which tends to a constant as time goes to infinity. See also section 3.3.

3.1 Reservation Wages

We have seen above that the reservation wage is defined according to equation (9). Adding a subscript i to our notation to allow for cross-group differences, this becomes:

$$\theta_i\left(t\right) = rQ - \frac{S_i^2\left(t\right)}{2\sigma_{x,i}^2}\frac{\partial^2 V}{\partial w^2}\left(\theta_i\left(t\right),t\right),\ i = A, B \tag{10}$$

Are there any cross-group differences in the reservation wages? We already know that both reservation wages converge in the long run from below to rQ. But, at $t = 0,$ we have:

$$\theta_B\left(0\right) = rQ - \sigma_\mu^4/\sigma_{x,B}^2\frac{\partial^2 V}{\partial w^2} > rQ - \sigma_\mu^4/\sigma_{x,A}^2\frac{\partial^2 V}{\partial w^2} = \theta_A\left(0\right) \tag{11}$$

Thus we recover the result already stated by Pissarides, among others (see *e.g.* PISSARIDES [2000, chapter 8]), that the reservation wage of the group where the instant variance is highest is also highest.

The intuition for this result is as follows. Workers know that they are insured against too important a drop in their wage by the option of quitting and going for the best alternative offer (that always yields initially $\overline{\mu}$). In addition, workers facing a higher degree of volatility in their wage also have a higher chance of seeing their wage attaining a very high value, through $\sigma_x Z\left(t\right)$. The effect of an increase in the volatility of wages is therefore asymmetric. Group B workers will take advantage of this by optimally increasing their reservation wage relative to workers belonging to group A (given that $\sigma_{x,B} > \sigma_{x,A}$). Stretching somehow the argument, this implies that group B workers will quit more often, all else equal, than workers belonging to group A.

3.2 Separation Probabilities

Before turning to wage-tenure profiles, we need to find a way to obtain a closed-form solution for a worker's separation probability as a function of the parameters of the model. This is unfortunately not possible *as long as we let the reservation wage be a function of time*. Indeed all closed-form results on the first-passage time of Brownian motions through a barrier hold for *fixed barriers*, and not for *time-dependent ones*. In what follows we therefore *explicitly adopt the strategy espoused by JOVANOVIC [1979] and fix $\theta\left(t\right) = rQ$*, for all t.

We are therefore imposing that every worker chooses a reservation wage equal to its limit value as time goes to infinity, that is, *its maximum possible value*. The results of the previous sub-section, where it was shown that, at $t = 0,$ group B workers have a higher reservation wage than workers belonging to group $A,$ should be discarded from now on and any difference in the separation probabilities is to be ascribed solely to the different values of the Brownian motion's instant variances.

Why did we arbitrarily decide to fix the reservation wage at rQ? The reasons are twofold: 1) it simplifies the reader's comparison of our results with those of JOVANOVIC; and 2) the model naturally gives us a specific constant value for the reservation wage, which is its value when time goes to infinity, that is, rQ.

With this restriction at hand, what we are looking for is the first passage time of a Brownian motion through a *constant* barrier equal to, given our assumptions (see JOVANOVIC [1978]), rQ. Applying Feller's formula for the first passage-time of a Brownian motion through a fixed barrier (see FELLER [1966], p. 171) this probability follows a normal distribution and the cumulative distribution function we are looking for is given by:[10]

$$F_i\left(t \mid \{w\}, Q\right) = 2\Phi\left(\frac{-\left(\overline{\mu} - rQ\right)}{p_i\left(t\right)^{1/2}}\right), \; i = A, B \tag{12}$$

where $\{w\}$ is the wage contract offered to the worker, $p_i\left(t\right)$ is equal to $\sigma_{\mu}^2 - S_i\left(t\right)$, Φ stands for the cdf of a Standard Normal distribution and the equality uses the symmetry property of the Normal distribution about its mean.

Are there any cross-group differences as far as F_i is concerned? It is easy to show that:

$$\frac{\partial F_i\left(t \mid \{w\}, Q\right)}{\partial \sigma_{x,i}^2} < 0 \tag{13}$$

The group with the lowest instant variance is also the one for which the cumulative distribution function of the separation probability is highest. Again, this is fairly intuitive. Given that for this group the uncertainty surrounding the true quality of the match will disappear fastest with time, we can expect these workers to quit faster as the uncertainty surrounding the true productivity of their match unfolds more rapidly.

As an alternative explanation, suppose that the instant variance were to be close to infinity. Then workers would not learn *anything* about the productivity of any match they might form as time goes by. Quitting their current match and starting afresh a new relationship would be useless given that the noise surrounding the true quality of all their matches is overwhelming. This in turn would imply a very low value of $F_i(\cdot)$ for these workers.

10. FELLER [1966], p. 171 shows that the probability that the first-passage time T_a for a Standard Brownian motion Z through a fixed barrier having a value $a > 0$ *from below happens before* t is given by

$$P\left(T_a < t\right) = 2\left[P\left(Z\left(t\right) > a\right)\right] = 2\left[1 - P\left(Z\left(t\right) < a\right)\right]$$

which can finally be rewritten as:

$$2\left[1 - \Phi\left(\frac{a}{\sqrt{t}}\right)\right].$$

Given that in our problem the Brownian motion has mean $\overline{\mu}$ and variance $p\left(t\right) = \sigma_{\mu}^2 - S\left(t\right)$, and that we are looking for the first-passage time of this Brownian motion through the barrier given by rQ *from above*, equation (12) is the result we are looking for. For a more recent reatment of these issues, see for example GRIMMETT and STIRZAKER [1992].

Before continuing, notice that the above result seems to go against the result of the previous sub-section. But this is entirely driven by our having imposed that reservation wages be fixed and equal across groups in order to derive a closed-form solution for the separation probability. Choosing fixed but *different* cross-group reservation wages (with that of group B higher than that of group A, *i.e.* $\theta_B(t) = \theta_B > \theta_A(t) = \theta_A, \forall t$) would allow us to obtain that $F_B(\cdot)$ gets closer to or even becomes greater than $F_A(\cdot)$,[11] a result more in line with that of section 3.1.

3.3 Wage-Tenure Profiles

Sticking to the previous approximation, $\theta(t) = rQ$, we can also calculate an approximation for the wage-tenure profiles. What we are looking for is the probability that a Brownian motion will not cross a linear boundary by a certain time t and that at that time it will end up at a particular value. That is, we are looking for the conditional expectation of the wage of a worker at t, conditional on this worker not having ever quit his job by then. This will give us a measure, at any point in time, of what a worker can be expected to earn, provided he sticks to his match.

Following COX and MILLER [1965, p. 221] or GRIMMETT and STIRZAKER [1992, pp. 500-509], the wage-tenure profile is given by:[12]

$$(14) \qquad \widehat{w_i}(t) = \overline{\mu} + \frac{(\overline{\mu} - rQ)\, 2\Phi\left(\dfrac{-(\overline{\mu} - rQ)}{p_i(t)^{1/2}}\right)}{1 - 2\Phi\left(\dfrac{-(\overline{\mu} - rQ)}{p_i(t)^{1/2}}\right)}, \; i = A, B$$

Remark too that $\widehat{w}(0) = w(0) = \overline{\mu}$, and that $\widehat{w}(\cdot)$ increases monotonically with time up to:

$$(15) \quad \widehat{w_i}(\infty) = \widehat{w}(\infty) = \overline{\mu} + \frac{(\overline{\mu} - rQ)\, 2\Phi\left(\dfrac{-(\overline{\mu} - rQ)}{\sigma_\mu}\right)}{1 - 2\Phi\left(\dfrac{-(\overline{\mu} - rQ)}{\sigma_\mu}\right)}, \; i = A, B$$

Equations (14) and (15) tell us that the conditional expectation of a worker's expected wage increases steadily trough time. Why is that? First of all, notice

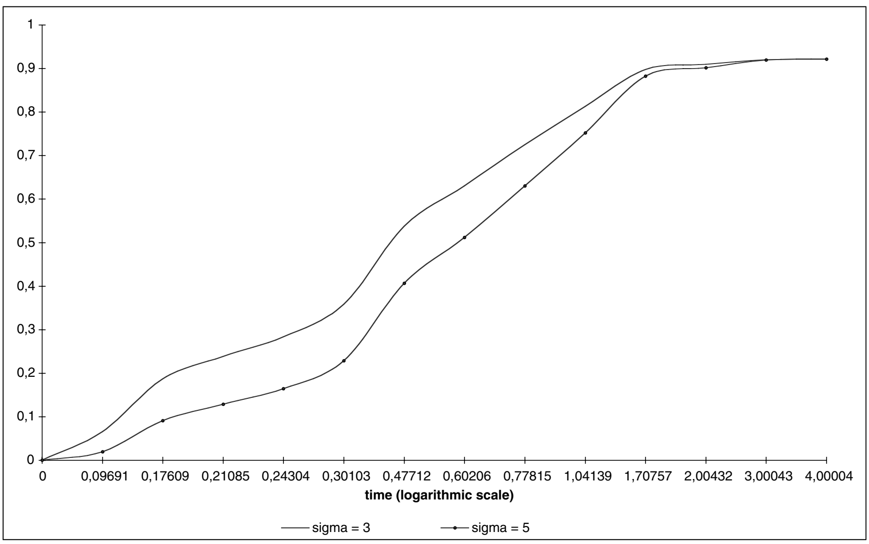
11. See equation (12).

12. We know that all wage functions start at $\overline{\mu}$, hence the first term on the right hand side of (14). The second term is an aplication of Bayes rule. The denominator of the second term of the right hand side of (14) is the probability that the worker will not have quit by t. But we know that the separation probability is $F(\cdot) = 2\Phi(\cdot)$. Therefore the denominator is given by $1 - 2\Phi(\cdot)$. The difficult bit is the numerator of the second term. We first need to compute the density at t of a Standard Brownian motion in the $p(t)$ scale facing an absorbing barrier at rQ. Once we have this density, we can compute its mathematical expectation, and this gives us the numerator. The fact that in the formula we have $\overline{\mu} - rQ$ instead of $\overline{\mu}$ *tout court* is due to the fact that, in order to solve for the density function we needed here, we re-parametrised our problem as that of finding the density of a Brownian motion starting at $\overline{\mu} - rQ$ instead of $\overline{\mu}$ and facing a barrier at 0 instead of rQ.

that the wage-tenure profile always exceeds $\overline{\mu}$, the initial value of the wage for any match. This is not surprising. The expectation of the wage at $t = 0$ cannot differ from $\overline{\mu}$ (see (3)). This explains why $\overline{\mu}$ is included on the right hand side of equations (14) and (15). Now remember that for any t the wage is distributed according to a Normal with mean $\overline{\mu}$ and variance $\sigma_{\mu}^{2} - S(t)$. Then, as time goes by, the portion of this Normal that falls under the reservation wage will increase, implying (because $\overline{\mu} > rQ > 0$) that the expectation of the wage for workers who have not quit their job will increase. This asymmetric effect is captured in the second term on the right hand side of (14) and (15).

Equation (14) is reported graphically hereafter, where we have normalised the initial wage $\overline{\mu}$ to 0 and have chosen a value of 3 for $\sigma_{x,A}$ and of 5 for $\sigma_{x,B}$:[13]

FIGURE 1
Wage-tenure profiles



As is clear from the graph, group A workers are such that their wage-tenure profile is everywhere above that of workers belonging to group B, implying that, except at $t = 0$ and for $t \to \infty$, among the workers who will never quit their initial match, the expected earnings of those workers facing a higher information cost are always lower.

Turning to *the shape* of the expected earnings reported in the figure, note that the different values for $\sigma_{x,A}$ and $\sigma_{x,B}$ do explain why the wage-tenure profiles in the graph below *first diverge and then converge*. This is due to the differing cross-group speeds at which information on the true value of the quality of the match is revealed. Indeed, it is easy to show that $S(t)$ declines

13. Changing the values of any of the parameters does not affect qualitatively any of our results

at a strictly decreasing speed and that this speed is initialy slower the higher is σ_x. Loosely speaking, one could say that the higher is σ_x, the more uniform through time will the information revelation be. Thus, the group that is discriminated against in this model will see its wage-schedule profile first increase at a slower pace relative to that of the other group, but will then catch-up (very much) later on, until the gap is completely closed and the two schedules are exactly equal (at infinity).

3.4 Discussion

In the previous three subsections, a series of results have been derived as regards the optimal behaviour of workers. It was shown that the optimal reservation wage (at $t = 0$) of workers belonging to group B will be higher than that of group A workers, leading the former, in expectation at least, to quit their job more often. This goes *against* the elimination of discrimination, even in the long run.

Imposing the reservation wage to be constant and equal across both groups, when comparing wage-tenure profiles, we also found that the profile of group A workers is everywhere above (or equal) to that of workers belonging to group B. This too implies that discrimination will not disappear, if not in the very long run.

Finally, one sees that the differential first increases and then decreases. Depending on when the quits are most likely to happen, one could even have that discrimination is likely to increase through time. But, all in all, market forces do not seem to be strong enough for discrimination to be eradicated in this context.

The following table reports our main results for the reservation wage at $t = 0$, $\theta(0)$; the separation probabilities, F; and the wage-tenure profile (for any t), $\widehat{w}(t)$:

	$\theta(0)$	F	$\widehat{w}(t)$
Group A relative to group B	lower	higher	higher

4 Conclusion and General Discussion

In his seminal contribution, Lang [1986] suggested that discriminatory behaviour could be due, among other things, to differential language or communication costs between agents. Focusing on dynamics, this paper proposed a formalisation based on Jovanovic [1979] which addresses some of the issues raised in Lang [1986].

Allowing for the speed of information revelation regarding the quality of the match to differ among two groups of otherwise identical workers, the paper showed that the discriminated group will be characterised by a lower wage-tenure profile. The analysis also suggested that this type of discrimination is unlikely to disappear, even in the long run.

Turning to possible extensions and avenues for further research, an interesting extension of this analysis, as suggested by JOVANOVIC himself in his [1978] Ph.D. thesis, would be to include business cycle fluctuations into the model. We indeed expect discrimination to be stronger when markets are slack, because of the relative scarcity of outside opportunities for workers (and the potentially higher relative readiness of employers to fire workers whose productivity is less certain) during economic downturns.[14] ▼

14. CRUTZEN [2003] tries to provide such an extension.

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