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An Equilibrium Model of the Labour Market with Endogenous Capital and Two-Sided Search

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ABSTRACT. – In this paper we extend the equilibrium search models of BURDETT and MORTENSEN [1998], BURDETT and VISHWANATH [1988] and MORTENSEN and VISHWANATH [1994] to allow for endogenous matching and endogenous capital determination. In our model, in order to attract a positive measure of workers, firms must produce a specific hiring effort which is by itself costly (cost of advertising posts, training new employees). Workers then draw firms in proportion to their hiring effort. Moreover, as in the model of ACEMOGLU and SHIMER [1997], upon entering the market firms must choose a determined amount of capital which is then fixed for ever and indexes labour productivity. We characterize the equilibrium and derive expressions for the endogenous equilibrium wage distributions. In particular, we show that with convex or concave hiring costs, the Nash equilibrium of the equilibrium search game is such that all operating firms must choose a different amount of capital from a continuous distribution, and a one-to-one mapping exists between capital and wages. We calibrate the model on French firm data and proceed to various simulations of tax reforms. We thus show that a reform which transfers labour taxes from low wages to high wages, by reducing the monopsony power of large firms, is welfare improving: unemployment is reduced, total output is increased as well as government revenue.

Un modèle d'équilibre sur le marché du travail avec capital endogène et recherche de la part des travailleurs et des entreprises

RÉSUMÉ. – Dans cet article les modèles de recherche d'emploi d'équilibre de BURDETT et MORTENSEN [1998], BURDETT et VISHWANATH [1988] et MORTENSEN et VISHWANATH [1994] sont étendus de façon à rendre endogènes le processus d'appariement et la détermination du capital des entreprises. Dans ce modèle, pour attirer une mesure non nulle de travailleurs, les entreprises doivent fournir un effort de recherche spécifique qui a un certain coût (de publicité des postes disponibles, de formation des nouveaux salariés, ...). Les entreprises sont alors contactées par les travailleurs en recherche d'emploi avec une probabilité proportionnelle à leur effort de recherche. Par ailleurs, comme dans le modèle de ACEMOGLU et SHIMER [1997], le niveau de capital mobilisé pour la production est déterminé de façon endogène à un niveau stationnaire et influe sur la productivité du travail. Nous caractérisons alors l'équilibre et dérivons l'expression de la distribution de salaires d'équilibre. En particulier, nous montrons que, à l'équilibre, lorsque les coûts de recherche sont strictement concaves ou convexes, toutes les firmes actives choisissent un niveau différent de capital, tiré dans une distribution d'équilibre non dégénérée, et que la relation entre salaire et capital au niveau des firmes est bijective. Le modèle est ensuite estimé sur données françaises d'entreprises et nous procédons à la simulation de diverses réformes fiscales. Nous montrons ainsi qu'une réforme consistant à alléger le coût des salariés les plus faiblement rémunérés améliore le bien être global car elle réduit le pouvoir de monopsone des grandes firmes : le chômage est réduit, la production totale augmente, de même que les recettes de l'État.

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1 Introduction

Job search theory has early been confronted to DIAMOND's [1971] critique that there is no wage dispersion at the equilibrium in a labor market with identical workers:[1] all firms should offer the same wage, called monopsony wage and equal to workers' common reservation wage, because no greater wage would increase the labour force of the firm. Firms offer different wages only if workers have different opportunity costs of employment (see, for example, ALBRECHT-AXELL [1984], for the theory, and ECKSTEIN and WOLPIN [1990], for an empirical application). The model of BURDETT and MORTENSEN ([1998]; but see also MORTENSEN [1990]), extends *Diamond*'s model to allow for on-the-job search. They show that equilibrium wage distribution is necessarily dispersed, although neither workers nor firms are heterogenous 'by birth'. Intuitively if a mass of firms offer the same wage, it is optimal for a single firm of this mass do deviate: by offering a slightly better wage they only marginally increase their cost but significantly increase their workforce by attracting a significant proportion from the mass of competing firms. The equilibrium wage offer distribution is therefore continuous. Unfortunately, its density is an increasing function. Consequently, in this framework, additional exogenous heterogeneity is required for a good fit with wage data. Nevertheless, the model is able to reproduce a number of stylized facts, as it predicts a positive correlation between wage and tenure, as well as between the offered wage and the level of employment.

The existing literature seems to show that *Burdett* and *Mortensen*'s dispersed wage equilibrium is very dependent of the assumed matching technology. They assume that all workers have the same probability of drawing any firm, whatever its size (let us call it 'uniform matching'). Yet, many alternative forms of matching are possible. In a rather unnoticed paper, BURDETT and VISHWANATH [1988] postulate 'balanced matching', by which firms are sampled with a probability proportional to their size. They show that in this case the equilibrium wage distribution is degenerate: all firms offer the same wage and have the same size.

This is a particularly distressing characteristic of equilibrium search models with on-the-job search that the existence of wage dispersion in equilibrium so closely depends on the assumed matching technology. The main theoretical contribution of our paper is to show that the degeneracy that was found by *Burdett* and *Vishwanath* is most likely a limit case, the rule being equilibrium wage dispersion. Instead of assuming an *ad hoc* form of matching, whereby th sampling probabilities of firms by workers are arbitrarily related to firm sizes, in this paper, we postulate that the probability for a worker of selecting a given firm (given that the wirker has made a contact) is a component of the

1. In the whole paper, equilibrium on the labor market is considered. It is not general equilibrium since we do not say anything about equilibrium on the goods market.

firm's strategy.[2] Specifically, we make it be proportional to the number of job offers posted by the firms. The optimal number of posted offers will be set so as to compensate the flow of workers continuously leaving the firm, because of exogenous lay-offs or job-to-job mobility, in order to maintain at a stationary level the optimal stock of employment. Since large firms incur large out-flows, they will need to either advertise many job offers or offer high wages in order to attract a sufficient number of workers. These large inflows will increase the firms' wage costs because of additional posting cost (advertising) and hiring costs (training).

We shall show how influential the existence of posting costs, however small, is for the equilibrium wage distribution to be non degenerate. Another consequence of the endogenous matching process is that job offer arrival rates positively depend on the aggregate hiring effort of all firms. The unemployment rate is therefore endogenous in a way that is similar to so-called 'matching models' (see *eg*, PISSARIDES [1990], and MORTENSEN and PISSARIDES [1994]).[3]

As *Burdett*, *Mortensen* and *Vishwanath*, we consider production functions with decreasing returns to labour scale, but we depart from their set-up by assuming that firms do not necessarily incorporate the same amount of capital. We adapt to our context an idea developed by ACEMOGLU and SHIMER [1997] who assume that the decision for a firm to enter the labour market involves a decision about capital. They show that the dispersed wage equilibrium that is generated by firm-to-firm worker mobility has its counterpart in term of capital. *Ex ante* identical firms will in equilibrium choose different levels of capital. The equilibrium distribution of capital then induces an equilibrium distribution of labour productivities.

We derive a number of interesting theoretical results. If hiring costs (*ie*, fixed costs of adapting worker skills to the firm specificity) are linear then firms' optimal wage offers are independent of their production technologies. Consequently, the wage offer distribution is independent of the form of heterogeneity in labour productivities which prevails in the economy. This distribution is shown to be uniform. Since the wage offer is independent of marginal labour productivity, it then follows that all firms choose the same amount of capital. When marginal hiring costs are strictly monotonous, heterogeneity in firms' labour productivities matters again and we derive the unique equilibrium distribution of capital across active firms which prevails at the equilibrium. We also characterize firms' optimal wages, hiring efforts and employment conditional on a the level of capital.

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2. We should here note that the generalisation of the *Burdett-Mortensen* and *Burdett-Vishwanath* setups within one unique model has already been tempted before us. MORTENSEN and VISHWANATH [1994] assume that workers randomly draw wage offers from a mixture of the distribution of wages offered by employers and the distribution of wages earned by employees. They show that Burdett and Vishwanath's equilibrium is the only equilibrium if the probability of balanced matching is large enough. Otherwise, the equilibrium wage distribution is non degenerate with maybe a mass point at the maximal wage offer. The matching technology considered by *Mortensen* and *Vishwanath*, although more general, is yet not less arbitrary than the uniform and balanced matching assumptions.
3. The difference between equilibrium search models and matching models lies in the wage-setting process. Equilibrium search models are *Bertrand* equilibrium models where firms set wages monopsonistically, whereas in *Pissarides*-like matching models wages result from a Nash bargaining process. Note that MORTENSEN [1998, 1999] proposes another very elegant and promising attempt at synthetising the two search approaches.

Adding features of matching models and endogenizing productivity along the line of *Acemoglu* and *Shimer* to the *Burdett-Mortensen* model allows us to construct a self-contained equilibrium model of the labour market in which both the unemployment rate and the wage offer distribution are endogenous. We thus obtain a rather rich theory of unemployment. On the supply side, a shock on non labour income or expected wage changes reservation wages and therefore affects employment. On the demand side, a productivity shock or a change in the legal minimum wage modifies the demography of active firms and therefore also affects employment. To better understand those mechanisms we investigate in details the comparative static responses of the model to various policy reforms (legal minimum wage, labour taxes, taxes on value-added). Our most interesting result supports the idea of a progressive labour tax which can be tuned to allow the entry into the market of low productivity firms.[4]

To make these simulations possible, we need a calibrated model. The calibration of French accounting firm data will be first performed using simple statistical inference techniques, the aim there being to construct a crude, but realistic description of the data.

The plan of the paper is as follows. The theory is developed in section 2 of this paper. Section 3 describes the data, the estimation methodology and presents the estimation of the results. And section 4 proceeds to the simulation of various policy reforms.

2 The Model

2.1 Firms' Strategies

We assume that a firm's strategy is two-fold. First of all, it offers a wage w. Yet, offering a wage is not enough to attract workers. For a positive measure of workers to reach the firm requires a specific recruiting activity. We shall assume that drawing a job offer for a worker is like drawing a colored ball (with replacement) from an urn with many balls of many different colors. Let E be the total number of balls in the urn and let e be the number of balls of a given color. Then the probability of drawing a ball of this color is e/E. Here, e is the number of job offers posted by a firm. We define firms' strategies as couples (w,e).

We use general measures instead of the *Lebesgue* measure, for instance, because we do not *a priori* exclude the existence of mass points in the distribution of (w,e). Thus, we assume that the steady-state distribution of (w,e) in the population of firms realizing nonnegative profit (active firms) can be described by a probability density function (pdf) $h(w,e)$ with respect to a product measure $\mu = \mu_1 \times \mu_2$ on a measurable product space $(\Omega,\mathcal{A},\mu) = (\mathbb{R}_+,\mathcal{R}_+,\mu_1) \times (\mathbb{R}_+,\mathcal{R}_+,\mu_2)$. We also let $N < +\infty$ define the measure of active firms.

4. We do not develop exercises in which the tax revenue is constant as our main goal here is to understand the mechanisms and not to evaluate which policy would be the most efficient, whatever the main objective seeked. This is a very naive representation of the reality that we here consider. For this reason, it is not possible to put much weight on the *quantitative* predictions of the model.

Let E define the overall sum of firms' job offers. Since $e\,Nh(w,e)$ is the number of job offers posted by all firms with strategy (w,e), it follows that

$$E = \int_{\mathbb{R}_+^2} e\,Nh(w,e)\,d\mu_1(w)\,d\mu_2(e). \tag{1}$$

We assume that the arrival rate of job offers to unemployed and employed workers are respectively $\lambda_0 E$ and $\lambda_1 E$. Job offers should indeed reach workers at a quicker rate if they are more numerous. Parameters λ_0 and λ_1 are exogenously given and characterize the relative efficiency of job search according to the occupational state of workers. Presumably, λ_0 and λ_1 are decreasing functions of the total measure of workers; but we neglect this feature since in the sequel the number of active workers is taken as exogenous (contrary to the measure of active firms) and normalized to one.

Conditional on receiving an offer, workers draw type-(w,e) offers with probability density function $\frac{eN\,h(w,e)}{E}$ w.r.t. measure μ, and the marginal probability density of wages w (with respect to μ_1) is obtained by marginalization of the offer sampling density:

$$f(w) = \int_{\mathbb{R}_+} \frac{eNh(w,e)}{E}\,d\mu_2(e). \tag{2}$$

We denote as $F(w)$ the cumulative distribution function and $\overline{F}(w) = 1 - F(w)$.

2.2 Workers' Behaviour

Workers seek to maximize the expected steady-state discounted (at rate σ) future income. We assume that they do not draw satisfaction from the recruiting effort of a firm, only from the wage offered. The opportunity cost of employment is denoted by b and is assumed to be constant across individuals. Whenever an offer arrives, the decision has to be made whether to accept it, or to reject it and search further for a better offer. Layoffs occur at the constant rate δ. The distribution of wage offers F is independent of the current state of the job searcher (employed or unemployed).

PROPOSITION 1. *(MORTENSEN and NEUMANN [1988]) The optimal strategy when unemployed is to accept any wage offer w* greater or equal *to w_R, where w_R, the reservation wage, is implicitly defined as*:

$$\begin{aligned} w_R &= b + (\lambda_0 - \lambda_1)E\int_{w_R}^{\overline{w}} \frac{\overline{F}(x^-)}{\sigma + \delta + \lambda_1 E\overline{F}(x^-)}dx \\ &= b + (\kappa_0 - \kappa_1)\int_{w_R}^{\overline{w}} \frac{E\overline{F}(x^-)}{\mu + 1 + \kappa_1 E\overline{F}(x^-)}dx, \end{aligned} \tag{1}$$

where $\overline{w}$ is the upper bound of the support of the wage offer distribution, and with $\kappa_0 = \lambda_0/\delta$, $\kappa_1 = \lambda_1/\delta$ and $\mu = \sigma/\delta$.

The optimal strategy when employed is to accept any wage offer strictly greater *than the present wage contract.*

Above, we use the convention that $F(w)$ denotes the probability that the wage offer *is smaller than or equal to* w. Moreover, $\overline{F}(x^-)$ is a notation for $\lim_{\varepsilon\downarrow 0}\overline{F}(x-\varepsilon)$.

Note that at this stage it is already clear that firms do not offer a wage $w < w_R$, because they would hire nobody. A firm with zero employment is said inactive and not participating to the market. Thus, the lower bound of the support of the wage offer distribution is $\underline{w} \geqslant \max\{w_R, w_{\min}\}$ where $w_{\min}$ is any legal minimum wage.

2.3 Steady-State Equilibrium Worker Flows

The measure of workers (employed or not) is normalized to one and the unemployment rate is u. Let $G(w)$ be the fraction of individuals with a wage lower than or equal to w in the stock of employed workers. $G(w)$ defines the cumulative density function of a probability measure with density function $g(w)$ with respect to measure μ_1. Consider all individuals working at a wage lower than or equal to w. In a steady-state demographic equilibrium, the flow of layoffs in an interval $(t, t+dt]$ is $\delta(1-u)G(w)\,dt$. Moreover, the measure of workers moving to jobs paying more than w is $\lambda_1 E\overline{F}(w)(1-u)G(w)\,dt$, while the measure of unemployed individuals accepting a wage smaller than or equal to w is $\lambda_0 EF(w)u\,dt$, if $w \geqslant w_R$, given that $F(w_R^-) = 0$. At the equilibrium, one must have equal flows in and out of the stock of workers employed at a wage lower than w. Hence,

$$\lambda_0 EF(w)u = \left[\delta + \lambda_1 E\overline{F}(w)\right](1-u)G(w) \tag{4}$$

if $w \geqslant \max\{w_R, w_{\min}\}$, and flows are zero if $w < \max\{w_R, w_{\min}\}$. It follows that:

PROPOSITION 2. *In a steady-state demographic equilibrium, one must have that:*

$$u = \frac{1}{1+\kappa_0 E}, \quad \text{and} \tag{5}$$

$$G(w) = \frac{F(w)}{1+\kappa_1 E\overline{F}(w)}. \tag{6}$$

2.4 Firms' Employment

Firms' employment can be obtained by considering local flows of workers. Consider a small measure $Nh(w,e)d\mu(w,e)$ of firms offering wage w and posting e job offers. Let $l(w,e)$ be the steady-state employment of each of these firms. They totalize employment $l(w,e)Nh(w,e)d\mu(w,e)$ and workers will draw them, if they draw a new job offer, with probability $\frac{eNh(w,e)d\mu(w,e)}{E}$. The inflow and outflow of workers are then equal if, and only if,

$$\left[\lambda_0 E u + \lambda_1 E(1-u)G(w^-)\right]\frac{e}{E} = \left[\delta + \lambda_1 E\overline{F}(w)\right] l(w,e) \tag{7}$$
$$= \varphi(w,e) \quad \text{(say)}.$$

Hence,

$$\begin{aligned} l(w,e) &= \frac{e}{E}\frac{\lambda_0 E u + \lambda_1 E(1-u)G(w^-)}{\delta + \lambda_1 E\overline{F}(w)} \\ &= \frac{e}{E}(1-u)\frac{1+\kappa_1 E}{\left[1+\kappa_1 E\overline{F}(w)\right]\left[1+\kappa_1 E\overline{F}(w^-)\right]} \\ &= e l_1(w) \quad \text{(say)}. \end{aligned} \tag{8}$$

This equation shows that firms' labour force is the product of recruiting effort e and of a function, $l_1(w)$, which only depends on the wage offer w. Function $l_1(w)$ is the employment yield *per* unit recruiting effort, which, because it does not depend on e, is also the overall employment of all firms offering wage w divided by the overall hiring effort:[5]

$$\begin{aligned} l_1(w) &= \frac{(1-u)g(w)}{E\ f(w)} \\ &= \frac{A}{\left[1+\kappa_1 E\overline{F}(w)\right]\left[1+\kappa_1 E\overline{F}(w^-)\right]} \end{aligned} \tag{9}$$

with,

$$\begin{aligned} A &= \frac{1-u}{E}(1+\kappa_1 E) \\ &= \kappa_0\frac{1+\kappa_1 E}{1+\kappa_0 E}. \end{aligned} \tag{10}$$

Function $l_1(w)$ is increasing. Hence, employment can be increased either by increasing hiring effort e or by increasing wage w.

Finally, the steady-state in-flow of new workers in a firm (w,e) is also multiplicatively separable in e and w:

$$\begin{aligned} \varphi(w,e) &= \left[\delta + \lambda_1 E\overline{F}(w)\right] l(w,e) \\ &= e\frac{\delta A}{1+\kappa_1 E\overline{F}(w^-)} \\ &= e\, l_2(w) \quad \text{(say)}. \end{aligned} \tag{11}$$

5. By equation (2), the hiring effort of firms offering wage w is:

$$E f(w) = N\int e\, h(w,e)\, d\mu_2(e),$$

and the measure of workers employed at wage w is the sum of $l(w,e)$ over all firms offering wage w yields:

$$\begin{aligned} N\int l(w,e)\, h(w,e)\, d\mu_2(e) &= N\int e\frac{(1-u)g(w)}{E\ f(w)}\, h(w,e)\, d\mu_2(e) \\ &= (1-u)g(w). \end{aligned}$$

2.5 Firms' Objective Function

Firms may differ by labour productivity. Different labour productivities are generated by different asset values. Let $q(k,l)$, or $q_k(l)$, denote firms' production function conditional on capital k and labour force l. A natural benchmark specification for $q(k,l)$ is the *Cobb-Douglas*:

$$q(k,l) = \theta k^{\alpha} l^{\beta},$$

with $\theta \in \mathbb{R}_+$, $0 < \alpha < 1$, and $0 < \beta < 1$. Capital has a cost that we specify as a general increasing function $r(k)$. For example, we could assume that,

$$r(k) = rk$$

where r is the interest rate plus the depreciation rate of capital.[6]

Let $c(\varphi(w,e))$ denote *hiring costs*, that is the steady-state cost of hiring a steady-state in-flow $\varphi(w,e)$ of new workers (essentially fixed costs necessary to train the newcomers so that they can work efficiently for their new employer). As a first-order approximation, hiring costs are supposed to be independent of firms' capital. Apart from training costs, there may exist specific *posting costs* $p(e)$, like advertising costs, which are likely to be negligible with respect to other hiring costs which are functions of the effective flow of new workers entering the firm in each period. Yet, they will be shown to have considerable importance in the equilibrium determination.

Firms seek to maximize their steady-state profit flow:

$$\begin{aligned} v(k,w,e) &= q(k,l(w,e)) - wl(w,e) - c(\varphi(w,e)) - p(e) - r(k) \\ &= \pi_k(w,e) - r(k) \quad \text{(say)}, \end{aligned}$$

where $v(k,w,e)$ can be understood as the steady-state profit flow that can be expected before entering the market when the firm has to decide which structural investment to make, and,

$$\pi_k(w,e) = q_k(l(w,e)) - wl(w,e) - c(\varphi(w,e)) - p(e)$$

is the *ex-post* current operating surplus.

We make the following assumptions:

6. Acemoglu and Shimer [1997] use the following specification, with constant returns to scale:

$$q(k,l) = f(k)l,$$
$$r(k) = k,$$

where f is continuously differentiable, increasing and strictly concave and there is a unique k_0 solution to $f'(k_0) = f(k_0)/k_0$ with $f'(k_0) > 1$.

ASSUMPTION 3. *(i)* $q(k,l), r(k),\ c(\varphi)$ *and* $p(e)$ *are two times continuously differentiable and increasing functions of their arguments. (ii) There exists an increasing function* $\ (\cdot)$ *such that, for all positive* $k, w,\ l_1$ *and* l_2, *the functions* $q(k,\ (\cdot)l_1) - w\ (\cdot)l_1 - c(\ (\cdot)l_2) - p(\ (\cdot))$ *are* strictly *concave. (iii) There exists an increasing function* $\phi(\cdot)$ *such that, for all* l, *the functions* $q(\phi(\cdot),l) - r(\phi(\cdot))$ *are* strictly *concave. (iv) For all* k,l, $\frac{\partial^2 q(k,l)}{\partial k \partial l} > 0$.

Part *(ii)* of assumption 3 is enunciated in order to exclude the case where there is no solution in e to profit maximization except if e is bounded above, in which case all firms would produce the maximal hiring effort. This case is obviously unattractive because it brings us back to *Burdett* and *Mortensen*'s standard equilibrium search model. Under assumption 3 *(ii)*, for all $k,\ w,\ l_1$ and l_2, there exists a unique profit-maximizing value for e which is the unique solution to the first-order condition of the problem of maximizing profit with respect to e. Let $\widehat{e_k}(w,l_1,l_2)$ be the unique maximizer of $e \mapsto q(k,el_1) - wel_1 - c(el_2) - p(e)$. Then, for all k and w, there exists a unique function:

$$e_k(w) = \widehat{e_k}(w,l_1(w),l_2(w))$$

of w such that,

$$\begin{aligned}\max_{e>0} \pi_k(w,e) &= \pi_k\left[w,e_k(w)\right] \\ &= \pi_k^*(w) \quad \text{(say)}.\end{aligned}$$

Assumption 3 *(iii)* plays a similar role as assumption 3 *(ii)* and ensures that, for all l, there exists a unique value $k^*(l)$ for k, increasing in l, and maximizing $q(k,l) - r(k)$ with respect to k given l.

To see how useful these changes in variables can be, consider the case of:

$$\begin{aligned}q(k,l) &= \theta k^\alpha l^\beta, \\ c(\varphi) &= \varphi^\gamma, \\ r(k) &= rk^\rho.\end{aligned}$$

Condition *(ii)* of assumption 3 is satisfied if $\beta < 1$ and $\gamma > 1$ ($q(k,l)$ concave in l and $c(\varphi)$ convex), but it is also satisfied with $\beta < \gamma$ and $\gamma < 1$ ($c(\varphi)$ concave and $q(k,l)$ concave enough to compensate for the concavity of $c(\varphi)$). This latter result is a particular case of 3 *(ii)*, with $\ = c^{-1}$. Similarly, condition 3 *(iii)* is verified if either $\alpha < 1$ and $\rho > 1$ ($q(k,l)$ concave in k and $r(k)$ convex) or if $\alpha < \rho$ and $\rho < 1$. It thus follows that the convexity of the costs functions $c(\varphi)$ and $r(k)$ is by no means necessary.

Finally, assumption 3 *(iv)* implies that labour productivity increases with capital. This also implies that $k^*(l)$ increases with the labour force l. Capital and labour are thus complementary.

The equilibrium distribution of firms' optimal strategies, $h(w,e)$, is completely characterized by the joint distribution of optimal wages w and capital intensities k. In the sequel, we characterize this joint distribution by the marginal distribution of capital intensities, $\Gamma(k)$, and the conditional distribution of wages, $F_k(w)$.

2.6 Market Steady-State Equilibrium

The equilibrium definition we use is closely related to the one proposed by ACEMOGLU and SHIMER [1997], which is basically the same as that of BURDETT and MORTENSEN [1997] except that they allow for free entry as a mechanism for generating productivity dispersion.

DEFINITION 1. *An equilibrium consists of a measure of active firms N, a distribution* Γ *(cdf) with support* $K \subset \mathbb{R}_+$ *of capital k for active firms, a reservation wage* w_R *for unemployed workers, wage distributions* F_k *with support* $\mathcal{X}_k \subset \mathbb{R}_+$ *for each firm type* $k \in K$*, such that:*

1. *(Optimal Stopping Rule for Unemployed) Unemployed workers accept any wage offer greater or equal to* w_R *of equation (3);*
2. *(Quitting Rule for Employees) Employees at wage* w *accept any alternative wage offer* $w' > w$*;*
3. *(Optimal Wage Offers)* $\forall k,\ \forall w \in \mathcal{X}_k,\ \forall w' \in \mathbb{R}_+,\ \pi_k^*(w) \geqslant \pi_k^*(w')$;
4. *(Optimal Choice of Capital)* $\forall k \in K,\ \forall k' \in \mathbb{R}_+,\ \forall w \in \mathcal{X}_k,\ \forall w' \in \mathcal{X}_{k'},$ $\pi_k^*(w) - r(k) \geqslant \pi_{k'}^*(w') - r(k')$;
5. *(Free Entry)* $\forall k \in K,\ \forall w \in \mathcal{X}_k,\ \pi_k^*(w) = r(k)$.

Note that the aggregate equilibrium wage distribution which workers sample from is then:

$$F(w) = \int_K F_k(w)\, d\Gamma(k).$$

The equilibrium is sequential: in the first stage, firms decide to enter or not the market, and, if yes, which capital investments to make. In the second stage, a measure N of firms with a distribution Γ of capital have entered the market and choose a wage offer so as to maximize the intertemporal flow of operating surplus.

The next proposition shows that the main result of MORTENSEN [1990] and BURDETT-MORTENSEN [1997], that any equilibrium wage distribution has no mass point, does not apply here as straightforwardly. The proof of this proposition and all the proofs which require more than a few lines of mathematics are in the appendix.

PROPOSITION 4. *(a) If employees do not search on the job (*$\lambda_1 = 0$*), then the equilibrium wage offer distribution is a mass point at* $\max(w_{\min}, w_R)$ *when* $w_{\min}$ *is any legal minimum wage.*

(b) If employees do search on the job ($\lambda_1 > 0$) and there is no posting costs ($p(e) = 0$), then the gross operating surplus $\pi_k^*(w)$ is a continuous function of wage w whether F is continuous or not. There is a multiplicity of equilibrium wage distributions with and without mass points.
(c) If employees do search on the job ($\lambda_1 > 0$) and there exists specific posting costs, then any equilibrium wage distribution has no mass point.
(d) Finally, if F is continuous then its support is connected and for each mass point there exists a right neighbourhood which is not in the support of the wage offer distribution.

If there is no search on the job, $\lambda_1 = 0$ and functions $l_1(w)$ and $l_2(w)$ degenerate to constants. Then, firms freely choosing (w, e) so as to maximize:

$$q_k(el_1) - wel_1 - c(el_2) - p(e)$$

will post e job offers such that the first order condition is satisfied:

$$\left[q_k'(\widehat{e}l_1) - w\right] l_1 - l_2 c'(\widehat{e}l_2) - p'(\widehat{e}) = 0,$$

and will offer the minimum wage that workers are willing to accept, *ie*, $\max(w_{\min}, w_R)$. In this case, as in DIAMOND [1971], the equilibrium wage distribution is degenerate and all firms propose the same monopsony wage.

If employees do receive alternative offers, then proposition 4 shows that for *Burdett* and *Mortensen*'s argument to apply, there needs to exist positive posting costs $p(e)$. If posting e offers cost nothing, then the profit function $\pi_k^*(w)$ is a continuous function of wage offer w. Therefore, if there is a mass of firms offering the same wage w, then not a single firm can exploit the right discontinuity of employment because both $l_1(w)$ and $l_2(w)$ jump at w^+, so that it is true that increasing l_1 increases employment and profit, but it also increases the flow of new workers entering the firm at each instant and increases hiring costs in the same proportion as it increases value added minus wage costs. On the other hand, if there exists a specific cost of posting a job offer (advertising for example), then proposition 4 shows that they will thwart the preceding adjustment of e and allow a fraction of firms to profit from the fact that a significant proportion of firms offer the same wage. Presumably, these specific posting costs are negligible in comparison to hiring costs, but they are enough to make symmetric equilibria unstable. In the sequel, we shall neglect posting costs but, nevertheless, we shall concentrate on equilibria with no mass points in the wage distribution.

ASSUMPTION 5. *Employees do search on the job, posting costs $p(e)$ are nil and the equilibrium wage distribution is continuous.*

Consider a firm with capital k. It posts $e_k(w)$ wage offers w such that the first-order condition:

$$\left[q_k'(e_k(w)l_1(w)) - w\right] l_1(w) - l_2(w)c'(e_k(w)l_2(w)) = 0 \tag{12}$$

is satisfied. Moreover, all profit maximizing wages in $\mathcal{X}_k$ yield the same optimal current operating surplus:

(13)
$$q_k(e_k(w)l_1(w)) - we_k(w)l_1(w) - l_2(w)c(e_k(w)l_2(w)) = \pi^*(k) \quad \text{(say)}.$$

Write these two conditions as $B(e_k(w), F(w)) = 0$. It is straightforward to show that under assumption 3 the gradient of $B(\cdot,\cdot)$ at any point $(e_k(w), F(w))$ corresponding to any choice of w in the interior set of $\mathcal{X}_k$, is non singular. The implicit-function theorem then implies that $e_k(w)$ and $F(w)$ exist, and are continuous and continuously differentiable in the interior set of $\mathcal{X}_k$. The next proposition builds on this property to show that under assumptions 3 and 5, more productive firms should produce a bigger hiring effort and profit maximizing wage sets should not overlap.

PROPOSITION 6. *(a) For any two firms $i = 0, 1$, such as $q_0(l) < q_1(l)$ and $q_0'(l) < q_1'(l)$ for all l, then the optimal hiring effort is such that for all w, $e_1(w) > e_0(w)$.*
(b) Let $\mathcal{X}_0$ and $\mathcal{X}_1$ denote the profit maximizing wage sets of firms 0 and 1. If $\mathcal{X}_0$ and $\mathcal{X}_1$ have non empty interiors, let w_1 be in the interior of $\mathcal{X}_1$ and w_0 be in the interior of $\mathcal{X}_0$. If $c(\varphi)$ is strictly convex then $w_1 \geqslant w_0$, if $c(\varphi)$ is strictly concave then $w_1 \leqslant w_0$, and if $c(\varphi)$ is linear then $\mathcal{X}_1 = \mathcal{X}_0$.

Given assumption 3(iv) any couple of firms verifies the conditions of proposition 6 if firm 0 has capital k_0 and firm 1 has capital k_1 with $k_1 > k_0$. Proposition 6 implies that, when the hiring cost function is either strictly concave or strictly convex, then, optimal wage sets $\mathcal{X}_k$ intersect at most at one point. If $c(.)$ is linear, proposition 6 shows that the wage supports are independent of predetermined capital.

In the next proposition, we apply the same argument as in BONTEMPS, ROBIN, VAN DEN BERG [1996] to deduce from the preceding proposition that if the distribution of firms' types is absolutely continuous w.r.t. *Lebesgue* measure (*ie*, Γ is continuous) then there exists a function $w(\cdot)$ such that $\mathcal{X}_k = \{w(k)\}$ Γ-almost surely. It is remarkable that proposition 6 does not at all show that $w(k)$ is then non decreasing, because the ordering in proposition 6 only applies to sets $\mathcal{X}_k$ which are **not** reduced to singletons.

PROPOSITION 7. *For any equilibrium distribution of capital across firms Γ exhibiting no mass points, if the hiring cost function is either strictly concave or strictly convex, then the capital-specific sets of profit maximizing wages $\mathcal{X}_k$ are singletons with Γ-probability one.*

We can now state and show the main result of this section:

PROPOSITION 8. *If the hiring cost function $c(\varphi)$ is strictly convex or concave, any equilibrium capital distribution has no mass point.*

The preceding propositions have established that it is crucial to distinguish the case where $c(\varphi)$ is linear from that where it is strictly convex or concave. The objective of the next two sections is to derive the precise form of the equilibrium in both cases.

2.7 Linear Hiring Costs

We first consider the case of a linear function $c(\varphi) = c\varphi$. The next lemma establishes firms' wage strategies.

LEMMA 9. *With a linear hiring cost function, profit maximizing wages are the wages which minimize marginal labour cost* $w + \frac{c}{l_1(w)}$, *under the constraint* $w \geqslant w_R$.

This proposition has an immediate consequence: **the optimal wage strategy is independent of capital**. Thus, contrary to the *Burdett-Mortensen* model, the equilibrium wage distribution is independent of the form of the heterogeneity in labour productivities which prevails in the economy. The next proposition establishes the form of the equilibrium wage offer distribution.

PROPOSITION 10. *With a linear hiring cost function, the wage offer distribution takes the form:*

$$F(w) = \frac{w - \underline{w}}{c\lambda_1 E}, \quad w \in \left[\underline{w}, \overline{w}\right], \tag{14}$$

where $\underline{w} = \max\{w_R, w_{\min}\}$ *and,*

$$\overline{w} = \underline{w} + c\lambda_1 E. \tag{15}$$

Moreover, a firm with capital k produces hiring effort:

$$e_k(w) = \frac{1}{l_1(w)}(q_k')^{-1}\left(\underline{w} + c\delta + c\lambda_1 E\right),$$

The optimal labour force of a firm offering wage w is then independent of the offered wage:

$$\begin{aligned} l\left[w, e_k(w)\right] = (q_k')^{-1}\left(\underline{w} + c\delta + c\lambda_1 E\right) \\ = l(k) \quad \text{(say)}. \end{aligned}$$

Proposition 10 states a remarkable result: in a market with homogenous workers and linear hiring costs, in which firms may differ in their production function, there is a unique continuous equilibrium wage offer distribution F and it is uniform. The reason why the equilibrium wage distribution is not affected by the specification of the production function is that the objective function to minimize is the marginal labour cost $w + \frac{cl_2(w)}{l_1(w)}$ which does not depend on the firm's specific production function.

The next proposition deduces from the previous one that any equilibrium distribution of capital must be degenerated.

PROPOSITION 11. *At the equilibrium, each of the N active firms employs a measure l^* of workers and uses capital k^*, the unemployment rate is u, the aggregate recruiting effort is E and the reservation wage of unemployed workers is w_R, where the vector (k^*, l^*, N, u, E, w_R) is an interior solution, if it exists, to the system:*

$$\begin{cases} \dfrac{\partial q(k^*, l^*)}{\partial k} - r'(k^*) = 0, \\ l^* = \dfrac{1-u}{N}, \\ q\left(k^*, l^*\right) - \left(\underline{w} + c\delta + c\lambda_1 E\right) l^* - r(k^*) = 0, \\ u = \dfrac{1}{1+\kappa_0 E}, \\ \underline{w} + c\delta + c\lambda_1 E = q'_{k^*}\left(l^*\right), \\ w_R = b + c\delta \dfrac{\kappa_0 - \kappa_1}{\kappa_1} \left\{ \kappa_1 E + (\mu+1) \ln \left[\dfrac{\mu+1}{\mu+1+\kappa_1 E} \right] \right\} \\ \qquad + \dfrac{(\kappa_0 - \kappa_1) E}{\mu + 1 + \kappa_1 E} \left(w_{\min} - w_R\right)^+, \end{cases}$$

where $(w_{\min} - w_R)^+ = w_{\min} - w_R$ if $w_{\min} > w_R$ and $= 0$ otherwise.

PROOF. Straightforward deduction from the fact that since all firms must employ the same labour force, per firm labour force is uniformly: $l^* = (1-u)/N$. The first equation then just states the optimality of capital k^*. The third equation is the free entry condition which gives the condition for N. The fourth is the equilibrium flow condition from and into unemployment. The fifth equality expresses the optimality of employment l^* as in proposition 10 and, when solved for E, yields the aggregate hiring effort of active firms. The last equation is the equation for the reservation wage of unemployed given F when the formula for F of proposition 10 is used to compute the integral in equation 3.

There is a lot of empirical evidence that the wage offer distribution is not of the kind predicted by this model (densities are not upward sloping). In particular, it should be affected by firms' heterogeneity. In the next section, we examine in details the situation when hiring costs are strictly convex or strictly concave.

2.8 **Strictly Convex or Strictly Concave Hiring Costs**

This section considers the case where $c(\varphi)$ is not linear but strictly convex or concave. We already know from proposition 8 that no homogenous equilibrium (*ie,* all active firms choose the same value of k) can arise if firms can freely choose their capital structure. We let Γ define the distribution of capital across active firms. Proposition 8 implies that Γ must be continuous. It therefore admits a density γ with respect to *Lebesgue* measure.

Let $w(k)$ be the wage strategy of a firm with capital intensity k and let $e(k) = e_k(w(k))$ be the corresponding recruitment effort. Let us rewrite workers' job sampling distribution (*ie,* the probability for a given worker to receive an offer from a firm with capital k conditional on receiving an offer from *a* firm) as:

$$F(w(k)) = Z(k). \tag{16}$$

It follows from equation (2) that distribution Z is related to distribution Γ by the equation:

$$z(k) = \frac{e(k)}{E} N\gamma(k), \tag{17}$$

where z is the density function of distribution Z: $z(k) = Z'(k)$. (We also denote $\overline{Z}(k) = 1 - Z(k)$.) Moreover, a firm with asset value k employs $l(k)$ employees such that:

$$\begin{aligned} l(k) &= e(k) l_1 [w(k)] \\ &= e(k) \frac{A}{\left[1 + \kappa_1 E \overline{Z}(k)\right]^2} . \end{aligned} \tag{18}$$

To simplify the notations, we shall write:

$$l_1(k) = \frac{A}{\left[1 + \kappa_1 E \overline{Z}(k)\right]^2}. \tag{19}$$

The next proposition considers the family of equilibria defined by Burdett and *Mortensen*. They correspond to constrained market equilibria where firms choose their optimal wage offer and their optimal recruiting effort conditional on an exogenously given distribution Γ of capital in the population of active firms and a measure of active firms N.

PROPOSITION 12. *If the distribution of capital in the population of active firms is some continuous distribution Γ with density γ and if the measure of active firms is N, then each firm characterized by capital $k \in K = [\underline{k}, \overline{k}]$ chooses to produce hiring effort $e(k)$ and to offer wage $w(k)$ satisfying the following system:*

$$
(20) \quad \begin{cases}
q_k' \quad \dfrac{Ae(k)}{\left[1+\kappa_1 E\overline{Z}(k)\right]^2}\Bigg) - w(k) - \left[\delta + \lambda_1 E\overline{Z}(k)\right] \dfrac{w'(k)}{\kappa_1 E z(k)} = 0, \\
c'\left(\dfrac{Ae(k)}{\delta + \lambda_1 E\bar{Z}(k)}\right) = \dfrac{w'(k)}{\kappa_1 E z(k)}, \\
z(k) = \dfrac{e(k)}{E} N\gamma(k), \\
E = \int e(k) N\gamma(k) dk, \\
Z(\underline{k}) = 0, \\
Z(\overline{k}) = 1, \\
w(\underline{k}) = \max(w_R, w_{\min}),
\end{cases}
$$

where $w_{\min}$ is any legal minimum wage and where w_R is given by equation (3).

The resolution of system (20) yields the optimal behaviour of active firms conditional on the variables determining firms' entry into the market. It is a system of differential equations defining functions $w(\cdot)$, $e(\cdot)$ and $Z(\cdot)$ as a function of the distribution of capital Γ and the measure of active firms N. The determination of Γ and N is then obtained by adding to the set of conditions yielding (20) the zero profit condition. One can then show the following proposition.

PROPOSITION 13. *Let functions $\psi_1(k),;\psi_2(k),;\psi_3(k),\ \psi_4(k)$ and $\psi_5(k)$ be recursively defined by the system:*

$$
(21) \quad \begin{cases}
\dfrac{\partial q\left[k,\psi_1(k)\right]}{\partial k} - r'(k) = 0, \\
\psi_2(k) = d^{-1}\left[q\left[k,\psi_1(k)\right] - \dfrac{\partial q\left[k,\psi_1(k)\right]}{\partial l}\psi_1(k) - r(k)\right], \\
\psi_3(k) = \dfrac{q\left[k,\psi_1(k)\right] - c\left[\psi_2(k)\right] - r(k)}{\psi_1(k)}, \\
\psi_4(k) = \dfrac{\psi_2(k)}{\psi_1(k)}, \\
\psi_5(k) = \dfrac{1}{\psi_2(k)}\left[\dfrac{\psi_1'(k)}{\psi_1(k)} - \dfrac{\psi_2'(k)}{\psi_2(k)}\right],
\end{cases}
$$

where $d(\varphi) = c(\varphi) - \varphi c'(\varphi)$. If they exist for all k, then, any unconstrained market equilibrium must be such that:

1. *Wage offer:* $w(k) = \psi_3(k)$;
2. *Labour in-flow:* $\varphi(k) = \psi_2(k)$;
3. *Labour force:* $l(k) = \psi_1(k)$;
4. *Sampling capital distribution:* $\delta + \lambda_1 E\overline{Z}(k) = \psi_4(k)$;
5. *Minimal capital:* $\delta + \lambda_1 E = \psi_4(\underline{k})$;

6. *Maximal capital:* $\delta = \psi_4(\overline{k})$;

7. *Capital distribution:* $\gamma(k) = \dfrac{\psi_5(k)}{\int_{\underline{k}}^{\overline{k}} \psi_5(k)dk}$;

8. *Wage distribution:* $F(w) = Z\left[\psi_3^{-1}(w)\right]$ *(where* $\psi_3^{-1}(w)$ *is the set of* k *such that* $w(k) \leqslant w$ *and* $Z\left[\psi_3^{-1}(w)\right]$ *is the probability of this set);*

9. *Market size:* $N = \dfrac{A}{\lambda_1} \int_{\underline{k}}^{\overline{k}} \psi_5(k)dk$;

10. *Reservation wage:*

$$w_R = b + \frac{\lambda_0 - \lambda_1}{\lambda_1} \int_{\underline{k}}^{\overline{k}} \frac{\psi_4(k) - 1}{\delta\mu + \psi_4(k)} \psi_3'(k)dk + \frac{(\kappa_0 - \kappa_1)\, E}{\mu + 1 + \kappa_1 E} (w_{\min} - w_R)^+ ;$$

11. *Overall sum of recruiting efforts* E *or, equivalently, constant* A *(note that* $E = \dfrac{1}{\kappa_0} \dfrac{\kappa_0 - A}{A - \kappa_1}$*), is chosen such that:*

$$\min_{k \in \left[\underline{k}, \overline{k}\right]} \psi_3(k) = \max\{w_R, w_{\min}\}.$$

For $c(0) = 0$ and $c(\varphi)$ convex (resp. concave), then $d(\varphi) \leqslant 0$ and $d'(\varphi) \leqslant 0$ (resp. $\geqslant 0$) for all $\varphi \geqslant 0$. The condition for these functions ψ_2 to exist for all k is that

$$q\,[k,\ \ _1(k)] - \frac{\partial q\,[k,\ \ _1(k)]}{\partial l}\ \ _1(k) - r(k) < 0 \text{ (resp. } > 0),$$

for all k. This condition is necessary to guaranty that there exists an interior solution in hiring effort to *ex post* profit maximization. If it were not the case, then it is easy to see that there would be a value for k above (or below) which the lower bound for φ (*ie,* 0) would be binding. Very productive firms would produce minimal hiring effort. Allowing for this case to occur is a source of additional complication but of no additional clarity. We thus preferred to assume that an interior solution to *ex post* profit maximization exists for all k.

Moreover, $q\left[k, \psi_1(k)\right] - c\left[\psi_2(k)\right] - r(k)$ or $\psi_3(k)$ must be positive for at least some k for an equilibrium to exist. Otherwise, any profit maximizing triple (k, w, e) would yield a negative profit and no firm would decide to enter the market.

Finally, notice that if $q\left[k, \psi_1(k)\right] - \frac{\partial q\left[k, \psi_1(k)\right]}{\partial l} \psi_1(k) - r(k)$ is constant for all k, then the distribution of wage offers takes the same form as with linear hiring costs. Indeed, this condition together with the equation defining $\psi_1(k)$ imply that $\frac{\partial q\left[k, \psi_1(k)\right]}{\partial l}$ is a constant, which in turn implies that the wage offer function $\psi_3(k)$ is an affine function of the labour force $\psi_1(k)$, and that $1+\kappa_1 E\overline{F}(w)$ is a linear function of w. This occurs whenever the technology is with constant returns to scale and the user cost of capital is affine. For example, let $q(k,l) = k^\alpha l^\beta$, $r(k) = r_0 + r_1 k$, and $\alpha + \beta = 1$.[7] We have then that:

$$\begin{aligned}
l(k) &= \psi_1(k) = \left(\frac{r_1}{\alpha}\right)^{1/\beta} k,\\
\varphi(k) &= \psi_2(k) = d^{-1}(-r_0),\\
w(k) &= \psi_3(k) = \frac{\beta}{\alpha}\left(\frac{r_1}{\alpha}\right)^{-1/\beta} - \left[r_0 + c(d^{-1}(-r_0))\right]\left(\frac{r_1}{\alpha}\right)^{-1/\beta} k^{-1},\\
\psi_4(k) &= d^{-1}(-r_0)\left(\frac{r_1}{\alpha}\right)^{-1/\beta} k^{-1}\\
&= \frac{d^{-1}(-r_0)}{r_0 - c(d^{-1}(-r_0))}\left[\frac{\beta}{\alpha}\left(\frac{r_1}{\alpha}\right)^{-1/\beta} - w(k)\right].
\end{aligned}$$

And it follows from $Z(k) = F(w(k))$ and $\delta + \lambda_1 E\overline{Z}(k) = \psi_4(k)$ that:

$$\delta + \lambda_1 E\overline{F}(w) = \frac{d^{-1}(-r_0)}{r_0 - c(d^{-1}(-r_0))}\left[\frac{\beta}{\alpha}\left(\frac{r_1}{\alpha}\right)^{-1/\beta} - w\right],$$

where $E = Ne(k) = Nd^{-1}(-r_0)$.

3 Estimation

3.1 Data

The estimation of the production and the hiring cost functions are conducted using a sample of the "*Bénéfices industriels et commerciaux*" (BIC) data collected by the French National Statistical Institute (INSEE) in 1994. The BIC data provide yearly accounting informations on all establishments employing more than 20 workers or with a cash flow greater than a given minimal amount. This corresponds to about 60% of all the firms in France. We deleted from this sample the observations for which the observed capital was over the 5th centile (the 6th for the Consulting sector) and below the 95th centile. This trimming was performed in order to correct for measurement error at the extremes of the distribution,[8] and it happened that the capacity of the model to describe the data proved extremely sensitive to this selection. To put it clearly, the estimated model is able to account for

7. Or equivalently $r(k) = r_0 + r_1 k^\rho$, with $\frac{\alpha}{\rho} + \beta = 1$.
8. There is clearly a lot of measurement errors in the data, in particular in the lower extreme of the distribution of capital. A significant number of firms declare negative values for employment. Note also that many more may well declare non erroneous negative added value or current operating surplus values which our static model is unable to account for.

thick distribution tails but not to the extent of what is present in the data. Moreover, we deleted all observations for which the total compensation costs, employment, value added or current operating surplus (value added minus compensation costs) were negative. In the sequel, all money values are expressed in thousands of francs per month.

There is clear empirical evidence that the between-industry mobility of workers is limited. Moreover, industries are different with their production functions or their skill requirements. It was therefore natural to stratify the data by industry. On the other hand, lack of place makes it impossible to account for all industries, even at a high aggregation level. We thus chose to analyze the case of only three different activities: Food, Consulting and Hotels, Restaurants.

These three industries have very different labour market structures, as is apparent from the summary statistics presented in Table 1. The Food industry is characterized by a higher level of capital, relatively low wages and by an average firm size 50% higher than in the two other sectors. Consulting gathers firms where capital investments are relatively small but paying high wages (nearly twice as in the other sectors). Hotels and restaurants are slightly more capitalistic than consulting, and slightly more heterogenous in firm sizes. The labour share is somewhat lower in the Food industry. Generally speaking, it was interesting to compare an industrial activity like Food, with a lot of capital heterogeneity, to two service activities characterized by very different skilled/unskilled labour ratios.

Figure 1 provides some information about the cross-sectional distributions of six firm variables which corresponding macro aggregates are usually found as particularly informative: the capital stock (here measured by gross productive assets), the labour force, the capital - output ratio (gross productive assets divided by value added), the labour share (wage costs divided by value added), wage costs per employee (including labour taxes) and the profit rate (ratio of the current operating surplus divided by gross productive assets). Each column of figure 1 corresponds to a different productive activity.

The first row of figure 1 shows kernel density estimators for capital stock, in log coordinates so as to emphasize the *Pareto* tail of this distribution (a Pareto density is a straight line in log coordinates). The last five rows provide a picture of the conditional distribution of each of the last five variables given capital. The top line indicates the conditional third quartile, the second line indicates the conditional median and the bottom line indicates the conditional first quartile.[9] All graphs are in log-coordinates and the horizontal and vertical lines indicate the 10th, 25th, 50th, 75th and 90th percentiles of the x and y variables.

It is plain clear from these pictures that the shapes of these distributions do not fundamentally differ according to activity: employment, capital – output ratios and mean wage costs increase with capital, whereas labour shares and profit rates have the opposite monotonicity. These quite remarkable cross-sectional patterns point out that some underlying structural relation between capital, labour and output should be at work.

---

9. We have split the total sample in 50 subsamples ordered by the stock of capital. For each of these subsamples we have computed the three quartiles, which we have then plotted.

TABLE 1

Summary Statistics on BIC Data

Sector 1: Food

Variables	Mean	Std Deviation	Minimum	Maximum
Capital	27263,71	40191,59	1369,00	237429,00
Log(Capital)	9,46	1,21	7,22	12,38
Added Value	1682,13	2548,58	31,25	35653,25
Log(Added value)	6,87	0,97	3,44	10,48
Firm Size	70,39	100,11	1,00	1691,00
Log(Firm Size)	3,82	0,84	0,00	7,43
Compensation costs	1042,19	1525,23	0,00	26312,08
Compensation costs /Added Value	0,69	0,17	0,00	1,00
Mean Offered Wage	14,65	4,19	0,00	43,22
Number of Firms	2 902			
Number of Employed Workers	204 265			

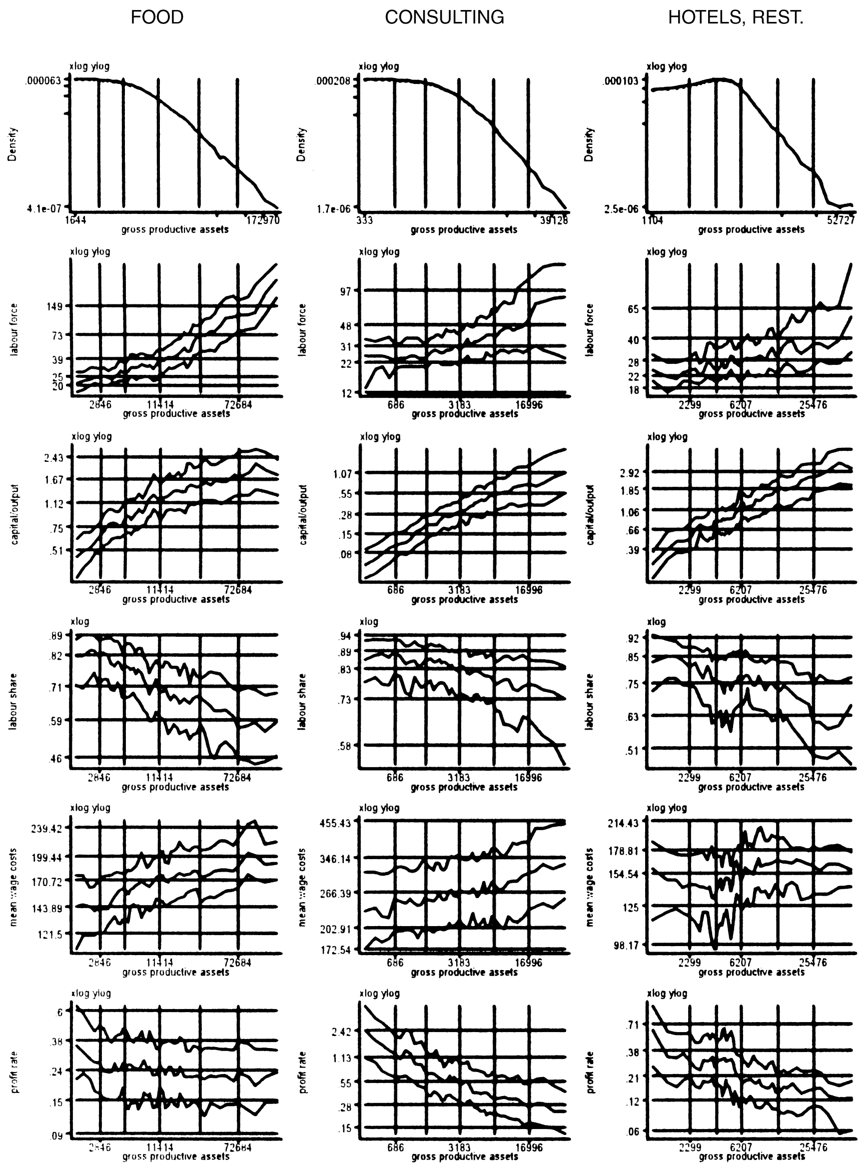
Sector 2: Consulting

Variables	Mean	Std Deviation	Minimum	Maximum
Capital	6698,82	9239,70	286,00	59159,00
Log(Capital)	8,13	1,16	5,66	10,99
Added Value	1533,99	2228,27	8,67	30123,33
Log(Added value)	6,87	0,90	2,16	10,31
Firm Size	50,01	74,35	1,00	2186,00
Log(Firm size)	3,47	0,93	0,00	7,69
Compensation costs	1177,78	1730,15	4,50	26487,58
Compensation costs /Added Value	0,79	0,16	0,00	1,00
Mean Offered Wage	24,91	12,56	1,89	168,57
Number of Firms	4 452			
Number of Employed Workers	222 623			

Sector 3: Hotels, Restaurant

Variables	Mean	Std Deviation	Minimum	Maximum
Capital	10735,33	11749,68	814,00	65169,00
Log(Capital)	8,84	0,91	6,70	11,08
Added Value	821,91	2380,39	36,75	61129,00
Log(Added value)	6,30	0,71	3,60	11,02
Firm Size	48,95	175,56	4,00	4631,00
Log(Firm size)	3,45	0,67	1,39	8,44
Compensation costs	615,57	2153,40	0,00	55311,25
Compensation costs /Added Value	0,73	0,15	0,00	1,00
Mean Offered Wage	13,04	4,28	0,00	47,17
Number of Firms	2 022			
Number of Employed Workers	98 971			

FIGURE 1
1994 BIC data (upper and lower 5% of capital trimmed)



(The first row of that figure shows kernel density estimates of the distribution of capital. In the last five rows, the three lines display the conditional quartiles of the distribution of the y-variable given capital. All graphs are in log-coordinates and the horizontal and vertical lines indicate the 10th, 25th, 50th, 75th, and 90th percentiles of the x- and y-variables.)

All these different arguments make us confident that the choice of these three industries, albeit arbitrary, is representative of the variety of ways the sector of activity conditions labour market.

3.2 Identification of Firms' Strategy and Arrival Rates from Firm Data

Given data on value-added q, capital k, employment l and wage costs wl, the production function $q(k,l)$, the hiring cost function $c(\varphi)$ and the user cost of capital $r(k)$ can be identified and straightforwardly estimated using the formulas of proposition 13.

For example, suppose that:

$$\begin{aligned} q = q(k,l) &= \theta k^{\alpha} l^{\beta}, \\ c(\varphi) &= \varphi^{\gamma}, \\ r(k) &= r_0 + r_1 k^{\rho}, \end{aligned}$$

with $\rho > \alpha$ and with $\gamma > \beta$ for $q(k,l) - r(k)$ and $q(k,el_1) - wel_1 - c(\varphi)$ to admit global maxima with respect to k and e respectively.[10] The employment-capital relation becomes:

$$l(k) = \psi_1(k) = \left(\frac{r_1\rho}{\theta\alpha}\right)^{\frac{1}{\beta}} k^{\frac{\rho-\alpha}{\beta}} \tag{22}$$

yielding added-value:

$$q(k) = \frac{r_1\rho}{\alpha} k^{\rho}. \tag{23}$$

Inflows are then:

$$\begin{aligned} \varphi(k) &= \psi_2(k) \\ &= \left[\frac{1}{\gamma-1}\left(\left(\frac{\alpha}{\rho} - 1 + \beta\right) q(k) + r_0\right)\right]^{1/\gamma} \end{aligned}$$

and wage costs follow as:

$$\begin{aligned} (wl)(k) &= \psi_3(k) \cdot \psi_1(k) \\ &= \frac{\beta}{\gamma-1}\left[\beta - \frac{\gamma}{\gamma-1}\left(\frac{\alpha}{\rho} - 1 + \beta\right)\right] q(k) - \frac{\gamma}{\gamma-1} r_0. \end{aligned} \tag{24}$$

Note that, for $\varphi(k)$ and $(wl)(k)$ to be defined for all k, it is needed that $\frac{1}{\gamma-1}\left(\frac{\alpha}{\rho} - 1 + \beta\right) > 0$ and that $\beta - \frac{\gamma}{\gamma-1}\left(\frac{\alpha}{\rho} - 1 + \beta\right) > 0$, or, equivalently,

10. The specification of the capital cost function $r(k)$ is here more flexible than the specification presented in the theoretical section.

that $\frac{\rho-\alpha}{\beta}$, the power of capital in the employment equation, belongs to the interval $[\rho, \rho/\gamma]$ or $[\rho/\gamma, \rho]$, depending on whether γ is lower or greater than 1.

Equations (22), (23) and (24) identify 6 parameters out of the 7, the last one being identified from the capital density:

$$\gamma(k) \propto \frac{1}{\psi_2(k)} \left[\frac{\psi_1'(k)}{\psi_1(k)} - \frac{\psi_2'(k)}{\psi_2(k)} \right].$$

In the particular case where $r_0 = 0$, then the capital distribution can be shown to be Pareto, its density being proportional to k to the power of $-1 - \frac{\rho}{\gamma}$.

We estimated equations (22), (23) and (24) by applying generalized least square to the reparameterised system:

(25)
$$\begin{cases} \log(q) = a_0 + a_1 \log(k), \\ \log(l) = b_0 + b_1 \log(k), \\ \dfrac{wl}{q} = c_0 + \dfrac{c_1}{q}. \end{cases}$$

Parameter γ was estimated by maximum likelihood setting all other parameters to their preestimated values.

We have seen how to identify the parameters of the firms' profit function. Estimates of parameters δ, $\lambda_0 E$ and $\lambda_1 E$ are obtained from firm data as follows: δ and $\lambda_1 E$ immediately follow from the conditions:

$$\psi_4\left(\underline{k}\right) = \delta + \lambda_1 E,$$
$$\psi_4(\overline{k}) = \delta.$$

And $\lambda_0 E$ is easily obtained from the value of the unemployment rate: $u = \frac{\delta}{\delta + \lambda_0 E}$.

However, it is not possible to separately identify E from $\lambda_0 E$ and $\lambda_1 E$. That is, without any more details on workers' search process (for example, we could say that $\lambda_0 dt$ is the probability that the unemployed goes to an employment agency and $\lambda_0 E/\lambda_0 = E$ is the probability that upon reaching the agency the worker does find a job offer), we may as well set $\lambda_0 = 1$.

3.3 Estimation Results

How good a description of the data equations (25) are? It is clear from figure 1 (second and third panels) that the linear approximation is correct only to the first order. The graphs indeed show that the relations between $\ln q$, $\ln l$ and $\ln k$ are slightly non linear. On the other hand the labour share is approximately a linearly decreasing function of logged capital, and we found that replacing $\ln k$ by $1/q$ gives good results if the smallest values of the added

TABLE 2
Reduced-form Parameters' Estimates
(Standard errors in parentheses)

Coefficients	Food	Consulting	Hotels, Rest.
a0	0,30	3,67	2,66
	(0,08)	(0,08)	(0,13)
a1	0,69	0,39	0,41
	(0,01)	(0,01)	(0,01)
b0	– 1,68	0,83	0,81
	(0,08)	(0,08)	(0,13)
b1	0,58	0,33	0,30
	(0,01)	(0,01)	(0,01)
c0	0,65	0,79	0,68
	(0,00)	(0,00)	(0,01)
c1	24,17	0,33	21,11
	(1,71)	(0,71)	(1,95)

TABLE 3
Structural Parameters' Estimates

	Food	Consulting	Hotels, Rest.
Alpha	0,16	0,04	0,09
Beta	0,92	1,10	1,07
Rho	0,69	0,39	0,41
r1	0,30	3,55	3,25
r0	– 13,49	– 0,20	– 16,15
Gamma	2,26	2,54	4,26
Teta	6,36	15,80	6,03
Ek0	7,00	7,00	7,00
Ek1	2,21	1,48	1,30
Delta	0,08	0,13	0,05

value variable are deleted from the sample. Yet, we keep in mind that more flexible specifications (for example, a CES for the production function) may yield better approximations. However, this model will never be able to account for the fact that an important share of the variance of $\ln q$, $\ln l$ and wl/q is not accounted for by the capital variable. Figure 1 shows that capital does explain a significant share of the variance of logged wages, logged employment and labour share in the sample, but it is not reasonable to assume that the residual variance is only due to measurement errors. There is a lot of heterogeneity that this model does not account for, like employee's skill heterogeneity, for example.

The estimation results of the reduced form (25) are presented in table 2 and table 3 shows the values of the structural parameters that can be obtained from the former using the following restrictions:

$$a_1 = \rho,\ a_0 = \log\left(\frac{\rho r_1}{\alpha}\right),$$
$$b_1 = \frac{\rho - \alpha}{\beta},\ b_0 = \frac{1}{\beta}\log\left(\frac{\rho r_1}{\alpha\theta}\right),$$
$$c_0 = \frac{\left(1 - \dfrac{\alpha}{\rho} - \dfrac{\beta}{\gamma}\right)}{1 - \dfrac{1}{\gamma}},\ c_1 = \frac{-r_0}{1 - \dfrac{1}{\gamma}}.$$

The structural restrictions imposed by the theory: $\gamma > 1$, $\left(1 - \dfrac{\alpha}{\rho} - \dfrac{\beta}{\gamma}\right) > 0$, $\rho > \alpha$, are satisfied by the estimated parameters.

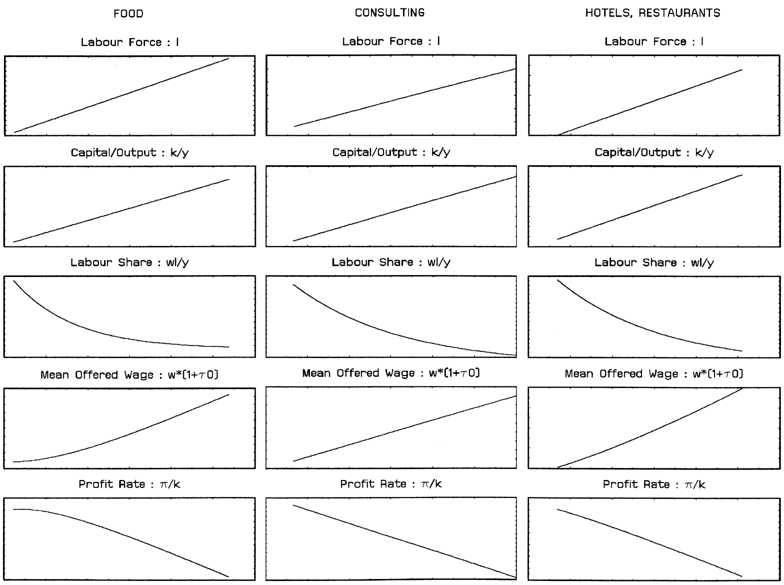
We estimate a negative value for r_0. This may indicate (for $r(k)$ to be positive) that there exists a minimum capital needed to install a new firm. This negative value of r_0 has also for consequence that the relationship between wage w and capital k for small values of k can be non monotonous, being first decreasing and then increasing. This is clearly not a very good feature of the model. Moreover, ρ is always lower than 1. This implies that capital user cost is predicted to be concave. An interpretation may be that large firms face less liquidity constraints. On the other hand, training costs are found convex ($\gamma > 1$). The more convex training costs are, the more concentrated in its lower part the labour force distribution will be. This explains why we obtain a bigger value for the Hotels and Restaurants, where firms are very heterogenous in size and where the mean firm size is rather small. Finally, β is not found very different from 1, which indicates that the assumption of a production function linear in labour that was always maintained in the empirical literature on equilibrium search may well be a reasonable assumption.

Table 3 also gives the estimated values for $\delta, \lambda_0 E$ and $\lambda_1 E$. $\lambda_0 E$ was obtained by postulating the same unemployment rate of 12.5% in all three industries,[11] whereas δ and $\lambda_1 E$ were obtained by fitting the maximal and minimal capital predicted by the model. The values of $\lambda_1 E$ that we thus estimate are of the same magnitude as what BONTEMPS, ROBIN and VAN DEN BERG [1998a,b] found using the transition data of the Labour Force Survey. The value of δ is one order of magnitude bigger.

Figure 2 provides a similar picture as figure 1 using the values predicted from the preceding estimations. It is only for the Food industry that the model is able to generate a bump in the distribution of capital that is similar to what was non parametrically obtained from the data. For the two other industries, the predicted distribution is entirely *Pareto*-like. The most remarkable difference occurs for profit rate which for Food and Hotel/Restaurants was found pretty much independent of capital. The predicted feature is that of a decreasing relationship as the one that was non parametrically obtained in the data for the Consulting industry.

11. It is likely that there is a sectoral component in the unemployment rate. Taking it into account would not qualitatively change the results we obtain.

FIGURE 2
Model's Predictions (continued)



Finally, figure 2 also displays the wage offer density and the earnings density (in level coordinates). Although the wage offer density is monotonously decreasing, firm size increases with offered wage at such a fast rate that the earnings density g (the density of wages in the population of employed workers) is monotonously *increasing*. Whether this is a feature of the model or of the particular specification we have used for estimation and simulation, we cannot say.

4 Policy Analysis

The preceding estimation allows us to calibrate a model that describes the firm data reasonably well. From a strict econometric point of view, one might be worried about the effect of trimming on the estimation of the rate coefficients based on distribution support extrema. Also the assumption that the unemployment rate is the same in all three sectors is certainly disputable, if only because the share of skilled/unskilled workers are obviously different. We claim that these worries are not justified as the main idea of this empirical analysis is 1) to evaluate the capacity of the model to replicate the main features of the data, and 2) to explore the qualitative effects on employment and on the wage distribution of various labour policies. As long as the estimated parameters are not unreasonable and as long as the fit of the model is relatively satisfactory, we are indeed confident that the direction of the effects that we now describe are likely to be correct, if not their magnitude.

The structural parameters in table 3 are used to simulate the economy according to proposition 13. In these simulations, we assume that the legal minimum wage (about 5000FF *per* month) is not binding (below reservation wage). This assumption is made in order to evaluate the opportunity cost of employment, since then the minimal wage offer is equal to the reservation wage. We also assume that wages are taxed at constant rate $\tau_0 = 1$ (employer plus employee labour taxes), so that the cost of a worker payed w is $\widetilde{w} = (1 + \tau_0)w = 2w$. This roughly corresponds to the actual taxation scheme in France in 1994. The product of labour taxes is $M(1-u)\int_{\underline{w}}^{\overline{w}} \tau_0 w dG(w)$, where G is the cdf of net earnings among workers and M is the measure of workers.

A number of descriptive statistics of the simulated economy are presented in table 4. In general, the predicted means of capital, value-added, wage offers, total compensation costs and employment *per* firm have the right order of magnitude (compare with table 1). This is not surprising given that the estimation method is designed to fit the distribution of capital and the means of value-added, firm size and labour share across firms. What is more interesting is that the number of firms which is predicted by the model is also very close from its true value in the data. Note, however, that the model is not able to generate distributions of value-added, wages and employment as dispersed as the true ones. The predicted standard deviations are indeed far from being

TABLE 4
Model's Predictions

Sector	Food	Consulting	Hotels, Rest.
Taxation coefficient on wages	1	1	1
Unemployment rate	12,50%	12,50%	12,50%
Mean Earnings	8,11	12,30	6,56
Standard Deviation	0,98	1,24	0,74
Maximal Earnings	9,69	14,28	7,79
Reservation Wage	6,34	9,93	5,26
Opportunity Cost of Unemployment	3,68	4,65	2,06
Minimum Capital	1363,00	285,00	801,00
Maximum Capital	238036,90	59783,00	66055,97
Mean Capital	32129,74	8410,00	13323,20
Standard Deviation	47691,58	12731,35	15820,04
Mean firm size	62,88	34,42	32,89
Standard Deviation	54,52	17,78	12,53
Mean compensation cost	1020,30	846,57	431,33
Standard Deviation	1036,79	530,63	214,74
Mean gross wage offer	14,70	23,35	12,56
Standard Deviation	1,80	2,45	1,45
Average Added Value	1523,96	1077,73	601,73
Standard Deviation	1586,18	675,78	315,00
Number of firms	2487,13	4951,73	2303,72
Total output	3,79E+06	5,34E+06	1,39E+06
Taxes	1,27E+06	2,10E+06	4,97E+05

similar to their true empirical counterparts. It is only for capital that the fit is good at the first and second order. This clearly indicates that another source of heterogeneity is required for a better description of the data. An obvious candidate is worker skill heterogeneity.

The model can be used to predict the minimal wage offer, which, if the legal minimum wage is not binding, is equal to reservation wage. The estimated opportunity cost of employment is roughly equal to a half of the minimal wage offer. This is consistent which the French legislation on unemployment insurance which provides the unemployed with an income that is roughly a half of the previous wage, on average, for a limited period of time.

We now analyze the effects on employment and wage of various economic policy reforms: a raise in the legal minimum wage, a raise in the marginal rate of labour taxation, the introduction of non linear labour taxation (lower marginal rates for small wages), a raise in the taxation of value-added, a raise in the taxation of capital.

4.1 Legal Minimum Wage Increases

We first analyze the effect of increasing the legal minimum wage progressively from 5000FF/month to 5500, 6000 and 6500FF/month. Note that, because the reservation wage was estimated at 9900FF/month for workers in the Consulting industry, this reform has no effect in this industry, and it is only when the legal minimum wage is greater than 6300FF/month that the reform has an effect in the Food industry.

Table 5 shows the results of the model's simulation. An increase of the legal minimum wage in this model has an unambiguously dramatic effect on unemployment which raises because the small, less capitalistic firms are not productive enough to maintain themselves in this new market configuration and must stop their activity. The total number of active firms therefore diminishes, the minimal capital necessary to enter the market increases whereas the capital of the biggest firms stays unchanged. Consequently, the mean wage offer increases while its standard deviation decreases. The average firm size tends to be bigger than before and the total output of each industry decreases as well as the revenue of labour taxes. Employees earn more and there is less inequality in earnings.

Making capital and hiring effort endogeneous enriches the role of equilibrium effects in comparison to the *Burdett-Mortensen* model. In the standard B-M model, rising the legal minimum wage shifts the wage offer distribution to the right (the mean wage offer increases) and thus pushes reservation wages up. Our model incorporates to the B-M model some aspects of matching models *à la Pissarides*. When the number of active firms decreases then so does the total hiring effort E. The reduction of firm opportunities forces unemployed workers to demand lower wages. We find that this effect dominates the first and opposite effect.[12]

12. Fershtman and Fishman [1994] examine the effects of wage control in search markets on the other wages. They come to the counterintuitive effect that a rise in the minimum wage may conduct to a diminution of the mean wage. This effect is mainly driven by the workers' search effort being dependent on the minimum wage. Unlike them, our model posit that firm's hiring effort is endogeneous, but worker's hiring effort is exogeneous.

Figure 3 shows how the whole distribution of capital, wage offers and earnings are affected by the minimum wage increase. The effect of the reform is similar to a left-truncature: the lower bound is shifted to the right and the density is uniformly increased.

All ratios (employment rate (labour force divided by capital), capital/output, labour share, profit rate) remain unchanged.

4.2 Proportional Wage Taxation

In order to analyze the effect of an increase of the marginal tax rate on wages, we choose to set the legal minimum wage to 6,000, to make it binding in at least one industry. The initial state of the economy is therefore that of the second column of each vertical panel of table 5. We consider an increase of the marginal tax rate on wages from 1, initially, to 1.02, 1.05 and 1.1. The results of the simulations are in table 6.

Increasing labour tax, as for legal minimum wage, tends to increase the unemployment rate. A smaller number of firms remain active after the reform and their size is bigger on average. Labour costs increase but, contrary to the previous case, the employees earn less. Total output is reduced and the tax revenue increases if the marginal rate increase is not too big. Indeed, for Hotels/Restaurants, when the marginal tax rate goes from 1.05 to 1.1, the legal minimum wage becomes binding; firms cannot respond to the increased marginal tax rate by reducing net wages as much as they would want and, consequently, many small firms cannot enter the market due to excessive labour costs. There, thus exists an optimal level of taxation above which an increase of marginal tax rate only lowers labour taxes because the number of firms stopping activity increases too rapidly.

FIGURE 3
Increasing the Legal Minimum Wage

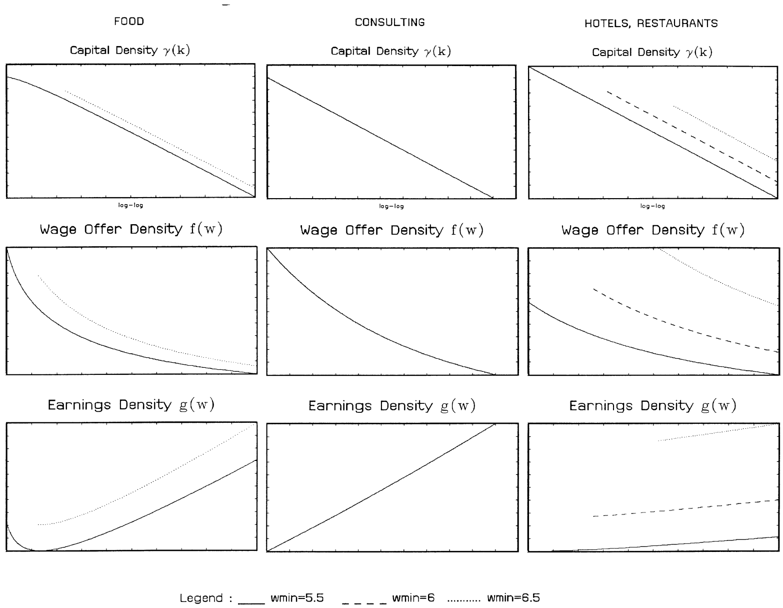


TABLE 5

Simulation of Legal Minimum Wage Increases

	Food			**Consulting**			**Hotels, Restaurant**		
Legal Minimum Wage	5,5	6	6,5	5,5	6	6,5	5,5	6	6,5
Unemployment Rate	12,50%	12,50%	14,24%	12,50%	12,50%	12,50%	14,96%	21,30%	30,36%
Average Earnings	8,11	8,11	8,20	12,30	12,30	12,30	6,67	6,92	7,16
Standard Deviation	0,98	0,98	0,92	1,24	1,24	1,24	0,66	0,52	0,37
Maximal Earnings	9,69	9,69	9,69	14,28	14,28	14,28	7,79	7,79	7,79
Reservation Wage	6,34	6,34	6,33	9,93	9,93	9,93	5,25	5,17	4,95
Minimum Capital	1363,00	1363,00	3202,00	285,00	285,00	285,00	1521,84	4465,61	10753,46
Maximum Capital	238036,90	238036,90	238037,00	59783,00	59783,00	59783,00	66055,95	66056,00	66056,00
Mean Capital	32129,81	32129,81	38824,61	8410,04	8410,04	8410,04	15745,73	21790,66	29760,16
Standard Deviation	47691,65	47691,65	50396,37	12731,37	12731,37	12731,37	16264,28	16481,29	15393,65
Mean firm size	62,88	62,88	73,27	34,42	34,42	34,42	35,82	41,55	47,08
Standard Deviation	54,52	54,52	55,21	17,78	17,78	17,78	11,66	9,66	7,37
Mean compensation cost	1020,30	1020,30	1201,50	846,57	846,57	846,57	478,21	574,86	673,99
Standard Deviation	1036,79	1036,79	1066,09	530,63	530,63	530,63	204,86	177,46	140,94
Mean gross wage offer	7,35	7,35	7,56	11,68	11,68	11,68	6,46	6,80	7,10
Standard Deviation	0,90	0,90	0,87	1,22	1,22	1,22	0,66	0,52	0,37
Average Added Value	1523,97	1523,97	1801,18	1077,73	1077,73	1077,73	670,49	812,27	957,67
Standard Deviation	1586,18	1586,18	1631,01	675,78	675,78	675,78	300,50	260,31	206,74
Number of firms	2487,13	2487,13	2091,98	4951,70	4951,70	4951,70	2055,91	1640,15	1281,08
Total output	3,79E+06	3,79E+06	3,77E+06	5,34E+06	5,34E+06	5,34E+06	1,38E+06	1,33E+06	1,23E+06
Taxes	1,27E+06	1,27E+06	1,26E+06	2,10E+06	2,10E+06	2,10E+06	4,92E+05	4,71E+05	4,32E+05

TABLE 6

Simulation of Proportional Taxes on Wages (Legal Minimum Wage = 6)

	Food			**Consulting**			**Hotels, Restaurant**		
Taxation Coefficient	1,02	1,05	1,1	1,02	1,05	1,1	1,02	1,05	1,1
Unemployment Rate	12,68%	12,93%	13,32%	12,57%	12,67%	12,84%	22,21%	23,65%	26,29%
Average Earnings	8,04	7,94	7,77	12,18	12,01	11,74	6,88	6,82	6,73
Standard Deviation	0,96	0,94	0,91	1,23	1,20	1,17	0,50	0,46	0,41
Maximal Earnings	9,59	9,45	9,23	14,14	13,93	13,60	7,72	7,60	7,42
Reservation Wage	6,29	6,22	6,11	9,84	9,72	9,52	5,11	5,02	4,86
Minimum Capital	1647,86	1927,68	2321,90	291,16	300,66	317,20	5002,61	5904,08	7695,62
Maximum Capital	238036,95	238036,98	238036,99	59783,00	59783,00	59783,00	66055,93	66055,99	66055,99
Mean Capital	32888,10	33895,11	35433,33	8458,03	8530,73	8653,84	22629,99	23943,73	26290,62
Standard Deviation	48031,75	48470,85	49111,42	12753,57	12786,85	12842,29	16430,71	16318,29	16024,04
Mean firm size	64,11	65,72	68,13	34,54	34,72	35,02	42,22	43,22	44,88
Standard Deviation	54,62	54,75	54,93	17,77	17,75	17,71	9,40	9,00	8,32
Mean compensation cost	1041,44	1069,20	1111,07	849,88	854,87	863,27	586,55	604,18	633,83
Standard Deviation	1040,71	1045,75	1052,88	530,44	530,14	529,60	173,57	167,48	156,67
Mean gross wage offer	7,30	7,22	7,10	11,57	11,42	11,17	6,77	6,72	6,65
Standard Deviation	0,89	0,87	0,84	1,21	1,18	1,15	0,49	0,46	0,41
Average Added Value	1556,31	1598,78	1662,83	1081,95	1088,31	1099,00	829,42	855,27	898,76
Standard Deviation	1592,18	1599,88	1610,80	675,54	675,16	674,47	254,60	245,67	229,81
Number of firms	2434,39	2368,06	2273,94	4930,98	4899,99	4848,61	1595,50	1529,72	1422,34
Total output	3,79E+06	3,79E+06	3,78E+06	5,34E+06	5,33E+06	5,33E+06	1,32E+06	1,31E+06	1,28E+06
Taxes	1,28E+06	1,30E+06	1,32E+06	2,12E+06	2,15E+06	2,19E+06	4,73E+05	4,73E+05	4,72E+05

Reservation wages decrease when the marginal tax rate increases according to the same mechanisms as when the legal minimum wage is changed. A smaller number of active firms makes the overall hiring effort E lower and pushes the reservation wage down (other things being held constant). On the other hand, firms try to lift the increased tax burden onto workers' back by reducing net wages. If lower wage offers are expected, then unemployed are willing to accept lower wages because the return to rejecting an offer, and waiting for another one, hoping that it will be greater, is lower. Note that the same equilibrium argument also explains why firms cannot entirely report the increased tax burden on workers, even when the legal minimum wage is far from being binding. Consequently, more capital is required to enter the market because the reservation wage decrease is not big enough to prevent labour costs from increasing.

Figure 4 shows how, when the legal minimum wage is not binding, the density of wage offers and earnings is shifted to the left. As far as the density of capital is concerned, the effect of the marginal tax increase has the same consequence on the distribution of capital as for a legal minimum wage increase: it is truncated to the left.

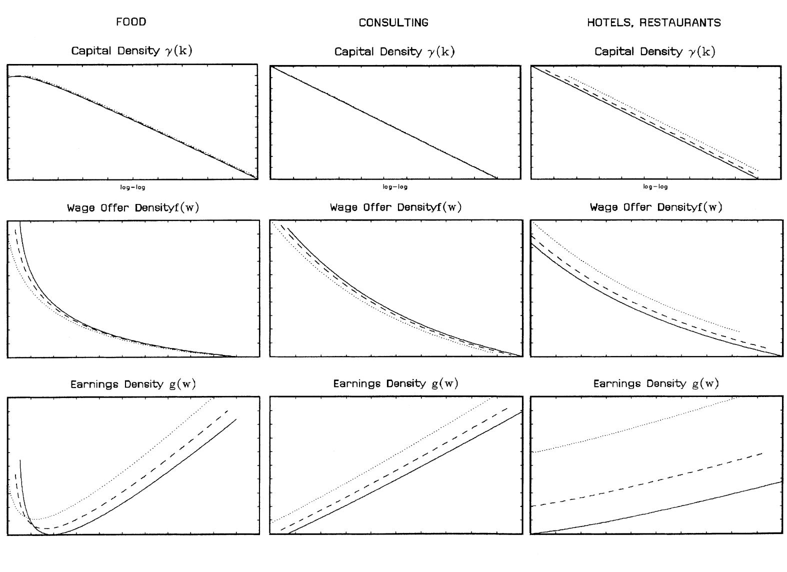
All ratios (employment rate, capital/output, labour share, profit rate) stay unchanged.

4.3 Progressive Wage Taxation

In this subsection, we consider the case of a progressive tax system which taxes high wages proportionally more than small wages. The labour cost *per* hour of work $\widetilde{w}$ is related to the net wage w by the following formula:

$$\widetilde{w} = w\,[1 + \tau\,(w)] = w\,[1 + \tau_0 + \tau_1\,(w - w_c)]$$

FIGURE 4
Simulation of Proportional Taxes on Wages



where τ_0, τ_1 and w_c are policy parameters. We set $\tau_0 = 1.1$ and analyze the effect of increasing τ_1 from 0 to 0.002, 0.005 and 0.01 for the Food industry and from 0 to 0.02, 0.05 and 0.1 for the two other industries.[13] Moreover, we set $w_c = 8{,}000$ and the legal minimum wage to 6,000.

The simulation results are reported in table 7. In all cases, making taxes more progressive tends to reduce unemployment. The number of active firms increases because the labour costs of small firms are reduced and smaller firms can now enter the market. The industry as a whole produces more output and the tax revenue is higher. The effect on earnings is different according to whether the reservation wage is above or below the legal minimum wage. For the Consulting industry (see figure 5), the whole density of wage offers and earnings is shifted to the left, so that employees earn less than before on average and earnings dispersion is reduced. For Hotels/Restaurants, the legal minimum wage is binding. The earnings density is shifted to the right. Earnings are bigger on average and more dispersed.

All ratios (capital/output, labour share, profit rate) stay unchanged.

4.4 Proportional Taxation of Value-Added

We now introduce a tax on value-added. Let τ_3 be the marginal rate. The output of the firm is:

$$(1 - \tau_3)\,\theta k^{\alpha} l^{\beta}$$

and the government income is:

$$N \int_{\underline{k}}^{\overline{k}} \tau_3 \theta k^{\alpha} l\,(k)^{\beta}\, \gamma\,(k)\, dk$$

The basic tax rate on wages has been set to 1.1 so that simulations could be done on the Food sector. The government tax revenue is thus:

$$N\tau_3 \int_{\underline{k}}^{\overline{k}} \theta k^{\alpha} l\,(k)^{\beta}\, \gamma\,(k)\, dk + M\,(1-u)\,\tau_0 \int_{\underline{w}}^{\overline{w}} wg\,(w)\, dw$$

Table 8 presents the results of the simulations using three different values for τ_3.

Increasing τ_3 is like decreasing the productivity of each couple of production factors (k, l). It makes firms reduce their production and capital investments, and they pay smaller wages on average. As long as the legal

13. In trying to simulate this new tax system we encountered the following problem. For the Food industry, in certain configurations of the tax system parameters and for small wage values, the relation between wage offers and capital (ψ_3) becomes decreasing and there may not exist solutions to the equation determining market size:

$$\min_{k \in [\underline{k}, \overline{k}]} \psi_3(k) = \max\{w_R, w_{\min}\}$$

To avoid this technical difficulty, we selected tax system parameters for which $w(k)$ is increasing for all w greater than the legal minimum wage. We do not fully understand what is happening when ψ_3 is not monotonously increasing. This requires further theoretical investigations which are outside the scope of this exploratory paper.

TABLE 7
Simulation of Progressive Taxes on Wages (Legal Minimum Wage = 6)

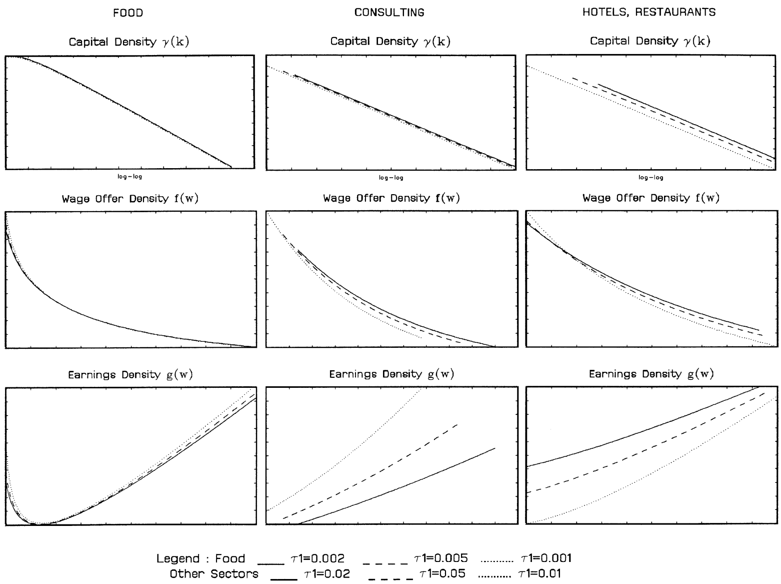
	Food			**Consulting**			**Hotels, Restaurant**		
Taxation Coefficient	1,1	1,1	1,1	1,1	1,1	1,1	1,1	1,1	1,1
Progressivity of the Taxation	0,002	0,005	0,01	0,02	0,05	0,1	0,02	0,05	0,1
Unemployment Rate	13,21%	13,03%	12,73%	12,14%	11,32%	10,27%	24,16%	21,30%	17,27%
Average Earnings	7,76	7,75	7,74	11,31	10,85	10,33	6,75	6,78	6,82
Standard Deviation	0,90	0,90	0,89	1,06	0,95	0,83	0,42	0,44	0,46
Maximal Earnings	9,21	9,20	9,17	12,99	12,33	11,61	7,46	7,51	7,58
Reservation Wage	6,11	6,11	6,11	9,27	8,99	8,67	4,95	5,08	5,27
Minimum Capital	2210,26	2037,47	1716,68	253,21	189,86	126,03	6232,09	4465,61	2419,68
Maximum Capital	238036,99	238036,98	238036,96	59783,00	59782,99	59783,00	66055,99	66056,00	66056,00
Mean Capital	34993,27	34317,06	33121,94	8151,34	7565,04	6824,57	24396,90	21790,66	17982,00
Standard Deviation	48931,91	48650,23	48135,07	12608,59	12309,81	11888,09	16270,52	16481,29	16476,00
Mean firm size	67,44	66,38	64,49	33,78	32,25	30,21	43,55	41,55	38,16
Standard Deviation	54,88	54,80	54,65	17,86	18,01	18,16	8,87	9,66	10,90
Mean compensation cost	1099,15	1080,74	1047,91	828,51	786,27	730,09	610,08	574,86	516,87
Standard Deviation	1050,91	1047,77	1041,90	531,56	532,94	532,99	165,39	177,46	194,97
Mean gross wage offer	7,09	7,08	7,06	10,77	10,34	9,85	6,66	6,68	6,69
Standard Deviation	0,84	0,84	0,83	1,05	0,94	0,82	0,42	0,43	0,45
Average Added Value	1644,60	1616,44	1566,21	1054,73	1000,94	929,40	863,93	812,27	727,20
Standard Deviation	1607,78	1602,98	1594,00	676,97	678,73	678,79	242,60	260,31	286,00
Number of firms	2300,04	2341,42	2418,62	5067,27	5356,15	5786,90	1508,06	1640,15	1877,54
Total output	3,783E+06	3,785E+06	3,788E+06	5,345E+06	5,361E+06	5,378E+06	1,303E+06	1,332E+06	1,365E+06
Taxes	1,324E+06	1,325E+06	1,327E+06	2,262E+06	2,337E+06	2,419E+06	4,768E+05	4,808E+05	4,816E+05

TABLE 8

Simulation of Proportional Taxes on Value Added (Legal Minimum Wage = 6)

	Food			**Consulting**			**Hotels, Restaurant**		
Taxation Coefficient	0,01	0,05	0,1	0,01	0,05	0,1	0,01	0,05	0,1
Unemployment Rate	13,48%	17,08%	23,85%	12,93%	13,34%	13,93%	28,01%	37,22%	59,14%
Average Earnings	7,66	7,34	6,98	11,60	11,03	10,34	6,67	6,45	6,19
Standard Deviation	0,89	0,73	0,54	1,15	1,07	0,97	0,38	0,26	0,11
Maximal Earnings	9,09	8,53	7,86	13,43	12,74	11,89	7,32	6,90	6,38
Reservation Wage	6,04	5,76	5,40	9,42	9,01	8,51	4,76	4,30	3,43
Minimum Capital	2291,94	4866,93	10559,47	309,36	280,32	248,94	8522,22	12840,37	21597,28
Maximum Capital	228657,37	193848,24	156037,23	56655,67	45444,98	34036,81	62975,78	51757,59	39997,36
Mean Capital	34430,81	39251,42	46136,36	8264,69	6851,97	5377,88	26474,53	27528,83	29774,22
Standard Deviation	47350,71	43321,34	36737,71	12198,66	9880,60	7499,69	15046,85	10991,47	5288,47
Mean firm size	67,88	81,15	99,98	34,89	34,38	33,79	45,63	48,81	53,28
Standard Deviation	54,32	51,95	46,16	17,54	16,86	15,98	7,87	5,86	2,83
Mean compensation cost	1091,62	1251,53	1464,78	849,67	796,70	733,68	639,16	661,64	692,95
Standard Deviation	1026,15	943,40	797,46	518,22	473,86	420,94	147,01	106,16	49,21
Mean gross wage offer	7,01	6,90	6,74	11,04	10,53	9,90	6,61	6,42	6,19
Standard Deviation	0,83	0,70	0,52	1,13	1,05	0,95	0,38	0,26	0,11
Average Added Value	1633,07	1877,72	2203,97	1081,69	1014,22	933,96	906,59	939,57	985,49
Standard Deviation	1569,90	1443,30	1220,02	659,98	603,48	536,09	215,65	155,72	72,19
Number of firms	2277,95	1826,21	1361,42	4861,51	4910,11	4962,28	1366,27	1113,77	664,13
Total output	3,758E+06	3,610E+06	3,334E+06	5,312E+06	5,242E+06	5,150E+06	1,251E+06	1,102E+06	7,272E+05
Taxes	1,340E+06	1,378E+06	1,378E+06	2,217E+06	2,311E+06	2,422E+06	4,699E+05	4,411E+05	3,138E+05

FIGURE 5
Simulation of Progressive Taxes on Wages



minimum wage is not binding, smaller firms enter the market and inflate the number of active firms, which allows tax revenue to increase. But, when wage cuts are forbidden by the existence of a legal minimum wage, the number of active firms falls instead, to such an extent that they generate less tax income. Unemployment always increases.

Figure 6 shows the effects of the tax changes on the densities of capital, wage offers and earnings. In the Consulting industry, the support of the distribution of capital is shifted to the left but the density at any point that remains feasible does not change. The largest firms are just replaced by small firms. The densities of wage offers and earnings are also shifted to the left but in no remarkable way. The case of the last two industries is different. The support of capital shrinks instead of being translated to the left and the distributions of wage offers and earnings are compressed to the left due to binding legal minimum wage.

All ratios (capital/output, labour share, profit rate) stay unchanged.

4.5 Proportional Taxation of Capital

We introduce a tax on capital that is equivalent to changing parameter r_1. Specifically, the cost of capital $r(k)$ becomes:

$$(1 + \tau_4)\, r_1 k^{\rho} + r_0$$

where τ_4 is the tax rate. The total government income from capital and wages is thus:

$$N \int_{\underline{k}}^{\overline{k}} \tau_4 r_1 k^{\rho} \gamma\,(k)\, dk + M\,(1-u)\,\tau_0 \int_{\underline{w}}^{\overline{w}} wg\,(w)\, dw$$

FIGURE 6
Simulation of Proportional Taxes on Value Added

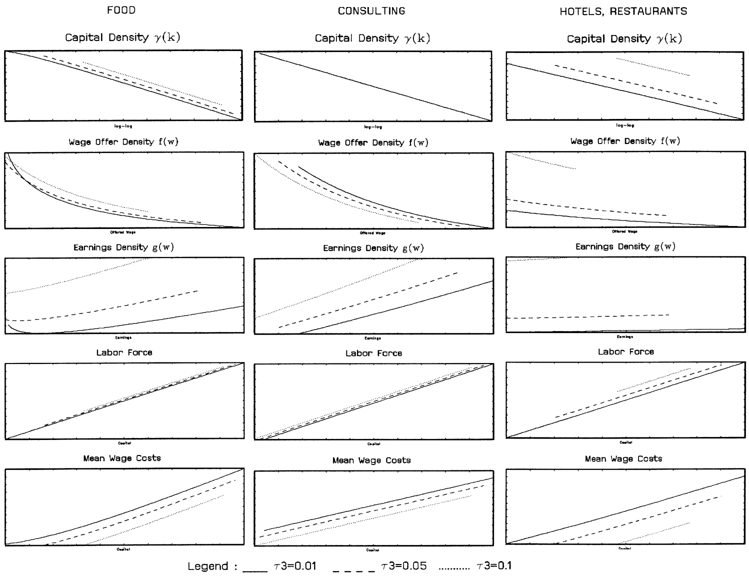


FIGURE 7
Simulation of Proportional Taxes on Capital (continued)

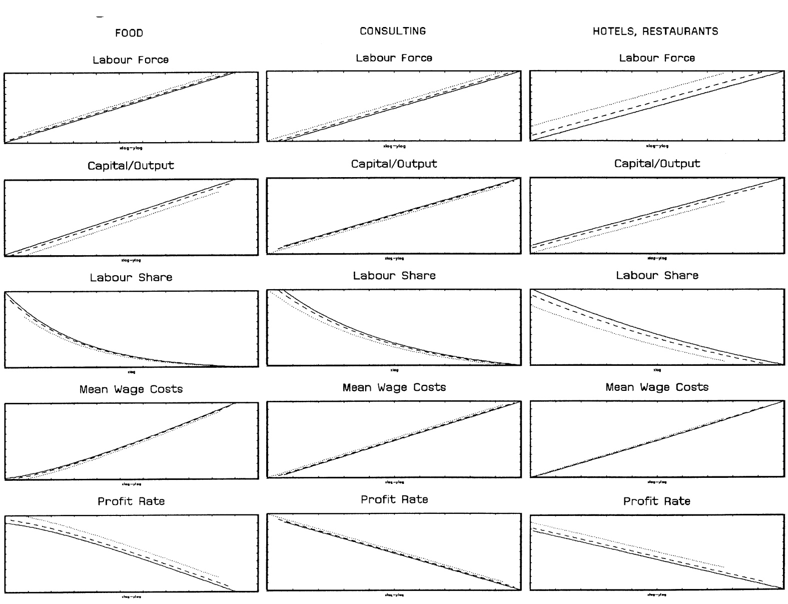


TABLE 9

Simulation of Proportional Taxes on Capital (Legal Minimum Wage = 6)

	Food			**Consulting**			**Hotels, Restaurant**		
Taxation Coefficient	0,05	0,1	0,2	0,05	0,1	0,2	0,05	0,1	0,2
Unemployment Rate	13,50%	14,15%	16,05%	12,88%	12,92%	12,99%	28,21%	30,24%	34,68%
Average Earnings	7,65	7,56	7,41	11,68	11,62	11,51	6,66	6,61	6,50
Standard Deviation	0,88	0,85	0,77	1,16	1,15	1,13	0,38	0,35	0,29
Maximal Earnings	9,07	8,93	8,66	13,53	13,45	13,32	7,31	7,20	6,99
Reservation Wage	6,03	5,96	5,83	9,48	9,43	9,35	4,75	4,64	4,42
Minimum Capital	2133,94	2356,08	3147,32	277,11	243,64	191,53	7653,56	7605,97	7499,45
Maximum Capital	212299,44	190352,01	155200,94	51575,41	44801,42	34428,22	55650,03	47257,67	34806,38
Mean Capital	32005,17	30494,14	29159,02	7491,01	6528,28	5047,59	23535,24	21201,09	17491,54
Standard Deviation	43979,75	40129,55	34101,01	11090,40	9643,13	7423,95	13271,43	11032,78	7677,80
Mean firm size	67,86	70,28	77,63	34,96	34,91	34,80	45,71	46,51	48,05
Standard Deviation	54,26	53,77	52,68	17,63	17,56	17,43	7,82	7,33	6,36
Mean compensation cost	1089,71	1115,24	1208,66	857,26	851,57	841,07	639,74	645,44	656,28
Standard Deviation	1023,52	1002,69	965,37	524,57	519,81	511,02	145,96	135,62	115,93
Mean gross wage offer	7,00	6,96	6,93	11,11	11,06	10,96	6,60	6,55	6,47
Standard Deviation	0,82	0,80	0,73	1,14	1,13	1,11	0,38	0,34	0,29
Average Added Value	1552,53	1517,47	1510,12	1039,37	985,55	892,28	864,23	832,54	776,41
Standard Deviation	1491,30	1394,56	1230,76	636,25	601,82	542,34	203,91	180,85	141,71
Number of firms	2278,31	2183,23	1932,87	4854,34	4859,72	4869,59	1360,10	1298,78	1177,20
Total output	3,714E+06	3,644E+06	3,503E+06	5,298E+06	5,268E+06	5,214E+06	1,234E+06	1,189E+06	1,097E+06
Taxes	1,341E+06	1,350E+06	1,356E+06	2,203E+06	2,211E+06	2,224E+06	4,692E+05	4,638E+05	4,464E+05

where τ_0 has been set to 1.1 so that simulations can be done with the same changes on the Food sector. The simulation results are presented in table 9. The effect of increasing τ_3 is similar to the effect of an increase of the tax rate on value added. Unemployment goes up, reservation wage and wage offers decrease. Smaller, less capitalistic firms are operating and inflate the number of active firms. The tax revenue increases except when the legal minimum wage is far above reservation wage (Hotels/Restaurants).

Figure 7 shows that the support of the distributions of capital, wage offers and earnings are translated to the left, except in the cases where the minimum wage is binding (Food and Hotels/Restaurants). Firms substitute labour to capital and for a given amount of capital the labour force increases and the capital/output ratio decreases. Yet, the labour share also falls and the profit rate increases.

5 Conclusion

In this paper, we have considered an extension of the *Burdett-Mortensen* equilibrium search model with two-sided search and an endogenous productivity distribution. We showed that a degenerate equilibrium wage distribution can never be a valid equilibrium solution if job posting is a costly process. Moreover, by allowing firms to freely enter the market and to freely choose their capital, we were able to generate more flexible wage offer distributions than in the model with homogenous firms. So doing, we generated a model of the labour economy sharing all the main features of a *Burdett-Mortensen* economy but flexible enough to be fitted on individual firm data on wages, labour force, capital assets and added-value. We thus obtained a reasonably good description of firm accounting data with a minimal number of exogenous parameters. An important limit of the exercise that we perform is yet that we rule out any worker ability heterogeneity.

Hoping that such a limitation is not enough to bias its predictions so as to make them the contrary to what they should be, we then used the model to study the effect of various policy reforms. An increase of the legal minimum wage, by increasing the minimal amount of capital necessary to operate, generates both unemployment and tax revenue losses. It makes employees (insiders) better off but raises the number of unemployed workers (outsiders) by evicting low productive firms from the market. Increasing the tax rate on wages, added-value or capital, in this model where it is not specified how the government revenue is spent, not surprisingly increases unemployment, decreases total output and may well decrease government revenue if the legal minimum wage is so far above reservation wage that it considerably reduces the possibility of downward wage adjustments. Only one reform had a positive effect: a reform which transfers a part of the tax burden from low wages to high wages (progressive wage taxation). Because labour costs are reduced for small firms, more firms can enter the market, unemployment is reduced, total output and government revenue increase. The fundamental reason why it is so is that a progressive tax reduces the monopsony power of large firms and therefore increases market efficiency.

The main interest of the model we develop in this paper is therefore to explain both why taxation is indeed a valid policy tool to increase market efficiency if employers have some monopsony power and why taxation can have the opposite effect to the desired one if it is uniformly applied to all firms without taking into account the counterproductive effect of a policy which would force too many low productive firms to retire from the market. We thus strongly support progressive tax policies against proportional tax and high legal minimum wage. ▼

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APPENDIX

Mathematical Appendix

PROOF OF PROPOSITION 4: We first proceed to show that when $p(e) = 0$ for all e, current operating product $\pi_k^*(w)$ is a continuous function of wage whether F is continuous or not. Let us define, for all $k \in K$, the function:

$$\widehat{\pi}_k(w,l_1,l_2) = \max_{e>0} q_k(el_1) - wel_1 - c(el_2).$$

The optimal number of posted job offers is $\widehat{e}_k(w,l_1,l_2)$ (or $\widehat{e}$ in short notation) which satisfies the first order condition:

$$\left[q_k'(\widehat{e}l_1) - w\right] l_1 - l_2 c'(\widehat{e}l_2) = 0.$$

The function $\widehat{\pi}_k(w,l_1,l_2)$ is continuous and differentiable, with continuous partial derivatives. Moreover, using the envelope theorem, we obtain:

$$\left.\frac{\partial \widehat{\pi}_k}{\partial l_1}\right|_{e=\widehat{e}_k(w,l_1,l_2)} = \widehat{e}\left[q_k'(\widehat{e}l_1) - w\right] = c'(\widehat{e}l_2)\frac{\widehat{e}l_2}{l_1},$$

and,

$$\left.\frac{\partial \widehat{\pi}_k}{\partial l_2}\right|_{e=e_k(w,l_1,l_2)} = -\widehat{e}c'(\widehat{e}l_2).$$

Now let w^* be a mass-point of F. There must exists some k such that w^* is profit maximizing for firms with capital k. We now show that one cannot choose $w > w^*$ arbitrarily close to w^*, yet yielding a profit $\widehat{\pi}_k(w,l_1(w),l_2(w))$ strictly greater than $\widehat{\pi}_k(w^*,l_1(w^*),l_2(w^*))$. Indeed,

$$\begin{aligned}
&\widehat{\pi}_k\left(w,l_1(w),l_2(w)\right) - \widehat{\pi}_k(w^*,l_1(w^*),l_2(w^*)) \\
&= (w - w^*)\frac{\partial \widehat{\pi}_k(w^{**},l_1^{**},l_2^{**})}{\partial w} + \left[l_1(w) - l_1\left(w^*\right)\right]\frac{\partial \widehat{\pi}_k(w^{**},l_1^{**},l_2^{**})}{\partial l_1} \\
&+ \left[l_2(w) - l_2\left(w^*\right)\right]\frac{\partial \widehat{\pi}_k(w^{**},l_1^{**},l_2^{**})}{\partial l_2},
\end{aligned}$$

where $w^{**} \in \left[w^*,w\right]$, $l_1^{**} \in \left[l_1\left(w^*\right),l_1(w)\right]$ and $l_2^{**} \in \left[l_2\left(w^*\right),l_2(w)\right]$ follow from the mean value theorem. Now, consider the sum of the last two terms of the sum in the right hand side of the last equality. It is equal to $\widehat{e}^{**}c'(\widehat{e}^{**}l_2^{**})$ (with $\widehat{e}^{**} = \widehat{e}_k(w^{**},l_1^{**},l_2^{**})$) times:

$$\left[l_1(w) - l_1\left(w^*\right)\right]\frac{l_2^{**}}{l_1^{**}} - \left[l_2(w) - l_2\left(w^*\right)\right].$$

Observe that when w tends to w^* from above, $l_1(w) - l_1\left(w^*\right)$ tends to:

$$\frac{A}{\left[1+\kappa_1 E\overline{F}(w^*)\right]^2} - \frac{A}{\left[1+\kappa_1 E\overline{F}(w^*)\right]\left[1+\kappa_1 E\overline{F}(w^{*-})\right]}$$

and that $\frac{l_2^{**}}{l_1^{**}}$ tends to:

$$\delta + \lambda_1 E\overline{F}(w^*)$$

and $l_2(w) - l_2\left(w^*\right)$ tends to:

$$\frac{\delta A}{1+\kappa_1 E\overline{F}(w^*)} - \frac{\delta A}{1+\kappa_1 E\overline{F}(w^{*-})}.$$

It thus follows that when w tends to w^* from above, then $\widehat{\pi}_k(w,l_1(w),l_2(w))$ must tend to $\widehat{\pi}_k(w^*,l_1(w^*),l_2(w^*))$. There is therefore no way of increasing profit for a fraction of firms offering wage w^* by slightly increasing the wage offer.
Now, let us add to hiring costs an additive component $p(e)$, continuously differentiable with $p'(e) > 0$, which represents the cost of posting e job offers (advertising costs for example). The first order condition for $\widehat{e}$ becomes:

$$\left[q_k'(\widehat{e}l_1) - w\right]l_1 - l_2 c'(\widehat{e}l_2) - p'(\widehat{e}) = 0,$$

and,

$$\left.\frac{\partial \widehat{\pi}_k}{\partial l_1}\right|_{e=\widehat{e}_k(w,l_1,l_2)} = \widehat{e}\left[q_k'(\widehat{e}l_1) - w\right] = \frac{\widehat{e}l_2 c'(\widehat{e}l_2) + p'(\widehat{e})}{l_1}.$$

It is then straightforward to verify that $\widehat{\pi}_k(w,l_1(w),l_2(w)) - \widehat{\pi}_k(w^*, l_1(w^*),l_2(w^*))$ does no go to zero when w goes to w^* from above. There is a remainder term which is equal to:

$$\left[l_1(w) - l_1\left(w^*\right)\right]\frac{p'(\widehat{e}_k(w^{**},l_1^{**},l_2^{**}))}{l_1^{**}} > 0$$

and which does not vanish when w goes to w^* since function l_1 is right-discontinuous at any mass point of F.

Third, we show that the support of F is connected or there must exist mass points in the distribution. Let $w_1 > w_2$ be such that (w_1,e_1) and (w_2,e_2) are profit maximizing strategies of two firms. Suppose that the interval $]w_1, w_2[$ is not in the support of F. If $F(w_1) = F(w_2)$ then, $l_1(w_1) = l_1(w_2)$ and $l_2(w_1) = l_2(w_2)$. Let π_1 be the profit function of the first firm. Then,

$$\pi_1(w_1,e_1) < \pi_1(w_2,e_1),$$

which contradicts the fact that (w_1,e_1) is profit maximizing for firm 1. Therefore, it must be that $F(w_2) > F(w_1)$. Note that the argument also applies to the case where (w_1,e_1) and (w_2,e_2) are profit maximizing strategies of the same firm. It follows that the support of the wage offer distribution must be an interval if F is continuous and that a right-neighbourhood of w must not be in the support of F is w is a mass point of F.

Proof of Proposition 6: Let $w_1 \in \mathcal{X}_1$ and $w_0 \in \mathcal{X}_0$. First, $\pi_1^*(w_1) \geqslant \pi_1^*(w_0)$ because w_1 is profit maximizing for type-1 firms, and $\pi_1^*(w_0) = \pi_1[w_0, e_1(w_0)] \geqslant \pi_1[w_0, e_0(w_0)]$ because $e_1(w_0)$ maximizes $\pi_1(w_0,e)$. Then,

$$\begin{aligned}\pi_1[w_0,e_0(w_0)] &= q_1[e_0(w_0)l_1(w_0)] - w_0e_0(w_0)l_1(w_0) - c(e_0(w_0)l_2(w_0))\\ &> q_0[e_0(w_0)l_1(w_0)] - w_0e_0(w_0)l_1(w_0) - c(e_0(w_0)l_2(w_0))\\ &= \pi_0[w_0,e_0(w_0)] = \pi_0^*(w_0)\end{aligned}$$

because $q_0(l) < q_1(l)$ for all l. Finally, $\pi_0^*(w_0) \geqslant \pi_0^*(w_1)$ because w_0 is profit maximizing for type-0 firms. We have thus shown that:

$$\pi_1^*(w_1) \geqslant \pi_1^*(w_0) > \pi_0^*(w_0) \geqslant \pi_0^*(w_1),$$

which implies that:

$$\pi_1^*(w_1) - \pi_0^*(w_1) \geqslant \pi_1^*(w_0) - \pi_0^*(w_0). \tag{A1}$$

For any given w, we now proceed to show that under assumption 3, $e_1(w) > e_0(w)$. There exists an increasing, concave function $\psi : \mathbb{R}_+ \to \mathbb{R}_+$ such that, for all k and w, $\pi_k[w, \ (u)]$, $k = 0, 1$, is a strictly concave function of u. Let u_k be the unique solution to the first-order condition:

$$\psi'(u_k)\frac{\partial \pi_k\left[w,\psi(u_k)\right]}{\partial e} = 0, ; k = 0,1,$$

with,

$$\frac{\partial \pi_k[w,e]}{\partial e} = l_1(w)q_k'(el_1(w)) - wl_1(w) - l_2(w)c'(el_2(w)).$$

The optimal hiring effort (conditional on k and w) is then $e_k(w) = \psi(u_k)$. Since $q_1'(l) > q_0'(l)$ for all l,

$$\begin{aligned} 0 &= \psi'(u_1)\frac{\partial \pi_1 [w,\psi(u_1)]}{\partial e} \\ &> \psi'(u_1)\frac{\partial \pi_0 [w,\psi(u_1)]}{\partial e} \\ &= \psi'(u_1)\frac{\partial \pi_0 [w,\psi(u_1)]}{\partial e} - \psi'(u_0)\frac{\partial \pi_0 [w,\psi(u_0)]}{\partial e}. \end{aligned}$$

The concavity of $u \mapsto \pi_k [w,\psi(u)]$ implies that $u \mapsto \psi'(u)\frac{\partial \pi_k \left[w,\psi(u)\right]}{\partial e}$ is decreasing. Hence, it must be that $u_1 > u_0$ and thus $e_1(w) > e_0(w)$ since ψ is increasing.

We then show that $\pi_1^* (w) - \pi_0^* (w)$ is a non decreasing function of w if c is convex and non increasing if c is concave. Proposition 4 implies that F is a continuous, differentiable function of w. Hence, so are $l_1(w)$ and $\pi_i^*(w)$. Moreover, the envelope theorem implies that:

$$\begin{aligned} \frac{d\pi_k^*(w)}{dw} &= e_k(w)\left\{l_1'(w)\left[q_k'(e_k(w)l_1(w)) - w\right] - l_1(w) \right. \\ &\qquad \left. - l_2'(w)\, c'(e_k(w)l_2(w))\right\} \\ &= e_k(w)\left\{\left[l_1'(w)\frac{l_2(w)}{l_1(w)} - l_2'(w)\right]c'(e_k(w)l_2(w)) - l_1(w)\right\} \end{aligned}$$

for all w, where the last equality follows from the first-order condition defining $e_k(w)$: $\frac{\partial \pi_k[w,e_k(w)]}{\partial e} = 0$. Hence, for any w_1 in the interior of the set $\mathcal{X}_1$ of profit-maximizing wages for firm 1, that is not profit maximizing of firm 0 ($w_1 \notin \mathcal{X}_0$),

$$\begin{aligned} \frac{d\pi_1^*(w_1)}{dw} - \frac{d\pi_0^*(w_1)}{dw} &= 0 - \frac{d\pi_0^*(w_1)}{dw} \\ &= -e_0(w_1)l_1(w_1)\left[\frac{c'(e_0(w_1)l_2(w_1))}{c'(e_1(w_1)l_2(w_1))} - 1\right]. \end{aligned}$$

And, for any w_0 in the interior of the set of profit-maximizing wages for firm 0 that is not profit maximizing of firm 1 ($w_0 \in \mathcal{X}_0,\ w_0 \notin \mathcal{X}_1$),

$$\begin{aligned} \frac{d\pi_1^*(w_0)}{dw} - \frac{d\pi_0^*(w_0)}{dw} &= \frac{d\pi_1^*(w_0)}{dw} - 0 \\ &= e_1(w_0)l_1(w_0)\left[\frac{c'(e_1(w_0)l_2(w_0))}{c'(e_0(w_0)l_2(w_0))} - 1\right]. \end{aligned}$$

Moreover,

$$\begin{aligned} \frac{d\pi_1^*(w_1)}{dw} - \frac{d\pi_0^*(w_1)}{dw} &= \frac{d\pi_1^*(w_0)}{dw} - \frac{d\pi_0^*(w_0)}{dw} \\ &= 0 \end{aligned}$$

if both w_1 and w_0 are interior optimal wage offers of both firms.

The monotonicity of $\pi_1^*(w) - \pi_0^*(w)$ on the interior set of $\mathcal{X}_0 \cup \mathcal{X}_1$ is therefore the same as that of c' since we have shown that $e_1(w) > e_0(w)$ for all w.
The monotonicity of $\pi_1^*(w) - \pi_0^*(w)$, together with inequality (1) above, show that (1) if $c(\varphi)$ is convex then $w_1 \geqslant w_0$, and (2) if $c(\varphi)$ is concave then $w_1 \leqslant w_0$. Moreover, if $c(\varphi)$ is linear, then it must be that $\pi_1^*(w_1) = \pi_1^*(w_0)$ and $\pi_0^*(w_0) = \pi_0^*(w_1)$; that is: w_1 and w_0 are profit maximising wages for both firms.

PROOF OF PROPOSITION 7: For all integers $n > 0$, let K_n be the subset of values of $k \in \text{supp}(\Gamma)$ for which $\mathcal{X}_k$ is of Lebesgue measure greater than $\frac{1}{n}$. Since the sets $\mathcal{X}_k$ do not intersect, it follows that K_n is a countable set. Any value of k for which $\mathcal{X}_k$ is not a singleton, necessarily belongs to $\bigcup_{n>0} K_n$. Then, the σ-additivity of probability measure Γ and the fact that it is continuous, together imply that $\bigcup_{n>0} K_n$ has Γ-probability zero.

PROOF OF PROPOSITION 8: We first show the following result.

LEMMA 14: *The optimal employment $l[w, e_k(w)] = e_k(w) l_1(w)$ is increasing (resp. decreasing, constant) on $\mathcal{X}_k$ for all k, increasing if (and only if) the hiring cost function $c(\varphi)$ is convex (resp. concave, constant).*

PROOF: Consider the set of firms with capital k. Since they are all identical, they must realize the same profit at the equilibrium. Hence, for all $w \in \overset{\circ}{\mathcal{X}}_k$ (if ever the interior of $\overset{\circ}{\mathcal{X}}_k$ is non empty), $\frac{d\pi_k^*(w)}{dw} = 0$, that is:

(A2)
$$l_1'(w)\left[q_k'(e_k(w)l_1(w)) - w\right] - l_1(w) - l_2'(w)\, c'(e_k(w)l_2(w)) = 0.$$

Moreover, conditional on k and w, optimal hiring effort is $e_k(w)$ such that

$$l_1(w)\left[q_k'(e_k(w)l_1(w)) - w\right] - l_2(w)\, c'(e_k(w)l_2(w)) = 0. \quad \text{(A3)}$$

It follows from differentiation that:

$$\begin{aligned} \frac{dl(w, e_k(w))}{dw} &= e_k'(w)l_1(w) + e_k(w)l_1'(w) \\ &= -\frac{l_2(w)^2 c''(e_k(w)l_2(w))}{l_1(w)^2 q_k''(e_k(w)l_1(w)) - l_2(w)^2 c''(e_k(w)l_2(w))} \\ &\quad \times \frac{\kappa_1 E f(w)}{1 + \kappa_1 E \overline{F}(w)} l(w, e_k(w)) \end{aligned}$$

has the sign of $c''(e_k(w)l_2(w))$. Hence, employment is increasing on $\overset{\circ}{\mathcal{X}}_k$ if $c(\varphi)$ is convex and is decreasing if it is concave. Moreover, $l(w, e_k(w))$ is constant on $\overset{\circ}{\mathcal{X}}_k$ if $c(\varphi)$ is linear.

We then proceed to show by contradiction that there is no equilibrium distribution of capital across firms with a mass point of capital at k^*. If it is not so, if there is a mass of firms with capital k^*, we show that any of these firms have interest to deviate from this choice of capital if all others behave according to the postulated equilibrium.

Let $\left[\underline{w}^*,\overline{w}^*\right]$ be the support of F_{k^*}. Note first that for $\left[\underline{w}^*,\overline{w}^*\right]$ to be profit maximizing for firms with capital k^*, then one must have:

$$\forall w \in \left[\underline{w}^*,\overline{w}^*\right],\ \pi^*_{k^*}(w) = \pi^*_{k^*}(\overline{w}^*).$$

Let $\underline{k}$ and $\overline{k}$ be such that,

$$\begin{aligned}\underline{k} &= \arg\max_k \pi^*_k(\underline{w}^*) - r(k) \quad \text{and} \\ \overline{k} &= \arg\max_k \pi^*_k(\overline{w}^*) - r(k).\end{aligned}$$

or equivalently:

$$\begin{aligned}\underline{k} &= \arg\max_k q\left(k,l\left[\overline{w}^*,e_k(\overline{w}^*)\right]\right) - r(k) \quad \text{and} \\ \overline{k} &= \arg\max_k q\left(k,l\left[\underline{w}^*,e_k(\underline{w}^*)\right]\right) - r(k).\end{aligned}$$

If c is strictly convex, then the previous lemma shows that $l\left[\overline{w}^*,e_k(\overline{w}^*)\right] > l\left[w,e_k(w)\right] > l\left[\underline{w}^*,e_k(\underline{w}^*)\right]$ for all $w \in \left]\underline{w}^*,\overline{w}^*\right[$. Second, by assumption 3 (part iii), any $k(l)$ such that:

$$k(l) = \arg\max_k q\ (k,l) - r(k)$$

equivalently satisfies the first order condition:

$$\frac{\partial q(k,l)}{\partial k} - r'(k) = 0$$

and is strictly increasing since $\frac{\partial^2 q(k,l)}{\partial k \partial l}$ is positive (part (iv) of assumption 3). It follows that $\underline{k} < \overline{k}$.

Now, if k^* is different from both $\underline{k}$ and $\overline{k}$ then any of these two values of capital yields greater profit than k^*. If $k^* = \overline{k}$ then,

$$\pi^*_{\underline{k}}(\underline{w}^*) - r(\underline{k}) > \pi^*_{k^*}(\underline{w}^*) - r(k^*),$$

and $\underline{k}$ yields bigger profit than $\overline{k}$. A similar argument shows that $\overline{k}$ would do better than $k^* = \underline{k}$.

The same argument applies to the case of concave hiring costs with $l\left[w,e_k(w)\right]$ decreasing instead of increasing.

PROOF OF LEMMA 9: With linear hiring costs, the profit function becomes:

$$\pi_k(w,e) = q_k\left[el_1(w)\right] - \left[w + c\frac{l_2(w)}{l_1(w)}\right]el_1(w).$$

The optimal labour force of a firm offering wage w is then:

$$\begin{aligned} l(w, e_k(w)) &= e_k(w) l_1(w) \\ &= (q_k')^{-1}\left(w + c\frac{l_2(w)}{l_1(w)}\right). \end{aligned}$$

Now consider the concentrated profit function $\pi_k^*(w) = \pi_k(w, e_k(w))$.

$$\pi_k(w, e_k(w)) = \Pi_k\left(w + c\frac{l_2(w)}{l_1(w)}\right),$$

with,

$$\Pi_k(x) = q_k \circ (q_k')^{-1}(x) - x(q_k')^{-1}(x).$$

We have that $\Pi_k'(x) = -(q_k')^{-1}(x) < 0$. It thus follows that profit maximizing wages are those wages which minimize function $w + c\frac{l_2(w)}{l_1(w)}$.

PROOF OF PROPOSITION 10: It follows from lemma 9 that, for any firm, whatever its characteristics, profit maximizing wages are those wage values greater than w_R which minimize

$$w + c\frac{l_2(w)}{l_1(w)}.$$

An equilibrium is a distribution F such that each element of its support is profit maximizing of some firm:

$$\text{supp}(F) \subset \arg\min_{w \geqslant w_R}\left(w + c\frac{l_2(w)}{l_1(w)}\right).$$

All $w \in \text{supp}(F)$ must yield the same marginal labour cost. Hence,

$$\begin{aligned} w + c\frac{l_2(w)}{l_1(w)} &= w + c\left[\delta + \lambda_1 E\overline{F}(w)\right] \\ &= \underline{w} + c\left[\delta + \lambda_1 E\right], \end{aligned}$$

where $\underline{w}$ is the minimal wage offer ($\overline{F}(w) = 1$) which minimizes marginal labour cost when it is equal to $\max(w_R, w_{\min})$.

Now, let us consider the firm which is offering the minimum wage $\underline{w}$. For this firm:

$$\underline{w} + \frac{c}{l_1(\underline{w})} = \underline{w} + \frac{c(1 + \kappa_1 E)^2}{A}$$

is minimized, under the constraint $\underline{w} \geqslant w_R$, at the corner, *ie*, when $\underline{w} = w_R$. The maximum wage $\overline{w}$ is then defined as the wage value such that $F(\overline{w}) = 1$, and verifies equation (15).

PROOF OF PROPOSITION 12: The first order conditions of firms' profit maximization programme now become:

$$\begin{cases} l_1'(w)\left[q_k'\left(el_1(w)\right) - w\right] - l_1(w) - l_2(w)c'(el_2(w)) = 0, \\ l_1(w)\left[q_k'\left(el_1(w)\right) - w\right] - l_2(w)c'(el_2(w)) = 0, \end{cases}$$

Substituting out $q_k'\left(el_1(w)\right) - w$ from these two equations yields:

$$\kappa_1 E f(w) c'\left(e_k(w) l_2\left(w\right)\right) = 1.$$

With,

$$f(w)w'(k) = z(k)$$

it thus follows that optimal hiring effort, as a function of k, is then such that:

$$c'\left(\frac{Ae(k)}{\delta + \lambda_1 E\bar{Z}\left(k\right)}\right) = \frac{w'(k)}{\kappa_1 E z(k)}.$$

And the first first-order condition can also be rewritten as:

$$q_k'\left(\frac{Ae(k)}{\left[1 + \kappa_1 E\overline{Z}(k)\right]^2}\right) - w(k) - \left[\delta + \lambda_1 E\overline{Z}(k)\right]\frac{w'(k)}{\kappa_1 E z(k)} = 0.$$

PROOF OF PROPOSITION 13: Since all firms are *ex ante* identical, any equilibrium must be such that all firms $k \in K = \left[\underline{k}, \overline{k}\right]$ make the same profit:

(A4) $$v = q\left[k, l(k)\right] - w(k)l(k) - c\left[e(k)l_2(w(k))\right] - r(k),$$

where,

(A5) $$l(k) = e(k)l_1\left[w(k)\right]$$

and where wage $w(k)$ and recruiting effort $e(k)$ are chosen so as to maximize value added:

$$\pi_k(w,e) = q\left[k, el_1(w)\right] - wel_1(w) - c(\varphi(w,e)).$$

First, the envelope theorem implies that:

$$\frac{d\pi_k\left[k,l(k)\right]}{dk} = \frac{\partial q\left[k,l(k)\right]}{\partial k}$$

and differentiating equation (A4) with respect to k yields:

$$\text{(A6)} \qquad 0 = \frac{\partial q\,[k, l(k)]}{\partial k} - r'(k).$$

Since $\frac{\partial q[k,l]}{\partial k \partial l} > 0$ for all couples (k,l), the implicit function theorem implies that there exists at most one increasing function $\psi_1(k)$ such that $l(k) = \psi_1(k)$ for all k.

Moreover, consider the first order condition for e:

$$\frac{\partial q\,[k, el_1(w)]}{\partial l} l_1(w) - wl_1(w) - l_2(w)c'(el_2(w)) = 0$$

evaluated at $w = w(k)$ and $e = e(k)$. Multiplying by $e(k)$ both sides of this equation yields the following equivalent expression:

$$\text{(A7)} \qquad \frac{\partial q\left[k, \psi_1(k)\right]}{\partial l} \psi_1(k) - w(k)\psi_1(k) - \varphi(k)c'(\varphi(w)) = 0,$$

with $\varphi(k) = e(k)l_2(w(k))$. Then, substitute out $w(k)\psi_1(k)$ from equations (A4) and (A7) to obtain:

$$\text{(A8)} \qquad \begin{aligned} & q\left[k, \psi_1(k)\right] - \frac{\partial q\left[k, \psi_1(k)\right]}{\partial l} \psi_1(k) - r(k) - v \\ & = c(\varphi(w)) - \varphi(k)c'(\varphi(w)) \\ & = d\,[\varphi(k)] \quad \text{(say)}. \end{aligned}$$

Note that $d(\varphi) = c(\varphi) - \varphi c'(\varphi)$ is a strictly monotonous function, provided that c is strictly convex or concave. It follows that there exists at most one solution $\psi_2(k)$ for $\varphi(k)$ to equation (A8).

Finally, the wage offer $w(k)$ is then obtained by solving for $w(k)$ equation (A4):

$$\begin{aligned} w(k) &= \frac{q\left[k, \psi_1(k)\right] - c\left[\psi_2(k)\right] - r(k) - v}{\psi_1(k)} \\ &= \psi_3(k) \quad \text{(say)}, \end{aligned}$$

and profit v is such that,

$$\text{(A9)} \qquad \begin{aligned} \underline{w} &= \max\{w_R, w_{\min}\} \\ &= \frac{q\left[\underline{k}, \psi_1(\underline{k})\right] - c\left[\psi_2(\underline{k})\right] - r(\underline{k}) - v}{\psi_1(\underline{k})}. \end{aligned}$$

We now proceed to derive the sampling distribution and the density of capital. First,

$$\begin{aligned} l(k) &= e(k)l_1\,[w(k)] \\ &= \varphi(k)\frac{l_1\,[w(k)]}{l_2\,[w(k)]} \\ &= \frac{\varphi(k)}{\delta + \lambda_1 E\overline{Z}(k)}. \end{aligned}$$

Hence,

$$\begin{aligned} \delta + \lambda_1 E \overline{Z}(k) &= \frac{\psi_2(k)}{\psi_1(k)} \\ &= \psi_4(k) \quad \text{(say)}. \end{aligned}$$

and the support bounds $\underline{k}$ and $\overline{k}$ are such that $\overline{Z}(\underline{k}) = 1$ and $\overline{Z}(\overline{k}) = 0$, *ie,*

(A10) $$\delta + \lambda_1 E = \psi_4(\underline{k}),$$

(A11) $$\delta = \psi_4(\overline{k}).$$

The distribution of capital across active firms follows from equation:

$$z(k) = E^{-1} N e(k) \gamma(k)$$

ie,

$$\begin{aligned} N\gamma(k) &= \frac{A}{\lambda_1 \psi_2(k)} \left[\frac{\psi_1'(k)}{\psi_1(k)} - \frac{\psi_2'(k)}{\psi_2(k)} \right] \\ &= \frac{A}{\lambda_1} \psi_5(k) \quad \text{(say)}. \end{aligned}$$

The overall recruiting effort E (or equivalently $A = \kappa_0 \frac{1+\kappa_1 E}{1+\kappa_0 E}$) must then be such that $\gamma(k)$ defines a proper density:

$$\frac{A}{\lambda_1} \int_{\underline{k}}^{\overline{k}} \psi_5(k) dk = N$$

and the measure of active firms is such that equilibrium profit v is nil (v depends on N via $\underline{k}$).