Modelling Economic Policy Responses with an Application to the G3

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ABSTRACT. – There is a range of formal approaches to the modelling of policy responses. This paper will give a very short summary of the broad approaches, which are currently used, then a new approach will be proposed. The main objective of this paper is to explain the motivation and workings of a new algorithm, which allows the calculation of optimal feedback rules designed to minimise the variance of the models responses to economic shocks. This algorithm is tractable even when the model being used is very large. We will place the new algorithm in context by very briefly outlining the development of the treatment of policy formation in econometric models. We will then outline the new algorithm. We will then present an illustration of how the algorithm can be used to calculate optimal feedback rules under various game structures (*Nash* or co-operative for example).

Modélisation de la politique économique avec une application au pays du G3

RÉSUMÉ. – Il existe diverses approches de la modélisation des fonctions de réaction de la politique économique. Ce papier propose d'abord un bref résumé des approches qui sont actuellement utilisées, puis développe une nouvelle approche. L'objectif principal de ce papier est d'expliquer la motivation et le fonctionnement d'un nouvel algorithme qui permet le calcul de réactions optimales destinées à minimiser la variance des réponses du modèles aux chocs économiques. Cet algorithme peut être mis en œuvre même pour un modèle de grande taille. Nous appliquons cet algorithme en exposant brièvement le développement du traitement de la formation de la politique économique dans un modèle macro-économétrique. Puis, nous exposons ce nouvel algorithme. Nous présentons alors une illustration de la façon dont cet algorithme peut être utilisé pour calculer la fonction de réaction optimale en fonction de différentes structures de jeux (*Nash* ou coopération par exemple).

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1 Introduction

Within the framework of the large macroeconometric models there are two overriding reasons for modelling the process of policy response to economic events; the first is the obvious importance of this topic to the policy formation process itself. The second is the need to provide a basic model closure so that the model is a sensible forecasting tool.

The use of models in policy formulation has been one of the prime uses of these models since they were first developed. This can vary from the traditional technique of simply specifying exogenous fiscal and monetary policy and then using the model to forecast conditional on these assumptions to a complex range of optimal control techniques or endogenous feedback rules. The underlying objective is always the same; to help to specify a 'better' set of economic policies through the use of a fully specified model of the economy. Even in the most basic approach of exogenous economic policy variables, we should still think of policy as being an endogenous response, although it is not formally modelled as such. The model is used to investigate the effects of a particular, given, set of policies. These policies may be respe cified many times by the model user until a satisfactory result for the economy emerges. In more complex modelling exercises this process is made formal by actually specifying the mechanism the policy maker uses to respond to economic events.

The model closure aspect of policy formulation is primarily a technical issue of interest to modellers. It is simply that the policy formulation process is so important to the economic properties of a model that it has become increasingly obvious that it is almost meaningless to consider a models properties in isolation from the assumption made about policy formulation. As models have become more sophisticated the 'old' exogenous economic policy assumption has become increasingly untenable. For example a model which includes rational expectations will often simply not solve under the assumption of fixed policy settings, and indeed we know that often it should not. The particular specification of the policy response also may have enormous effects on a models response to any given shock and so it has become increasingly obvious that any model comparison exercise must be done on the basis of similar economic response assumptions (*eg,* BRYANT *et al.* [1988, 1993]). If this is not done the exercise simply reduces to the finding that different policies give rise to different effects and nothing can be said about the actual differences between the models.

There is a range of formal approaches to the modelling of policy responses. This paper will not attempt to survey these but a very short summary of the broad approaches will be given to put the novel approach proposed here into a proper context. The main objective of this paper is to explain the motivation and workings of a new algorithm, which allows the calculation of optimal feedback rules designed to minimise the variance of the models responses to economic shocks. This algorithm is tractable even when the model being used is very large.

The plan of the paper is as follows: The next section will place the new algorithm in context by very briefly outlining the development of the treatment of policy formation in econometric models. Section 3 will then outline the new algorithm proposed here. Section 4 will then present an illustration of how the algorithm can be used to calculate optimal feedback rules under various game structures (*eg, Nash* or co-operative for example). Finally, section 5 will conclude.

2 A Brief Review of Policy Formulation in Models

The basic underpinning of all policy work is the optimal control framework where we specify an objective function and an economic system and choose the optimal setting for the policy instruments in the light of the these two elements. Even the specification of exogenous policy variables may be interpreted in this way, with the optimisation being done informally 'off model'. Optimal control has been carried out on large models for a considerable time, early examples are CHOW [1975] and BRAY [1975], a survey of techniques used may be found in FAIR [1984] or HALL and HENRY [1988]. The fully optimal rule makes use of the entire state vector of the model including all future values. This rule is therefore likely to be quite complex and this has lead to a number of criticisms of the use of full optimal control.

Firstly, as optimal control exploits the full information about the structure of the model, what tends to happen in practice is that the rule will exploit the dynamic characteristics of the system being controlled or else it may find some facet of the model that does not truly reflect the real world. This might simply be some odd quirk in the model, an odd non-linearity, a corner solution or even an extreme assumption such as rational expectations, which the optimal policy rule is able to exploit. Optimal policy rules therefore tend to be highly model specific, as is demonstrated by BRAY *et al.* [1995]. The second problem also identified by the Ball Report related to the *Lucas critique*. LUCAS [1976] was the first to raise doubts about the usefulness of macroeconomic models for policy making when economic agents formed expectations that are forward looking. The *Lucas critique* then gave rise to the whole problem of time inconsistency and credibility effects, which stemmed from the seminal work of BARRO and GORDON [1983].

The alternative to the full optimal rule is to design rules that exploit only the information, which we believe to be useful and to de-emphasise the less reliable elements of the model's structure. Thus, simple feedback rules were developed which are generally a restricted form of the full optimal control solution. These rules limit the amount of information drawn from the structure of the model to those areas which are of special relevance to the policy question at hand (see for example, VINES *et al.* [1983]; CURRIE and LEVINE, [1985]; TAYLOR [1985] and EDISON *et al.* [1988]). By implicitly excluding much of the model, simple rules are supposedly robust to uncertainty. Furthermore, if articulated publicly, they meet *Kydland* and *Prescott*'s criteria of being simple and easy to interpret, and therefore useful when it comes to monitoring the authorities.

Simple feedback rules were first developed in the engineering literature and later applied to economic systems. PHILLIPS [1954, 1957] discusses the relative merits of proportional, integral, and derivative control although many of the later applications adopted rather simpler formulations. Within the context of a simple rule there is an important distinction based on how the parameters of the rule are chosen. When *Phillips* first introduced such rules into economics in 1954 the parameters were selected in a largely informal way, mainly by trial and error. Later work was undertaken to parameterise the simple rules in a more formal way and so optimal simple rules were developed. This is done quite simply be specifying an objective function and then treating the parameters of the control rule as the control variables which are to be determined in an optimal way. Of course, it is not necessarily the case that a particular simple rule will behave well and part of the skill of a modeller is in knowing the form of simple rule which will work satisfactorily in any given model.

Recently a number of papers have attempted to directly estimate policy feedback rules by examining actual data. The question thus posed is whether over the past the policy actions of the central bank (or the fiscal authority) can be represented by an estimated rule linking instruments to targets. Examples include CLARIDA *et al.* [1998] applied to G6 and E3 (UK, France and Italy), and WALTON and MASSONE [1999], for the UK. *Clarida et al.* provides a good account of the motivation for this approach.

We argue that there are two fundamental problems with this approach. The first is the problem of the econometric identification of the reaction function; the second is the economic interpretation of this function once it is derived. We will discuss these two in turn.

The econometric problems of identifying a reaction function where none of the variables are weakly exogenous are profound. *Clarida et al.* interpret their equations as characterizing monetary policy over the period, showing that there was a concerted move towards inflation targeting, albeit what the authors call ‘soft-hearted’ targeting, *ie*, inflation targets with some stabilisation element in policy too (SVENSSON [1998] refers to this as ‘flexible’ inflation targeting). That is, the response to a rise in expected inflation is to push up nominal rates by a sufficient amount to increase real interest rates. The results for the G3 show that the baseline specification proposed in their paper works best. In other words, the addition of additional variables like money aggregates or exchange rates does not add significantly to the explanatory power of the equation. Also adding lagged inflation does not significantly improve the equation, and the authors interpret this as confirming the forward-looking specification used in the model. In sum, the estimated equations are advanced as a plausible description of how central banks have conducted policy. There is also the suggestion that the results may be interpreted as showing what policies were actually desirable. (see CLARIDA *et al.* [1998] p. 1037). We discuss such an optimal interpretation of these equations below. Even interpreting the results as a description of what determined policy actions of the authorities is highly problematic. Fitting econometric equations to instruments and objectives and interpreting the result as an actual reaction function is almost certainly inappropriate. As there is a fundamental identification problem involved in exercises of this sort: there are at least two relations between these variables – the ‘true’ policy reaction function (which we assert is not what the authors identify with their equations), and the relationships of the economy itself. The fitted equations combine these two in some unknown way. The authors argue that the benefit of their 'weakly restricted' version of the reaction function is that it is sensible for a wide range of different macroeconomic frameworks (models). It is hard to know how we would establish this. Estimating by allowing for the regressor variables to be stochastic, as the authors do, does not deal with this issue. A Full Information method is required as a means of identifying both the responses of the economy to policy and other exogenous shocks on the one hand, and the policy responses to developments in the economy on the other. Furthermore, there is almost certainly structural change affecting both of these basic relations – the model and the policy reaction equations – in different ways. Thus, we would strongly suspect that equations of this sort, although apparently well fitting, would exhibit structural instability. This suggests a simple way to test the validity of the *Clarida et al.* approach, is to test its structural stability. Our argument suggests these estimates must be structurally or parameter unstable.

The second problem is that even assuming the estimation has correctly identified the authorities reaction function, the interpretation of this equation is profoundly difficult. The best that can be said is that it represents what the authorities were actually doing. But were they behaving correctly, or optimally, or were they in fact following a completely erroneous set of policies. Even if we can assert that the authorities have been operating a good set of policies are they based on a particular form of co-operative structure? How would the policy have changed if the form of co-operation changed? All these questions are completely unanswerable from the perspective of this methodology and hence represent a sever limitation on its use for policy analysis.

Policy analysis is often carried out as if the government was the only agent in the economy who is able to exercise any degree of discretion. Even in a closed economy this is not obviously the case, the monetary and fiscal policy makers may not always co-operate perfectly and many sectors of the economy may be able to make there own well-informed decisions. When we consider an open economy then there is obviously a need to investigate the form of co-operation between policy makers in different countries. We need therefore to draw on the game theory literature to structure our analysis of how co-operation may take place and how to investigate the consequences of these structures. A formal exposition may be found in INTRILIGATOR [1971]. Two primary sources of particular importance are LUCE and RAIFFA [1957] and VON NEUMANN and MORGERNSTERN [1944]. In very broad terms game theory allows us to begin to evaluate the effects of a range of structures of cooperation between decision-makers, this would include *Nash* equilibria, cooperative equilibria and stackelberg (leader follower) type solutions.

When we are dealing with large models all these basic forms of game solution may be solved numerically, although the computational burden of solving for the full *Nash* solution may be high. Within this structure the players in the game may be optimising an unconstrained objective function or they may be optimising the parameters of a simple feedback rule as discussed above.

A further branch of the policy literature has recognised that econometric models are by their very nature subject to uncertainties. So, it is often inappropriate to think only of the point forecast given by a conventional deterministic model solution. We may wish to consider two important consequences of the stochastic nature of models. First, the deterministic forecast of a model is not generally the mean forecast of the model. Second, we may wish to choose a set of policies which are deliberately designed to reduce the uncertainty of the actual outcome. That is we may wish to give some weight to reducing the variance of the models outcome when we design the policy regime.

The general technique for analysing stochastic models is called stochastic simulations and it is surveyed fully in HALL and HENRY [1988]. A number of approaches have been adopted to the analysis of policy setting within a stochastic regime which broadly parallel the analysis of deterministic models given above. First, optimal control algorithms have been developed for dealing with the correctly specified expected value of the model and also for allowing an optimal policy rule to be chosen which also minimises the variance of a particular outcome. Second, a great deal of work has been undertaken to investigate the effect of simple rules on the stochastic model and to select rules on the basis of minimising the resultant uncertainty.

HALL and STEPHENSON [1990] proposed an algorithm which allowed stochastic optimal control to be carried out where the primary focus was on the expected value of the level of the variables in the model (rather than their variance). RUSTEM [1993] addressed the minimum variance problem by adding a term to the standard objective function, which allowed the variance to be approximated.

In a non optimal setting the late 1980's and early 90's saw a number of studies which evaluated a range of policy rules or policy options by using stochastic simulation to investigate which rule gave rise to the smallest variance in outcomes. BRYANT *et al.* [1988] included a number of studies looking at world models which focused on the stochastic nature of models and selected policy rules on the basis of minimum variance outcomes. Subsequently BRYANT *et al.* [1993] extended this comparative work to consider a wider range of models. The work of TAYLOR [1985] is based solely around choosing between a range of policy rules based on a minimum variance criteria, this work has had considerable impact on the policy making community. A common problem with all this work is however that the parameters of the rule are selected in an arbitrary way, a range of different rules are selected to represent different policy regimes. Stochastic simulations are carried out to investigate the size of the resulting variances of the endogenous variables and then the 'best' rule is selected as the one which gives the smallest variance in outcomes. However the parameters of each rule are simply chosen on an '*ad hoc*' basis and it is possible that different parameters in the same rules could have produced a different ordering of results or at the very least performed in a very different way.

So to summarise the above, the policy literature has developed 4 major strands; deterministic optimal simple rules, deterministic games, stochastic optimal control designed to target either the expected values of a model solution or the variance of the solution and simple non-optimal rules in a stochastic context. Clearly to objectively evaluate the relative performance of a number of different rules we would want each of them to be performing at their most efficient level. So each should have the parameters of the rule selected so as to minimise a common loss function. This would ensure comparability between the rules. This however raises a further question as to what type of game is being played out between and how the optimisation should be carried out when there is more than one policy maker being considered. Ultimately this leads to the conclusion that what we should be doing is to select the parameters of the rule by optimal control so as to minimise the variance of the outcomes based on a range of solutions assumptions such as *Nash* or stackelberg games. Taken at face value this would imply that we should be carrying out iterating optimisations over the full stochastic model, this would be hugely expensive in computer time (a single stochastic simulation may take a day of computer time).

In the next section, we will outline a new algorithm which will allow us to solve this very general problem in a tractable way.

3 Optimal Rules to Minimise the Variance of Economic Variables in a Game Context

Given the preceding discussion, we propose an algorithm which will allow us to choose the parameters of a set of rules so as to minimise the variance of selected variables in the economy when it is subject to a particular set of stochastic shocks. Moreover, this often needs to be done allowing for possible strategic interaction between different policy makers, so the analysis has to allow for game playing which will involve successive optimisations over a number of players to achieve a range of different forms of solution, *eg*, *Nash*, stackelberg, co-operative etc. We therefore specify the problem, in compact notation as,

$$\min \operatorname{var}(C) = \operatorname{var}\left(\sum_{t=1}^{T} \Phi_t \sum_{i=1}^{n} \Theta_i Y_{it}\right) \qquad (1)$$
$$Y_{it} = g(e,u)$$

Where e is a matrix of k stochastic terms over the T periods of the model solution which have a given covariance matrix Ω, Y is the vector of endogenous variables in the model, Φ and Θ are weights in the cost function and u is a vector of control variables which in our case are the parameters of a control rule. In a policy game each player would have an objective function of this form.

The computational burden of this form of problem is considerable; to evaluate the variance alone needs a stochastic simulation involving thousands of conventional model solutions. This kind of solution would have to be calculated many hundreds of time during a conventional numerical optimisation. It seems that, for this reason alone, researchers have not pursued this approach to policy formulation. The innovation we propose is a simplification of the problem, which will yield an identical solution for most forms of nonlinearity, which are observed in the large macro models. The idea here is based on the notion that any monotonic transformation of the cost function will yield an identical solution for the control variables. So if we minimise the variance of the cost function ($V(.)$) with respect to a set of variables u then we will have exactly the same solution for u if we minimized a monotonic transformation of V (*eg*, $\log(V)$ or V^2). We use these propositions to substantially reduce the computational problem in minimising $V(.)$, using a special transformation based on two elements: the first is the technique of anti-thetic errors used in stochastic simulation, the second constructs a minimum set of replications which exactly reproduce the covariance matrix of the stochastic process.

The essence of the proposed technique is that we state a new objective function (C^*) which for a wide range of model structures will be related to the general objective function above (VAR(C)) by a monotonic transformation. This means that if we optimize the relatively simply objective function C^* we will theoretically obtain the same solution values for the control variables as would solve the much more complex problem in (1) without ever actually evaluating VAR(C). The construction of our C^* function rests on the following 2 aspects.

Anti-thetic Errors

Anti-thetic errors simply mean that instead of drawing a sequence of completely random sets of shocks, the sets of shocks are chosen in symmetric pairs so that two replications from a stochastic simulation represent an exactly symmetric pair, in terms of the shocks being applied to the model. This technique increases the efficiency of stochastic simulations enormously but even one pair gives a lot of information. For example, if the model is linear then the resulting average of the endogenous variables from the two solutions will be identical to the deterministic model solution, hence any divergence from the deterministic solution is an absolute sign of non-linearity.

Minimum Set of Replications

For the moment let's assume that we are dealing with a single stochastic error term. In that case the following objective function would be our monotonic transformation of (1).

$$(2) \quad \min\ C^* = \sum_{t=1}^{t} \Phi_t \sum_{i=1}^{n} \Theta_i (|g(e,u) - g(0,u)| + |g(-e,u) - g(0,u)|)$$

This objective function minimizes the absolute deviation from the no shock solution after applying an arbitrary size shock to the model. The antithetic errors are represented by the two terms with plus the shock and minus the shock. Our claim is that there is a monotonic transformation between this objective function and (1). Hence, the resulting optimal u will also be the solution to (1).

If we were dealing with a single error this would obviously be sufficient to give the solution we require. However, there is a further complication when the vector of errors is larger than a single scalar. The problem is that any single draw of the error vector cannot be representative of the whole distribution of errors, so it cannot represent the covariance matrix. A scalar error can have a value equal to its standard error but a vector cannot have both variances and covariance's equal to the full covariance matrix. This point can be seen by considering the bivariate case. Let the covariance matrix be,

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \tag{3}$$

Now any single pair of shocks cannot give both the variance and covariance's simultaneously. For example $(1,1)$ has unit variances for both errors but a unit covariance, $(1,0)$ would have a zero covariance but the variance on e_2 would also be zero. In fact, in this case it takes two sets of shocks to exactly replicate the covariance matrix. The required shocks are $(1,-1)$, $(1,1)$, which have unit variances for both errors and zero covariance. The anti-thetic pair corresponding to this would be $(-1,1),(-1,-1)$. So, if we were interested in solving the problem for a vector of two stochastic shocks we could do this by evaluating:

$$\min\ C^* = \sum_{j=1}^{k} \left(\sum_{t=1}^{t} \Phi_t \sum_{i=1}^{n} \Theta_i (|g(e^j,u) - g(0,u)| \right. \\ \left. + |g(-e^j,u) - g(0,u)|) \right) \tag{4}$$

Where $k = 2$ and where the two vectors of shocks (e^j) are given as above. So, in this case, instead of carrying out many thousand replications to estimate the variance of c, (VAR(C)) we can achieve the same solution values for the control variables by minimising C^* based on only four model solutions. This clearly brings the possibility of using optimal control within the bounds of computational feasibility, even in a game context.

The above case is an example of how the proposed procedure would work for a case of two shocks. In the general procedure we chose a set of k vectors of shocks such that:

$$\Omega = \sum_{i=1}^{k} e_i e_i \tag{5}$$

This will generally involve approximately $n = k$ sets of shocks where n is the number of stochastic elements in the model being examined. The reason why this is only approximate is that the relationship is different for an even and odd number of shocks. The above formulae gives an exact determination of the shocks when n is odd but when it is even we need some extra conditions to uniquely determine the shocks. In the bivariate case above, for example, there are actually an infinite number of pairs, which would give the required covariance matrix. This can be seen by writing out the problem in full.

N	K	R
1	1	0
2	2	1
3	3	3
4	4	6
5	5	10
6	6	15
7	7	21

$$\begin{aligned} \Omega_{11} &= e_{11}^2 + e_{21}^2 \\ \Omega_{12} &= e_{11}e_{12} + e_{21}e_{22} \\ \Omega_{22} &= e_{12}^2 + e_{22}^2 \end{aligned} \tag{6}$$

This yields 3 equations in four unknowns and so we need to impose an extra condition to uniquely determine the shocks, we propose simply setting $e_{11}^2 = \Omega_{11}$ as the extra required restriction. For an odd number of shocks we exactly determine the k vectors of errors.

The table above gives the relationship between n, the dimension of the covariance matrix, k the minimum number of sets of shocks and r, the number of extra sets of restrictions required.

So, in general, given the extra effect of the antithetic errors, we will need approximately twice the number of replications as the dimension of the covariance matrix. If we wish to calculate an optimal policy rule for a countries monetary policy given shocks to both the exchange rate in that country and shocks to the exchange rate in two other countries we would therefore need six model solutions to evaluate the objective function we need to maximise.

The Monotonic Transformation

This proposed technique will not always give exactly the same answer as (1) above, it is possible that for a sufficiently non-linear model the mapping between (1) and (4) would cease to be monotonic and hence they would have different solutions. However, our argument is that this would require an extremely perverse and unusual form of non-linearity to be present which is not typical of any macroeconomic model.

The essence of the monotonicity assumption is that if we have any two sets of control variables, u^1 and u^2, such that:

$$C^*(u^1) > C^*(u^2) \tag{7}$$

That is, a deviation in C from its deterministic value is larger for the set of control variables u^1 than u^2. Then, monotonicity between the two objective functions means that:

$$\mathrm{var}(C(u^1)) > \mathrm{var}(C(u^2)) \tag{8}$$

This simply amounts to the assumption that if one set of control produces larger deviations in the model variables from their deterministic values then it will also lead to a larger variance. In our view, it is almost inconceivable to think of an economic model where this would not be true. It is obviously true, for example, for a linear model or a model where the non-linearity is limited a set to bijective mappings from the control variables to the endogenous variables (such as log functions). A failure of the monotonicity assumption would require, at the very least, a non uniqueness in the mapping from the control variables to the endogenous variables so that a single value of the control variable could give rise to more than one value of the endogenous variables. One plausible case where this might arise is where a model has more than one solution and they are only locally stable.

4 An Illustration of the Algorithm

This section illustrates the new algorithm by undertaking a range of policy co-ordination game exercises for a medium size econometric model of the G3 economies. In these exercises the objective function is always to minimize the equally weighted variance of inflation and output in the presence of a stochastic demand shock in the USA by setting interest rates using a simple PID feedback rule which responds to deviations in inflation from a target value. The parameters of this feedback rule form the control variables, so we are calculating optimal simple feedback rule parameters for a range of game settings in a stochastic environment. Four quite separate forms of solutions are then compared. In the first, the US reacts optimally in terms of its own interest rate responses but there is no response from the other two countries. The same exercise is repeated for each country, giving a 'national' optimal rule for each country assuming zero response from the other countries. In the second, every country uses its 'national' rule from the first case; this does not require any further policy optimization. In effect, it is illustrating what would happen if each country set its policy in the belief that the other countries would not react but in fact they all behave in the same way. The third case is where each country optimizes in the light of, and the knowledge of the optimal behavior in each other country – A *Nash* solution. In the last case, we have a fully cooperative solution in the sense that the countries optimize a joint objective function so they are all co-operating to produce the best possible outcome. In what follows we refer to the first as single country optimizing, the second as multicountry I (where each of the country assumes no policy reaction from the other). The third is multicountry II (*Nash*) and the last is multicountry III which is a fully co-operative solution. All solutions are for the period 1984-1994.

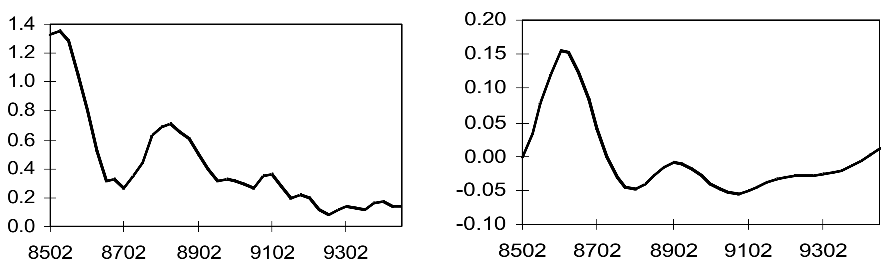
We can not of course show a full stochastic solution in any convenient way so instead we will demonstrate the properties of each optimization by showing how the model responds to a simple shock given the optimal parameters derived from our algorithm for the respective game structure.

Single Country Optimizing

In this alternative, each national authority optimizes the weights of its PID monetary rule; in order to minimise the deviations of inflation from its base following the US demand shock. But in these exercises, in each country, policy actions are governed by the national monetary rule, and there is no policy reaction from the other countries. There are consequences for each country, which flow from the actions of the others nevertheless. These take the form of standard spillover effects *via* trade and exchange rate movements but there are no positive policy responses from the other country.

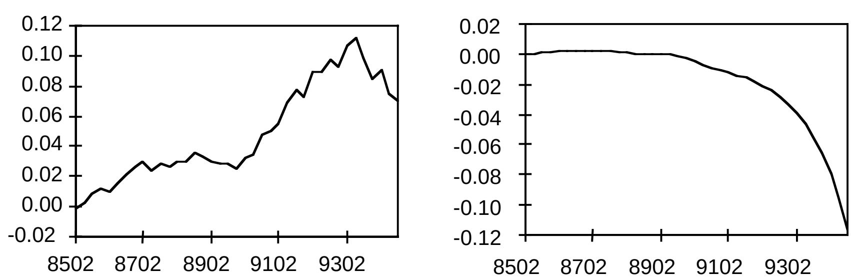
Figures 1a and 1b show the output and inflation effects of the US fiscal shock accompanied by optimal single country monetary policy response in that country Output growth increases by over 1% initially, but reduces to around 0.5% over the next 4 years as monetary policy is tightened (Figure 1a). As Figure1b shows, the policy correction is successful in reducing the inflationary impulse, and by the end of the simulation the rate of inflation has reached its base value.

FIGURE 1
Single Country Optimizing: Effects on US of US Fiscal Shock



(a) Output (b) Inflation

FIGURE 2
Single Country Optimizing: Effects on Japan of US Fiscal Shock



(a) Output (b) Inflation

If we reverse the roles next, and let Japan's monetary policy react optimally to the US shock we get the effects shown in Figures 2a and 2b. Output growth picks up steadily, but by small amounts, to reach 0.1% higher after about 8 years (Figure 2a) before falling. This stimulus operates through familiar net trade effects. There is a very small inflation effect from this. However, as Figure 2b shows, the authorities reduce inflation by a small amount below base (0.1% after 10 years), entailing a small gain in output with slightly lower inflation by the end of the simulation. A similar pattern emerges when Germany optimizes monetary policy following the US expansion, with similar orders of magnitude but with some differences in timing. The differences overall are not significant enough to warrant separate treatment though. (Figures for Germany are therefore not included.) The optimal weights obtained in this set of single country exercises are then used in the next exercise, which begins the multicountry analysis proper.

Multicountry I

We are now in a position to analyse in a preliminary way the optimal responses to the US fiscal shock on a proper multicountry basis. In this next exercise, all countries responds together, each country according to its own optimal monetary policy rule derived from the single country optimizing exercise above. It is a limited form of multicountry response: although each country follows a (national) optimal rule, it assumes there will be no policy reaction in the other countries. This is an incorrect assumption to make. This means that the present exercise introduces further forms of spillover compared with the traditional case (which came in *(i)* above). Firstly, there are effects between interest rates across countries due to the workings of interest arbitrage. Second, there are policy-induced effects on interest rates, as each national authority seeks to offset the inflation consequences of the US fiscal expansion, using its own monetary policy rule. For both reasons, there will be inflation and unemployment effects due to the effects of changing interest rates on expenditures, including investment, and thence the capital stock.

Even though each country is (in this limited sense) making an optimal response to the US shock, the effects of it on Germany and Japan are striking. Figure 3 shows the effects on growth, Figure 4 the effects on inflation. Growth in the US expands more than in the previous case. Inflation rises there too, but by only a small amount (Figure 4), peaking at about 0.5% above base after 1.5 years. The repercussions in the other countries are profound, especially in Japan. Inflation picks up markedly over the first two years, and remains stubbornly high for a further 4 years, at something under 2% above base. The source is the appreciating dollar, and rising import price inflation in the two other countries. In consequences, monetary policy has to be tightened very sharply in both Japan and Germany.

This inflationary effect is compounded initially by the expansionary effect of the fiscal stimulus in the US on Japanese and German growth. After 2 years in the case of Germany, and 3.5 years in Japan, the strongly corrective monetary policy reduces growth. It proves difficult to reduce inflation in Japan and growth there is reduced substantially over most of the simulation period in the effort to contain the inflationary effects of the US shock

FIGURE 3
MultiCountry I: Inflation Effects of US Fiscal Shock

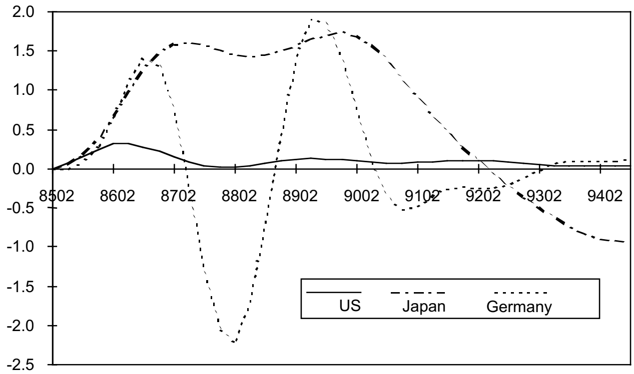


FIGURE 4
MultiCountry I: Inflation Effects of US Fiscal Shock

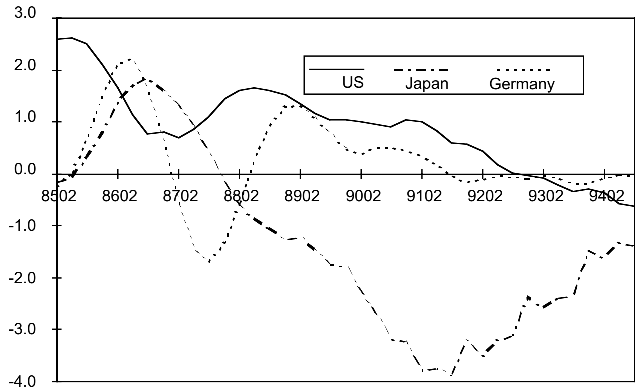
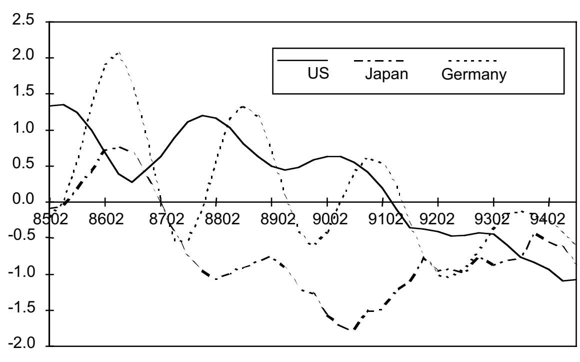


FIGURE 5
MultiCountry II: Output Effects of US Shock



Although this exercise is obviously limited –it assumes that each country assumes the others will not react to its own policy changes, incorrectly– it indicates that the spillover effects of uncoordinated fiscal expansion can be very substantial indeed. In effect, each country tries to export its domestic inflation with no regard to the response of the other countries and hence a spiral of rising interest rates set in.

Multicountry II

One of the limiting assumptions in the previous exercise is now dropped, and we proceed to implement a full *Nash* solution on the optimisation. Allowing for each country to optimize, given that it assumes (correctly) that each of the other does the same, has evident consequences for the outcomes following the US shock. Figures 5 and 6 show the growth and inflation differences from base in this regime. As compared with the previous exercise, the growth effects – with one exception – are more constrained. The expansion in the US is less initially, and more stable. While Japan's experience is also much less severe, although it again has the same sort of prolonged growth recession as in the previous case, the fall in growth being about half that of the previous case at its worst. For both countries, the major gain in this exercise is on inflation. Unlike the Multicountry I, inflation in Japan is reasonably well contained, rising between 0.3-0.5% until the end of 1990, but is effectively squeezed out thereafter (Figure 6). Inflation in the US is broadly the same as in the earlier case. Hence, this case may be characterised as showing that better inflation can be achieved with smaller output losses when adopting *Nash*-type optimal strategic policies compared with single country optimizing.

Multicountry III

Once a fully cooperative international policy regime is instituted the situation is transformed, showing substantial gains over the full *Nash* solution. Figures 7 and 8 give the growth and inflation differences from base for this case.

The most conspicuous effect is upon US growth, which now is positive throughout and much more stable. Although not as high as initially in case *(ii)* where there is no policy reaction at all from other country, in this case, there is a positive increment to growth throughout the period (and by the end of 1994, it is still 0.75% above base). Similarly, in the other two countries, the adverse effects on growth are minimized in this regime. The adverse effects on growth in Japan are much shorter lived than in both of the non-cooperative exercise, and are much less severe. Germany also has a short-lived fall in growth compared with base, but positive effects there after. (Figure 7). Inflation is effectively contained in each country after some cycling in Germany. At the end, each has inflation some 0.3-0.4% above base. (Figure 8). This represents a considerable improvement over the *Nash* game.

FIGURE 6
MultiCountry II: Inflation Effects of US Fiscal Shock

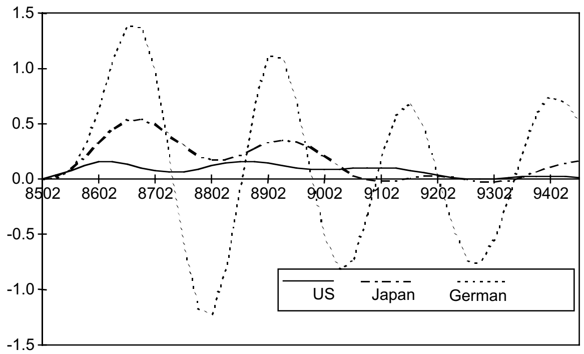


FIGURE 7
MultiCountry III: Output Effects of US Shock

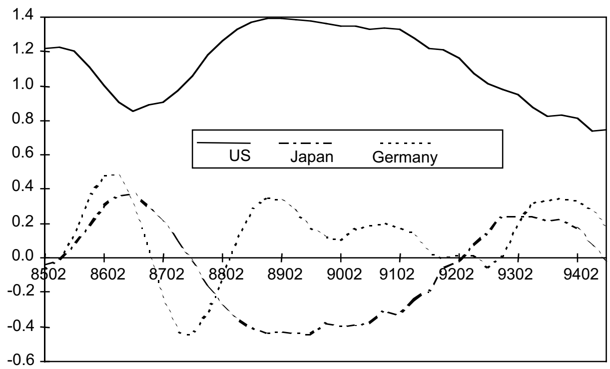
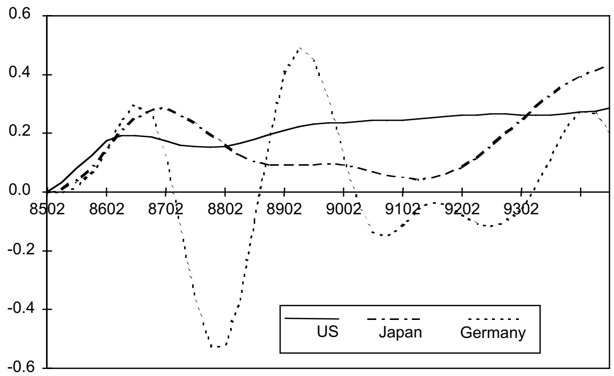


FIGURE 8
MultiCountry III: Inflation Effects of US Shock



7 Conclusion

In this paper, we have briefly outlined the conventional approaches to the modelling of policy formulation. Traditionally this has fallen into one of two groups, fully optimal responses or the use of some simple feedback rule to represent the policy maker's behaviour. These two approaches may be carried out either for a single agent or in a game setting between a number of policy makers. There are a number of conceptual reasons for favouring the use of simple rules and modern economics tends to favour the idea of policy stabilisation rather than trying to change the permanent behaviour of the economy. This leads us towards designing optimal policy rules, which minimise the variance of the economy when it is subject to shocks. Computationally this has been very difficult with existing algorithms. We have proposed an algorithm here which makes this a feasible computational task even in a full game context. We have then illustrated the use of this technique with a medium size three-country model, which allows a range of game structures between the countries. ▼

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