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Using Firm Data to Assess the Performance of Equilibrium Search Models of the Labor Market

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ABSTRACT. – Equilibrium search models are useful tools for the evaluation of labor market policies. Recently developed equilibrium search models of the labor market are able to fit the wage distribution perfectly with longitudinal labor supply data, by estimating an appropriate distribution of labor productivity across firms. This paper formally compares such structural estimates to their directly observed counterparts in firm data. More generally, we investigate the extent to which these models are able to explain the observed distributions of wages, productivities and firm sizes across firms, as well as the extent to which they are able to explain the observed relationships between these variables across firms. The parameters that capture search frictions are estimated with worker data that are matched to the firm data.

Usage des données d'entreprises pour évaluer les performances de modèles d'équilibre de recherche d'emploi

RÉSUMÉ. – Les modèles de recherche d'emploi d'équilibre sont des outils utiles pour évaluer les politiques de l'emploi. Des modèles récemment développés ont permis de relier précisément les distributions de salaires à des données longitudinales sur les offres d'emploi en estimant une distribution appropriée de la productivité du travail parmi les entreprises. Cet article compare formellement ces estimations structurelles à leurs contreparties empiriques directement observées dans les données d'entreprises. Plus généralement, nous étudions l'aptitude de ces modèles à expliquer les distributions observées de salaires, de productivités et de tailles parmi les entreprises ainsi que les relations observées qui existent entre ces variables. Les paramètres qui captent les frictions liées à la recherche sont estimés sur des données de salariés qui sont appariées aux données d'entreprises.

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1 Introduction

During the past two decades, equilibrium search models have become important tools for the analysis of labor market issues. These models deal with wage determination in the presence of informational frictions or search frictions (see MORTENSEN and PISSARIDES [1999], and VAN DEN BERG [1999], for overviews of the theoretical and empirical literature, respectively). As such they explain wages, individual transitions between different labor market states, the durations spent in those states, and, last but not least, aggregate numbers like the unemployment rate and worker reallocation rates (RIDDER and VAN DEN BERG [2000]). Equilibrium search models are particularly useful to study policy changes, like changes in the minimum wage or the unemployment benefits level. Structural estimates of these models enables policy inference that is not subject to the *Lucas critique*.

Unfortunately, most existing equilibrium search models have difficulty explaining the cross-sectional distribution of wage data in a given labor market (see the above surveys; see also the discussions in ECKSTEIN and WOLPIN [1990], and VAN DEN BERG and RIDDER [1998]).[1] Recently, however, BONTEMPS, ROBIN and VAN DEN BERG [2000] (BRVdB) developed and estimated an equilibrium search model that is able to provide a perfect fit to the wage distribution. The model extends the BURDETT and MORTENSEN [1998] model with homogeneous workers and firms by allowing firms acting in a given market to have different labor productivities (or revenue products). Equilibrium search models are typically estimated with longitudinal labor supply data, covering unemployment and job durations, transitions from one job to another or to unemployment, and wages of employed workers. With such data, firm heterogeneity in the BRVdB model is unobserved heterogeneity, and BRVdB show that a large set of wage distributions can be supported by an appropriate underlying continuous productivity distribution. The set of wage distributions that can be generated is a subset of the set of all distributions of nonnnegative random variables. This means that the empirical wage distribution can be used for a nonparametric specification test. However, if the test accepts the specification then there are basically no overidentifying restrictions on the wage distribution anymore (unless the data contain sufficiently rich additional wage data, for example on wages that are accepted by employed workers moving to another job in due course).

The BRVdB model has become rather popular in the literature. Moreover, most other recent empirical analyses also adopt heterogeneity of firms' productivity in order to obtain an acceptable fit to the wage data (see *eg*, BOWLUS, KIEFER and NEUMANN [2001]). In this paper we examine the performance of the BRVdB model when confronted to *firm data*. In firm data, the firm's labor productivity is observed, and the distribution of these observations can be compared to the estimated distribution from more traditional data

1. The literature on equilibrium search models uses different words for the 'cross-sectional distribution of wage data', such as the 'distribution of earned wages' and 'wage data from employed workers'. We will also use these in the remainder of this paper.

(on wages and individual labor market spells). More generally, we provide a comprehensive formal investigation of the extent to which the BRVdB model is able to explain the observed distributions of wages, productivities and firm sizes across firms, as well as the extent to which the model is able to explain the observed relationships between these variables across firms.

This paper thus provides the first formal analysis of the performance of such models when confronted to firm data.[2] Given the lack of overidentifying information in the data that are traditionally used, firm data may be useful to assess the performance of these models. A close inspection of the fit to firm data may generate research questions for future theoretical research. For example, the theoretical relation between the productivity and the wage reflects the first-order condition of the optimization problem of the firm, given the behavior of all other firms and all workers. If the observed relation differs from the predicted relation in a certain data interval, then this may indicate that important aspects of the optimization problem of the corresponding firms are ignored in the model.

We use data from Dutch manufacturing firms. Since we are not able to describe our results for all different industries, we restrict ourselves to three different industries: *(1)* textiles, *(2)* publishing and printing, and *(3)* electrical machinery. These industries are quite different from each other with respect to the product markets. Although the differences in product market do not necessarily imply that the labor markets of these industries are different, we expect this to be the case in the remainder of this paper. In addition, we find that the results do not differ very much for the industries that we do not describe in this paper. Therefore, these industries may also represent a lot of the aspects of other industries not included in our analysis. We use two surveys. The Production Survey covers the firm data, and these are matched to the Wage and Employment Survey, which contains information on job mobility and wages of individual workers. Analogously to BRVdB, the parameters that capture search frictions are estimated with the latter type of data. These parameters serve as inputs in the main empirical analyses. We show that these parameters suffice to characterize the distribution of firm sizes. BRVdB show that the frictional parameters together with data on the wage distribution enable one to back out the underlying productivity distribution. More precisely, if the frictional parameters and one of these distributions are known, then for each given wage (productivity) level, the corresponding productivity (wage) level and firm size can be calculated. In this paper, we use all of this to compare predictions to observations.

The paper is organized as follows. The next section summarizes the model framework as introduced by BRVdB. Section 3 discusses the data used in this paper. The empirical implementation is discussed in section 4. Results follow in section 5. Finally, we give our conclusions in section 6.

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2. It should be noted that BRVdB make a graphical comparison between the distributions of wages, productivities and firm sizes as estimated from labor force survey data on the one hand, and the observed distribution of these variables in firm data on the other hand. They also graphically compare the corresponding relations between the variables. ROBIN and ROUX [1998] and POSTEL-VINAY and ROBIN [2000] estimate particular equilibrium search models with joined data sets on workers and firms.

2 Equilibrium Search Theory

Since the model we use in this paper is already discussed in detail by BRVdB, the exposition of this model will be brief.

Workers seek to maximize their expected steady-state discounted future income. The opportunity cost of employment is denoted by b and is assumed to be constant across individuals. Job offers arrive according to a *Poisson* process with a constant rate λ_0 when unemployed. When employed, job offers arrive at the constant rate λ_1. Whenever a job offer arrives, the decision has to be made whether to accept it or to reject it and search further for a better offer. Layoffs arrive at the constant rate δ. The distribution of wage offers F is independent of the current state of the job searcher (employed or unemployed).[3] Let $\text{supp}(F)$ denote the support of F. We denote $\underline{w} = \inf[\text{supp}(F)]$ and $\overline{w} = \sup[\text{supp}(F)]$. The legal minimum wage, not necessarily positive, is denoted by w_{min}. Necessarily then, $\underline{w} \geqslant w_{min}$. A well-known result is that the optimal strategy when unemployed is to accept any wage offer w greater or equal to the reservation wage (BURDETT and MORTENSEN [1998]). For convenience, we assume the reservation wage to be lower than the minimum wage, which means that any wage offer is acceptable for the unemployed. This assumption seems to be reasonable in light of other studies on the Dutch labor market. Employed workers accept any job that pays them above their present wage.

The measure of employed workers is equal to M and the measure of firms is equal to N.[4] Since the arrival of job offers for an unemployed worker as well as the job separations follow a *Poisson* process, the steady-state distribution of being unemployed is a *Bernoulli* distribution with parameter U, where U is equal to:

$$U = \frac{\delta}{\delta + \lambda_0}$$

For the remainder of this paper, we refer to U as the (expected) unemployment rate.

We make the additional assumption that the distribution of offered wages (denoted before by F) is the same as the distribution of wage costs among firms. We use the convention $\overline{F} = 1 - F$. This implies that, whatever their size, the number of vacancies is the same across firms. In an early discussion of equilibrium search models, BURDETT and VISHWANATH [1988] introduce a model in which the meeting rate is dependent on firm size. They find that the

3. The literature on equilibrium search models uses different words for the ‘distribution of wage offers’, such as ‘firm wages’, ‘wage of the firm’ and ‘wages across firms’. We will also use these in the remainder of this paper.

4. The values of M and N are exogenous and independent of the labor market frictions. This implies that we do not allow for free entry of firms in this model. In addition, the allocation of workers across sectors is exogenous. PISSARIDES [1984] describes an equilibrium in which there is free entry of both firms and workers and PISSARIDES [1990] describes a model with free entry of firms and a fixed number of workers.

wages offered to workers follows the same distribution as the distribution of wages among a cross-section of these workers (denoted before by G). Contrary to our analysis, the distribution of wage costs differs from the wage offer distribution. ROBIN and ROUX [1998] generalize the analysis of BURDETT and VISHWANATH [1988] by allowing for endogenous search effort of firms.

Let $G(w)$ be the fraction of individuals with a wage lower than or equal to w. In a steady-state equilibrium, the flow of layoffs in an interval $(t,t+dt]$, $\delta(M-U)G(w)dt$, plus the flow of individuals who find a better job, $\lambda_1\overline{F}(w)(M-U)G(w)dt$, is equal to the flow of unemployed individuals accepting a wage smaller than or equal to w, $\lambda_0 F(w)dt$, if $w \geqslant w_{min}$. Thus,

$$G(w) = \frac{F(w)}{1+\kappa_1\overline{F}(w)} \tag{1}$$

where $\kappa_1 = \lambda_1/\delta$ denotes the expected number of job offers during a spell of employment. Likewise, we use the notation $\kappa_0 = \lambda_0/\delta$. We assume that workers draw job offers by randomly picking firms using a uniform sampling scheme. Consider a firm that offers only one wage w. The rate at which workers flow into this firm is equal to $(\delta+\lambda_1 G(w))\frac{M-U}{N}$. The rate in which workers flow out of the firm is equal to $\delta+\lambda_1\overline{F}(w)$. MORTENSEN [1999] shows that the steady-state distribution of workers employed in a firm is *Poisson* where the single parameter is equal to the inflow divided by the outflow. By using equation (1), we find:

$$l(w) = \frac{A}{(1+\kappa_1\overline{F}(w))^2}$$

where $A = \frac{1+\kappa_1}{1+\kappa_0}\kappa_0\frac{M}{N}$. This parameter can be interpreted as the steady-state number of workers employed within a firm, conditional on the wage being offered. We denote the number of individuals employed within a firm by L. The distribution of L conditional on the wage offer is given by:

$$\mathrm{P}(L \leqslant l|W=w) = \sum_{k=0}^{l}\frac{l(w)^k}{k!}e^{-l(w)} \tag{2}$$

Firms may differ in their labor productivity p, which is not dependent on the number of workers. The distribution of p of the participating firms is $\Gamma(p)$, with $\underline{p} \geqslant 0$ as the infimum point of its support and $\overline{p}$ as the supremum. We assume that p has a finite mean, *ie,* $\mathrm{E}_\Gamma(p) < \infty$. The assumption that the productivity level is a firm characteristic is maintained throughout the paper. Workers are homogenous in their productivity levels as well as their opportunity costs of employment.

We assume that firms seek to maximize their long-run profits, defined as:

$$\pi = \mathrm{E}_L\left(L(p-w)|W=w\right) = (p-w)l(w)$$

where the second step uses the result that the distribution of L conditional on $W = w$ is *Poisson* with parameter $l(w)$. The derivation of this equation is based on two implicit assumptions. First, it implies that firms are risk neutral since they are maximizing the profits of the stationary state distribution. Second, we use a linear production function. Therefore, although firms can have more or less workers than expected, this does not result in a change in the expectation of the stationary state distribution of the profits. This does not necessarily happen when the specification of the production function is not linear.

It follows that firms participating in the labor market must have marginal productivity levels that are at least as high as the mandatory minimum wage. This means that the productivity distribution is actually a truncated distribution of the distribution of productivity levels of firms that can potentially enter the market. Note that a firm that sets a wage w smaller than its productivity p is always willing to expand, and so will accept all job applicants and will always have a vacancy.

We make the assumption that $\Gamma(p)$ is continuous. BRVdB show that for this continuous case, the equilibrium strategies of firms are pure strategies: only one wage can be profit maximizing, given the firm type. We use the notation $w = K(p)$ for this profit maximizing wage, where K is an increasing and continuous function on $[\underline{p},\overline{p}]$. It thus follows that the distribution of wage offers can be derived from this mapping by the relationship $F(w) = \Gamma(K^{-1}(w))$.

The optimal $w = K(p)$, given p and F, follows from the first order conditions: by taking $\partial\pi(p,w)/\partial w = 0$, we obtain:

$$-(1+\kappa_1\overline{F}(w)) + 2\kappa_1 f(w)(p-w) = 0 \tag{3}$$

under the restriction that $w \geqslant w_{min}$, where $f(w)$ is the density associated with $F(w)$. This is an important equation, since it derives an implicit function of a firm's wage offer given the productivity and the distribution of wage offers. We use this assumption to test the restriction that high productivity firms offer higher wages. Firms with the lowest possible p (*ie*, with $\underline{p}$) offer a wage equal to w_{min}. BRVdB show that the equilibrium profits for firms with productivity levels p are equal to:

$$\begin{aligned}\Pi(p) &= \pi(p,K(p)) \\ &= \frac{M-U}{N}(1+\kappa_1)\left[\frac{\underline{p}-w_{min}}{(1+\kappa_1)^2} + \int_{w_{min}}^{p}\frac{dx}{(1+\kappa_1\overline{\Gamma}(x))^2}\right]\end{aligned} \tag{4}$$

From $\Pi(p) = (p-K(p))l(K(p))$, it follows that $K(p) = p - \Pi(p)/l(K(p))$. Thus, we find that the wage offer $w \equiv K(p)$ of a firm with productivity p equals:

$$K(p) = p - \left[1+\kappa_1\overline{\Gamma}(p)\right]^2 \int_{w_{min}}^{p}\frac{dx}{(1+\kappa_1\overline{\Gamma}(x))^2} \tag{5}$$

This equation is the central equation of the model. In our empirical implementation, this equation is used to non-parametrically estimate the wage offers from the distribution of productivity levels.

Clearly, job durations are exponentially distributed where the single parameter equals $\delta + \lambda_1 \overline{F}(w)$. This is explained intuitively as follows: the hazard from leaving the job can be decomposed into two components. One component is the hazard of being fired, which is equal to δ and the other is the hazard of a quit. The contribution of a quit to the total hazard is equal to the rate in which job offers arrive, multiplied by the probability that a job offer is acceptable (see, for further details, BURDETT and MORTENSEN [1998], and VAN DEN BERG and RIDDER [1998]). Hence, individual workers with higher wage levels stay longer in their job than those with lower wages. This result is used in the empirical implementation of the paper.

3 The Data Set

We use two different surveys of Statistics Netherlands for our research. These are the *Production Survey* (PS) and the *Wage and Employment Survey* (WES). The data sets are based on two consecutive years of observation: 1993 and 1994. Firms are legally obliged to respond and give sufficient answers to the different questions of these surveys when sampled.

In the *Production Survey*, all firms with 20 or more employees are surveyed and from the firms with less than 20 employees a sample is drawn. We use the following variables of this survey for our analysis

1. *Registration number of the firm.* This is a unique identifier, which makes it possible to match this data set with the Wage and Employment Survey.
2. *Sector classification of the firm based on the Standard Industry Classification (SIC).* Our sample contains firms in the manufacturing industry, who have 2-digit SIC numbers, ranging between 15 and 37. As stated in the introduction, we focus on three industries: the textiles industry (SIC number 17), the publishing industry (SIC number 22) and the electrical machinery industry (SIC number 31).
3. *Total employment in the firm.* This is the number of individuals who were working in the firm at the end of September of the year of observation.
4. *Total value added of the firm.* This is the total sales of the firm minus the monetary value of all purchases. Corrections are made for fluctuations in the stock of primary goods. The value is based on the accumulation over the whole year.
5. *Total wage costs of the firm.* This is the total of the wage bill of the firm. It includes taxes and social security payments for both employers and employees. The value is based on the accumulation over the whole year.
6. *Depreciation costs of the firm.* These are the depreciation costs as they appear in the firms' accounting systems. They are roughly equal to a fixed percentage of the historical price of the long lived assets. The value is based on the accumulation over the whole year.

We assume that the total wage costs of firms divided by total employment levels equal the wage offers of vacancies at firms. Hence, the distribution of F can be derived from these wage costs per worker. The productivity level is defined as the total value added of the firm divided by total employment.

We make three additional notes concerning the data set. First, the aggregate numbers of firms are yearly figures, while the number of employees is based on the number of workers within the firm at the end of September. This is a source of measurement error, since it implies that firms that have more than their average number of workers employed in September have relatively low wage costs and value added per worker. Second, we use the depreciation costs as a measure for the firms' capital stocks in our analysis. A drawback of the use of these costs is that they are based on the life span of capital goods used by accountants which may be much shorter than the actual life span of these goods. This leads to a wrong measure of the stock of capital goods. Nevertheless, we expect that firms with relatively high depreciation costs also have a high value of their capital stock. This is precisely what we need for our analysis. Third, our measures of the productivity levels and the wage costs per worker as well as the deprecation costs per worker are not corrected for part-time work. This has consequences in our analysis if the relative number of part-time jobs is very different among the firms in each of the particular industries we consider. The use of full-time equivalents would have been more convincing, but these are not available in the data set.

The *Wage and Employment Survey (WES)* is based on a two-stage sample design. Statistics Netherlands takes a sample of firms to ask questions about total employment of the firm in the first stage. The sample is not random but takes account of firm size.[5] Questions about individual workers employed in firms are asked in the second stage. The sample of firms in this second stage is a subsample of the ultimate sample of the first stage.[6] The number of employees being sampled within the firm depends on firm size as well. The actual percentage decreases in the number of employees. We take account of the sampling device in our empirical implementation. The total number of individuals in the WES is about 78,000 per year. We use the following variables of the wage and employment survey.

1. *Registration number of the firm*. This is the same unique identifier as is used for the production survey.

2. *The sector classification of the firm based on the Standard Industry Classification (SIC)*. Again, we use data of the three industries of analysis.

3. *The total number of workers within the firm*. This is the number of individuals working in the firm at September, 30th. Note that this includes full as well as part-time workers.

4. *The total number of employees in the sample*. This is the number of workers from whom the firm has provided answers to the questions of the survey.

5. All firms with 20 or more employees are drawn, while a sample is taken from the firms with less than 20 employees. The latter sample is purely random and does not depend on firm size.
6. All firms with 100 or more employees are drawn, while a sample is taken of the firms with less than 100 employees.

5. *Individual hourly wage including extra payments for overtime hours*.

6. *Occupational classification of the particular worker.*

7. *Year in which the worker started his job*. This is informative on the elapsed job duration.

We use data of three different industries that are classified by their 2-digit SIC code. We note that the firms in the industries are still heterogeneous in the characteristics of the workers they employ. However, based on the number of observations that we have and the difficulties related to stratification with respect to other variables of the firm (for example regional differences), we decided not to stratify the firm data further.

Table 1 summarizes some descriptive statistics of the industries that we use in our analysis: the textiles industry, the publishing and printing industry and the industry for electrical machinery. First, we summarize the productivity levels per worker. The publishing industry has the highest productivity level among the industries and the productivity levels of the other industries are quite similar. We summarize the average wage bill per worker in the second row of table 1. The same patterns as with the productivity levels are found, where the publishing and printing industry pays the highest wages. We report the mandatory minimum wages in the third row. The textiles industry and the publishing and printing industry have quite similar mandatory minimum wages, while the mandatory minimum wage of the electrical machinery industry is much lower. Productivity differences may be the result of differences in the use of physical capital. As such, we would expect the publishing

TABLE 1
Descriptive Statistics of the Three Industries of Analysis

	Textiles	Publishing and printing	Electrical machinery
Business statistics			
Productivity level[a]	6808	8106	6635
Wage offer[a]	5335	5991	5135
Mandatory minimum wage[a]	2516	2510	2164
Depreciation costs per worker[a]	559	1087	519
Number of firms and employer size			
Total number of firms	900	4512	728
Percentage of firms $>$ 20 employees	24.0%	3.5%	19.2%
Percentage of firms $>$ 100 employees	5.6%	2.1%	4.9%
Employer size when $>$ 20 employees	85	94	130
Characteristics of workers			
Job durations[b]	123.1	126.7	126.5
Percentage of skilled workers	25.3	61.4	37.9

Note: [a] *In Dutch guilders per month per worker*
[b] *In months*

industry to have the highest depreciation costs. Looking at the fourth line of table 1, we see that this is indeed the case.

The next four rows of table 1 summarize the number of firms and the differences in firm size. First, we have the total number of firms within an industry. It is found that the publishing industry has by far the highest number of firms. Second, we find that most firms in the publishing industry are quite small. Only 3.5% of the firms employ more than 20 workers. Additionally, we find that although the number of firms with more than 20 employees in the publishing industry is quite small, the number of very big firms is relatively high. This is also found when we look at the average employer size, which we find to be the lowest in the textiles industry and the highest in the electrical machinery industry. The relatively large average firms size of the electrical machinery industry is due to a few large firms with over 500 employees.[7]

The final two rows of table 1 summarize some of the characteristics of workers. First, we find job durations to be quite similar between the different industries. The textiles industry has the lowest elapsed job durations. Additionally, we find that there are important differences between the skills decomposition of the industries. The highest percentage of skilled workers is found in the publishing industry, while the lowest percentage is found in the textiles industry. This fact could explain a part of the observed differences in the productivity levels. We do not look at this effect in the present paper.

To sum up the results from the paragraphs above, we conclude that the publishing and printing industry is quite different from the other two industries. Especially, the percentage of skilled workers and depreciation costs per worker differ a lot. The textiles industry and the electrical machinery industry are rather close in their observed characteristics. The only main difference between them consists in the mandatory minimum wage. However, there are not many workers in the electrical machinery industry that are paid below the minimum wage of the textiles industry. Therefore, the main difference between these two sectors may not be that important.

4 Preliminary Issues in the Empirical Implementation

For a job with a given time-invariant wage w, the exit rate out of the present job equals

$$\theta = \delta + \lambda_1(1 - F(w)) \tag{6}$$

This is the hazard rate of the distribution of the duration an individual spends in a job given the wage w. As a result, the duration of a job with a given wage

7. We did not delete these firms in our analysis. Instead, we did some sensitivity analysis to investigate the impact of these outliers. It was found that the effect was negligible.

w has an exponential distribution with this parameter θ.[8] RIDDER and VAN DEN BERG [1999] show that the tenure (*ie*, the elapsed job duration at the survey date), given the current wage, also has an exponential distribution with parameter θ, provided that two additional conditions are met: worker flows are in a steady state, and unemployed workers accept all wage offers. These assumptions are valid for our analysis. In addition, we can use equation (1) to find the relationship between F and G. Consequently, the joint density of a worker's wage and tenure at the survey date can be expressed in terms of λ_1, δ and G.

The distribution of G within a market is estimated by using the empirical distribution of the wage data:

$$\widehat{G}(w) = \sum_i s_i 1(w_i \leqslant w) \quad \text{with} \sum_i s_i = 1$$

where the w_i's are the observed wages and the s_i's are the weights for the observations in the market under consideration. These weights are used to correct for the oversampling of the larger firms in the data set.

The log likelihood function for λ_1 and δ within a market is based on the conditional distribution of tenure given the wage,

$$\log L = \sum_i -(\delta + \lambda_1 \overline{F}(w_i))t_i + \log(\delta + \lambda_1 \overline{F}(w_i))$$

where $\overline{F} := 1 - F$ can be expressed in terms of G, δ and λ_1, and where t_i denotes the tenure of individual i. We have to take account of the fact that we only observe the year in which the employee started to work for his firm, so we have to aggregate over time intervals. Estimates of λ_1 and δ are obtained by maximization of this likelihood. One may argue that the likelihood does not take account of the earnings distribution G. However, it is possible to show that taking this into account only adds terms that are dependent on G but not on λ_1 or δ.

Our analysis is based on job transitions between firms. Internal promotions within firms are not taken into account. This is simply because the data do not allow us to observe internal promotions within firms. Instead, a firm may be thought to consist of a number of sub-firms, between which job-to-job transitions are possible. If the latter are governed by the same parameters as the transitions between firms, then presumably our estimation methods leads to biased estimates of λ_1 and κ_1. On the one hand, the data would show relatively many workers with high job durations and high wages due to internal promotions. This may lead to over-estimation of λ_1. On the other hand, the data would also show many workers with low wages and high durations, and this mitigates the former.

The original observations in the data set for employers are likely to be affected by measurement error. For example, firms may report the same values as reported in their financial accounts or they may use the value

8. It would have been more realistic to assume that job-to-job transitions are costly. The empirical analysis in VAN DEN BERG [1992] shows that these costs are only a minor determinant of the optimal strategy, and that they only have a small effect on θ.

reported to the tax officer. These values may be manipulated by firms in order to minimize taxes or to increase the confidence of the shareholders. In addition, as discussed in the data section, the aggregate numbers of the firms are based on the accumulation over the whole year, while the number of employees is based on the number employed in September. This problem of measurement error may have an impact on the final estimates of the productivity and wage offer densities. Therefore, it is better to use averages over the years available (1993 and 1994), rather than using figures of the separate years.

5 The Results

We summarize the results of the estimates of λ_1's and δ in table 2. The labor market frictions have similar order of magnitude across industries. They seem to be of less importance in the publishing industry, while the highest frictions are found in the textiles industry. The standard errors are computed by using bootstraps.[9] [10]

The values of the estimated λ_1 are rather low in comparison to earlier studies of equilibrium search models in the Netherlands (KONING, RIDDER and VAN DEN BERG, [1995], VAN DEN BERG and RIDDER, [1998] and VAN VUUREN, VAN DEN BERG and RIDDER, [2000]). For example, VAN DEN BERG and RIDDER find average estimates of λ_1 to be equal to 0.047, which is more than twice as high as our estimates. This may be due to differences in the definition used for job durations. The values of the estimated δ are more in line with these studies.

TABLE 2

Estimates of the Labor Market Friction Parameters for the Different Industries

	λ_1	δ	κ_1
Textiles industry	0.0141	0.0054	2.630
	(0.0017)	(0.0002)	(0.372)
Publishing industry	0.0157	0.0050	3.160
	(0.0013)	(0.0001)	(0.258)
Electrical machinery	0.0131	0.0045	2.908
	(0.0023)	(0.0001)	(0.592)

9. For our bootstrap analysis we proceed as follows. First, we sample a wage and corresponding duration from the empirical distribution of our data set. Then, we estimate our distribution G. Finally the parameters λ_1 and δ are estimated by maximization of the log likelihood and using equation (1). This process is repeated and every step results in one bootstrap estimate of both λ_1 and δ. The standard errors of these parameters are obtained by computation of the standard errors of the sample of bootstrap estimates.

10. Although it is possible to find the exact asymptotic distribution of the estimators of λ_1 and δ, by using the same techniques as introduced in for example WOOLDRIDGE [2002] and NEWEY and MCFADDEN [1994], we do not compute it in this paper. We leave this interesting research area for future research.

5.1 The Relationship between Earned and Offered Wages

One of the most important predictions of the model is the relationship between the wage offer distribution and the distribution of earned wages of employed workers. Reformulation of equation (1) and taking the first order derivative leads to:

$$f(w) = \frac{(1+\kappa_1)g(w)}{(1+\kappa_1 G(w))^2} \tag{7}$$

We can estimate this density by using a kernel density estimate of g and the empirical distribution function of G. This distribution can be compared with the kernel density estimate of the observed wages across firms from the data set of the firms.

A problem that arises when comparing the earned wages and the average wage costs is that the social security payments made by firms are included in the wage costs but not in the gross wages earned by workers. Hence, the earned wages are lower than the average wage costs per worker. It is possible to calculate the amounts of social security payments by applying the corresponding rules.[11] We correct for these in our empirical analysis. It implies that we increase the offered wages with the calculated amounts of social security payments.

The relationship between earned and offered wages is illustrated in figure 1. The smoothing parameter is obtained by using least-squares cross validation (SILVERMAN [1986]). Least-squares cross validation is a completely automatic method to find the smoothing parameter that minimizes the mean integrated square error.[12] The method is superior to the rules of thumb, because it does not require any assumption on the population distribution.

The modus of the wage offer distribution of the textiles industry is at the right location. However, the distribution of directly observed wage offers (wages across firms) is much more dispersed than the wage offer distribution estimated from the worker data. In addition, we only find one modus in our estimated density function while there are three modi in the kernel density function. The performance of the estimated wage offer densities of the other two sectors is rather poor. We find that the modi in both estimated densities are much to the left of the modi in the kernel density estimates. This implies that the relationship between f and g is poor. This relationship represents that, in the theoretical model, workers climb the job ladder. Therefore, there are relatively many high wages in a cross section of workers. Although the earned wages (including social security payments of firms) are on average a little bit higher than the wages across firms, we find that the difference is much too low to be explained by the theoretical model. This may imply that workers do not necessarily only accept wages that are higher than their current wage. It

11. The rules of 1993 are summarized in CPB [1995].
12. The mean integrated square error of a distribution f is defined as $\int (f(x) - \widehat{f}(x))^2 dx$, where $\widehat{f}$ can be any estimate of f.

FIGURE 1

Estimates of the wage offer distribution using kernel density estimates of the directly observed average wage costs of firms, from the firm data (kernel), and using estimates of earned wages of workers using equation (7) (estimated).

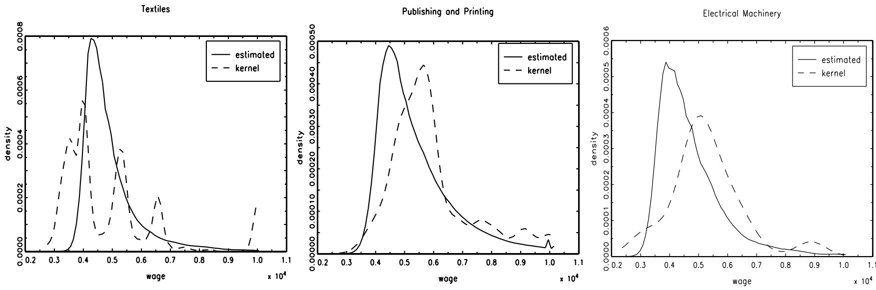
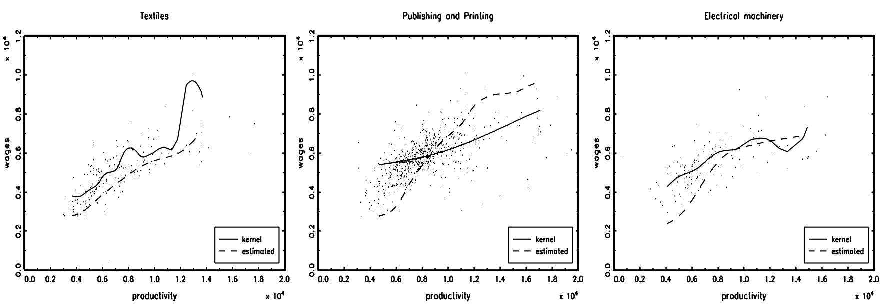


FIGURE 2

Estimated wage levels given the productivity levels by using kernel regression estimation (kernel) and by using equation (8) (estimated)



may also imply that our definition of a labor market is not right. In particular, we do not look at differences in skill levels within an industry. Note that our estimation technique to derive λ and δ is based on the relationship between F and G. Hence, the performance of these estimates may be poor as well.

5.2 The Relationship between the Productivity and Wage Level

We now compare the model predictions with the observed data of firms by deriving wage offers from productivity levels and *vice versa*. Using the estimates of the labor market friction parameters, we can estimate the wage levels, given the productivity levels,

$$(8) \qquad \widehat{w}_i = K(p_i|\widehat{\kappa}_1) = p_i - \left[1 + \kappa_1 \widehat{\overline{\Gamma}}(p_i)\right]^2 \int_{\underline{w}}^{p_i} \frac{dx}{(1 + \kappa_1 \widehat{\overline{\Gamma}}(x))^2}$$

which is the counterpart of equation (5). We estimate the distribution function Γ by using the empirical distribution of the productivity levels. Figure 2 plots the estimated wage levels for the different productivity levels. We also plotted *Nadayara-Watson* kernel regression estimates of the productivity levels for comparison. Again, we use least-squares cross validation to obtain the smoothing parameter. We refer to BLUNDELL and DUNCAN [1998] for least-squares cross validation with *Nadayara-Watson* estimates. We find that the estimated relationship by using equation (8) is able to capture some aspects of the observed relationship. However, there are important differences when we look at these pictures in more detail.

We can derive the estimated productivity levels from the wage offers of firms by using a reformulation of equation (3):

$$K^{-1}(w) = \widehat{p}_i = w_i + \frac{1 + \widehat{\kappa}_1 \widehat{\overline{F}}(w_i)}{2\widehat{\kappa}_1 \widehat{f}(w_i)} \tag{9}$$

As before, we can use the estimates of F, f and κ_1 to estimate this relationship. Results of these estimates are summarized in figure 3. We also plotted the results using kernel regression, being equal to the mirror image of the results presented in figure 2. The estimated relationship is downward sloping for very small wages. This is not possible according to the theory, since it means that higher productivity firms offer lower wages. It implies that the firms at the lower tail of the productivity distribution could increase their profits by decreasing their wages. Evidently this does not happen, so the theory fails to capture this behavior. There are a few explanations for this. First, workers are actually different in their marginal productivity and firms who lower their wages run the risk that they can only attract the low productive workers. The efficiency wage literature indicates that this is an important aspect. Second, the level of the unemployment benefits may not be the same among workers. In the discussion of the model, we explicitly assumed that the mandatory minimum wage is above the workers' reservation wage. This is not a very plausible assumption for particular workers, while dropping this assumption affects firm behavior. In particular, firms do not lower their wages because they cannot attract all workers when their offered wages are too low. BONTEMPS, ROBIN and VAN DEN BERG [1999] discuss the model with different unemployment benefits in detail. Although it should be possible to obtain somewhat comparable derivations as we did in the previous sections, we do not elaborate on this in our paper. Finally, the matching technology between workers and firms can also explain why firms do not reduce their wages. In particular, when workers meet firms according to the size of these firms, then the wage offer distribution is degenerate even in the case when there are differences in the productivity levels among firms (see BURDETT and VISHWANATH [1988]).[13]

In section 5.4 we look at the issue of profit maximization by firms in more detail.

---

13. Although they assume that there is only one productivity level, it is not hard to find that the distribution among productivity levels does not affect the results.

FIGURE 3

Estimates productivity levels given the wage levels by using kernel regress-sion (kernel) and by using equation (9) (estimated)

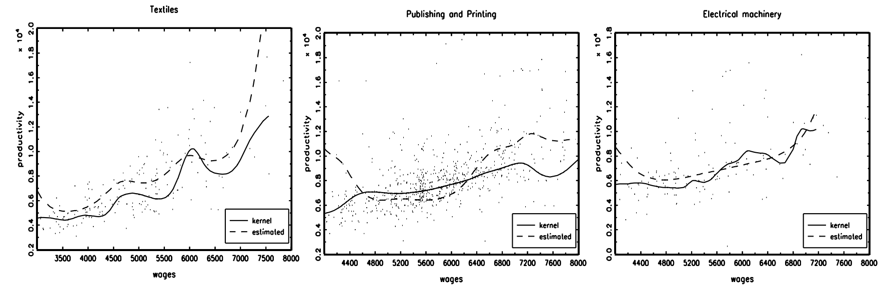


FIGURE 4

Estimates of the wage offer density by using kernel density estimates of the wage data (kernel) and by using the estimates wage offers from section 5.2 (estimated)

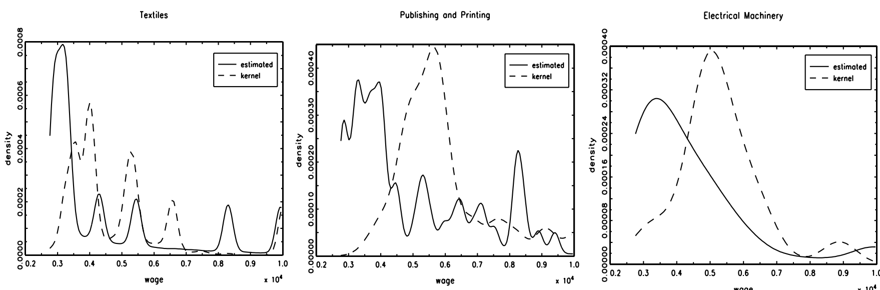
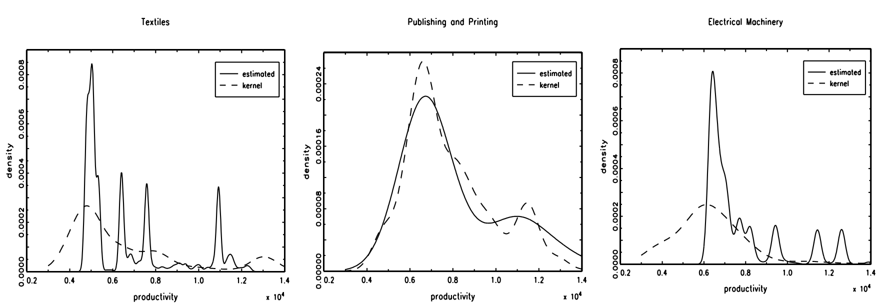


FIGURE 5

Estimates of the productivity density by using kernel density estimates of the productivity data (kernel) and the estimated productivity levels from section 5.2 (estimated)



5.3 **The Predicted Wage Offer and Productivity Distribution**

From the results of section 5.2, we can derive the estimated wage and productivity densities. We obtain these by calculation of the $\widehat{w}_i$'s and $\widehat{p}_i$'s and use them in kernel density estimates. These can be compared with the wage and productivity densities using kernel density estimates directly from the wage and productivity data.

The densities of the wage offers are plotted in figure 4.[14] The performance of the estimated wage offer density of the textiles industry is not too bad. We find that the estimated wage offer density has the same spikes as are found in the kernel density estimates, although there are some problems with the estimation of the left tail of this distribution. The estimated wage offer distributions of the other two industries do not describe the data very well. In both industries, the modus of the kernel density estimates are located to the right of the estimated distributions.

Figure 5 summarizes the estimated and observed density of the productivity distribution.[15] We find some problems with the estimation of these densities for the different industries. The performance of the productivity density of the textiles industry is poor. We find that the estimated wages are concentrated in a few narrowly defined intervals. The problems that exist in the productivity distribution of the textiles industry do not seem to exist in the distribution of the publishing industry. We find that the modus of the estimated productivity distribution using wage offer data corresponds with that of the productivity distribution using a kernel density estimate directly from the productivity data. Additionally, the estimated right tail of the distribution performs quite well. On the other hand, the estimated probability mass just to the right of the modus is not very good. We find that the modus of the electrical machinery industry is at the good location of the density. However, the left tail of the productivity distribution of this industry does not fit the data very well.

The results from the figures above provide insights into the performance of the equilibrium search models. From the data we know that the variance of productivity levels is much higher than the variance of wage offers. This is actually an interesting result, since it is also predicted by the equilibrium search model. However, from the figures of the publishing and printing industry and the electrical machinery industry, it seems that the wages are still too far apart to predict the wage offer density very good. This means that from the data of the productivity levels, the calculation of the wage offers using equation (8) leads to wages that are much more concentrated than the original data. Additionally, it is predicted that the wage offer density is situated to the left of the density estimation by using kernel estimates directly from the data of wage offers. This means that firms pay higher wages than what the model predicts given their productivity levels. We find the complete opposite picture from the productivity distribution. Especially, from the figure of the electrical machinery industry, it is clear that the model cannot explain the low productivity levels that are observed from the data. Additionally, the observed variance of the productivity levels is much higher than that of the predicted productivity levels.

14. The value of the bandwidth of the estimated distribution of the publishing and printing industry becomes very small when using least squares cross-validation. Instead, we use $\frac{1}{4}\sigma n^{-\frac{1}{5}}$, where σ is the standard error of the observations.

15. Again, we use the a bandwidth equal to $\frac{1}{4}\sigma n^{-\frac{1}{5}}$ for the publishing and printing industry.

5.4 The Restrictions on the Wage Offer Distribution

Equation 9 can also be used to investigate whether the distribution of wage offers is in the set of possible distributions that can be derived from the model. From BRVdB, we have that the first order derivative of $K^{-1}(w)$ is equal to:

$$
\begin{aligned}
(K^{-1})'(w) &= \frac{\kappa_1 f(w)^2 - f'(w)(1+\kappa_1 \overline{F}(w))}{2\kappa_1 f(w)^2} \\
&= \frac{3\kappa_1 g(w)^2 - g'(w)(1+\kappa_1 G(w))}{2\kappa_1 g(w)^2} > 0
\end{aligned}
\tag{10}
$$

Since high productivity firms pay higher wages, the right hand side has to be positive and hence the numerators in equation (10) have to be positive. This means that we have a restriction on both the wage offer data and the wage data sampled from a cross section of workers. Figure 6 illustrates the confidence intervals of the numerators for the wage offers from firms. Figure 7 illustrates confidence intervals for the wage data from employed

FIGURE 6

Illustration of 95% confidence intervals for the numerator of the first line of equation (10)

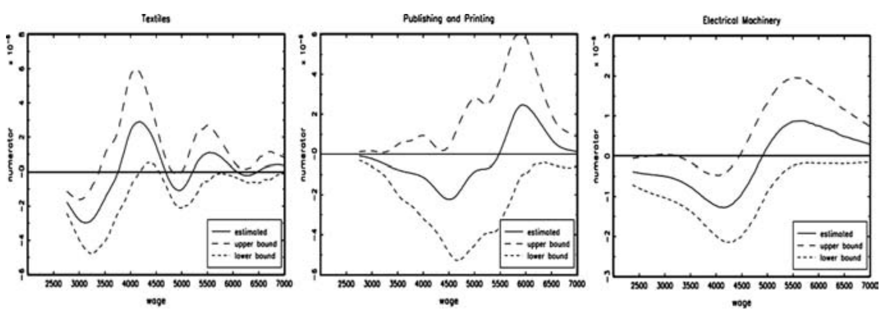
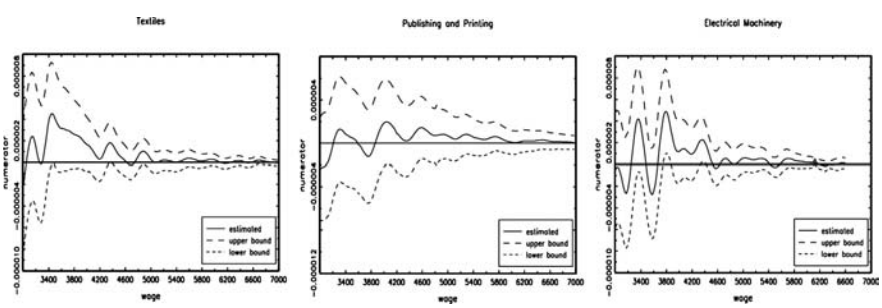


FIGURE 7

Illustration of 95% confidence intervals for the numerator of the second line of equation (10)



workers. We use the methods introduced in *Härdle*, MARRON and WAND [1990] to calculate the least-squares cross validation function. We calculated the standard errors by using bootstraps.[16]

The point estimates and confidence intervals for different wage levels are plotted in figure 6. We find that the point estimates are negative for low wages. The confidence intervals are completely below zero for a range of wages of the textiles industry and the electrical machinery industry. Hence, for these wages the null hypothesis that the the right hand side of equation (10) is positive, is rejected. Since the confidence intervals of the publishing and printing industry are wide, it is not possible to draw any conclusions from figure 6 for that industry.

Narrower confidence intervals are found for the wages earned by employed workers in figure 7. The point estimates of the numerator are positive for most of the wages, but there are still some ranges of the wages where we find negative values. In addition, we do not find that the numerator is significantly positive for any of the wages analyzed.

5.5 The Distribution of Firm Size

The distribution of the firm size is given in equation (2). Although this is a useful theoretical representation, it is not applicable for our empirical work, since there is just one observation for every single wage level. Instead, it would be more valuable to find the marginal distribution of the firm size, which can be obtained by integration over the wages:

$$\mathrm{P}(L = l) = \int_{\underline{w}}^{\overline{w}} \mathrm{P}(L = l | W = w)\, f(w) dw$$

Using equation (2), we obtain:

$$\mathrm{P}(L = l) = \int_{\underline{w}}^{\overline{w}} \left(\frac{A}{(1 + \kappa_1 \overline{F}(w))^2} \right)^l \frac{1}{l!} \exp \left(- \frac{A}{(1 + \kappa_1 \overline{F}(w))^2} \right)$$

The right hand side of this equation can be simplified using the transformation:

$$y = l(w) = \frac{A}{(1 + \kappa \overline{F}(w))^2}$$

16. We are aware of the fact that our method derives confidence intervals of $(K^{-1})'(w|\widehat{f}, \widehat{f}')$ and $(K^{-1})(w|\widehat{g}, \widehat{g}')$ rather than $(K^{-1})'(w|f, f')$ and $(K^{-1})'(w|g, g')$. There are techniques to derive confidence intervals based on the density functions rather than their estimates. HALL [1992] suggests to use the method of undersmoothing. This implies that we use a bandwidth that is proportional to $n^{-1/3}$ instead of the usual $n^{-1/5}$ of the optimal bandwidth for the minimum mean integrated square error. Since our bandwidths using least-squares cross validation are already very small bandwidths, we do not elaborate on this.

We obtain:

$$\text{(11)} \qquad \mathrm{P}(L=l) = \frac{\sqrt{A}}{2\kappa_1 l!} \int_{\frac{A}{(1+\kappa_1)^2}}^{A} y^{l-\frac{3}{2}} e^{-y} dy \qquad \forall\, l \in \mathbb{N}$$

The integral can be computed quite efficiently by noting that it is an incomplete gamma function. Note that this distribution does not depend on F and G and hence not on the strategy of firms. The parameter A can be estimated easily by:

$$\text{(12)} \qquad \widehat{A} = \bar{l}(1+\widehat{\kappa}_1)$$

where $\bar{l}$ is a consistent estimator of the mean firm size in the industry (*ie*, $\mathrm{E}(L)$). This can be seen by using:

$$M - U = \frac{A}{1+\kappa_1} N$$

The left hand side of this equation is equal to the total number of workers employed within the industry. Using the law of large numbers we find that this is equal to $N\mathrm{E}(L)$ for large N. The same result is obtained when we use:

$$\mathrm{E}(L) = \int_{\underline{w}}^{\overline{w}} \mathrm{E}(L|w) f(w) dw$$

$$= A \int_{\underline{w}}^{\overline{w}} \frac{1}{(1+\kappa_1 \overline{F}(w))^2} f(w) dw$$

$$= \frac{A}{1+\kappa_1}$$

Table 3 gives estimates of the variable A for the three different industries. Note that based on these estimates, it is possible to calculate the number of individuals that are working in the segment. These are 29 281, 125 701 and 27 701 workers for the textiles, publishing and electrical machinery industry. Based on this, the publishing industry is the most important industry from a macro perspective.

We can make comparisons between the distribution in equation (11) and the actually observed distribution of employment levels within the industry. This is illustrated in figure 8. The global shape of the distribution is quite well estimated, although it seems not possible to explain the spikes in the distribution. These spikes may be caused by measurement errors, since there are a lot of individual firms that report employment values dividable by 5.

We can also compare estimates of $\mathrm{E}(l|w)$ and $\mathrm{E}(l|p)$ with kernel regression estimates of l on the levels of the wage offers and productivity levels. Results

TABLE 3
Estimates of the Variable $A = \frac{M-U}{N}(1+\kappa_1)$

	A
Textiles industry	117.5
	(18.1)
Publishing industry	120.1
	(12.1)
Electrical machinery	148.8
	(28.8)

of the estimated relationship between the employment level and the wage offers are illustrated in figure 9. The kernel regression estimates are quite messy, although we already used quite a large bandwidth.

We illustrated the relationship between the firms' productivity levels and the employment level in figure 10. These results are similar to the results found in figure 9.

5.6 Using χ^2 Goodness of Fit Tests

We use χ^2-tests to test the goodness of fit of the productivity and wage offer distributions. For convenience, we only use the data from the firms that have 20 employees or more. The wage offer distribution, conditional on this observation is equal to:

$$F(w|L \geqslant 20) = \frac{\mathrm{P}(L \geqslant 20; W \leqslant w)}{\mathrm{P}(L \geqslant 20)} = \frac{\mathrm{P}(L \geqslant 20|W \leqslant w)F(w)}{\mathrm{P}(L \geqslant 20)} \tag{13}$$

The numerator can be rewritten as follows:

$$\begin{aligned}
\mathrm{P}(L \geqslant 20|W \leqslant w)F(w) &= \int_{\underline{w}}^{w} \mathrm{P}(L \geqslant 20|W = x)\, f(x|x \leqslant w)dx F(w) \\
&= \int_{\underline{w}}^{w} \mathrm{P}(L \geqslant 20|W = x)\, f(x)dx \\
&= 1 - \sum_{k=0}^{19} \int_{\underline{w}}^{w} \mathrm{P}(L = k|W = x)\, f(x)dx \\
&= 1 - \sum_{k=0}^{19} \frac{\sqrt{A}}{2\kappa_1 k!} \int_{\frac{A}{(1+\kappa_1)^2}}^{\frac{A}{(1+\kappa_1 \overline{F}(w))^2}} y^{k-\frac{3}{2}} e^{-y} dy
\end{aligned}$$

FIGURE 8

Estimates of the employment density (where the bars represent the number of observations)

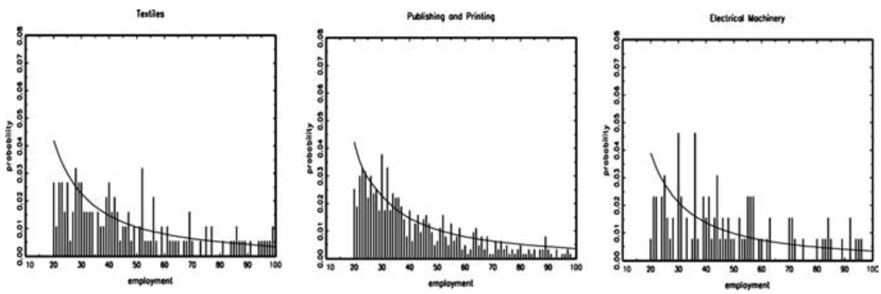


FIGURE 9

Estimates of the relationship between the wage level and the employment level by using kernel regression estimates (kernel) and the estimation of $\mathrm{E}(l|w)$ ***(estimated)***

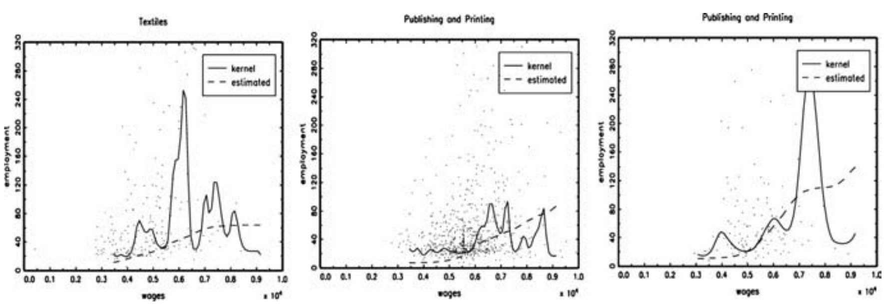
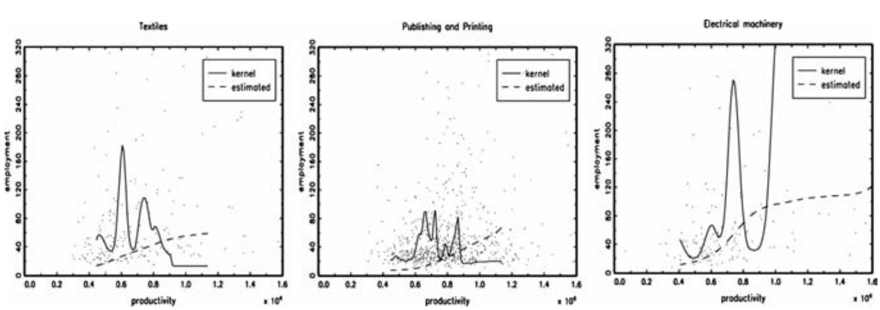


FIGURE 10

Estimates of the relationship between the productivity level and the employment level by using kernel regression estimates (kernel) and the estimation of $\mathrm{E}(l|p)$ ***(estimated)***



The final step of this equation is obtained by using the same steps to obtain equation (11). Substitution of this into the second line of equation (13) results in:[17]

$$F(w|L \geqslant 20) = \frac{1 - \sum_{k=0}^{19} \frac{\sqrt{A}}{2\kappa_1 k!} \int_{\frac{A}{(1+\kappa_1)^2}}^{\frac{A}{(1+\kappa_1 \overline{F}(w))^2}} y^{k-\frac{3}{2}} e^{-y} dy}{1 - \sum_{k=0}^{19} \frac{\sqrt{A}}{2\kappa_1 k!} \int_{\frac{A}{(1+\kappa_1)^2}}^{A} y^{k-\frac{3}{2}} e^{-y} dy}$$

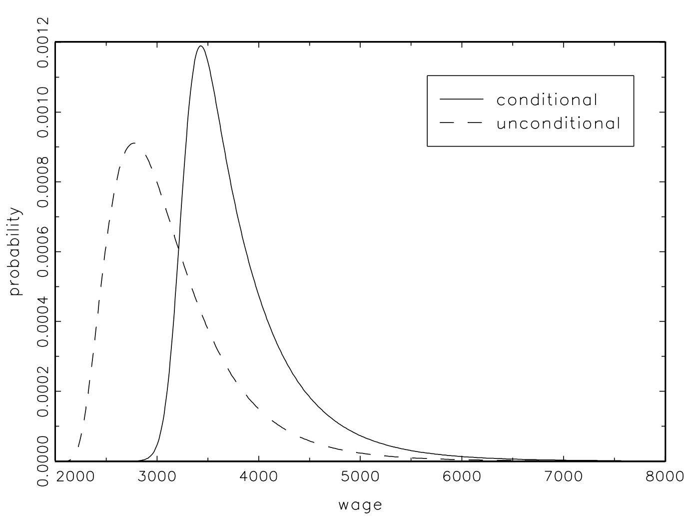
Likewise, the conditional productivity distribution is equal to:

$$\Gamma(p|L \geqslant 20) = \frac{1 - \sum_{k=0}^{19} \frac{\sqrt{A}}{2\kappa_1 k!} \int_{\frac{A}{(1+\kappa_1)^2}}^{\frac{A}{(1+\kappa_1 \overline{\Gamma}(p))^2}} y^{k-\frac{3}{2}} e^{-y} dy}{1 - \sum_{k=0}^{19} \frac{\sqrt{A}}{2\kappa_1 k!} \int_{\frac{A}{(1+\kappa_1)^2}}^{A} y^{k-\frac{3}{2}} e^{-y} dy}$$

The differences between the conditional and the unconditional productivity density are illustrated in figure 11. The picture is based on the assumption that the productivity distribution is log normal. It is possible to see that the conditioning on large firms has effects on the left tail of the distribution.

FIGURE 11

Illustration of the differences between the conditional and unconditional density when Γ ***is a log normal distribution with*** $\mu = 6.9$, $\sigma = 0.5$, $p_{min} = 2000$***. The other parameters in the model are equal to*** $\lambda_0 = 0.1$***,*** $\lambda_1 = 0{,}03$***,*** $\delta = 0.01$, $M = 40000$***,*** $N = 2000$ ***and*** $w_{min} = 1600$



17. Note that the denominator is equal to $\mathrm{P}(L \geqslant 20) = \mathrm{P}(L \geqslant 20|W \leqslant w)F(w)$ for $w \to \infty$.

The calculated probabilities are compared to the observed probabilities from the multinomial distribution:

$$\widehat{P_i} = \sum_{i=1}^{n} \mathbf{1}(w_i \in A_i)$$

for $i = 1, \ldots k$ and where A_i is the set of classes with $\bigcup_{i=1}^{k} A_i \supseteq \operatorname{supp}(F)$, $\operatorname{supp}(\Gamma)$ The estimated probabilities are equal to:

$$\begin{aligned} P_i(\widehat{\kappa_1}) &= \mathrm{P}_{\widehat{\kappa_1}}(W \in A_i | L \geqslant 20) \\ &= F_{\widehat{\kappa_1}}(\sup(A_i)|L \geqslant 20) - F_{\widehat{\kappa_1}}(\inf(A_i)|L \geqslant 20) \end{aligned}$$

for the wage offer distribution. These probabilities are similarly defined for the productivity distribution. Let $\widehat{P}$ and $P(\widehat{\kappa_1})$ be the vectors containing the elements $\widehat{P_i}$ and $P_i(\widehat{\kappa_1})$; $i = 1, \ldots k$. To correct for the fact that we estimate κ_1, we use the *Rao-Robson-Nikolin* instead of the conventional *Pearson* test statistic (Van der Vaart [1998])

(14)
$$T = n\left(\frac{\widehat{P} - P(\widehat{\kappa_1})}{\sqrt{P(\widehat{\kappa_1})}}\right)^T \left(I_k - \frac{n}{m}\sigma^2(\widehat{\kappa_1})C(\widehat{\kappa_1})^T C(\widehat{\kappa_1})\right)^{-1} \left(\frac{\widehat{P} - P(\widehat{\kappa_1})}{\sqrt{P(\widehat{\kappa_1})}}\right)$$

where,

$$C(\widehat{\kappa_1})_i = \frac{\frac{1}{n}\sum_{j=1}^{n} I_{A_i}(w_j)\frac{\partial \log f(w_j, \widehat{\kappa_1})}{\partial \kappa_1}}{\sqrt{P(A_i; \widehat{\kappa_1})}}$$

Here, m is the number of observations on which our maximum likelihood estimates are based and n is the number of observations that we use to compare the estimated distribution with. This statistic can be shown to follow a χ^2-distribution with $k-1$ degrees of freedom under the null hypothesis that the data are drawn from the distribution being estimated. Actually, this statement is based on the condition that $\widehat{\kappa}_1$ is an asymptotically efficient estimator of κ_1. Since we use a maximum likelihood procedure containing an empirical distribution function for G, asymptotic efficiency can not be guaranteed using the standard procedure. Therefore, the null hypothesis may be incorrectly rejected in some cases (*ie*, the actual distribution of T stochastically dominates the χ^2 distribution with $k-1$ degrees of freedom).

Another possibility to test the theory is by using minimum χ^2-estimates, *ie*, the minimization of the statistic:

$$T = \min_{\kappa_1 > 0}\left\{\left(\frac{\widehat{P} - P(\widehat{\kappa_1})}{\sqrt{P(\widehat{\kappa_1})}}\right)^T \left(\frac{\widehat{P} - P(\widehat{\kappa_1})}{\sqrt{P(\widehat{\kappa_1})}}\right)\right\}$$

This statistic can be shown to follow a χ^2-distribution with $k-2$ degrees of freedom (Van der Vaart [1998]). The problems with the asymptotic distri-

TABLE 4

Results of the χ^2 Goodness of Fit Tests

	χ^2_F	χ^2_Γ	χ^2_L
Textiles industry	52.12	320.71	86.02
Publishing industry	348.99	584.32	157.37
Electrical machinery	21.10	206.97	54.37

TABLE 5

Results of the Minimum χ^2-Estimates and the Function Value of the Criterion Function

	Wage offers		Productivity levels		Employment levels	
	κ_1	χ^2	κ_1	χ^2	κ_1	χ^2
Textiles industry	2.423	45.54	1.410	51.08	2.507	68.36
Publishing industry	2.620	190.06	0.864	187.11	1.876	105.39
Electrical machinery	3.458	4.15	1.843	54.61	2.708	41.95

FIGURE 12

Estimates of the conditional wage offer density (estimated) together with the kernel density estimate of the firms with 20 or more employees (kernel)

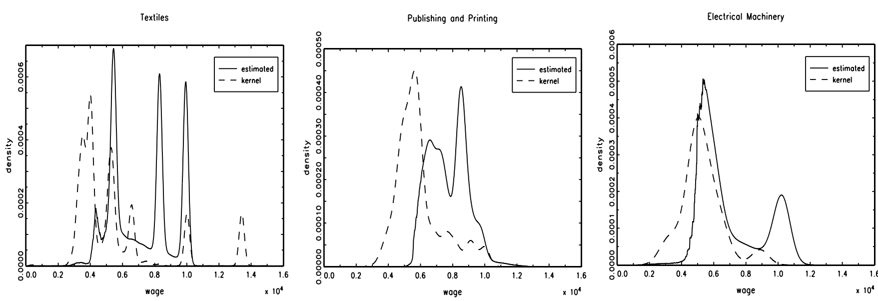
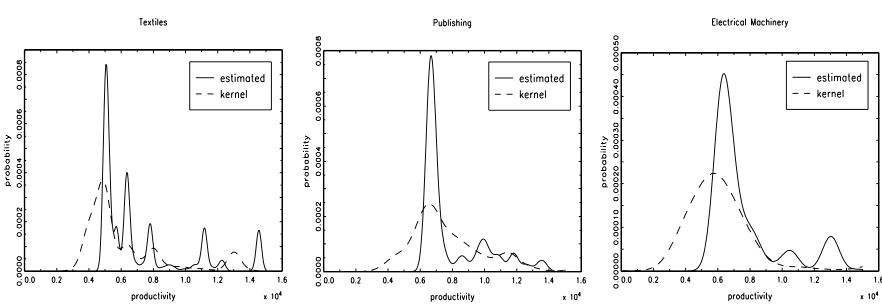


FIGURE 13

Estimates of the conditional productivity density (estimated) together with the kernel density estimate of the firms with 20 or more employees (kernel)



bution of the *Rao-Robson-Nikolin* test do not occur here. Note that both statistics do not have to result in the same conclusions, since the latter statistic is based on employer data only. This means that when the first method leads to a rejection of the theory, while the second does not lead to such a rejection, we expect problems to occur in the translation between employer and employee data.

Figures 12 and 13 illustrate the conditional distributions of the wage offers and productivity levels. The kernel density estimates are based on the firms with 20 employees or more. We find some differences with the results found from the unconditional distribution.

The results of the χ^2 goodness of fit tests are found in table 4. These tests are based on 10 classes, which means that the number of degrees of freedom of these tests is equal to 9. The levels of $\sigma^2(\widehat{\kappa_1})$ can be found in table 2 and the levels of m in equation (14) are equal to 1878, 5613 and 1841 for the textiles, the publishing and the electrical machinery industry. It is possible to see that all tests are rejected under a 5% significance level. Using a 1% significance level leads to the failure to reject the null hypothesis for the wage offer distribution of the electrical machinery industry.

Table 5 summarizes the results of the minimum χ^2-estimates of κ_1 using the classification based on 10 different intervals of the wage offer and productivity distribution. Minimizing the χ^2-statictic based on the distribution of wage offers, the resulting then the resulting values of κ_1 decrease for the textiles and publishing industry, while the value of this parameter increases for the electrical machinery industry. Based on the statistic for the different productivity levels, the estimates of κ_1 decrease for the textiles and electrical machinery industry. It increases a little for the publishing industry. The κ_1's for the employment levels are similar to their original values, except for the publishing industry.

The minimized χ^2-statistics resulting from the estimation method are presented in the second and fourth column of table 5. We find similar results for the χ^2-test statistics as we saw in table 4. The χ^2-statistic of the publishing industry is decreased quite a lot for the wage offer distribution, but is still far from a failure to reject the null hypothesis. Note that the degrees of freedom of this test is now decreased by 1 to a level of 8. The resulting value of the test statistic for the electrical machinery industry leads to a failure to reject the null hypothesis for any usual significance level. The values of the test statistic for the productivity distribution do not differ that much from the results found in table 4. The values of the χ^2-test statistic for the employment levels are all high enough to reject the null hypothesis.

5.7 The Relationship between Productivity Levels and Depreciation Costs

So far, we have used the total value added divided by the number of workers as the measure of the labor productivity of workers in a particular firm. It can be argued that this simple measure is not robust when we allow for more realistic assumptions with respect to the production function. For example, there are large differences between capital stocks of firms, even within a narrowly defined market, and the value added of a firm may need to

FIGURE 14

Illustration of the Relationship between Depreciation Costs and Productivity Levels of Firms

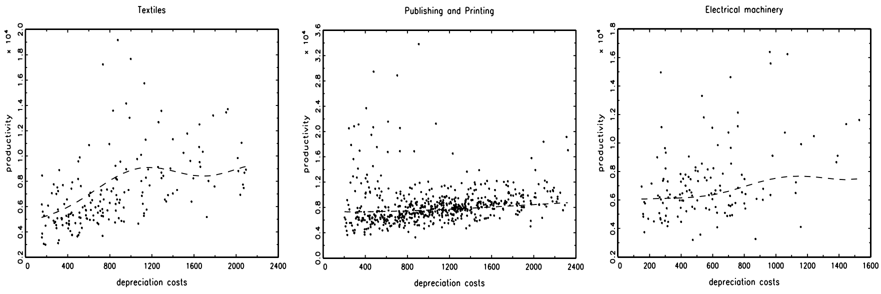


TABLE 6

Ordinary Least Squares Estimates of the Relationship between Depreciation Costs and Productivity Levels

	Textiles	Publishing and Printing	Electrical Machinery
constant	4091	7292	5213
	(392)	(315)	(544)
depreciation costs	4.440	0.795	4.424
	(0.631)	(0.363)	(1.296)
(depreciation costs)^2 (x1000)	– 0.717	0.195	– 0.068
	(0.195)	(0.085)	(0.561)
R^2	0.336	0.099	0.284

cover capital costs, so that the profit per worker is smaller than the value added minus the wage. This may have effects on the wage offering behavior of firms, because some firms may only appear to be productive because of their high capital investments. It is not obvious that these firms offer much higher wages in a wage posting framework. Hence, the poor performance of the wage posting model as described in the previous sections may be due to an incorrect observation of labor productivity. We investigate this in the remainder of this section, by examining the relationship between our definition of labor productivity and the capital investments (per worker) made by the firm. If our definition of labor productivity is correct then labor productivity can not be influenced by capital investments per worker. A strong (positive) relationship implies that our definition of productivity was misspecified. Such a result would suggest that capital investments should be made endogenous in an empirical equilibrium search model (like in ROBIN and ROUX [1998]).

One problem is that we do not have data on the capital investments of firms. Instead, we use depreciation costs. Figure 14 shows the relationship between productivity levels and depreciation costs of firms by using *Nadayara-Watson* kernel regression estimates. We find that the relationship is indeed upward sloping and that the relationship is not linear. However, it seems possible to approximate the relationship by ways of a second order polynomial. The estimation results using ordinary least squares are summarized in table 6. We find that the depreciation costs as well as the depreciation costs squared are highly significant (one exception is the electrical machinery industry).[18] The values of the R^2 differ remarkably between the different segments, where it is quite large for the textiles industry and low for the publishing and printing industry.

6 Conclusions

In this paper, we used employer data to test the predicted equilibrium outcomes of search models. For this purpose, we used the modelling framework developed in BONTEMPS, ROBIN and VAN DEN BERG [2000] (BRVdB). We estimated the model using data on individually earned wages and elapsed unemployment durations in a similarly flexible way as they did. Based on the estimates of search frictions, we thus obtained estimates of the productivity distribution, the wage offer distribution and the employment distribution among firms. Additionally, we analyzed the relationships between the wage offer, the productivity level and the employment levels. Formal tests were used to investigate whether the employer data could be sampled from the distributions being estimated.

We found that there are problems with the estimation of the distributions of wage offers and productivity levels. It is especially hard to explain the low productivity levels of the larger firms. Additionally, it is found that the wages that are offered by some firms are sub-optimal, given the productivity levels of these firms. The standard deviations of the predicted productivity distributions are much smaller than those of the observations, while the opposite is true for the wage offer distribution. Formal tests for the goodness of fit of the distributions find that the null hypothesis is rejected in almost all cases, where the null hypothesis states that the observations from the data are sampled from the distribution that is predicted by the model.

The fundamental relation in the model of BRVdB is that between the wage offer and the productivity level. We tested the fit of this relationship in this paper and the problems we encountered have to be interpreted as a failure of the specific model to explain all phenomena of importance. This provides information about the origins of the problems and hence give us suggestions for future research. The difficulties to explain firm behavior indicate that the differences between individual workers play an important role in the determination of the wage offers by firms. This is in line with the recently published results of ABOWD, KRAMARZ and MARGOLIS [1999] and ABOWD, FINER and KRAMARZ [1999], who also find that the individual specific component is

18. As noted in section 3, the depreciation costs are likely to be measured with error. It is well known that the estimated regression coefficients are biased downwards in such a situation. Hence, our test of capital investments not to have any influence on productivity, is likely not to reject in many cases where there is some influence. Since our results indicate that, despite the possible existence of measurement error, there is a strong relationship between productivity and depreciation costs, we do not elaborate on this issue.

important in determining wage levels.[19] Additionally, other aspects that we ignored in our analysis may play a role as well, like capital intensity, the distribution of benefits among workers and the matching technology between workers and firms. ▼

• References

ABOWD J.M., CREECY R., KRAMARZ F. (2002). – Computing Person and Firm Effects Using Linked Longitudinal Employer-Employee Data, *Working Paper*, Cornell University, Ithaca.

ABOWD J.M., FINER H., KRAMARZ F. (1999). – Individual and Firm Heterogeneity in Compensation: An Analysis of Matched Longitudinal Employer and Employee Data for the State of Washington, in: J. Haltiwanger, J. Lane, J. Spletzer, K. Troske (eds.), *The Creation and Analysis of Employer-Employee Matched Data*, North-Holland, Amsterdam.

ABOWD J.M., KRAMARZ F., MARGOLIS D.N., (1999). – High Wage Workers and High Wage Firms, *Econometrica* 67, p. 251-335.

ANDREWS D.W.K. (1988), – Chi-square Diagnostic Tests for Econometric Models: Theory, *Econometrica* 56, p. 1416-1452.

BLUNDELL R., DUNCAN A. (1998). – Kernel Regression in Empirical Microeconomics, *Journal of Human Resources* 33, p. 62-87.

BONTEMPS C., ROBIN J.M., VAN DEN BERG G.J. (1999). – An Empirical Equilibrium Job Search Model with Search on the Job and Heterogeneous Workers and Firms, *International Economic Review* 40, p. 1039-1075.

BONTEMPS C., ROBIN J.M., VAN DEN BERG G.J. (2000). – Equilibrium Search with Continuous Productivity Dispersion: Theory and Non-parametric Estimation, *International Economic Review* 41, p. 305-358.

BOWLUS A.J., KIEFER N.M., NEUMANN G.R. (2001). – Equilibrium Search Models and the Transition from School to Work, *International Economic Review* 42, forthcoming.

BURDETT K., MORTENSEN D.T. (1998). – Wage Differentials, Employer Size and Unemployment, *International Economic Review* 39, p. 257-273.

BURDETT K., VISHWANATH T. (1988). – Balanced Matching and Labor Market Equilibrium, *Journal of Political Economy* 96, p. 1048-1065.

CENTRAL PLANNING BUREAU (1995). – Replacement Rates, a Transatlantic Overview, CPB Working Papers 80, Central Planning Bureau, The Hague.

ECKSTEIN Z., WOLPIN K.I. (1990). – Estimating a Market Equilibrium Search Model from Panel Data on Individuals, *Econometrica* 58, p. 783-808.

HALL P. (1992).– *The Bootstrap and Edgeworth Expansion*, Springer-Verlag, New York.

HÄRDLE W., MARRON J.S., WAND M.P. (1990). – Bandwidth Choice for Density Derivatives, *Journal of the Royal Statistical Society Series B* 52, p. 223-232.

HOROWITZ J. (2001). – The Bootstrap, *in:* J.J. Heckman and E. Leamer, *Handbook of Econometrics* 5, North-Holland, 3159-3228.

19. ABOWD, KRAMARZ and MARGOLIS [1999] analyze use data of the French labor market, while ABOWD, FINER and KRAMARZ [1999] look at workers' compensation in the state of Washington. Both papers are based on approximation methods and find that the individual specific effects are important. Comparing the two papers, it is found that individual effects are more important in France than in the state of Washington. Although ABOWD, CREECY and KRAMARZ [2002] find that the approximation method used was somewhat inaccurate for the ABOWD, KRAMARZ and MARGOLIS [1999] paper, this result stays the same.

KONING P.W.C., RIDDER G., VAN DEN BERG G.J. (1995). – Structural and Frictional Unemployment in an Equilibrium Search Model with Heterogeneous Agents, *Journal of Applied Econometrics* 10, S133-S151.

MORTENSEN D.T., PISSARIDES C.A. (1999). – New Developments in Models of Search in the Labor Market, *in:* O. Ashenfelter and D. Card, *Handbook of Labor Economics* 3, North-Holland, Amsterdam.

NEWEY W., MCFADDEN D.L. (1994). – Large Sample Estimation and Hypothesis Testing, in: R.F. Engle and D.L. McFadden (eds.), *Handbook of Econometrics* 4, North-Holland, Amsterdam.

PISSARIDES C.A. (1984). – Efficient Job Rejection, *Economic Journal* 94, p. 97-108.

PISSARIDES C.A. (1990). – *Equilibrium Unemployment Theory*, Blackwell, Oxford.

POSTEL-VINAY F., ROBIN J.M. (2002). – Equilibrium Wage Dispersion with Worker and Employer Heterogeneity, *Econometrica* 70, p. 2295-2350.

RIDDER G., VAN DEN BERG G.J. (2000). – Estimating Measures of Labor Market Imperfection in Five OECD Countries, using Aggregate Data an Equilibrium Search Framework, *Working Paper*, Free University, Amsterdam.

ROBIN J.M., ROUX S. (1998). – Random or Balanced Matching: an Equilibrium Search Model with Endogenous Capital and Two-Sided Search, LEA-INRA and CREST-INSEE, Paris.

SILVERMAN B.W. (1986). – *Density Estimation*, Chapman and Hall, London.

VAN DEN BERG G.J. (1999). – Empirical Inference with Equilibrium Search Models of the Labor Market, *Economic Journal* 109, p. F283-F306.

VAN DEN BERG G.J., RIDDER G. (1998). – An Empirical Equilibrium Search Model of the Labor Market, *Econometrica* 66, p. 1183-1221.

VAN DER VAART A. (1998). – *Asymptotic Statistics*, Cambridge University Press, Cambridge.

VAN VUUREN A.P., VAN DEN BERG G.J., RIDDER G. (2000), – Measuring the Equilibrium Effects of Unemployment Benefits Dispersion, *Journal of Applied Econometrics* 15, p. 547-574.

WOOLDRIDGE J.M. (2001). – *Econometric Analysis of Cross Section and Panel Data*, The MIT Press, Cambridge.