Output and Inflation Dynamics under Price and Wage Staggering: Analytical Results

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ABSTRACT. – In this paper we construct a dynamic stochastic general equilibrium model combining price and wage staggered contracts. A closed form solution is given for the optimal contracts and for the resulting macroeconomic dynamics, for any average length of price or wage contracts. We then investigate whether output and inflation can have a persistent and hump shaped response to monetary shocks under reasonable values for contract durations. We find that they do.

Dynamique de l'output et de l'inflation avec contrats imbriqués en prix et en salaires : quelques résultats analytiques

RÉSUMÉ. – On construit dans cet article un modèle d'équilibre général stochastique intégrant des contrats échelonnés de prix et de salaires. On donne une solution explicite pour les contrats optimaux et pour la dynamique macroéconomique résultante, et ceci pour n'importe quelle durée des contrats. On se demande ensuite s'il est possible d'obtenir une réponse persistante et en cloche de l'output et de l'inflation aux chocs monétaires pour des valeurs raisonnables de la durée des contrats, et la réponse est positive.

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1 Introduction

The purpose of this paper is threefold: (a) construct a dynamic stochastic general equilibrium (DSGE) model integrating both wage and price staggered contracts; (b) derive an explicit solution for this model; (c) see whether the persistent and hump-shaped response of output and inflation to monetary shocks observed in the data can be obtained with reasonable parameter values for the length of the contracts.

Introducing wage or price contracts into DSGE models is quite a natural enterprise. The initial contributions by GRAY [1976], FISCHER [1977], PHELPS-TAYLOR [1977], PHELPS [1978], TAYLOR [1979, 1980] and CALVO [1983] showed that embedding staggered contracts into models that, at the time, were not fully structural, would generate some degree of persistence in output and employment. On the other hand rigorous models in the DSGE line[1] have often been criticized for their difficulties in generating a sizeable propagation mechanism (COGLEY and NASON [1993, 1995]). In particular a number of authors (see, for example, CHRISTIANO, EICHENBAUM and EVANS [1999, 2001], COGLEY and NASON [1995]) have shown that output and inflation have a hump-shaped response to monetary shocks, a feature that most traditional DSGE models fail to account for.

So a natural step was to include staggered contracts into DSGE models, and this has actually been done by a number of authors.[2] They constructed models including wage and price contracts either in isolation, or together, or in addition to other rigidities. The problem is that, although the answer might have looked intuitive at the outset, there is a bewildering and conflicting variety of answers. Some authors, like COLLARD and ERTZ [2000] find a persistent and hump-shaped response of output whereas others, like CHARI, KEHOE and MCGRATTAN [2000], conclude that contracts yield practically no persistence.

In such a situation it is particularly useful to complement the numerical explorations, which is the traditional strength of this line of research, by a theoretical approach which will allow to understand which parameters are important, and whether reasonable parameter values can produce a sizeable propagation mechanism.[3] We shall see that this is indeed the case.

1. Initially known as "real business cycles" models. See KYDLAND and PRESCOTT [1982] and LONG and PLOSSER [1983].

2. See notably, among others, AMBLER, GUAY and PHANEUF [1997, 2001], ANDERSEN [1998], ASCARI [2000], BÉNASSY [2000], CHARI, KEHOE and MCGRATTAN [2000], CHRISTIANO, EICHENBAUM and EVANS [2001], COLLARD and ERTZ [2000], HUANG, LIU and PHANEUF [2000], JEANNE [1998] and YUN [1996].

3. Earlier contributions giving explicit solutions with staggered contracts are ANDERSEN [1998], who compares price and wage contracts *à la Taylor*, ASCARI [2000], who studies wage contracts *à la Taylor*, and JEANNE [1998], who studies price contracts *à la Calvo*.

2 The Model

The economy studied is a monetary economy with markets for goods at the (average) price P_t and markets for labor at the (average) wage W_t. The goods and labor markets function under a system of imperfectly competitive staggered contracts, which we now describe.

2.1 Wage and Price Contracts

We shall use staggered wage and price contracts which were developed in BÉNASSY [2000], itself directly inspired by CALVO [1983].

Let us start with the wage contracts. As in CALVO [1983], in each period there is a random draw for all wage contracts, after which any particular contract will continue unchanged (with probability γ), or be terminated (with probability $1-\gamma$). In this last case the corresponding contract wage is renegotiated on the basis of all information currently available.

We denote by X_{st} the wage contract signed in period s for period $t \geqslant s$ (we will show below that all workers sign the same contracts at the same time, so that it does not need to be further indexed). The important difference with the *Calvo* contracts is that the values of the contracts X_{st}, $t \geqslant s$, may differ for all t, whereas they were identical in CALVO [1983].

The description of price contracts is totally symmetrical: We denote by P_t the average price, and by Q_{st} the price contract signed in period s for period t. Price contracts will continue with probability ϕ, and be terminated with probability $1-\phi$.

2.2 Firms and Households

We first describe the production side of the economy. Output Y_t is an aggregate of a continuum of output types, indexed by $j \in [0,1]$:

$$LogY_t = \int_0^1 LogY_{jt}dj \tag{1}$$

The main difference between these types, as will be seen below, is that they may have signed different price contracts at different times. Now each index Y_{jt} is itself the aggregate of another infinity of output types indexed by l:

$$Y_{jt} = \left(\int_0^1 Y_{jlt}^{\sigma} dl \right)^{1/\sigma} \qquad 0 < \sigma < 1 \tag{2}$$

The index j can be interpreted as representing sectors, while index l represents firms within these sectors. Quite naturally substitutability is greater within sectors than across sectors.[4] We assume that all firms with the same index j face exactly the same situation in terms of price contracts, which means in particular that their contracts are renewed at the same time.

The representative firm (we omit the indexes j or l) has a Cobb-Douglas technology:

$$Y_t = Z_t N_t^\alpha \tag{3}$$

The representation of labor markets is fully symmetrical to that of goods markets.[5] The labor index N_t is an aggregate of a continuum of labor types, indexed by $i \in [0,1]$:

$$LogN_t = \int_0^1 LogN_{it} di \tag{4}$$

Each index N_{it} is itself the aggregate of another infinity of labor types indexed by k:

$$N_{it} = \left(\int_0^1 N_{ikt}^\theta dk \right)^{1/\theta} \qquad 0 < \theta < 1 \tag{5}$$

All workers with the same index i face exactly the same situation in terms of wage contracts, and notably renegotiate their contracts at the same time.

The representative household (we omit the indexes i or k at this stage) works N_t, consumes C_t and ends the period with a quantity of money M_t. It maximizes the expected value of its discounted utility, with the following intertemporal utility:

$$U = \sum_t \beta^t \left[LogC_t + \omega Log \frac{M_t}{P_t} - V\left(N_t\right) \right] \tag{6}$$

where V is a convex function. At the beginning of period t there is a stochastic multiplicative monetary shock as in Lucas [1972]: money holdings carried from the previous period M_{t-1} are multiplied by the same factor μ_t for all agents, so that the representative household starts period t with money holdings $\mu_t M_{t-1}$. Its budget constraint in t is thus:

$$C_t + \frac{M_t}{P_t} = \frac{W_t}{P_t} N_t + \frac{\mu_t M_{t-1}}{P_t} + \Pi_t \tag{7}$$

where Π_t is profits, which are distributed uniformly to all households.

4. The reader may actually wonder why, besides the added realism, we use two levels of aggregation, sectors and firms. As it turns out, this will allow a much more elegant solution. It will appear indeed below that it is convenient to have a logarithmic aggregator like (1) for firms with different prices. But, as is wellknown in monopolistic competition theory, a logarithmic aggregator yields too much market power and an infinite markup. So the addition of a second "more competitive" aggregator like (2) allows to have a well defined profit maximization program with finite markups.

5. This representation of the labor market is due to Snower [1983].

3 The Walrasian Regime

We shall now, as a benchmark for what follows, compute the Walrasian equilibrium of this economy. In that case all workers have the same wage W_t, which clears the labor market. The real wage is equal to the marginal productivity of labor:

$$\frac{W_t}{P_t} = \frac{\partial Y_t}{\partial N_t} = \alpha \frac{Y_t}{N_t} \tag{8}$$

Now the households maximize the expected value of the utility function (6) subject to the budget constraints (7). The Lagrangean of this maximization program is:

$$\sum_t \beta^t \left[LogC_t + \omega Log \frac{M_t}{P_t} - V\left(N_t\right) + \lambda_t \left(\frac{W_t N_t}{P_t} + \frac{\mu_t M_{t-1}}{P_t} - C_t - \frac{M_t}{P_t} \right) \right] \tag{9}$$

and the first order conditions:

$$\lambda_t = \frac{1}{C_t} \tag{10}$$

$$\frac{\lambda_t}{P_t} = \frac{\omega}{M_t} + \beta E_t \left(\frac{\mu_{t+1} \lambda_{t+1}}{P_{t+1}} \right) \tag{11}$$

$$V'\left(N_t\right) = \frac{\lambda_t W_t}{P_t} \tag{12}$$

Using (10) and the fact that $\mu_{t+1} = M_{t+1}/M_t$, equation (11) is rewritten:

$$\frac{M_t}{P_t C_t} = \omega + \beta E_t \left(\frac{M_{t+1}}{P_{t+1} C_{t+1}} \right) \tag{13}$$

which solves as:

$$\frac{M_t}{P_t C_t} = \frac{\omega}{1-\beta} \tag{14}$$

We see further, combining (8), (10) and (12), that Walrasian employment is constant[6] and equal to N, which is given by:

$$N V'\left(N\right) = \alpha \tag{15}$$

6. The constancy of Walrasian employment and the neutrality of money are due notably to the logarithmic utility of consumption and the multiplicative money shocks. In that way employment fluctuations are directly related to the price and wage rigidities.

In the particular case where $V(N_t) = \xi N_t^\nu / \nu$, which we shall be using below, we find:

$$N = \left(\frac{\alpha}{\xi} \right)^{1/\nu} \tag{16}$$

4 The Demand for Labor and Goods

We shall now begin the study of our model under wage and price contracts. It is assumed that households, possibly through trade-unions, decide on the level of wages, and supply the amount of labor demanded by firms at these wages. Similarly the firms set prices and supply the goods demanded. In this section we will derive the demand for labor addressed to households, and the demand for goods adressed to firms. Optimal contracts themselves will be derived in sections 5 and 6.

4.1 The Demand for Labor

At any time firms are confronted with a multiplicity of wage contracts which have been signed at different points in time.

For a given index N_{it}, the corresponding firms choose the N_{ikt} that minimize cost, *i.e.,* they solve the following program:

$$\text{Min} \int_0^1 W_{ikt} N_{ikt} dk \qquad \text{s.t.} \qquad \left(\int_0^1 N_{ikt}^\theta dk \right)^{1/\theta} = N_{it}$$

whose solution is:

$$N_{ikt} = N_{it} \left(\frac{W_{ikt}}{W_{it}} \right)^{-1/(1-\theta)} \tag{17}$$

$$W_{it} = \left(\int_0^1 W_{ikt}^{-\theta/(1-\theta)} dk \right)^{-(1-\theta)/\theta} \tag{18}$$

The associated cost is $W_{it} N_{it}$. Now for a given aggregate labor index N_t, the corresponding firms will similarly minimize costs, *i.e.,* they will solve the following program:

$$\text{Min} \int_0^1 W_{it} N_{it} di \qquad \text{s.t.} \qquad \int_0^1 Log N_{it} di = Log N_t$$

whose solution is:

$$N_{it} = \frac{W_t N_t}{W_{it}} \tag{19}$$

$$Log W_t = \int_0^1 Log W_{it} di \tag{20}$$

and the cost is $W_t N_t$. Putting together equations (17) and (19) we obtain the expression of the demand for labor:

$$N_{ikt} = \frac{W_t N_t}{W_{it}} \left(\frac{W_{ikt}}{W_{it}} \right)^{-1/(1-\theta)} \tag{21}$$

An important thing to remember for what follows is that, in view of equation (19), wage income will be the same in all sectors:

$$W_{it} N_{it} = W_t N_t \qquad \forall i \tag{22}$$

4.2 The Demand for Goods

Let us now move to the demand for goods. For a given index Y_{jt} firms will choose the Y_{jlt} that minimize cost, *i.e.,* they will solve:

$$\text{Min} \int_0^1 P_{jlt} Y_{jlt} dl \qquad \text{s.t.} \qquad \left(\int_0^1 Y_{jlt}^{\sigma} dl \right)^{1/\sigma} = Y_{jt}$$

whose solution is:

$$Y_{jlt} = Y_{jt} \left(\frac{P_{jlt}}{P_{jt}} \right)^{-1/(1-\sigma)} \tag{23}$$

$$P_{jt} = \left(\int_0^1 P_{jlt}^{-\sigma/(1-\sigma)} dl \right)^{-(1-\sigma)/\sigma} \tag{24}$$

Similarly for a given aggregate output index Y_t, firms minimize costs, *i.e.,* they solve the following program:

$$\text{Min} \int_0^1 P_{jt} Y_{jt} dj \qquad \text{s.t.} \qquad \int_0^1 Log Y_{jt} dj = Log Y_t$$

whose solution is:

$$Y_{jt} = \frac{P_t Y_t}{P_{jt}} \tag{25}$$

$$Log P_t = \int_0^1 Log P_{jt} dj \tag{26}$$

Putting together equations (23) and (25) we obtain the expression of the demand for goods:

$$Y_{jlt} = \frac{P_t Y_t}{P_{jt}} \left(\frac{P_{jlt}}{P_{jt}} \right)^{-1/(1-\sigma)} \tag{27}$$

An important thing to remember for what follows is that, in view of (25), the value of sales will be the same in all sectors:

$$P_{jt} Y_{jt} = P_t Y_t \qquad \forall j \tag{28}$$

5 Optimal Wage Contracts

We shall derive the optimal labor contracts, assuming from now on the particular disutility of labor:

$$V\left(N_t\right) = \frac{\xi N_t^{\nu}}{\nu} \tag{29}$$

Wage contracts are characterized through the following proposition:

PROPOSITION 1: *The wage contract X_{st} signed in s for period t is given by:*

$$X_{st}^{\nu} E_s \left(\frac{W_t N_t}{M_t} \right) = \frac{\xi \left(1-\beta\right)}{\omega \theta} E_s \left(W_t N_t\right)^{\nu} \tag{30}$$

PROOF: We denote by X_{ikst} the contract signed at time s, to be in effect in time t, by household (i,k). In order to determine X_{ikst}, household (i,k) maximizes its discounted expected utility. We shall consider here only the terms corresponding to the wage contracts signed at time s and still in effect at time t. Since wage contracts have a probability γ to survive each period, the contract signed in s has a probability γ^{t-s} to be still in effect in period t, and the household will thus maximize the following expected utility:

$$E_s \sum_{t \geqslant s} \beta^{t-s} \gamma^{t-s} \left[Log C_{ikt} + \omega Log \frac{M_{ikt}}{P_t} - \frac{\xi N_{ikt}^{\nu}}{\nu} \right] \tag{31}$$

subject, in each period, to the budget constraint:

$$C_{ikt} + \frac{M_{ikt}}{P_t} = \frac{W_{ikt}}{P_t} N_{ikt} + \frac{\mu_t M_{ikt-1}}{P_t} + \Pi_t \tag{32}$$

and the equation giving the demand for labor (21):

$$N_{ikt} = \frac{W_t N_t}{W_{it}} \left(\frac{W_{ikt}}{W_{it}} \right)^{-1/(1-\theta)} \tag{33}$$

Now since we consider only the wage contracts signed in s and still in effect in t, the wage W_{ikt} in formulas (32) and (33) must be replaced by X_{ikst}. Similarly W_{it} is equal to the average X_{ist} of these contracts across households indexed by i, *i.e.,* in view of formula (18):

$$X_{ist} = \left(\int_0^1 X_{ikst}^{-\theta/(1-\theta)} dk \right)^{-(1-\theta)/\theta} \tag{34}$$

Inserting (33) into (31) and (32), and replacing, as we indicated, W_{ikt} by X_{ikst} and W_{it} by X_{ist}, the household's maximization program becomes:

$$\max E_s \sum_{t \geqslant s} \beta^{t-s} \gamma^{t-s} \left\{ Log C_{ikt} + \omega Log \frac{M_{ikt}}{P_t} - \frac{\xi}{\nu} \left[\frac{W_t N_t}{X_{ist}} \left(\frac{X_{ikst}}{X_{ist}} \right)^{-1/(1-\theta)} \right]^{\nu} \right\} \text{ s.t.}$$

$$C_{ikt} + \frac{M_{ikt}}{P_t} = \frac{W_t N_t}{P_t} \left(\frac{X_{ikst}}{X_{ist}} \right)^{-\theta/(1-\theta)} + \frac{\mu_t M_{ikt-1}}{P_t} + \Pi_t$$

Inspecting this maximization problem we first see that all households with the same index i solve exactly the same program, so that in equilibrium we have:

$$X_{ikst} = X_{ist} \qquad \forall k \tag{35}$$

Moreover, in view of equation (22), all households have the same wage income, and therefore the same consumption and money holdings (but they will differ, of course, in their wages and employment levels):

$$C_{ikt} = C_t \qquad M_{ikt} = M_t \qquad \forall i,k \tag{36}$$

We can now write the Lagrangean of the above maximization program. Taking into account (36), it is written:

$$E_s \sum_{t \geqslant s} \beta^{t-s} \gamma^{t-s} \left\{ Log C_t + \omega Log \frac{M_t}{P_t} - \frac{\xi}{\nu} \left[\frac{W_t N_t}{X_{ist}} \left(\frac{X_{ikst}}{X_{ist}} \right)^{-1/(1-\theta)} \right]^{\nu} \right\}$$
$$+ E_s \sum_{t \geqslant s} \beta^{t-s} \gamma^{t-s} \lambda_{ikt} \left[\frac{W_t N_t}{P_t} \left(\frac{X_{ikst}}{X_{ist}} \right)^{-\theta/(1-\theta)} + \frac{\mu_t M_{t-1}}{P_t} - C_t - \frac{M_t}{P_t} \right] \tag{37}$$

Maximization in C_t yields:

$$\lambda_{ikt} = \frac{1}{C_t} \tag{38}$$

so that the term in X_{ikst} is, suppressing unimportant constants:

$$E_s \left\{ \frac{W_t N_t}{P_t C_t} \left(\frac{X_{ikst}}{X_{ist}} \right)^{-\theta/(1-\theta)} - \frac{\xi}{\nu} \left[\frac{W_t N_t}{X_{ist}} \left(\frac{X_{ikst}}{X_{ist}} \right)^{-1/(1-\theta)} \right]^{\nu} \right\} \tag{39}$$

The first order condition in X_{ikst} is:

$$E_s \left\{ \frac{\theta W_t N_t}{P_t C_t} \left(\frac{X_{ikst}}{X_{ist}} \right)^{-1/(1-\theta)} - \xi \left(\frac{W_t N_t}{X_{ist}} \right)^{\nu} \left(\frac{X_{ikst}}{X_{ist}} \right)^{-\nu/(1-\theta)-1} \right\} = 0 \tag{40}$$

Now we know from above (35) that in equilibrium $X_{ikst} = X_{ist}$ for all k, so that the above first order condition first simplifies as:

$$E_s \left\{ \frac{\theta W_t N_t}{P_t C_t} - \xi \left(\frac{W_t N_t}{X_{ist}} \right)^{\nu} \right\} = 0 \tag{41}$$

We further see that the solution in X_{ist} is the same for all agents indexed by i and we denote it as X_{st}. So:

$$X_{st}^{\nu} E_s \left(\frac{\theta W_t N_t}{P_t C_t} \right) = \xi E_s \left(W_t N_t \right)^{\nu} \tag{42}$$

from which, using formula (14), we deduce formula (30). Q.E.D.

We shall now derive the average wage index W_t as a function of all contracts signed in the past, X_{st}, $s \leqslant t$. We first have the general formula (20) giving W_t:

$$Log W_t = \int_0^1 Log W_{it} di \tag{43}$$

Because of the law of large numbers, and in view of the survival rate γ, a proportion $1 - \gamma$ of the wage contracts will come from period t, a proportion $\gamma \left(1 - \gamma\right)$ from period $t - 1$, ..., a proportion $\gamma^{t-s} \left(1 - \gamma\right)$ from period s, and so on. As a result formula (43) is rewritten:

$$Log W_t = \left(1 - \gamma\right) \sum_{s=-\infty}^{t} \gamma^{t-s} Log X_{st} \tag{44}$$

6 Optimal Price Contracts

We shall now derive the optimal price contracts. They are characterized through the following proposition:

PROPOSITION 2: *The price contract Q_{st} signed in s for period t is given by:*

$$Q_{st}^{1/\alpha} = \frac{1}{\alpha\sigma}\left(\frac{1-\beta}{\omega}\right)^{(1-\alpha)/\alpha} E_s\left[W_t M_t^{(1-\alpha)/\alpha} Z_t^{-1/\alpha}\right] \tag{45}$$

PROOF: We denote as Q_{jlst} the price contract signed at time s, to be in effect in time t, by firm (j,l). In order to determine Q_{jlst}, firm (j,l) maximizes its discounted expected profits weighted by the marginal utility of goods. We will consider here only the terms corresponding to the price contracts signed at time s and still in effect at time t. Since price contracts have a probability ϕ to survive each period, the contract signed in s has a probability ϕ^{t-s} to be still in effect in period t, and the firm will thus maximize the following expected discounted profit:

$$E_s \sum_{t\geqslant s} \beta^{t-s}\phi^{t-s}\frac{1}{P_t C_t}\left(P_{jlt}Y_{jlt} - W_t N_{jlt}\right) \tag{46}$$

$$= E_s \sum_{t\geqslant s} \beta^{t-s}\phi^{t-s}\frac{1}{P_t C_t}\left[P_{jlt}Y_{jlt} - W_t\left(\frac{Y_{jlt}}{Z_t}\right)^{1/\alpha}\right]$$

subject to the equation giving the demand for goods (27):

$$Y_{jlt} = \frac{P_t Y_t}{P_{jt}}\left(\frac{P_{jlt}}{P_{jt}}\right)^{-1/(1-\sigma)} \tag{47}$$

Now since we consider only the price contracts signed in s and still in effect in t, P_{jlt} in formulas (46) and (47) must be replaced by Q_{jlst}. Similarly P_{jt} is equal to the average Q_{jst} of these price contracts across firms indexed by j, *i.e.,* in view of formula (24):

$$Q_{jst} = \left(\int_0^1 Q_{jlst}^{-\sigma/(1-\sigma)} dl\right)^{-(1-\sigma)/\sigma} \tag{48}$$

Inspecting the above maximization problem we first see that all firms with the same index j solve the same program, so that in equilibrium we have:

$$Q_{jlst} = Q_{jst} \qquad \forall l \tag{49}$$

Firms indexed by (j,l) maximize (46) subject to (47). Let us insert the value of Y_{jlt} (equation 47) into (46). Taking into account the fact that $C_t = Y_t$, the maximand is written:

$$(50) \qquad = E_s \sum_{t \geqslant s} \beta^{t-s} \phi^{t-s}$$

$$\left[\left(\frac{Q_{jlst}}{Q_{jst}} \right)^{-\sigma/(1-\sigma)} - \frac{W_t}{P_t Y_t} \left(\frac{P_t Y_t}{Z_t Q_{jst}} \right)^{1/\alpha} \left(\frac{Q_{jlst}}{Q_{jst}} \right)^{-1/\alpha(1-\sigma)} \right]$$

The part of this maximand concerning Q_{jlst} is:

$$(51) \quad E_s \left[\left(\frac{Q_{jlst}}{Q_{jst}} \right)^{-\sigma/(1-\sigma)} - \frac{W_t}{P_t Y_t} \left(\frac{P_t Y_t}{Z_t Q_{jst}} \right)^{1/\alpha} \left(\frac{Q_{jlst}}{Q_{jst}} \right)^{-1/\alpha(1-\sigma)} \right]$$

and the first order condition in Q_{jlst}:

$$(52) \qquad E_s \left[\frac{\sigma}{Q_{jst}} \left(\frac{Q_{jlst}}{Q_{jst}} \right)^{-1/(1-\sigma)} \right.$$

$$\left. - \frac{W_t}{\alpha P_t Y_t} \left(\frac{P_t Y_t}{Z_t Q_{jst}} \right)^{1/\alpha} \frac{1}{Q_{jst}} \left(\frac{Q_{jlst}}{Q_{jst}} \right)^{-1/\alpha(1-\sigma)-1} \right] = 0$$

Now we know from above (49) that in equilibrium $Q_{jlst} = Q_{jst}$, so that the first order condition (52) first simplifies as:

$$(53) \qquad Q_{jst}^{1/\alpha} = E_s \left[\frac{W_t}{\alpha \sigma P_t Y_t} \left(\frac{P_t Y_t}{Z_t} \right)^{1/\alpha} \right]$$

We further see that the solution in Q_{jst} is the same for all firms, independently of the index j, and we denote it as Q_{st}:

$$(54) \qquad Q_{st}^{1/\alpha} = \frac{1}{\alpha \sigma} E_s \left[\frac{W_t}{P_t Y_t} \left(\frac{P_t Y_t}{Z_t} \right)^{1/\alpha} \right]$$

which, using (14), yields formula (45). Q.E.D.

We will now derive the average price index P_t as a function of all contracts signed in the past, Q_{st}, $s \leqslant t$. We first have the general formula (26) giving P_t:

$$(55) \qquad Log P_t = \int_0^1 Log P_{jt} dj$$

Because of the law of large numbers, and with a survival rate ϕ, a proportion $1-\phi$ of the price contracts comes from period t, a proportion $\phi(1-\phi)$ from period $t-1$, ..., a proportion $\phi^{t-s}(1-\phi)$ from period s, and so on. As a result formula (55) is rewritten:

$$LogP_t = (1-\phi)\sum_{s=-\infty}^{t} \phi^{t-s} LogQ_{st} \tag{56}$$

7 Macroeconomic Dynamics

In order to derive the dynamics of the system, let us now rewrite in loglinearized form, and omitting irrelevant constant terms, equations (3), (14), (30), (44), (45) and (56):

$$y_t = \alpha n_t + z_t \tag{57}$$

$$m_t = p_t + c_t = p_t + y_t \tag{58}$$

$$\nu x_{st} = (\nu-1)\, E_s\,(w_t+n_t) + E_s\,(m_t) \tag{59}$$

$$w_t = (1-\gamma)\sum_{s=-\infty}^{t} \gamma^{t-s} x_{st} \tag{60}$$

$$q_{st} = E_s\left[\alpha w_t + (1-\alpha)\, m_t - z_t\right] \tag{61}$$

$$p_t = (1-\phi)\sum_{s=-\infty}^{t} \phi^{t-s} q_{st} \tag{62}$$

The resulting dynamics is described through the following proposition:

PROPOSITION 3: *Let us denote by $u_t = m_t - E_{t-1}m_t$ and $\varepsilon_t = z_t - E_{t-1}z_t$ the innovations in money and technology respectively. Then output is given by:*

$$y_t = z_t + \alpha\left[m_t - (1-\gamma)\sum_{j=0}^{\infty}\gamma^j E_{t-j} m_t\right] + \sum_{i=0}^{\infty}\left[\frac{\nu-(\nu-1)\left(1-\gamma^{i+1}\right)}{\nu-(\nu-1)\left(1-\gamma^{i+1}\right)\phi^{i+1}}\right]\left(\omega_i u_{t-i} + \lambda_i \varepsilon_{t-i}\right) \tag{63}$$

where the ω_i's and the λ_i's are given by:

$$\sum_{i=0}^{\infty} \omega_i u_{t-i} = \Omega_t = m_t - (1-\phi) \sum_{j=0}^{\infty} \phi^j E_{t-j} m_t - \alpha \left[m_t - (1-\gamma\phi) \sum_{j=0}^{\infty} \gamma^j \phi^j E_{t-j} m_t \right] \tag{64}$$

$$\sum_{i=0}^{\infty} \lambda_i u_{t-i} = \Lambda_t = - \left[z_t - (1-\phi) \sum_{j=0}^{\infty} \phi^j E_{t-j} z_t \right] \tag{65}$$

PROOF: appendix 1.

Now in order to compute some simple impulse response functions, we will make things a little more specific. First, it is traditionally assumed that money increments are autoregressive, so we will take the usual money process:

$$m_t - m_{t-1} = \frac{u_t}{1-\rho L} \tag{66}$$

On the other hand it is usually assumed that z_t is autoregressive, with possibly a unit root, so we will take:

$$z_t = \frac{\varepsilon_t}{1-\varphi L} \tag{67}$$

Now we can describe the dynamics as:

PROPOSITION 4: *If money and technology shocks follow the processes* (66) *and* (67)*, then output is given by:*

$$y_t = z_t + \frac{\alpha \gamma u_t}{(1-\gamma L)(1-\gamma\rho L)} + \sum_{i=0}^{\infty} \left[\frac{\nu - (\nu-1)\left(1-\gamma^{i+1}\right)}{\nu - (\nu-1)\left(1-\gamma^{i+1}\right)\phi^{i+1}} \right] \left(\omega_i u_{t-i} + \lambda_i \varepsilon_{t-i}\right) \tag{68}$$

where the ω_i's and the λ_i's are given by:

$$\sum_{i=0}^{\infty} \omega_i u_{t-i} = \Omega_t = \frac{\phi u_t}{(1-\phi L)(1-\phi\rho L)} - \frac{\alpha\gamma\phi u_t}{(1-\gamma\phi L)(1-\gamma\phi\rho L)} \tag{69}$$

$$\sum_{i=0}^{\infty} \lambda_i u_{t-i} = \Lambda_t = -\frac{\phi \varepsilon_t}{1-\phi\varphi L} \tag{70}$$

PROOF: insert the formulas of appendix 2 into proposition 3.

We may note a few things about formulas (68) to (70). We first see that the effects of price rigidities (corresponding to $\phi > 0$) are somehow dampened by high values of ν, *i.e.,* by a relatively inelastic supply of labor. The intuition for that effect has been already given by ANDERSEN [1998] for *Taylor* contracts: Consider indeed a positive shock which creates an additional demand for labor. On the subset of labor markets where the wage is flexible this leads to a wage increase, which is higher, the more inelastic the labor supply. This wage increase contributes to dampen the initial effect, and this dampening will thus be stronger if labor supply is more inelastic.

We also see from formulas (68) and (70) that a positive technological shock has a negative effect on employment. The intuition for this is particularly straightforward if we start with the extreme case of full price rigidity ($\phi = 1$). In that case for a given demand a positive technological shock mechanically reduces the quantity of labor necessary to produce that demand. With incomplete price rigidity ($\phi < 1$) the same effect is at work, but at a lower scale.

8 Impulse Response Functions and Humps

We will now investigate, with the help of proposition 4, whether various combinations of wage and price staggering can generate a substantial propagation mechanism. Since the debate has been particularly intense around the response to monetary shocks, we will concentrate on them, and ignore technology shocks in the discussion that follows.

We will actually not attempt to do any kind of proper numerical calibration, but rather investigate for which combinations of parameters one may obtain hump-shaped responses of both output and inflation to monetary shocks, since the absence of such a hump-shaped response was noted as a strong defect of traditional models.

In order to make the discussion more transparent, we will consider the case $\nu = 1$, corresponding to an elastic labor supply. For this parameter value formulas (68) and (69) simplify to:

$$(71) \qquad y_t = \frac{\alpha\gamma u_t}{(1-\gamma L)(1-\gamma\rho L)} + \frac{\phi u_t}{(1-\phi L)(1-\phi\rho L)} - \frac{\alpha\gamma\phi u_t}{(1-\gamma\phi L)(1-\gamma\phi\rho L)}$$

Now the response of output to monetary shocks will display a hump if the first period response to the shock is smaller than the second period response, *i.e.,* if:

$$(72) \qquad \alpha\gamma + \phi - \alpha\gamma\phi < (1+\rho)\left(\alpha\gamma^2 + \phi^2 - \alpha\gamma^2\phi^2\right)$$

The corresponding region in the parameter space (γ,ϕ) is represented in figure 1. It is the region above and to the right of the curve noted (1).

Let us now move to inflation, which is given by:

(73) $\pi_t = (1-L)\,p_t = (1-L)\,(m_t - y_t) = (1-L)\,m_t - (1-L)\,y_t$

Consequently, using formula (71):

$$\pi_t = \frac{u_t}{1-\rho L} - \frac{\alpha\gamma\,(1-L)\,u_t}{(1-\gamma L)\,(1-\gamma\rho L)} - \frac{\phi\,(1-L)\,u_t}{(1-\phi L)\,(1-\phi\rho L)} + \frac{\alpha\gamma\phi\,(1-L)\,u_t}{(1-\gamma\phi L)\,(1-\gamma\phi\rho L)} \tag{74}$$

Again there will be a hump if the first period impact is smaller than the second period one, *i.e.,* if:

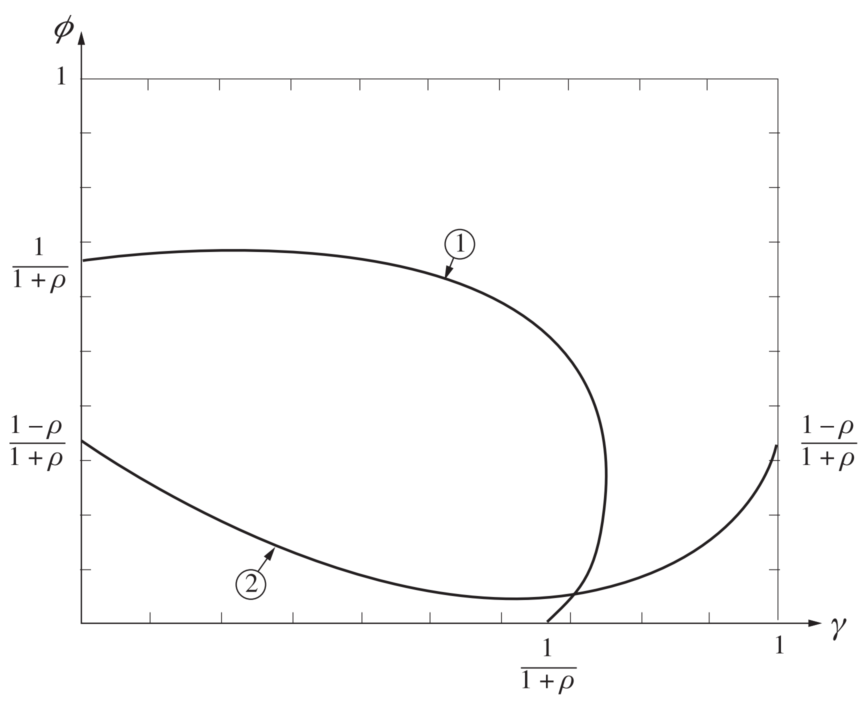
$$1 - \alpha\gamma - \phi + \alpha\gamma\phi < \rho - \alpha\gamma\,(\gamma + \gamma\rho - 1) - \phi\,(\phi + \phi\rho - 1) + \alpha\gamma\phi\,(\gamma\phi + \gamma\phi\rho - 1) \tag{75}$$

which can be simplified as:

$$(1+\phi)\,(1+\rho) > \frac{2\,(1-\alpha\gamma)}{1-\alpha\gamma^2} \tag{76}$$

FIGURE 1

Conditions for a Humpshaped Response of Output and Inflation



The corresponding region has also been drawn in figure 1. It is the region above the curve noted (2).

9 A Few Simulations

We can now run a few simulations to illustrate the above theoretical results. We will actually proceed in two steps:

(a) We will first show that we can reproduce with our model some results in the literature which looked somewhat contradictory. This will show that our model can effectively cover a wide spectrum of situations.

(b) We will then show that a realistic combination of price and wage rigidities enables us to obtain a hump shaped response of both output and inflation to monetary shocks.

As formula (71) shows, there are four central parameters,[7] α, ρ, γ, and ϕ. For α and ρ we take in all simulations the two traditional values $\alpha = 2/3$ and $\rho = 1/2$.

To have an idea about γ, let us first compute the average duration of wage contracts: a contract has a probability $(1-\gamma)\gamma^j$ to be still in effect in period j, so that the average duration is:

$$(1-\gamma)\sum_{j=0}^{\infty} j\gamma^j = \frac{\gamma}{1-\gamma} \tag{77}$$

Similarly the average duration of price contracts is:

$$(1-\phi)\sum_{j=0}^{\infty} j\phi^j = \frac{\phi}{1-\phi} \tag{78}$$

9.1 Price Rigidities Only

Let us start with price rigidities only ($\gamma = 0$). CHARI, KEHOE and MCGRATTAN [2000] studied such a case with a central hypothesis corresponding to an average duration of price contracts of one quarter ($\phi = 1/2$), and concluded that one should not hope to find persistence with such contracts. Our model very much predicts the same thing: there are two autoregressive roots, equal to $1/2$ and $1/4$ respectively, which clearly does not lead to a very persistent response. This appears indeed in figure 2, which pictures the impulse response function to a monetary shock, corresponding to formula (71) with $\gamma = 0$ and $\phi = 1/2$. There is no hump either, something that could be predicted directly from figure 1.

7. Let us recall that α is the labor elasticity of output, ρ the autocorrelation of money increases, γ and ϕ the survival probabilities of wage and price contracts respectively.

The lack of persistence in that case is fully predictable from our model, in view of the very short average duration of contracts. We may note that the same negative result would have obtained as well under one quarter wage contracts.

9.2 Wage Rigidities Only

Let us now move to the other polar case, wage contracts only ($\phi = 0$). In that case, a duration of one year ($\gamma = 4/5$) seems to be the most accepted figure (see for example TAYLOR [1999]). Such a duration, as well as two years, was actually studied by COLLARD and ERTZ [2000], who found a persistent and hump-shaped response. We see from figure 1 that our model also predicts a hump-shaped response, which is confirmed by direct simulation of formula (71) with $\gamma = 4/5$ and $\phi = 0$ (figure 3).

One might think at this stage that the introduction of wage contracts is sufficient to deliver the dynamic response we are looking for. If, however, we now look at the response of inflation with the same values of parameters ($\gamma = 4/5,\ \phi = 0$), a direct simulation shows that there will be no hump (figure 4). This was also predictable from looking at figure 1.

So the natural next step is to look at wage and price rigidities together.

FIGURE 2
Output Impulse Response Function: $\gamma = 0$, $\phi = 1/2$

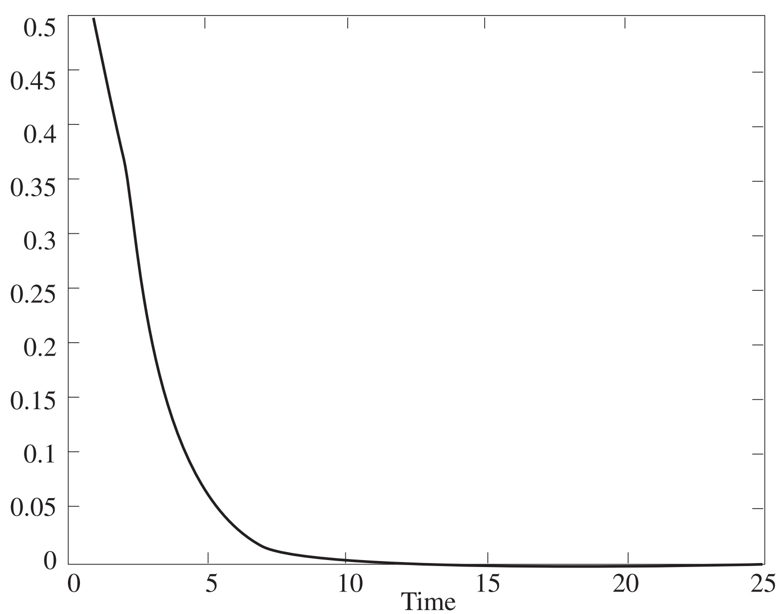


FIGURE 3
Output Impulse Response Function: $\gamma = 4/5$, $\phi = 0$

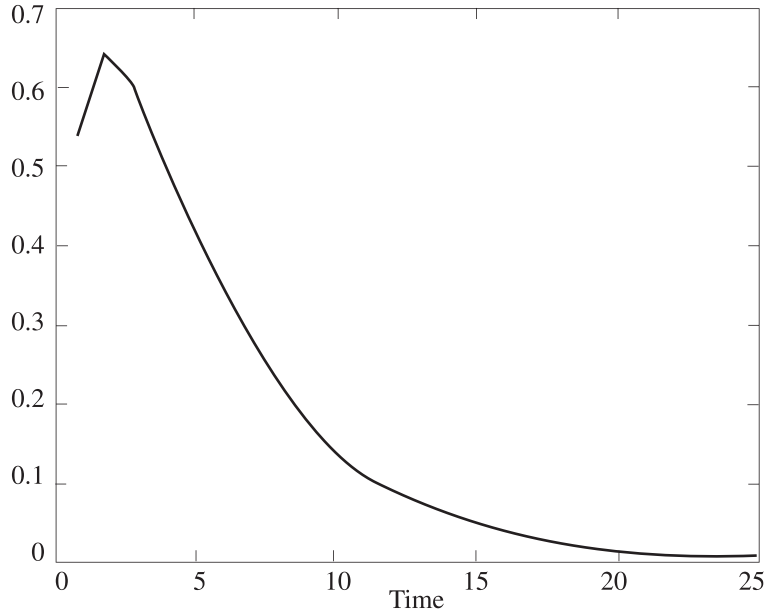
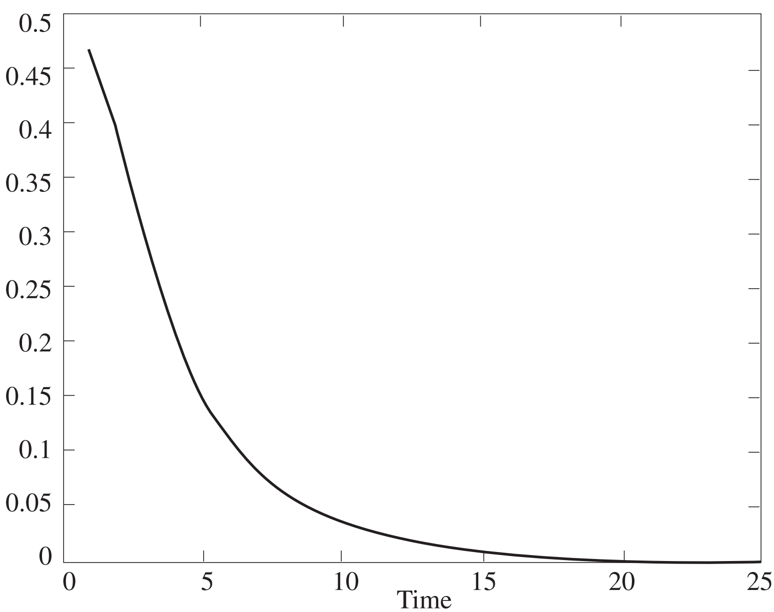


FIGURE 4
Inflation Impulse Response Function: $\gamma = 4/5$, $\phi = 0$



9.3 **Both Rigidities**

In order to show that the combination of the two rigidities can achieve what each in isolation could not, we take exactly the same values that we investigated previously, *i.e.,* $\gamma = 4/5$ (one year wage contracts) and $\phi = 1/2$ (one quarter price contracts). Figure 1 leads us to expect that there will be a hump in both output and inflation responses. This is clearly confirmed by the simulations (figures 5 and 6).

10 Conclusions

We constructed in this paper a dynamic stochastic general equilibrium model with staggered wage and price contracts together. We derived explicit solutions for both the value of the contracts and the resulting macroeconomic dynamics. This allowed us to show in particular that adequate combinations of wage and price contracts with reasonable durations could deliver a hump shaped response of both output and inflation to money shocks. This was obtained for parameter values such that each rigidity in isolation could not deliver the same result. This shows that it was indeed important to construct a model integrating both rigidities together. ▼

FIGURE 5
Output Impulse Response Function: $\gamma = 4/5$, $\phi = 1/2$

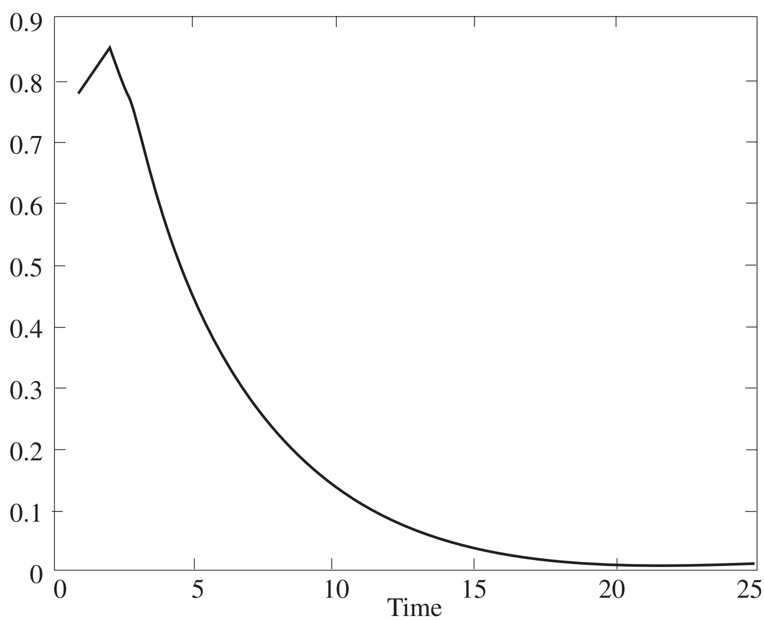
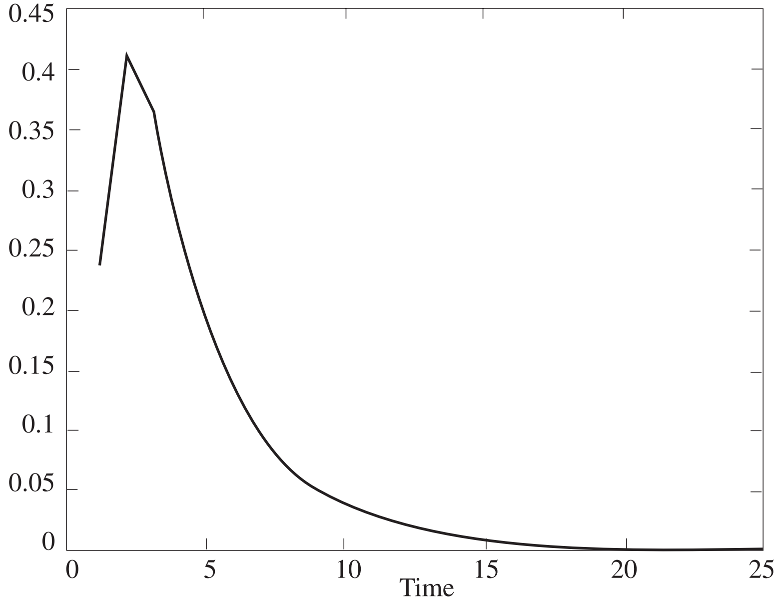


FIGURE 6
Inflation Impulse Response Function: $\gamma = 4/5$, $\phi = 1/2$



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APPENDIX 1

Proof of Proposition 1

Let us recall the dynamic equations:

$$y_t = \alpha n_t + z_t \tag{79}$$

$$p_t + y_t = m_t \tag{80}$$

$$\nu x_{st} = (\nu - 1)\, E_s\,(w_t + n_t) + E_s\,(m_t) \tag{81}$$

$$w_t = (1-\gamma) \sum_{s=-\infty}^{t} \gamma^{t-s} x_{st} \tag{82}$$

$$q_{st} = E_s\,[\alpha w_t + (1-\alpha)\, m_t - z_t] \tag{83}$$

$$p_t = (1-\phi) \sum_{s=-\infty}^{t} \phi^{t-s} q_{st} \tag{84}$$

Let us start from the hypothesis:

$$w_t + n_t - m_t = \nu \sum_{i=0}^{\infty} a_i u_{t-i} + \nu \sum_{i=0}^{\infty} b_i \varepsilon_{t-i} \tag{85}$$

Combining (81) and (82), and making a change of index, we obtain the expression of the wage:

$$w_t = (1-\gamma) \sum_{j=0}^{\infty} \gamma^j E_{t-j} m_t + \frac{\nu - 1}{\nu} (1-\gamma) \sum_{j=0}^{\infty} \gamma^j E_{t-j}\,(w_t + n_t - m_t) \tag{86}$$

Let us evaluate the second term, using (85):

$$\begin{aligned} &\frac{1-\gamma}{\nu} \sum_{j=0}^{\infty} \gamma^j E_{t-j}\,(w_t + n_t - m_t) \\ &= (1-\gamma) \sum_{j=0}^{\infty} \gamma^j E_{t-j} \left(\sum_{i=0}^{\infty} a_i u_{t-i} + \sum_{i=0}^{\infty} b_i \varepsilon_{t-i} \right) \\ &= (1-\gamma) \sum_{j=0}^{\infty} \gamma^j \left(\sum_{i \geqslant j} a_i u_{t-i} + \sum_{i \geqslant j} b_i \varepsilon_{t-i} \right) \\ &= \left(\sum_{i=0}^{\infty} a_i u_{t-i} + \sum_{i=0}^{\infty} b_i \varepsilon_{t-i} \right) (1-\gamma) \sum_{j \leqslant i} \gamma^j \\ &= \sum_{i=0}^{\infty} \left(1 - \gamma^{i+1}\right) \left(a_i u_{t-i} + b_i \varepsilon_{t-i}\right) \end{aligned} \tag{87}$$

so that combining (86) and (87):

$$w_t = (1-\gamma)\sum_{j=0}^{\infty}\gamma^j E_{t-j}m_t + (\nu-1)\sum_{i=0}^{\infty}\left(1-\gamma^{i+1}\right)\left(a_i u_{t-i} + b_i \varepsilon_{t-i}\right) \tag{88}$$

We can now compute the price level. Combining (83), (84), and making a change of index we obtain:

$$p_t = (1-\phi)\sum_{j=0}^{\infty}\phi^j E_{t-j}\left(m_t - z_t\right) + \alpha\left(1-\phi\right)\sum_{j=0}^{\infty}\phi^j E_{t-j}\left(w_t - m_t\right) \tag{89}$$

Let us compute the second term. Using (88) we have:

$$\begin{aligned}(1-\phi)\sum_{j=0}^{\infty}\phi^j E_{t-j}\left(w_t - m_t\right) &= (1-\phi)\sum_{j=0}^{\infty}\phi^j E_{t-j}\left[(1-\gamma)\sum_{i=0}^{\infty}\gamma^i E_{t-i}m_t - m_t\right] \\ &+ (\nu-1)(1-\phi)\sum_{j=0}^{\infty}\phi^j E_{t-j}\left[\sum_{i=0}^{\infty}\left(1-\gamma^{i+1}\right)\left(a_i u_{t-i} + b_i\varepsilon_{t-i}\right)\right]\end{aligned} \tag{90}$$

Let us compute the first term of (90):

$$\begin{aligned}&(1-\phi)\sum_{j=0}^{\infty}\phi^j E_{t-j}\left[(1-\gamma)\sum_{i=0}^{\infty}\gamma^i E_{t-i}m_t - m_t\right] \\ &= (1-\phi)\sum_{j=0}^{\infty}\phi^j\left[(1-\gamma)\sum_{i=0}^{j-1}\gamma^i - 1\right]E_{t-j}m_t \\ &+ \sum_{i=0}^{\infty}(1-\phi)\phi^j\left[(1-\gamma)\sum_{i\geqslant j}\gamma^i E_{t-i}m_t\right] \\ &= -(1-\phi)\sum_{j=0}^{\infty}\gamma^j\phi^j E_{t-j}m_t + (1-\gamma)\sum_{i=0}^{\infty}\left[(1-\phi)\sum_{j\leqslant i}\phi^j\right]\gamma^i E_{t-i}m_t \\ &= -(1-\phi)\sum_{j=0}^{\infty}\gamma^j\phi^j E_{t-j}m_t + (1-\gamma)\sum_{i=0}^{\infty}\left(1-\phi^{i+1}\right)\gamma^i E_{t-i}m_t \\ &= (1-\gamma)\sum_{j=0}^{\infty}\gamma^j E_{t-j}m_t - (1-\gamma\phi)\sum_{j=0}^{\infty}\gamma^j\phi^j E_{t-j}m_t\end{aligned} \tag{91}$$

Now let us compute the second term of (90):

$$(92)\quad (1-\phi)\sum_{j=0}^{\infty}\phi^{j}E_{t-j}\left[\sum_{i=0}^{\infty}\left(1-\gamma^{i+1}\right)\left(a_{i}u_{t-i}+b_{i}\varepsilon_{t-i}\right)\right]$$

$$=(1-\phi)\sum_{j=0}^{\infty}\phi^{j}\left[\sum_{i\geqslant j}^{\infty}\left(1-\gamma^{i+1}\right)\left(a_{i}u_{t-i}+b_{i}\varepsilon_{t-i}\right)\right]$$

$$=\sum_{i=0}^{\infty}\left(1-\gamma^{i+1}\right)\left(a_{i}u_{t-i}+b_{i}\varepsilon_{t-i}\right)\left[(1-\phi)\sum_{j\leqslant i}^{\infty}\phi^{j}\right]$$

$$=\sum_{i=0}^{\infty}\left(1-\gamma^{i+1}\right)\left(1-\phi^{i+1}\right)\left(a_{i}u_{t-i}+b_{i}\varepsilon_{t-i}\right)$$

So:

$$(93)\quad (1-\phi)\sum_{j=0}^{\infty}\phi^{j}E_{t-j}\left(w_{t}-m_{t}\right)$$

$$=(1-\gamma)\sum_{j=0}^{\infty}\gamma^{j}E_{t-j}m_{t}-(1-\gamma\phi)\sum_{j=0}^{\infty}\gamma^{j}\phi^{j}E_{t-j}m_{t}$$

$$+(\nu-1)\sum_{i=0}^{\infty}\left(1-\gamma^{i+1}\right)\left(1-\phi^{i+1}\right)\left(a_{i}u_{t-i}+b_{i}\varepsilon_{t-i}\right)$$

Inserting (93) into (89) we can compute the price:

$$(94)\quad p_{t}=(1-\phi)\sum_{j=0}^{\infty}\phi^{j}E_{t-j}\left(m_{t}-z_{t}\right)+\alpha(1-\gamma)\sum_{j=0}^{\infty}\gamma^{j}E_{t-j}m_{t}$$

$$-\alpha(1-\gamma\phi)\sum_{j=0}^{\infty}\gamma^{j}\phi^{j}E_{t-j}m_{t}$$

$$+\alpha(\nu-1)\sum_{i=0}^{\infty}\left(1-\gamma^{i+1}\right)\left(1-\phi^{i+1}\right)\left(a_{i}u_{t-i}+b_{i}\varepsilon_{t-i}\right)$$

Output is equal to $m_t - p_t$, *i.e.*,

$$(95)\quad y_{t}=m_{t}-(1-\phi)\sum_{j=0}^{\infty}\phi^{j}E_{t-j}\left(m_{t}-z_{t}\right)-\alpha(1-\gamma)\sum_{j=0}^{\infty}\gamma^{j}E_{t-j}m_{t}$$

$$+\alpha(1-\gamma\phi)\sum_{j=0}^{\infty}\gamma^{j}\phi^{j}E_{t-j}m_{t}$$

$$-\alpha(\nu-1)\sum_{i=0}^{\infty}\left(1-\gamma^{i+1}\right)\left(1-\phi^{i+1}\right)\left(a_{i}u_{t-i}+b_{i}\varepsilon_{t-i}\right)$$

and employment:

$$n_t = \frac{1}{\alpha}\left[m_t - (1-\phi)\sum_{j=0}^{\infty}\phi^j E_{t-j}m_t\right] + \frac{1}{\alpha}\left[(1-\phi)\sum_{j=0}^{\infty}\phi^j E_{t-j}z_t - z_t\right] + (1-\gamma\phi)\sum_{j=0}^{\infty}\gamma^j\phi^j E_{t-j}m_t - (1-\gamma)\sum_{j=0}^{\infty}\gamma^j E_{t-j}m_t - (\nu-1)\sum_{i=0}^{\infty}\left(1-\gamma^{i+1}\right)\left(1-\phi^{i+1}\right)\left(a_i u_{t-i} + b_i\varepsilon_{t-i}\right) \tag{96}$$

Combining (96) with equation (88) giving w_t we can compute $w_t + n_t - m_t$, which we equal to its hypothesized value (85):

$$w_t + n_t - m_t = \frac{m_t - z_t}{\alpha} - \frac{1-\phi}{\alpha}\sum_{j=0}^{\infty}\phi^j E_{t-j}\left(m_t - z_t\right) + (1-\gamma\phi)\sum_{j=0}^{\infty}\gamma^j\phi^j E_{t-j}m_t - m_t + (\nu-1)\sum_{i=0}^{\infty}\left(1-\gamma^{i+1}\right)\phi^{i+1}\left(a_i u_{t-i} + b_i\varepsilon_{t-i}\right) = \nu\sum_{i=0}^{\infty}a_i u_{t-i} + \nu\sum_{i=0}^{\infty}b_i\varepsilon_{t-i} \tag{97}$$

Let us call:

$$\Omega_t = m_t - (1-\phi)\sum_{j=0}^{\infty}\phi^j E_{t-j}m_t + \alpha\left[(1-\gamma\phi)\sum_{j=0}^{\infty}\gamma^j\phi^j E_{t-j}m_t - m_t\right] \tag{98}$$

$$\Lambda_t = (1-\phi)\sum_{j=0}^{\infty}\phi^j E_{t-j}z_t - z_t \tag{99}$$

Both Ω_t and Λ_t can be decomposed as sums in the innovations in m_t and z_t, respectively denoted as u_t and ε_t:

$$\Omega_t = \sum_{i=0}^{\infty}\omega_i u_{t-i} \qquad \Lambda_t = \sum_{i=0}^{\infty}\lambda_i\varepsilon_{t-i} \tag{100}$$

Combining (97) to (100) we find:

$$(101)\quad \alpha \sum_{i=0}^{\infty} \left[\nu - (\nu - 1) \left(1 - \gamma^{i+1}\right) \phi^{i+1} \right] \left(a_i u_{t-i} + b_i \varepsilon_{t-i}\right) = \Omega_t + \Lambda_t$$

$$= \sum_{i=0}^{\infty} \left(\omega_i u_{t-i} + \lambda_i \varepsilon_{t-i}\right)$$

so that, identifying the coefficients of u_{t-i} and ε_{t-i} we obtain:

$$(102)\qquad a_i = \frac{1}{\alpha} \frac{\omega_i}{\nu - (\nu - 1) \left(1 - \gamma^{i+1}\right) \phi^{i+1}}$$

$$(103)\qquad b_i = \frac{1}{\alpha} \frac{\lambda_i}{\nu - (\nu - 1) \left(1 - \gamma^{i+1}\right) \phi^{i+1}}$$

Let us insert the above formulas into formula (95) giving y_t. This yields:

$$(104)\qquad y_t = z_t + \Omega_t + \Lambda_t + \alpha \left[m_t - (1 - \gamma) \sum_{j=0}^{\infty} \gamma^j E_{t-j} m_t \right]$$

$$-\alpha (\nu - 1) \sum_{i=0}^{\infty} \left(1 - \gamma^{i+1}\right) \left(1 - \phi^{i+1}\right) \left(a_i u_{t-i} + b_i \varepsilon_{t-i}\right)$$

Inserting (100), (102) and (103) into (104), we obtain (63).

APPENDIX 2

Lemma 1: *Consider the monetary process:*

$$(105)\qquad m_t - m_{t-1} = \frac{u_t}{1 - \rho L}$$

Then:

$$(106)\qquad (1 - \chi) \sum_{j=0}^{\infty} \chi^j E_{t-j} m_t = m_t - \frac{\chi u_t}{(1 - \chi L)(1 - \chi \rho L)}$$

Proof: Let us rewrite m_t:

$$(107)\qquad m_t = m_{t-j} + \frac{u_{t-j+1}}{1 - \rho L} + ... + \frac{u_t}{1 - \rho L}$$

so that:

$$E_{t-j}m_t = m_{t-j} + \frac{\rho u_{t-j}}{1-\rho L} + ... + \frac{\rho^j u_{t-j}}{1-\rho L} \tag{108}$$

$$= m_{t-j} + \frac{\rho\left(1-\rho^j\right)u_{t-j}}{(1-\rho)(1-\rho L)}$$

$$\sum_{j=0}^{\infty}\chi^j E_{t-j}m_t = \sum_{j=0}^{\infty}\chi^j E_{t-j}\left[m_{t-j} + \frac{\rho\left(1-\rho^j\right)u_{t-j}}{(1-\rho)(1-\rho L)}\right] \tag{109}$$

$$= \frac{m_t}{1-\chi L} + \frac{\rho u_t}{(1-\rho)(1-\rho L)}\left[\frac{1}{1-\chi L} - \frac{1}{1-\chi\rho L}\right]$$

$$(1-\chi)\sum_{j=0}^{\infty}\chi^j E_{t-j}m_t = m_t + \frac{(1-\chi)m_t}{1-\chi L} - m_t \tag{110}$$

$$+\frac{(1-\chi)\rho u_t}{(1-\rho)(1-\rho L)}\left[\frac{1}{1-\chi L} - \frac{1}{1-\chi\rho L}\right]$$

$$= m_t - \frac{\chi u_t}{(1-\chi L)(1-\rho L)} + \frac{\chi\rho(1-\chi)Lu_t}{(1-\chi L)(1-\rho L)(1-\chi\rho L)}$$

$$= m_t - \frac{\chi u_t}{(1-\chi L)(1-\chi\rho L)}$$

Lemma 2: *Consider the technology process:*

$$z_t = \frac{\varepsilon_t}{1-\varphi L} \tag{111}$$

Then:

$$\sum_{j=0}^{\infty}\phi^j E_{t-j}z_t = \frac{\varepsilon_t}{(1-\varphi L)(1-\phi\varphi L)} \tag{112}$$

Proof: We have:

$$E_{t-j}z_t = \frac{\varphi^j\varepsilon_{t-j}}{1-\varphi L} \tag{113}$$

$$\sum_{j=0}^{\infty}\phi^j E_{t-j}z_t = \sum_{j=0}^{\infty}\frac{\phi^j\varphi^j\varepsilon_{t-j}}{1-\varphi L} \tag{114}$$

$$= \sum_{j=0}^{\infty}\frac{\phi^j\varphi^j L^j\varepsilon_t}{1-\varphi L} = \frac{\varepsilon_t}{(1-\varphi L)(1-\phi\varphi L)}$$

Long Summary

The purpose of this article is threefold: (a) construct a dynamic stochastic general equilibrium (DSGE) model integrating both staggered wage and price contracts; (b) derive an analytic solution for this model; (c) see whether the persistent and hump-shaped response of output and inflation to monetary shocks observed in the data can be obtained with reasonable parameter values for the length of the contracts.

Introducing wage or price contracts into DSGE models is quite a natural enterprise. Indeed the initial contributions by GRAY [1976], FISCHER [1977], PHELPS-TAYLOR [1977], PHELPS [1978], TAYLOR [1979, 1980] and CALVO [1983] showed that embedding staggered contracts into models that, at the time, were not fully structural, generates some degree of persistence in output and employment. On the other hand rigorous models in the DSGE line have often been criticized for their difficulties in generating a sizeable propagation mechanism (COGLEY and NASON [1993, 1995]). This is particularly bothering as a number of authors (see, for example, CHRISTIANO, EICHENBAUM and EVANS [1999, 2001], COGLEY and NASON, [1995]) have shown in particular that output and inflation have a persistent and hump-shaped response to monetary shocks, a feature that most traditional DSGE models fail to account for.

So a natural step was to include staggered contracts into DSGE models, and this has actually been done by a number of authors. They constructed models including wage and price contracts either in isolation, or together, or in addition to other rigidities. The problem is that, although the answer might have looked intuitive at the outset, there is a bewildering and conflicting variety of answers. Some contributions do find that contracts yield substantial persistence, but others conclude that contracts yield practically no persistence.

In such a situation it is particularly useful to complement the numerical explorations, which is the traditional strength of this line of research, by a theoretical approach which will allow to understand which parameters are important, and whether reasonable parameter values can produce the missing propagation mechanism. This is the line followed in this article.

The choice of an adequate price and wage contracts structure is of course central to the model. We use staggered contracts developed in BÉNASSY [2000], itself adapted from the wellknown contribution by CALVO [1983].

Let us start with the wage contracts. As in Calvo, in each period there is a random draw for all wage contracts, after which any particular contract will continue unchanged (with probability γ), or be terminated (with probability $1-\gamma$). In this last case the corresponding contract wage is renegotiated on the basis of all information currently available. The difference between Calvo contracts and ours is that the values of the contracts signed in period s for periods $t \geqslant s$ may differ for all t, whereas they were identical in CALVO [1983]. We may note that the average duration of wage contracts is $\gamma/(1-\gamma)$, which varies from zero to infinity as γ goes from zero to one, so that this specification is particularly flexible.

The description of price contracts is symmetrical: a particular price contract will continue unchanged (with probability ϕ), or be terminated (with probability $1-\phi$), in which case it is renegotiated on the basis of current information. The average duration of price contracts is $\phi / (1-\phi)$.

The contents of the article are the following: Section 2 presents the model. Section 3 computes, as a benchmark, the Walrasian equilibrium of this economy. Section 4 derives the demands for different types of labor and goods. Using these demands, section 5 derives the optimal wage contracts that will be chosen by optimizing workers. Symmetrically section 6 derives the optimal price contracts that the firms will choose. Section 7 puts these optimal price and wage contracts together, and derives the dynamics of output, employment and inflation as a function of technological and monetary shocks. Section 8 investigates whether the response of output and inflation to monetary shocks can be persistent and hump-shaped. Section 9 carries a few simulations to illustrate the above results. It is shown first that both the persistence and nonpersistence results obtained by various authors in the literature can be obtained in our model if we use the same parameters as these authors. We then show that a hump-shaped response of both output and inflation can occur with very reasonable parameter values, such as wage contracts lasting one year in average, and price contracts lasting one quarter in average. Moreover this is obtained for parameter values such that each rigidity in isolation would not deliver the same result. This shows that it was indeed important to construct a model integrating both rigidities together.