ANNALES D'ÉCONOMIE ET DE STATISTIQUE. – N° 67/68 – 2002

Monetary Policy Analysis in Backward-Looking Models

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ABSTRACT. – In this paper, I use a dynamic general equilibrium model to quantify how sensitive a typical backward-looking model used in monetary policy analysis is to the *Lucas critique*. The results show that the backward-looking model exhibit significant parameter instability that is economically important, but that a standard econometric test for detecting this instability fails to do so accurately in small samples. These findings suggest that the relative merits of alternative monetary policy rules should be checked in an equilibrium framework.

Analyse de la politique monétaire dans les modèles tournés vers le passé

RÉSUMÉ. – Dans cet article, on utilise un modèle dynamique d'équilibre général afin de quantifier la sensibilité à la *critique de Lucas* d'un modèle tourné vers le passé usuellement utilisé pour analyser les effets de la politique monétaire. Les résultats montrent que le modèle tourné vers le passé présente une instabilité significative de ses paramètres. En revanche, les tests usuels de stabilité ne permettent pas de détecter cette instabilité dans les échantillons de taille réduite. Ces résultats suggèrent que les effets de différentes règles de politique monétaire doivent être évalués dans un cadre de modèles d'équilibre.

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1 Introduction

Recently, the empirical relevance of the LUCAS [1976] critique has received increased attention. A possible explanation for this is the extensive use of backward-looking models in monetary policy analysis, *cf.* BALL [1997], SVENSSON [1997], RUDEBUSCH and SVENSSON [1999] and TAYLOR [1999]. In this class of models, where the structure of the model economy is assumed to be unaffected by changes in economic policy, a considerable amount of effort has been devoted to examining the relative merits of alternative proposed monetary policy rules. Now, if the *Lucas critique* is valid and quantitatively important, this type of policy experiments may produce misleading results.[1]

ESTRELLA and FUHRER [1999] argue that the *Lucas critique* is an empirically testable hypothesis. They provide evidence that when there is a change in monetary policy regime, a pure forward-looking model is less stable than the better fitting backward-looking model they consider (the model by RUDEBUSCH and SVENSSON [1999], which they argue is an observation inconsistent with the *Lucas critique*.[2] This finding is troublesome because the forward-looking model they consider has been a workhorse model in monetary policy analysis, see *eg*, CLARIDA, GALÍ and GERTLER [1999]. FUHRER [1997] also maintains that backward-looking behavior seems to be a better approximation of reality than forward-looking behavior. In addition, most –if not all– of the many papers which have used the concept of super-exogeneity presented in ENGLE, HENDRY and RICHARD [1983] to examine the *Lucas critique* empirically have found no evidence in favor of the proposition; see the survey by ERICSSON and IRONS [1995]. But in a recent study, LINDÉ [2001a] finds that the power of the super-exogeneity test may be very low in small samples.[3]

Thus, it is still an open issue whether the *Lucas critique* is empirically important or not.

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1. LUCAS's [1976] argument was that shifts in economic policy change how policy affects the economy since agents in the economy are forward-looking and adapt their expectations and behavior to the new policy stance. For this reason, *Lucas* concluded that reduced-form economic and econometric models cannot provide useful information about the actual consequences of alternative policies.
2. The findings in LINDÉ [2001b] suggest that only the true forward-looking model will have parameters invariant to the monetary regime. With this in mind, the results in ESTRELLA and FUHRER [1999] which suggest that the *Lucas critique* is more relevant for the forward-looking model than the backward-looking model may be due to model misspecification and that the stability tests have weak power in small samples.
3. FAVERO and HENDRY [1992] have also studied the small samples properties of the super exogeneity test, using a non equilibrium model as a data generating process. In contrast to the findings by LINDÉ [2001a], their results suggest that the *Lucas critique* lacks force in practice and that the super exogeneity test has satisfactory power in small samples. Presumably, the most important reason why the results differ is that *Lindé* consider a multiple shock framework, whereas *Favero* and *Hendry* have a very stylized single shock framework. Another important factor may be that *Favero* and *Hendry*'s analysis is restricted to a subset of the parameters in the monetary policy rule that is considered by *Lindé*, and the use of different data generating processes. More specifically, let the monetary policy *Taylor*-type interest rate rule be $R_t = R^* + \lambda_\pi (\pi_t - \pi^*) + \lambda_Y (\ln Y_t - \ln Y^*) + \rho_R R_{t-1} + \varepsilon_t$ where $\varepsilon \sim i.i.d.\ N\left(0, \sigma^2_{eR}\right)$. *Favero* and *Hendry* consider changes in R^* and σ^2_e one at a time, but not changes in λ_π, λ_Y or ρ_R.

Because the empirical relevance of the *Lucas critique* is still an open question and hard to test on data, I think it is useful, as a first step, to investigate how important the *Lucas critique* seems to be for a typical backward-looking model used in monetary policy analysis –the RUDEBUSCH and SVENSSON [1999] model. More specifically, it is of interest to examine, first, if changes in the monetary policy rule lead to changes in the reduced form parameters that are economically important for policy analysis, and second, if the observed changes in the reduced form parameters are significant in a statistical sense, and third, if the super-exogeneity test, used to identify the empirical relevance of the *Lucas critique,* has high power in small samples in order to assess if one possible explanation why ESTRELLA and FUHRER [1999] did not identified structural breaks was due to lack of power in the statistical testing.

My approach is to set up a modified version of COOLEY and HANSEN's [1995] real business cycle model with money.[4] The modification is that the model here includes government expenditures and a *Taylor*-type policy rule (see TAYLOR [1993]) for nominal money growth similar to the rule analyzed by MCCALLUM [1984, 1988]. The policy rule is then estimated using US data for the recent periods in office of Federal Reserve's chairmen *Arthur Burns, Paul Volcker*, and *Alan Greenspan* following JUDD and RUDEBUSCH [1998].[5]

By calibrating the equilibrium model with the different estimated monetary policy regimes, I study the properties of the reduced-form parameters in the RUDEBUSCH and SVENSSON [1999] model by means of simple Monte Carlo simulations. I also study the model implications for policy analysis of the changes in the model parameters. With policy analysis, I here mean the type of experiments that are often considered in the monetary policy literature, *ie*, the long run effects (impulse responses and volatilities of inflation and output) of changes in monetary policy rules. Recent work by LEEPER and ZHA [1999] suggest that for more modest policy interventions – 'within' a policy regime – the *Lucas critique* might be safely ignored. Finally, I study the small-sample properties of the super-exogeneity test.

The results in the paper are as follows. Firstly, it is shown that the reduced form parameters of the *Rudebusch* and *Svensson* model change in a statistically significant way when the monetary policy rule changes. Secondly, the changes in the parameters are not only important according to a statistical criteria, they are also very important from an economic point of view. For instance, in the aggregate supply curve, the coefficient for output varies between about 0 and 0.50 in the *Rudebusch* and *Svensson* model. The changes in individual parameters are also such that the quantitative implications of the backward-looking model as a whole are largely affected when there is a

4. The real business cycle (or equilibrium business cycle) literature initiated by LUCAS [1975], [1977], KYDLAND and PRESCOTT [1982] and others use models for policy analysis that are supposedly immune to the *Lucas critique* because they are equilibrium models with forward-looking behavior. However, later work has questioned the view that RBC-models with a representative agent are immune against the *Lucas critique*, see *eg,* GEWEKE [1985] and ALTISSIMO, SIVIERO and TERLIZZESE [2000].

5. JUDD and RUDEBUSCH [1998] start out by noting that there is instability in the Fed reaction function. They then find support for the hypothesis that the monetary policy rule has varied systematically with the different periods in office of Fed chairmen *Burns, Volcker,* and *Greenspan*. As in their analysis, the period with chairman *Miller* is omitted here because of his very short tenure.

regime shift. Thus, the *Lucas critique* is highly relevant when this type of model is used for policy analysis. The super-exogeneity test, however, does not have high power in small-sample to detect the empirical relevance of the *Lucas critique* in the model, confirming the results in LINDÉ [2001a].

The structure of the paper is as follows. In the next section, I introduce the monetary equilibrium model, and indicate how to compute the equilibrium. Estimation and calibration issues are addressed in Section 3. In Section 4, I present the RUDEBUSCH and SVENSSON [1999] backward-looking model. Next, in Section 5, results of the Monte Carlo simulations regarding the sensitivity of the backward-looking models to the *Lucas critique* are reported. Some results regarding the importance of the *Lucas critique* when using the backward-looking model in policy analysis is reported in Section 6. In Section 7, the properties of the super exogeneity test in small samples is examined. Finally, Section 8 concludes.

2 The Equilibrium Model

In this section, I describe and solve a slightly modified version of COOLEY and HANSEN's [1989, 1995] monetary equilibrium business cycle model. The model is a standard real business cycle model with some additional features. A stochastic nominal money supply interacts with a cash-in-advance technology and one-period nominal wage contracts, which creates short run real effects of nominal money supply shocks. As in COOLEY and HANSEN [1995], one period is one quarter.[6]

The difference between the model in this paper and the one in COOLEY and HANSEN [1995] is that the central bank is here assumed to use a policy rule when it decides on the nominal money supply growth in each period similar to that suggested by MCCALLUM [1984, 1988]. More specifically, the growth rate in nominal money supply in period t is assumed to follow a *Taylor*-type policy rule and depends on the output gap, the difference between actual and targeted inflation rate (hereafter named inflation gap), an uncontrollable shock, and the growth rate in nominal money in period $t-1$. This specification is intended to capture the real world phenomenon that central banks use money supply to affect inflation and output gaps, although they act gradually and do not have perfect control of the process. It is shown that this monetary policy rule for nominal money growth can be rewritten as a *Taylor* rule for the nominal interest rate.

In the model, I abstract from population and technological growth and represent all variables in *per capita* terms.

Finally, a notational comment; in the following, capital letters denote economy wide averages which the agent takes as given and small letters individual specific values which the agent internalizes.

6. I would like to emphasize that the qualitative aspects of the results in the paper are not at all dependent on whether I calibrate the model to match quarterly or yearly data.

2.1 An Equilibrium Monetary Business Cycle Model

Infinitely many identical infinitely lived agents maximize expected utility with preferences summarized by:

$$\mathrm{E}_0 \sum_{t=0}^{\infty} \beta^t u\left(c_{1t}, c_{2t}, h_t\right), \tag{1}$$
$$u\left(c_{1t}, c_{2t}, h_t\right) \equiv \alpha \ln(c_{1t}) + (1-\alpha)\ln\left(c_{2t}\right) - \gamma h_t$$

where c_{1t} is consumption of the 'cash-good' in period t, c_{2t} is consumption of the 'credit good', and h_t is the share of available time spent in employment which enters linearly in (1) because of the 'indivisible labor' assumption (see HANSEN [1985]). In (1), β is the subjective discount factor, γ the disutility the agent gets from working, while α reflects the trade-off between consumption of the cash and credit goods.

The flow budget constraint facing the agent is:

$$\begin{aligned} & c_{1t} + c_{2t} + i_t + \frac{m_{t+1}}{P_t} + \frac{b_{t+1}}{P_t} \\ & \quad = \left(\frac{W_t^c}{P_t}\right) h_t + R_t^K k_t + \frac{m_t}{P_t} + \left(1 + R_{t-1}\right)\frac{b_t}{P_t} + \frac{TR_t}{P_t} \end{aligned} \tag{2}$$

where i_t denotes the agent's investment, m_{t+1} and b_{t+1} the agent's holdings of nominal money and government bonds at the end of period t, P_t the aggregate price level, W_t^c the contracted nominal wage, R_t^K the gross real return on the capital stock k_t, R_{t-1} the nominal interest rate on government bonds between periods $t-1$ and t, and TR_t nominal lump-sum transfers (or taxes if negative) from the government.

The agent has the following cash-in-advance constraint for the cash-good c_{1t},

$$P_t c_{1t} = m_t + \left(1 + R_{t-1}\right) b_t + TR_t - b_{t+1} \tag{3}$$

which always holds with equality since the nominal interest rate will always be positive in this model.

The government's budget constraint is:

$$P_t G_t + TR_t = M_{t+1} - M_t + B_{t+1} - (1 + R_{t-1}) B_t \tag{4}$$

where G is exogenous public consumption expenditures, and M and B aggregate nominal money supply and government bonds. As in COOLEY and HANSEN [1995], I will assume that $B_t = 0$ for $t \geqslant 0$ and only use it to compute the nominal interest rate in the economy. By virtue of the logarithmic utility function in this model, it can be shown that the nominal interest rate in equilibrium is given by:

$$R_t = \frac{\alpha}{1-\alpha}\frac{C_{2t}}{C_{1t}} - 1 \tag{5}$$

where C_{1t} and C_{2t} are aggregate consumption of the cash and credit goods, respectively.

Government consumption, G, in (4) is assumed to be generated by the following stationary AR(1) process,

$$\begin{aligned} &\ln G_{t+1} = \left(1 - \rho^{\ln G}\right) \ln \bar{G} + \rho^{\ln G} \ln G_t + \varepsilon_{t+1}^{\ln G}, \\ &0 < \rho^{\ln G} < 1, \varepsilon^{\ln G} \sim i.i.d. N\left(0, \sigma_{\ln G}^2\right) \end{aligned} \tag{6}$$

Aggregate nominal money supply is assumed to evolve according to:

$$M_{t+1} = e^{\mu_t} M_t \tag{7}$$

where the growth rate in nominal money supply in period t, defined as $\Delta \ln M_{t+1}$ and denoted μ_t, is assumed to be determined by:

$$\begin{aligned} &\mu_t = \eta \mu_{t-1} - \lambda_\pi \left(\pi_t - \pi^*\right) - \lambda_Y \left(\ln Y_t - \ln Y^*\right) + \xi_t, 0 < \eta < 1, \\ &\xi \sim i.i.d.\ LogNormal, \mathrm{E}[\xi] = (1-\eta)\bar{\mu}, \mathrm{Var}(\xi) = \sigma_\xi^2 \end{aligned} \tag{8}$$

where π_t is defined as $\ln P_t - \ln P_{t-1}$, and λ_π and λ_Y measure how the central bank reacts to deviations in the inflation $\left(\pi_t - \pi^*\right)$ and the output gap $\left(\ln Y_t - \ln Y^*\right)$, respectively.[7] The implicit assumption underlying the specification in (8) is that the central bank tries to stabilize inflation and/or output, and one might think of (8) as an implementable monetary policy rule for a central bank which has been attached a conventional quadratic loss function in the inflation and output gaps. For simplicity, we will also set π^* and $\ln Y^*$ in (8) equal to steady state nominal money supply growth $(\bar{\mu})$ and log of output $(\ln \bar{Y})$, respectively. The error term, ξ, as can be thought of as policy shocks from the perspective of the private sector. By introducing the persistence component $\eta \mu_{t-1}$, it is also assumed that the central bank reacts gradually to shocks which hit the economy.

The policy rule in (8) is not optimal. One important reason for choosing it nevertheless, is that is possible to derive a standard *Taylor*-type rule (see TAYLOR [1993] and [1999] for the nominal interest rate within the equilibrium model given the functional form of (8). Log-linearizing (5), (3) and (19), and substituting these equations into (8), it is possible to derive:

7. Although, we assume that ξ is log normally distributed, we require that ξ has mean $(1-\eta)\bar{\mu}$, and variance σ_ξ^2 as seen in (8). By using that $\mathrm{E}[\xi] = e^{\mathrm{E}[\ln \xi] + \frac{1}{2}\mathrm{Var}(\ln \xi)}$ and that $\mathrm{Var}(\xi) = \mathrm{E}\left\{(\xi - \mathrm{E}[\xi])^2\right\} = \mathrm{E}\left[\xi^2\right] - [(1-\eta)\bar{\mu}]^2 = e^{2\mathrm{E}[\ln \xi] + \mathrm{Var}(\ln \xi)} - [(1-\eta)\bar{\mu}]^2$ since ξ is log-normally distributed, one can pin down the mean and the variance for $\ln \xi$ as $-\frac{1}{2}\ln\left(\sigma_\xi^2 + [(1-\eta)\bar{\mu}]^2\right)$ $+ 2\ln((1-\eta)\bar{\mu})$ and $\ln\left(\sigma_\xi^2 + [(1-\eta)\bar{\mu}]^2\right) - 2\ln((1-\eta)\bar{\mu})$ respectively.

$$
\begin{aligned}
R_t = &-\frac{\lambda_\pi \pi^* + \lambda_Y \ln Y^*}{\left(1-\bar{\hat{P}}\bar{G}\right)\kappa_3} + \frac{1+\lambda_\pi - \eta \mathrm{L}}{\left(1-\bar{\hat{P}}\bar{G}\right)\kappa_3}\pi_t \\
&+ \frac{\lambda_Y + \bar{\hat{P}}\bar{G}\left(1-\eta \mathrm{L}\right)\left(1-\mathrm{L}\right)\frac{\bar{Y}}{\bar{C}}}{\left(1-\bar{\hat{P}}\bar{G}\right)\kappa_3}\ln Y_t + \left(1+\eta-\eta \mathrm{L}\right) R_{t-1} + \varepsilon_t^R
\end{aligned} \tag{9}
$$

where $\varepsilon_t^R \equiv \left[\frac{-\xi_t + \left(\frac{\bar{C}-\bar{G}}{\bar{C}}\right)\bar{\hat{P}}\bar{G}(1-\eta \mathrm{L})(1-\mathrm{L})\ln G_t - \delta \frac{\bar{K}}{\bar{C}}\bar{\hat{P}}\bar{G}(1-\eta \mathrm{L})(1-\mathrm{L})\ln I_t}{\left(1-\bar{\hat{P}}\bar{G}\right)\kappa_3}\right]$,

$\kappa_3 = \frac{\bar{C}-\bar{C}_1}{\bar{C}} > 0$ (bar denotes steady state values) and L is the lag operator. Thus, it is possible to transform the *Taylor* inspired rule for nominal growth μ to a 'standard' rule for the nominal interest rate R in the model.[8]

The production function is assumed to have constant returns to scale and be of *Cobb-Douglas* type:

$$
Y_t = e^{\ln Z_t} K_t^{\theta} H_t^{1-\theta} \tag{10}
$$

where K_t and H_t are aggregate (average) capital stock and hours worked, respectively, and Z_t the technology level which is assumed to follow a stationary AR(1)-process (in natural logs):

$$
\ln Z_{t+1} = \rho^{\ln Z} \ln Z_t + \varepsilon_{t+1}^{\ln Z}, \varepsilon^{\ln Z} \sim i.i.d.N\left(0, \sigma_{\ln Z}^2\right). \tag{11}
$$

Individual and aggregate investment in period t produces productive capital in period $t+1$ according to:

$$
k_{t+1} = (1-\delta) k_t + i_t \tag{12}
$$

and,

$$
K_{t+1} = (1-\delta) K_t + I_t, \tag{13}
$$

where δ is the rate of capital depreciation.

The perfect competition zero profit maximizing conditions for the representative firm are:

$$
W_t^c = (1-\theta) e^{\ln Z_t} \left(\frac{K_t}{H_t}\right)^{\theta} P_t \tag{14}
$$

and,

$$
R_t^K = \theta e^{\ln Z_t} \left(\frac{K_t}{H_t}\right)^{\theta - 1}. \tag{15}
$$

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8. One potential problem with interpreting (9) as a standard *Taylor*-type rule is that the residual is presumably correlated with the arguments. However, this is not a specific issue for the model at hand, but rather a general problem that also has been acknowledged by many researchers, see *eg*, CLARIDA, GALI and GERTLER [1999] and MCCALLUM and NELSON [1999].

The nominal wage W_t^c is assumed to be set at the end of period $t-1$ (see Cooley and Hansen [1995] for further details on the nominal wage arrangement) as:

$$\ln W_t^c = \ln(1-\theta) + \mathrm{E}_{t-1}\ln Z_t + \theta\left(K_t - \mathrm{E}_{t-1}H_t\right) + \mathrm{E}_{t-1}P_t \tag{16}$$

where E_{t-1} denotes the conditional expectations operator on all relevant information in period $t-1$.[9] Moreover, households are assumed to transfer to the firms the right to choose aggregate hours worked in period t, H_t, to equate the marginal product of labor to the contracted wage rate. If we combine (14) and (16) in natural logarithms, using (11) below, we obtain:

$$\ln H_t = \mathrm{E}_{t-1}\ln H_t + \frac{1}{\theta}\left(\ln P_t - \mathrm{E}_{t-1}\ln P_t\right) + \frac{1}{\theta}\varepsilon_t^{\ln Z}. \tag{17}$$

Similarly, one realizes that the natural logarithm of h_t for an agent in equilibrium is given by:

$$\ln h_t = \mathrm{E}_{t-1}\ln H_t + \frac{1}{\theta}\left(\ln P_t - \mathrm{E}_{t-1}\ln P_t\right) + \frac{1}{\theta}\varepsilon_t^{\ln Z}. \tag{18}$$

The aggregate resource constraint:

$$Y_t = C_{1t} + C_{2t} + I_t + G_t \equiv C_t + I_t + G_t \tag{19}$$

also holds in every period where C_t is total consumption.

2.2 Equilibrium in the Model

The equilibrium in the model consists of a set of decision rules for the agents $\ln k_{t+1} = k\left(\mathbf{S}_t, \ln k_t, \ln \hat{m}_t\right)$, $\ln \hat{m}_{t+1} = \hat{m}\left(\mathbf{S}_t, \ln k_t, \ln \hat{m}_t\right)$ and $\ln h_t = h\left(\mathbf{S}_t, \ln k_t, \ln \hat{m}_t\right)$, and a set of aggregate decision rules $\ln K_{t+1} = K(\mathbf{S}_t)$, $\ln H_t = H(\mathbf{S}_t)$, $\ln \hat{P}_t = \hat{P}(\mathbf{S}_t)$ where $\mathbf{S}_t = \left[\ln Z_{t-1}, \varepsilon_t^{\ln Z}, \mu_{t-1}, \xi_t, \ln G_t, \ln K_t, \ln \hat{P}_{t-1}\right]'$ such that; *(i)* agents maximize utility, *(ii)* firms maximize profits, and *(iii)*, individual decision rules are consistent with aggregate outcomes. Equilibrium condition (iii) implies that $k(\mathbf{S}_t, \ln K_t, 1) = K(\mathbf{S}_t)$, $\hat{m}(\mathbf{S}_t, \ln K_t, 1) = 1$, and $h(\mathbf{S}_t, \ln K_t, 1) = H(\mathbf{S}_t)$ for all $\mathbf{S}_t$.

In Appendix A, I describe how to compute the equilibrium in this model.

3 Estimation and Calibration

The parameters in the equilibrium model are determined in two ways. About half of the parameters (η, $\bar{\mu}$, σ_ξ^2, λ_π, λ_Y, $\rho^{\ln G}$, $\sigma_{\ln G}^2$ and $\bar{g} \equiv \frac{\bar{G}}{\bar{Y}}$) are

9. Note that $\ln K_t$ is known at the end of period $t-1$ through the equilibrium decision rules (see Appendix A).

estimated on US data 1960-1997 with Instrumental Variables method (IV) and Ordinary Least Squares (OLS). The other half of the parameters (α, β, δ, γ, θ, $\rho^{\ln Z}$ and $\sigma^2_{\ln Z}$) are adapted from COOLEY and HANSEN [1995], and chosen so that the model's steady state properties are consistent with US growth facts.

To estimate the parameters η, $\bar{\mu}$, σ^2_{ξ}, λ_{π}, and λ_Y in the monetary policy rule (8) for different Fed chairmen periods, I collected quarterly data on real gross national product *per capita* in natural logarithms ($\ln Y_t$), growth rate in nominal money supply (μ_t) and the inflation rate in the consumer price index (π_t). To compute measures of $\ln Y_t - \ln Y^*$ and $\pi_t - \pi^*$, I simply filtered the series for output and inflation rate with the *Hodrick-Prescott* (H-P) filter (see HODRICK and PRESCOTT [1997]).[10] It is standard to use H-P filtered output as measure of the output gap, but is less clear how to compute an appropriate measure of π^* from historical data as discussed by JUDD and RUDEBUSCH [1998].[11] Since the model does not distinguish between money controlled by the Fed (the monetary base, M0) and money used in private transactions (M2), I compromise between them and use M1 as a measure of money as in COOLEY and HANSEN [1989, 1995]. The reason for estimating with IV rather than Ordinary Least Squares (OLS), is that OLS is likely to be a biased and inconsistent estimator due to the fact that we may have contemporaneous correlation between the error term and the regressors in (8). In terms of the theoretical model used in this paper, there will, *via* the equilibrium decision rules, be a positive correlation between the error term ξ_t and the regressors π_t and $\ln Y_t$ in (8). As instruments in the estimation, I therefore use $\left(\ln Y - \ln Y^*\right)_{t-1}$, μ_{t-1} and $\left(\pi - \pi^*\right)_{t-1}$ which are uncorrelated with the error term ξ_t in (8). In addition to that, the estimated λ_{π} and λ_Y will be correlated in general, why inference must be conducted with great care.

I estimate the monetary policy rule (8) with IV for the whole sample period (1970Q1-1997Q4), for chairman *Burns*' office period (1970Q1-1978Q1), chairman *Volckers*' office period (1979Q3-1987Q2), chairman *Greenspans*' office period (1987Q3-1997Q4), and omit chairman *Miller* as in JUDD and RUDEBUSCH [1998] because of his short tenure. The results of the estimations are reported in Table 1 (a constant is included in the regressions but is omitted from the table).

The D-W and *Breusch-Godfrey* statistics indicates presence of positive autocorrelation in the regressions, suggesting difficulties to interpret the significance levels of the estimates of η, λ_{π} and λ_Y. However, use of the asymptotic χ^2-distribution for the *Breusch-Godfrey* test is very likely to yield an oversized test (*ie*, an exaggerated probability of rejecting a true null hypothesis of no autocorrelation) for sample sizes as small as the present ones.

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10. I use the common value 1,600 (quarterly data) for the smoothness coefficient λ in the H-P filter. See Appendix B for a detailed description of the raw data and data transformations.

11. Although my approach regarding $\pi - \pi^*$ appears to be as good as any other considerable alternative (see JUDD and RUDEBUSCH [1998]), I have nevertheless experimented with other measures (such as the average inflation rate during a given chairmen's term), but it did not have any impact on the conclusions drawn in the paper.

TABLE 1
IV Estimation Results for the Monetary Policy Rule (8)

Estimation period	$\hat{\eta}$	$\hat{\lambda}_\pi$	$\hat{\lambda}_Y$	$\hat{\sigma}_\xi$	$\bar{R}^2$	D-W	B-G χ^2 (4)	J-B	T
Whole	0.931	0.181	0.083	0.0138	0.89	1.39	33.33	0.400	112
	(0.033)	(0.093)	(0.092)				(0.000)	(0.819)	
Burns	0.515	0.182	−0.166	0.0073	0.73	1.96	14.17	0.715	33
	(0.151)	(0.087)	(0.148)				(0.007)	(0.699)	
Volcker	0.717	0.377	−0.137	0.0158	0.73	1.59	11.86	1.289	32
	(0.116)	(0.203)	(0.249)				(0.019)	(0.525)	
Greenspan	0.919	0.532	0.013	0.0153	0.91	0.98	17.37	1.249	42
	(0.054)	(0.540)	(0.262)				(0.002)	(0.536)	

Note: Standard errors in parenthesis for $\hat{\eta}$, $\hat{\lambda}_\pi$ and $\hat{\lambda}_Y$, and p-values in parenthesis for the Breusch-Godfrey autocorrelation test (null hypothesis no autocorrelation up to 4 lags) and the Jarque-Bera normality test (null hypothesis normally distributed residuals). A constant, $(\ln Y - \ln Y^*)_{t-1}$, μ_{t-1} and $(\pi - \pi^*)_{t-1}$ have been used as instruments. T denotes the number of observations in the regressions.

Simulated small sample adjusted p-values for the *Breusch-Godfrey* test confirm the size problem, and result in a non-significant autocorrelation effect.[12] The p-value (*ie*, the nominal significance level) for a joint F-test of the null hypothesis $H_0 : \eta = \lambda_\pi = \lambda_Y = 0$ is around 0 for all regimes, as indicated by the model's satisfactory fit during the subsamples (measured by the multiple correlation coefficient $\bar{R}^2$). Also, as carefully documented in LINDÉ [2001b], the estimated parameter changes in the monetary policy rule between the regimes *Burns*, *Volcker* and *Greenspan* are highly significant. So, although the estimates seem imprecisely estimated, the estimated rules are significantly different as in JUDD and RUDEBUSCH [1998] which together with the observation that they produce non-local dynamics in the equilibrium model are sufficient observations for the analysis in this paper. Not surprisingly, we get the highest estimates of λ_π during chairmen *Volcker* and *Greenspan* periods of office, and the lowest for chairman *Burns*.

To examine if these parameter changes are in line with experiments conducted with interest rate rules, we insert the estimates of η, λ_π and λ_Y into the *Taylor* rule for the nominal interest rate in (9). It is then easy to verify that the resulting parameter changes in the *Taylor* rule for the nominal interest rate are well in line with typical parameter experiments considered in the interest rate rule literature.

To estimate $\rho^{\ln G}$ and $\sigma^2_{\ln G}$ in (6), I collected quarterly data series on real government expenditures on consumption and investment per capita in natural logarithms, and filtered the series with the *Hodrick-Prescott* (H-P) filter (see

12. The small sample adjusted B-G test statistics have been computed by: *(i)* estimating a VAR-model with 6 lags including the variables $\ln Y_t - \ln Y^*$, $\pi_t - \pi^*$ and μ_t (using likelihood ratio, autocorrelation and normality tests to determine the lag order) on data for the different periods; *(ii)* using the estimated VAR-model as a data generating process to simulate artificial samples of data; *(iii)* estimating the regression (8) on the simulated data with IV and then computing the associated B-G statistics. From the resulting distributions of B-G statistics, the small sample adjusted p-values are computed as the fraction of simulated B-G statistics that are larger than the estimated ones. The resulting p-values by this procedure are 0.683, 0.613, 0.663 and 0.458 for 'Whole sample', *Burns*, *Volcker* and *Greenspan* regimes respectively.

HODRICK and PRESCOTT [1997] to get a measure of $\ln G_t$. I then estimated (6) on the sample period $1960Q1$ to $1997Q4$ with OLS with the result (standard error in parenthesis):

$$\ln G_t = \underset{(0.0485)}{0.8019} \ln G_{t-1} + \hat{\varepsilon}_t^{\ln G},\ \hat{\sigma}_{\ln G} = 0.009844, \text{D-W} = 1.93, \bar{R}^2 = 0.64, \text{B-G}\chi^2(4) = \underset{(0.0003)}{20.912}. \tag{20}$$

Although the D-W statistic is satisfactory, the *Breusch-Godfrey* test for autocorrelation in (20) is significant and shows tendencies of positive autocorrelation. But when I augmented the estimation with more lags on the dependent variable to remove this autocorrelation, I found that the estimated parameters were largely unaffected.

To compute values for $\bar{\mu}$ and $\bar{g}$, I took averages of quarterly nominal money growth and the ratio of government expenditures to gross national product to get 0.01310 and 0.21038 respectively.

γ is calibrated in the same way as in COOLEY and HANSEN [1995] and set so that hours worked as share of available time in steady state, $\bar{H}$, equals 0.30.[13]

The remaining parameters are directly taken from *Cooley* and *Hansen*; α is set to 0.84, β is set to 0.989, δ is set to 0.019, θ is set to 0.40 and $\rho^{\ln Z}$ and $\sigma_{\ln Z}$ are set to 0.95 and 0.00721 respectively.

4 The Backward-Looking Model

In this Section, I will briefly present the backward-looking model that I have chosen to study – the RUDEBUSCH and SVENSSON [1999] model. The *Rudebusch* and *Svensson* model, which draws on the SVENSSON [1997] model, is intended to be a reasonable approximation of reality. It contains much richer dynamics than the simple *Svensson* model by allowing for four lags of inflation in the AS curve and two lags of output in the AD curve.

4.1 The *Rudebusch* and *Svensson* Model

The RUDEBUSCH and SVENSSON [1999] model is similar to many other models used for monetary policy analysis. It consists of aggregate supply (AS) and aggregate demand (AD) equations relating the output gap (the percentage deviation of output from its steady state level) and the inflation rate to each other and a monetary policy instrument, the (short-run) interest rate. Formally, the model economy is described by the following equations:

13. Formally, we have that $\gamma = \frac{(1-\theta)\left(\alpha\beta\frac{1}{e^{\bar{\mu}}}+1-\alpha\right)\left(\frac{1-\beta(1-\delta)}{\beta\theta}\right)}{\bar{H}\left((1-\bar{g})\left(\frac{1-\beta(1-\delta)}{\beta\theta}\right)-\delta\right)}$, which can be used to compute $\gamma = 3.404$ given the values for the other parameters. This value is higher than *Cooley* and *Hansen*'s value (2.53) since I have government expenditures in the model.

$$
(21) \qquad \begin{aligned} \pi_t &= \sum_{j=1}^{4} \alpha_{\pi,j}\pi_{t-j} + +\alpha_y y_{t-1} + \varepsilon_t^{\pi}, \\ y_t &= \beta_{y,1}y_{t-1} + \beta_{y,2}y_{t-2} + \beta_r \sum_{j=1}^{4} \frac{1}{4}(i-\pi)_{t-j} + \varepsilon_t^{y}. \end{aligned}
$$

In (21), the first equation is the AS curve (or *Phillips* curve), where the (annualized) inflation rate π depends on past inflation rates, the output gap in the previous period and an exogenous supply shock ε^{π} (i.i.d. with zero mean and variance σ_{π}^2). The second equation in (21) is the AD curve, where the output gap y_t is related to past output gaps y_{t-1} and y_{t-2}, the average *ex post* real interest rate in the four previous periods, $\sum_{j=1}^{4} \frac{1}{4}(i-\pi)_{t-j}$, and an exogenous demand shock ε_t^{y} (i.i.d.with zero mean and constant variance). The central bank, which is assumed to control the nominal interest rate i_t, thus affects the inflation rate with a two period lag. The monetary transmission mechanism is *via* output to the inflation rate. In the *Rudebusch* and *Svensson* framework, the sum of the estimated $\alpha_{\pi,j}$'s is restricted to equal 1 to get an accelerationist *Phillips* curve where long-run monetary neutrality holds.

Rudebusch and *Svensson* estimate (21) on quarterly US data for the sample period 1961Q1 to 1996Q2. They cannot reject the hypothesis that $\sum_{j=1}^{4} \alpha_{\pi,j}$ equals 1, so they maintain that assumption throughout their analysis. But in the model framework here –when we have the equilibrium model as a data generating process– this restriction only received econometric support in the estimations below for the monetary policy rule estimated on the 'whole sample' period, so I therefore only imposed this restriction for that rule and consequently allow for $\sum_{j=1}^{4} \alpha_{\pi,j}$ to differ from 1 for the rules estimated for *Burns*, *Volcker* and *Greenspan*. Although, *Rudebusch* and *Svensson* do not explicitly report any relevant statistics regarding the properties of ε^{π} and ε^{y}, they test for structural stability in the equations and cannot reject the null hypothesis of no instability.

Rudebusch and *Svensson* measure the inflation rate π_t, the output gap y_t and the *ex post* real interest rate $(i-\pi)_t$ in the following way. To get a measure of π, they compute $400\left(\ln p_t - \ln p_{t-1}\right)$ where p is the quarterly chain-weighted GDP price index. y_t is measured as the percentage gap between real output and potential output $100\left((Y_t - Y_t^*)/Y_t^*\right)$. The *ex post* real interest rate (in period t) included in the AD curve is measured as $\frac{1}{4}\sum_{j=1}^{4}(i-\pi)_{t-j}$, where i is the average quarterly federal funds rate and π is the inflation rate defined previously. All variables are then demeaned prior to estimation of the model economy; hence no constants are included in the regressions.

Assuming a quadratic objective function for the central bank over inflation and output (for example, $L_t \equiv \pi_t^2 + \lambda y_t^2 + \nu \Delta i_t^2$), it is possible solve the central bank's problem subject to the estimated model economy summarized by (21). The resulting decision rule for the nominal interest rate as a (linear) function of current inflation rate, output gap and lagged nominal interest rate are then used along with the estimated model economy to conduct policy analysis.[14]

5 Parameter Stability in the Backward-Looking Model

In this section, I present the results of some simulation experiments designed to examine how robust the parameters in the backward-looking model are when there is a change in the monetary policy rule. I will discuss this issue from both a statistical as well as an economic point of view. For further reference, I report some stability test results for the monetary policy rule. In the next subsection, I also present and motivate how the experiments have been carried out.

5.1 Testing Strategy

To investigate whether the *Lucas critique* seems to be significant in a statistical sense for the backward-looking model, I have simulated the equilibrium model for the estimated monetary policy rules for nominal money growth and estimated the backward-looking model (21) and the monetary policy rule (8) on the simulated data.

The procedure in the simulations has been as follows:

1. Generate an artificial data set by simulating the equilibrium model; for T periods under the assumption that the monetary policy rule changes completely unexpectedly after $T/2$ periods from one regime to another (for example, from *Burns* to *Volcker* and *Burns* to *Greenspan*).[15]

2. Estimate (21) and (8) with OLS on the first $1,...,T/2$ observations in the simulated sample. Denote the estimated parameter vectors $\hat{\beta}_{AS}$ (for aggregate supply), $\hat{\beta}_{AD}$ (aggregate demand) and $\hat{\beta}_{PR}$ (monetary policy rule) respectively.

3. Estimate (21) and (8) with OLS on the last $T/2+1,...,T$ observations in the simulated sample. Denote the estimated parameter vectors $\hat{\alpha}_{AS}$, $\hat{\alpha}_{AD}$ and $\hat{\alpha}_{PR}$ respectively.

14. Of course, the backward-looking model cannot be used for welfare analysis, since the connection between the typical loss function used in this literature and the social welfare function is not clear. Therefore, the optimization-based approach in *eg*, ROTEMBERG and WOODFORD [1997] has the advantage that welfare effects of alternative monetary policy rules can be evaluated.

15. The simulations are made in the GAUSS programming language, using the random number generator RDND with RDNDSEED set to $159425 + iter$ for $iter = 1,2,...,N$. To get a stochastic initial state in each simulation, the model is simulated for $T + 100$ periods, where the first 100 periods are then discarded.

4. **4.** Use a version of the F-test, often called the CHOW [1960] breakpoint test, to examine if the null hypotheses:

$$H_0 : \boldsymbol{\alpha}_{AS} = \boldsymbol{\beta}_{AS},$$
$$H_0 : \boldsymbol{\alpha}_{AD} = \boldsymbol{\beta}_{AD},$$
$$H_0 : \boldsymbol{\alpha}_{PR} = \boldsymbol{\beta}_{PR},$$

and,

$$H_0 : (\boldsymbol{\alpha}_{AS} = \boldsymbol{\beta}_{AS})\text{and}(\boldsymbol{\alpha}_{AD} = \boldsymbol{\beta}_{AD})$$

are rejected on appropriate significance levels.

5. Repeat Step 1 to Step 4 many (N) times to compute probabilities for how often the null hypotheses are maintained for the given significance level.

6. To get correct significance levels, Step 1 to Step 5 above are carried out twice. In the first round, small sample critical values are computed under the (true) null hypotheses $H_0 : \boldsymbol{\alpha}_{AS} = \boldsymbol{\beta}_{AS}$, $H_0 : \boldsymbol{\alpha}_{AD} = \boldsymbol{\beta}_{AD}$ and $H_0 : \boldsymbol{\alpha}_{PR} = \boldsymbol{\beta}_{PR}$ (that is, compute the distribution of F-statistics although there has been no regime shift). In the second round, these adjusted critical values are used in the F-test.

7. Now, if the computed probabilities in Step 5 (in the second round) of rejecting parameter stability are higher/lower than the given significance levels, the *Lucas critique* is/is not relevant in this model in a statistical sense.

The critical assumptions in Step 1 to Step 6 are clearly made in Step 1 to 3, and I would like to briefly comment on them. First, I have chosen to change monetary policy regime in the middle of the sample. The motivationbehind this choice is that it gives the highest possible power in the testing. Secondly, I have chosen to model the once and for all change in monetary policy regime as a completely unexpected shift in the estimated monetary policy rule, where I let the economy bring the state vector from the last period in the previous regime (period $T/2$) to the first period in the new regime (period $T/2+1$). Third, it is assumed that the monetary regime is perfectly credible and expected to last forever. By this procedure, I implicitly assume a first order *Markov* chain for the different monetary policy regimes, where I let the diagonal elements in the transition matrix approach unity. The second and third assumptions are very convenient since they allows me to use the same decision rules for the first $T/2$ periods, and then, change to new decision rules in the beginning of period $T/2+1$ for the remaining $T/2$ periods. Finally, I have chosen to use OLS as the estimation method – although it can be argued that it is an inconsistent estimator here – since it is a widely used method. I have made some experiments with a consistent estimator (the IV method), but the results were largely unaffected.

5.2 **Results**

The results of this exercise for the different estimated monetary policy rules for sample size $T = 200$ (corresponding to 50 years of quarterly data), are provided in Table 2. In the estimations, all the variables involved in the

TABLE 2

F-test Probabilities for Rejecting the Null Hypothesis or Parameter Stability in the RUDEBUSCH and SVENSSON [1999] Model in (21) at Various Significance Levels

	Significance level 10 percent				Significance level 5 percent			
	Comparison regime							
	WS	B	V	G	WS	B	V	G
Benchmark regime	*The aggregate supply function;* $H_0 : \boldsymbol{\alpha}_{AS} = \boldsymbol{\beta}_{AS}$							
Whole sample (WS)	0.100	0.851	0.749	0.691	0.050	0.766	0.653	0.576
Burns (B)	0.725	0.100	0.087	0.539	0.562	0.05	0.054	0.341
Volcker (V)	0.731	0.154	0.100	0.521	0.499	0.056	0.050	0.286
Greenspan (G)	0.735	0.739	0.563	0.100	0.594	0.576	0.427	0.050
Benchmark regime	*The aggregate demand function*; $H_0 : \boldsymbol{\alpha}_{AD} = \boldsymbol{\beta}_{AD}$							
Whole sample	0.100	0.426	0.360	0.140	0.050	0.262	0.216	0.065
Burns	0.294	0.100	0.106	0.145	0.168	0.050	0.060	0.074
Volcker	0.238	0.104	0.100	0.116	0.109	0.044	0.050	0.051
Greenspan	0.154	0.318	0.256	0.100	0.084	0.200	0.161	0.050
Benchmark regime	*Either AS-or AD-curve;* $H_0 : (\boldsymbol{\alpha}_{AS} = \boldsymbol{\beta}_{AS})$ *and* $(\boldsymbol{\alpha}_{AD} = \boldsymbol{\beta}_{AD})$							
Whole sample	N.C.	0.905	0.826	0.731	N.C.	0.820	0.717	0.600
Burns	0.825	N.C.	0.149	0.618	0.653	N.C.	0.082	0.384
Volcker	0.811	0.212	N.C.	0.591	0.566	0.079	N.C.	0.324
Greenspan	0.779	0.808	0.662	N.C.	0.627	0.642	0.501	N.C.
Benchmark regime	*The monetary policy rule;* $H_0 : \boldsymbol{\alpha}_{PR} = \boldsymbol{\beta}_{PR}$							
Whole sample	0.100	0.807	0.660	0.327	0.050	0.679	0.523	0.219
Burns	0.998	0.100	0.257	0.920	0.993	0.050	0.151	0.867
Volcker	0.915	0.086	0.100	0.709	0.854	0.039	0.050	0.623
Greenspan	0.268	0.460	0.379	0.100	0.167	0.352	0.277	0.050

Note: NC is shorthand notation for not computed. The CHOW [1960] statistic underlying the computation of the probabilities is defined as $\frac{\left(\hat{\sigma}_T^2 - \frac{T_1}{T}\hat{\sigma}_{T_1}^2 - \frac{T_2}{T}\hat{\sigma}_{T_2}^2\right)/k}{\left(\frac{T_1}{T}\hat{\sigma}_{T_1}^2 + \frac{T_2}{T}\hat{\sigma}_{T_2}^2\right)/(T-2k)}$ *and it follows the F-distribution with* $k, T-2k$ *degrees of freedom where k is the number of parameter restrictions that are being tested, T the total number of observations (*$T \equiv T_1 + T_2$*) and* $\hat{\sigma}_T^2$, $\hat{\sigma}_{T_1}^2$*, and* $\hat{\sigma}_{T_2}^2$ *denote the estimated standard error of the regression during both monetary regimes, the first monetary regime, and the second monetary regime, respectively. The small sample critical values are generated under the null hypothesis in a first round of* $N = 50{,}000$ *simulations, while the probabilities reported in the table are computed from a second round of simulations (again,* $N = 50{,}000$*), where the small sample critical values are used in the testing.*

TABLE 3

OLS Estimation of the Rudesbusch and Svensson Model (21) for Different Regimes on Simulated Data

Estimation output for the AS curve									
Regime	$\alpha_{\pi,1}$	$\alpha_{\pi,2}$	$\alpha_{\pi,3}$	$\alpha_{\pi,4}$	α_y	$\bar{R}^2$	D-W	$\hat{\sigma}$	B-G χ^2 (4)
Whole sample	0.559	0.293	0.129	0.019	0.052	0.77	1.99	3.46	3.87 (0.796)
Burns	0.062	0.133	0.062	0.041	0.496	0.36	2.03	4.47	1.68 (0.922)
Volcker	0.136	0.140	0.051	0.022	0.411	0.35	2.01	5.39	1.36 (0.956)
Greenspan	0.173	0.076	0.043	0.022	-0.003	0.10	2.00	2.65	5.14 (0.865)

Estimation output for the AD curve							
Regime	$\beta_{y,1}$	$\beta_{y,2}$	β_r	$\bar{R}^2$	D-W	$\hat{\sigma}$	B-G χ^2 (4)
Whole sample	0.8240	0.0990	-0.0146	0.81	2.01	2.24	5.10 (0.855)
Burns	0.4743	0.3319	0.0172	0.51	2.12	2.83	10.37 (0.488)
Volcker	0.4764	0.3267	-0.0405	0.50	2.11	3.32	10.44 (0.503)
Greenspan	0.6935	0.2141	-0.0136	0.76	2.03	2.00	4.83 (0.869)

Note: $\hat{\sigma}$ denotes standard error of regression in percent. The values within parentheses measure the likelihood that the computed test statistics are insignificant (significance level 5 percent) for the Breusch-Godfrey's χ^2-test (null hypotheses no 4th order autocorrelation). All the statistics reported are averages of $N = 50{,}000$ simulations of sample size $T = 200$.

regressions have been measured in precisely the same way as by *Rudebusch* and *Svensson*, whose measurement procedure was presented in Section 4.1.[16]

As seen in Table 2, the probabilities of rejecting the null hypothesis of parameter stability between regimes are clearly higher than the given significance levels in most cases. For the AS curve, we see that the probabilities of rejecting parameter stability are found to be low between the *Burns* and *Volcker* regimes and *vice versa* (0.054 and 0.056 respectively at the 5 percent level), indicating that the *Lucas critique* is not quantitatively important in for the AS curve in these cases in a statistically significant way. This is quite natural since we can see in Table 1 that the estimated monetary policy rules for *Burns* and *Volcker* are similar (lower η, negative λ_Y), which is also manifested by low probabilities for the monetary policy rule in Table 2 for these regime shifts. Turning to the AD curve, we find that the probabilities for rejecting parameter stability are in general lower than for the AS counterparts, implying that the AD curve is less sensitive to the *Lucas critique* than the AS curve. In

16. To be able to generate reliable small sample critical values under the null (when there is no regime shift) in the first round, the model has been simulated $N = 50{,}000$ times. Note that the probabilities in the diagonal (when there is no regime shift) for the AS- and AD-curves equal 0.10 (0.05) at the 10 (5) percent significance level since the same shock realizations have been used in the second round.

particular, the statistical significance of the *Lucas critique* between the *Burns* and *Volcker* regimes is again ambiguous. Looking at both, the parameter estimates in Table 1 and the probabilities in Table 2, one conclusion seems to be that, in particular, the parameters η and λ_Y are most important for the AS curve while η seems to be most important for the AD curve. Finally, it is important to note that the rather high probabilities reported in Table 2 do not imply that parameter stability tests are able to identify the regime shifts correctly, see Section 7. LINDÉ [2001b] also shows that when changing the monetary policy rule recursively from *Burns* → *Volcker* → *Greenspan* and use the same number of periods in each regime as in the data (that is, 33, 32 and 42, respectively, see Table 1), the parameter stability test used by ESTRELLA and FUHRER [1999] and RUDEBUSCH and SVENSSON [1999] is very unlikely to detect parameter instability in the backward-looking model.

However, the most interesting hypothesis to test - because both the AS and AD curve are used in policy analysis – is the null hypothesis of instability in either the AS or the AD curve. In Table 2, the results for this hypothesis clearly indicate that the parameters in the *Rudebusch* and *Svensson* model as a whole are not exogenous to the parameters in the monetary policy rule (and, thus, the central banks optimization problem).

From an economic point of view, it is of particular interest to examine whether this instability is economically meaningful. To shed light on this issue, Table 3 reports the OLS estimation results of the model (21) on simulated data.[17] Before turning to the results in Table 3, it should be noted that I have imposed the restriction $\sum_{j=1}^{4} \alpha_{\pi,j} = 1$ for the 'Whole sample' regime, since this restriction could not be rejected when estimating the AS-curve on simulated data.[18]

We see from Table 3 that the results in Table 2 are confirmed. From an economic point of view, the estimated parameters in the model, in particular for the AS curve, are heavily affected by changes in the monetary policy rule. The output parameter in the AS equation varies from about 0 to 0.50 and the real interest rate coefficient in the AD equation, although low in general, also alters in sign. From an econometric point of view, the; estimated equations often pass (in about 80-90 percent on average) statistical tests for autocorrelation, as indicated by the *Breusch-Godfrey* statistics for autocorrelation. The adjusted r-squares are also satisfactory in most cases, the exception being the low adjusted r-square for the AS curve for the *Greenspan* regime. The reason for this is the estimated high value for λ_π during the *Greenspan* regime, which drives down the autocorrelation (and the volatility) of the inflation rate. Consequently, all in all, an econometrician who evaluates this model from the data would not immediately reject it for statistical reasons in most cases.

To sum up, the simulation results for the null hypothesis of instability in either the AS or the AD curve clearly indicate that the parameters in the *Rudebusch* and *Svensson* model as a whole are not exogenous to the central

17. I have not been able to solve analytically for the reduced-form parameters as functions of the monetary policy rule parameters η, λ_π and λ_Y (and the other parameters in the equilibrium model). Therefore, I have been forced to estimate them on simulated data. I expanded the number of simulations until the (estimated) parameters coefficients converged in mean down to five digits, which required slightly less than 50,000 simulations for $T = 200$.

18. Note, however, that I do not impose this restriction when testing for parameter stability in Tables 2 and 4, because this restriction is not met in all simulations.

banks optimization problem (and, thus, the parameters in the monetary policy rule) using an equilibrium model with forward-looking agents as a data generating process. Thus, the *Lucas critique* applies strongly to this model according to the equilibrium model, and the super exogeneity test should be able to identify that.

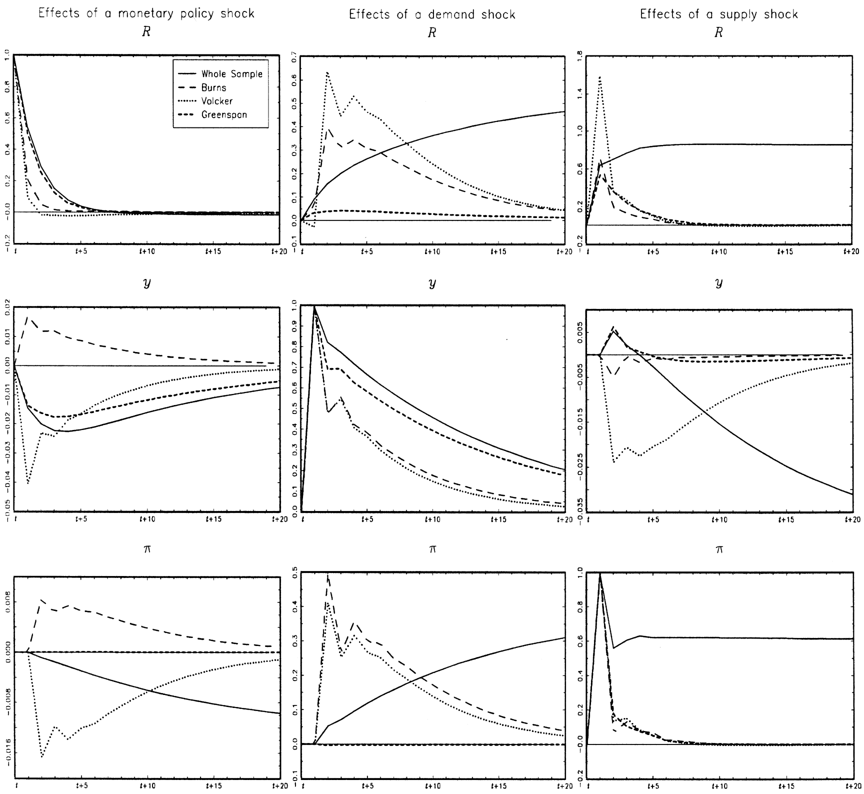
6 The Importance of the *Lucas critique* in Policy Analysis with the Backward-Looking Model

The results of the stability tests in the previous subsection do not necessarily imply that the *Lucas critique* is severe for the model considered as a system, they only imply the *Lucas critique* is highly relevant for individual parameters. When there is a monetary regime shift, there is a possibility that the resulting parameter changes in the AS- and AD-curves along with the interest rate *Taylor*-type rule (9) are such that the *Lucas critique* is not important for the model implications when it is considered as a system. This section examines this issue by computing impulse response functions using the estimated AS- and AD-curves from Table 3 along with estimates of the interest rate rule (9) .

Figure 1 shows the impulse response functions for the nominal interest rate R, the output gap y and the inflation gap π for the different shocks, in the model. The first column depicts the effects of a monetary policy, shock in (9) defined as $\varepsilon_t^R = 1$ and $\varepsilon_{t+j}^R = 0$ for $j > 0$, the second the effects of a demand shock to the AD-curve ($\varepsilon_t^y = 1$ and $\varepsilon_{t+j}^y = 0$ for $j > 0$), and the third the effects of a supply shock to the AS-curve ($\varepsilon_t^\pi = 1$ and $\varepsilon_{t+j}^\pi = 0$ for $j > 0$). From Figure 1, we see that the impulse response for the interest rate for a monetary policy shock are most persistent for the *Greenspan* and, in particular, the 'whole sample' regimes. Looking at the results in Table 1, this is also what one would expect since the estimated persistence coefficient η is highest for these regimes. For the backward-looking model estimated for the 'Whole sample' regime, the restriction that $\sum_{j=1}^{4} \alpha_{\pi,j} = 1$ has an important role, and if it is relaxed, the response of inflation to shocks is much less persistent. Another part of the explanation is that the estimated coefficient for the inflation rate in the *Taylor*-type rule (9) on simulated data is less than 1, when using the monetary policy rule (8) estimated on the 'whole sample' period. We also see that the effects on the output and inflation gap of a monetary policy shock are not too large. At least, not in comparison with demand shocks, which have much larger (and persistent) effects on inflation and output gaps. Interest rates are most sensitive to supply shocks, although the effects evaporate faster than demand shocks. A general impression is that the impulse responses in Figure 1 differ a lot, which is an indication that the

FIGURE 1

Impulse Responses in the Backward-Looking Model for Different Monetary Regimes



Lucas critique is also very relevant for the backward-looking model considered as a system.

In Figure 2, I have quantified the importance of the *Lucas critique* for the inflation rate and the output gap of a monetary policy shock as follows:

— Let $\mathbf{IRF}(\{AS_{\mathrm{WS}}, AD_{\mathrm{WS}}\} \,|\, TR_{\mathrm{WS}})$ denote the (true) impulse responses for the inflation rate and the output gap of a monetary policy shock when estimated of the AS- and AD-curves for the whole sample regime are being used along with estimates of the monetary policy rule (9) for the whole sample regime.

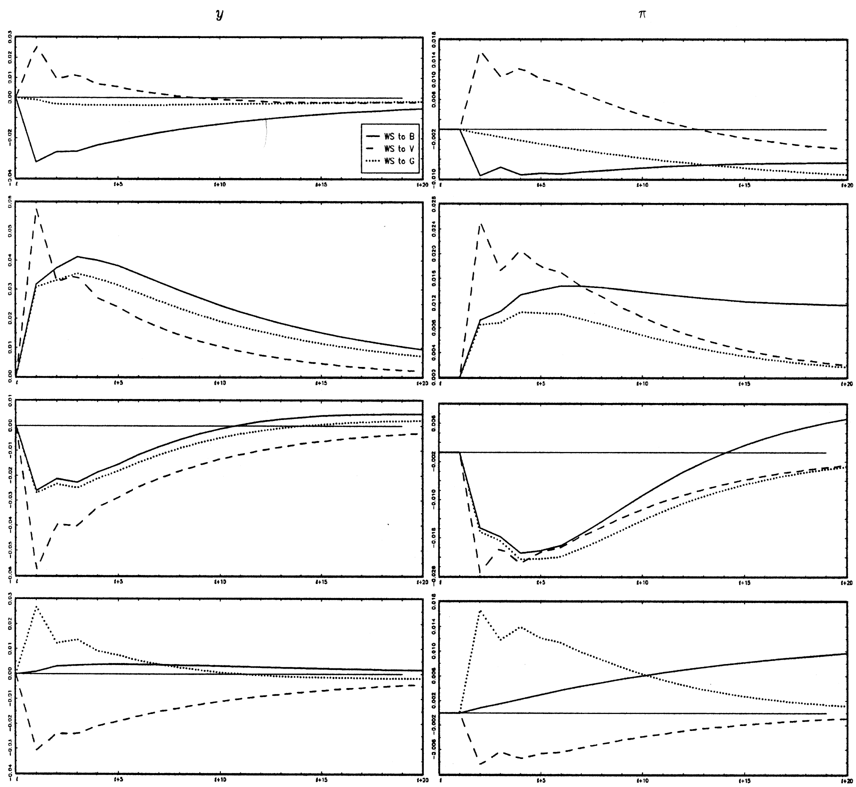
— Then $\mathbf{IRF}(\{AS_{\mathrm{WS}}, AD_{\mathrm{WS}}\} \,|\, TR_{\mathrm{B}}) - \mathbf{IRF}(\{AS_{\mathrm{B}}, AD_{\mathrm{B}}\} \,|\, TR_{\mathrm{B}})$ is the relevant quantitative measure of the *Lucas critique* when a policy regime shift from 'whole sample' to *Burns* is considered.

The assumptions behind this measure are that the policymaker knows the slopes of the AS- and AD-curves in the existing regime, and that the policymaker uses this information to compute the effects on the economy of applying alternative *Taylor*-type interest rate rules.

Figure 2 depicts this measure for the output and inflation gaps for all various combinations of regime shifts for a monetary policy shock. The diffe-

FIGURE 2

Deviations from True Impulse for Different Regime Shifts



rences are not large in absolute values, a result which we expected since the impulse responses for π and y for temporary monetary policy shock were found to be relatively small as can be seen in Figure 1. However, we clearly see that the differences in the impulse responses as share of the true impulse responses, *eg*, $(\mathbf{IRF}\,(\{AS_{\mathrm{WS}}, AD_{\mathrm{WS}}\}\,|TR_{\mathrm{B}}) - \mathbf{IRF}\,(\{AS_{\mathrm{B}}, AD_{\mathrm{B}}\}\,|TR_{\mathrm{B}})) \;\; / \,\mathbf{IRF}(\{AS_{\mathrm{B}}, AD_{\mathrm{B}}\}\,|TR_{\mathrm{B}})$, are very large (over 1 in most cases, indicating a 100 percent deviation in impulse response functions or more) in almost every case. This verifies the quantitative importance of the *Lucas critique* in the backward-looking model, at least on longer horizons when the agents have sufficient information to understand that a regime shift has occurred.[19]

19. The quantitative long-run importance of the *Lucas critique* can be verified by computing the impulse response functions for a permanent monetary policy shock, defined as $\varepsilon^{R}_{t+j} = 1$ for $j \geqslant 0$. Although not reported, it can be understood from the results in Figure 2 that the differences becomes very large over time (after 1 year, say).

7 Testing for Super Exogeneity in the Backward-Looking Model

In this section, we investigate the small-sample properties of the super-exogeneity test developed by ENGLE *et al*. [1983] to detect a monetary regime shift in the reduced-form parameters in the backward-looking model. The test implemented in this section corresponds precisely to Test I of super exogeneity which is described in detail by ERICSSON and IRONS [1995]. The purpose with the exercise is to investigate the power of the test to identify monetary policy regime shifts in the reduced-form model in the case when the *Lucas critique* actually is important which was established in Section 5.2. The testing procedure is the same as in LINDÉ [2001a], and involves the following steps:

1. Simulate the model for T periods under the assumption that the monetary policy rule changes completely unexpectedly after $T/2$ periods from one regime to another.

2. Estimate the AS- and AD-curves in (21) and monetary policy rule (8) with OLS on the first $1,...,T/2$ observations in the simulated sample. Denote the estimated parameter vectors $\hat{\boldsymbol{\beta}}_{AS}$, $\hat{\boldsymbol{\beta}}_{AD}$ and $\hat{\boldsymbol{\beta}}_{PR}$, respectively.

3. Estimate (21) and (8) with OLS on the last $T/2+1,...,T$ observations in the simulated sample. Denote the estimated parameter vectors $\hat{\boldsymbol{\alpha}}_{AS}$, $\hat{\boldsymbol{\alpha}}_{AD}$ and $\hat{\boldsymbol{\alpha}}_{PR}$, respectively.

4. Use a version of the F-test, the *Chow* breakpoint test, to examine if the null hypotheses $\boldsymbol{\alpha}_{AS}=\boldsymbol{\beta}_{AS}$ and $\boldsymbol{\alpha}_{AD}=\boldsymbol{\beta}_{AD}$ can be rejected when the null $\boldsymbol{\alpha}_{PR}=\boldsymbol{\beta}_{PR}$ is rejected at the 5 and 1 percent significance levels.

5. Repeat Steps 1–4 many (N) times to compute the power of the super exogeneity test, that is, probabilities for how often the null hypothesis $\boldsymbol{\alpha}_{PR}=\boldsymbol{\beta}_{PR}$ is rejected and the null hypotheses $\boldsymbol{\alpha}_{AS}=\boldsymbol{\beta}_{AS}$ and $\boldsymbol{\alpha}_{AD}=\boldsymbol{\beta}_{AD}$ are be rejected simultaneously for the given significance levels.

According to the *Lucas critique*, which we verified in Section IV was relevant in this model, the null hypothesis:

$$H_0\colon (\boldsymbol{\alpha}_{AS}=\boldsymbol{\beta}_{AS} \text{ and } \boldsymbol{\alpha}_{AD}=\boldsymbol{\beta}_{AD})\,|\boldsymbol{\alpha}_{TR}\neq\boldsymbol{\beta}_{TR}$$

is false, so if the computed probabilities in Step 5 of rejecting parameter stability in (8) and (21) at the same time are low, the power of the super-exogeneity test is low in small samples. On the other hand, if the computed probabilities are found to be high, then the power of the super-exogeneity test in small samples is satisfactory.[20]

20. Note that the power of a test is formally defined as $1-\beta$, where β, the type II error, is defined as $\Pr(\text{do not reject } H_0|H_0 \text{ is false})$. In our case, β is the probability of not rejecting stability in the AS- or the AD-curve, while rejecting stability in the monetary policy rule simultaneously.

TABLE 4

Power of the Super-Exogeneity Tests in Small Samples: Chow Test Probabilities of Rejecting Stability in the RUDEBUSCH and SVENSSON [1999] Model (21) when Stability in the Monetary Policy Rule (8) is Rejected at Various Significance Levels

		Comparison regime		
	Whole sample	Burns	Volcker	Greenspan
Benchmark regime		Significance level 5 percent		
Whole sample	NC	0.700	0.559	0.371
Burns	0.225	NC	0.296	0.263
Volcker	0.078	0.256	NC	0.099
Greenspan	0.159	0.413	0.301	NC
Benchmark regime		Significance level 1 percent		
Whole sample	NC	0.303	0.257	0.160
Burns	0.032	NC	0.060	0.055
Volcker	0.010	0.333	NC	0.025
Greenspan	0.027	0.088	0.043	NC

Note: NC is shorthand notation for not computed. Small sample critical values generated in Section 5 are used. The probability in each entry is formally defined as $Pr((F_{AS} > F_{0.95}^{AS} F_{AD} > F_{0.95}^{AD})|F_{PR} > F_{0.95}^{PR})$, where F_{PR} denotes the computed Chow statistic for the monetary policy rule, F_{AS} and F_{AD} the corresponding statistics for aggregate supply and demand, respectively, and $F_{0.95}^{AS}$, $F_{0.95}^{AD}$ and $F_{0.95}^{PR}$ the 95'th percentile in the simulated distributions for these equations under the null hypothesis. Under the null hypothesis, the Chow statistic follows the F-distribution with $k, T-2k$ degrees of freedom where k is the number of parameter restrictions and T is the number of observations. The probabilities reported are computed using $N = 50{,}000$ simulations.

In the *Chow* testing in Step 4, I use small-sample adjusted critical values generated in Section 5.2 rather than asymptotic values. The reason for doing so is to get the correct nominal significance level for each equation involved in the testing.[21]

The critical assumptions in Steps 1–5 are clearly made in Steps 1–3. The assumptions underlying Step 1 were discussed in Section 5.1, and since I already discussed them there, I will not spend time on discussing them here again. The assumptions made in Step 2 and 3 implies that both the breakpoint date and the policy rule is known to the econometrician. The setup used here thus gives the super-exogeneity test highest possible power in the testing.

The results of this exercise for the estimated monetary policy rules on the sample size $T = 200$ are provided in Table 4. As we can see from Table 4, the test has not high power to detect parameter instability, except for the regime shift between 'Whole Sample' and *Burns*. The average power estimate is 0.35 and 0.13 at the 5 and 1 percent significance levels, respectively. As explained in LINDÉ [2001a], the reason for lack of power for the super-exogeneity test in small samples is that there are other shocks that hit the economy at the same time as the monetary policy shocks (here, technology and government expen-

21. I would like to emphasize that the results in Table 4 are largely unaffected when increasing the sample size up to 400 observations, which implies a considerable amount of data (50 years) in each regime.

diture shocks), which makes it hard for the test to correctly identify the effects of the monetary regime shifts on the reduced-form parameters. It can be shown that if one conditions on the realizations on the other shocks that hit the economy, the power of the super-exogeneity test increase dramatically to about 0.90 on the 5 percent level.

8 Concluding Remarks

It seems as if the *Lucas critique* is potentially very important quantitatively, at least on longer horizons, for the type of backward-looking models that has been extensively used in the recent literature on monetary policy rules and, not least, for the type of questions that this literature addresses. However, the statistical test that has been widely used in the literature to examine the empirical relevance of the *Lucas critique*, the super-exogeneity test, is unfortunately found to have low power in small-samples. This finding offers one potential explanation of the parameter stability found in the RUDEBUSCH and SVENSSON [1999] model on real world data. More research, using other equilibrium models, *eg*, the limited participation model advocated by CHRISTIANO, EICHENBAUM and EVANS [1999], is warranted to examine the robustness of the results here which are based on the COOLEY and HANSEN [1995] monetary equilibrium model.[22] If the results here are valid, then the results from the large literature regarding the relative merits of alternative monetary policy rules using backward-looking models might be misleading. Thus, it seems to be a good research idea to check the robustness of the results in this literature in a dynamic general equilibrium framework.

One interesting extension of the paper would be to compute the welfare costs of the non-robustness of the RUDEBUSCH and SVENSSON [1999] model when it is used to design 'optimal' monetary policy rules. The method used in the paper would allow for such a calculation.[23]

One may argue that the results are driven by incorrectly specified monetary policy regimes (and thus rules). I have done some experiments with attaching the central bank a standard quadratic loss function in inflation and output volatility and solved numerically for the implied coefficients in the monetary

22. The other side of this argument is that there may be other backward-looking models that are less vulnerable to the *Lucas critique*. However, the impression of the results in this paper and in LINDÉ [2001a, 2001b] is that this possibility can be ruled out at least as long as the reduced-form models include both nominal and real variables when considering changes in monetary policy rules.

23. More specifically, one way to compute the welfare costs of using the backward-looking (BL-) model rather than the true (hypothesized) dynamic general equilibrium (GE-) model when analyzing optimal monetary policy rules is as follows: *(i)* Solve the GE-model for a given loss function L_A assigned to the monetary policy maker to get policy rule PR_{A^*}. *(ii)* Estimate BL-model on simulated data from the GE-model calibrated with PR_{A^*} or select parameters that give policy rule PR_{A^*} as the outcome in the BL-model when using loss function L_A. *(iii)* Change loss function to L_B and compute implied policy rule $PR_{\bar{B}}$ in the BL-model using the reduced-form parameters estimated in *(ii)*. *(iv)* Compute monetary policy rule PR_{B^*} in GE-model using loss function L_B. *(v)* Compute the welfare costs in the GE-model of using the rule $PR_{\bar{B}}$ rather than the optimal rule PR_{B^*}.

policy rule that are consistent with minimization for different relative weights on inflation and output volatility.[24] This approach, which is more standard in the literature, does not affect the qualitative conclusions in the paper.

Moreover, the paper may be criticized for using a monetary rule for nominal money growth rather than a rule for the nominal interest rate. But, I think that the results in this paper are robust against this critique for two reasons. First, it is possible to map analytically the rule for the nominal money growth to a 'standard' *Taylor*-type rule for the interest rate. Second, the parameter changes in the monetary policy rule for nominal money growth are not large in comparison with typical parameter changes in interest rate rules.

Another limitation of the paper is that I have implicitly assumed that the institutional design of the economy (that is, the one period nominal wage contacting assumption in the model) is unaffected by the monetary policy regime shifts. This assumption can be motivated by the real world observation that institutions, for example labor market arrangements, seems to change very slowly over time.[25] Consequently, the hope is that the effects of institutional changes not accounted for in the analysis are of 'second order', while the effects of monetary policy regime shifts examined in this paper are of 'first order'. ▼

24. The loss function (LF) apopted is $L_t = \frac{1}{2}\left[(\pi_t - \pi^*)^2 + \lambda\,(\ln Y_t - \ln Y^*)^2 + \nu\,(\Delta i_t)^2\right]$ and the coefficients η, λ_π and λ_Y in the monetary policy rule (8) are chosen so that they are consistent with minimization of LF for values of λ equal to 0, 0.2, 1 and 5 given $\nu = 1$, as in RUDEBUSCH and SVENSSON [1999].

25. If a sufficiently long period is covered (50 years or so), the effects of institutional changes may be larger; see, for instance, FREGERT and JONUNG [1998].

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APPENDICES

A. *Computation of Equilibrium*

In order to make all variables in the model above converge to a (constant)\ steady state, I transform the nominal variables by dividing m_{t+1} and P_t with M_{t+1}, and m_t with M_t. If we introduce the notation:

$$\hat{m}_{t+s} \equiv \frac{m_{t+s}}{M_{t+s}} \text{ and } \hat{P}_{t+s} \equiv \frac{P_{t+s}}{M_{t+s+1}}$$

and use the transformations to rewrite the equations (2), (3), (4), (8), (17) and (18), the representative agent's optimization problem can, following HANSEN and PRESCOTT [1995], be expressed as the recursive dynamic programming problem:

$$\text{(A1)} \quad \begin{aligned} V\left(\mathbf{S}_t, \hat{m}_t, k_t\right) \equiv & \max_{\{\hat{m}_{t+1}, h_t, k_{t+1}\}} [\alpha \ln (c_{1t}) \\ & + (1-\alpha) \ln\ (c_{2t}) - \gamma h_t + \beta \mathrm{E}_t V\left(\mathbf{S}_{t+1}, \hat{m}_{t+1}, k_{t+1}\right)] \\ & \quad s.t.\ (10), (6), (11), \end{aligned}$$

$$\begin{aligned} c_{1t} &= \tfrac{\hat{m}_t + e^{\mu_t} - 1}{e^{\mu_t} \hat{P}_t} - G_t, \\ c_{2t} &= (1-\theta) e^{\ln Z_t} \left(\tfrac{K_t}{H_t}\right)^{\theta} h_t + \left(1 + R_t^K - \delta\right) k_t - k_{t+1} - \tfrac{\hat{m}_{t+1}}{\hat{P}_t}, \\ \mu_t &= \tfrac{\eta}{1+\lambda_\pi} \mu_{t-1} - \tfrac{\lambda_\pi}{1+\lambda_\pi} \left(\ln \hat{P}_t - \ln \hat{P}_{t-1} - \pi^*\right) \\ &\qquad - \tfrac{\lambda_Y}{1+\lambda_\pi} (\ln Y_t - \ln Y^*) + \tfrac{1}{1+\lambda_\pi} \xi_t, \\ H_t - \mathrm{E}_{t-1} \ln H_t &= \tfrac{1}{\theta(1+\lambda_\pi)+(1-\theta)\lambda_Y} \left(\ln \hat{P}_t - \mathrm{E}_{t-1} \ln \hat{P}_t\right) \\ &\quad + \tfrac{1+\lambda_\pi - \lambda_Y}{\theta(1+\lambda_\pi)+(1-\theta)\lambda_Y} \varepsilon_t^{\ln Z} + \tfrac{\xi_t - (1-\eta)\bar{\mu}}{\theta(1+\lambda_\pi)+(1-\theta)\lambda_Y}, \\ h_t - \mathrm{E}_{t-1} \ln H_t &= \tfrac{1}{\theta(1+\lambda_\pi)+(1-\theta)\lambda_Y} \left(\ln \hat{P}_t - \mathrm{E}_{t-1} \ln \hat{P}_t\right) \\ &\quad + \tfrac{1+\lambda_\pi - \lambda_Y}{\theta(1+\lambda_\pi)+(1-\theta)\lambda_Y} \varepsilon_t^{\ln Z} + \tfrac{\xi_t - (1-\eta)\bar{\mu}}{\theta(1+\lambda_\pi)+(1-\theta)\lambda_Y}, \\ \ln K_{t+1} &= K\left(\mathbf{S}_t\right), \ln H_t = H\left(\mathbf{S}_t\right), \ln \hat{P}_t = \hat{P}\left(\mathbf{S}_t\right). \end{aligned}$$

In (A1), $\mathbf{S}_t$ is a 1×8 row vector which contains all the aggregate state variables $\ln Z_{t-1}$, $\varepsilon_t^{\ln Z}$, $\ln G_t$, μ_{t-1}, ξ_t, $\ln K_t$, $\ln \hat{P}_{t-1}$ and a constant term. If $\lambda_\pi = 0$, then $\ln \hat{P}_{t-1}$ vanishes in $\mathbf{S}_t$.[26] In maximization of (A1), the agent takes the economy-wide aggregate (average) variables as given. The functions K, $\hat{P}$ and H describe the relationship perceived by agents between the aggregate decision variables and the state of the economy. As the solution to the problem in (A1), we have the agent's decision rules $\ln k_{t+1} = k\left(\mathbf{S}_t, \ln k_t, \ln \hat{m}_t\right)$, $\ln \hat{m}_{t+1} = \hat{m}\left(\mathbf{S}_t, \ln k_t, \ln \hat{m}_t\right)$ and $\ln h_t = h\left(\mathbf{S}_t, \ln k_t, \ln \hat{m}_t\right)$. The

26. Note that the household budget constraint on line 4 in (A1) incorporates the fact that the contracted nominal wage divided by the price level equals the equilibrium marginal product of labor since firms unilaterally determine hours worked in period t.

competitive equilibrium is obtained when the individual and average decision rules coincide for $\ln k_t = \ln K_t$ and $\ln \hat{m}_{t+1} = \ln \hat{m}_t = 0$.

Since it is impossible to derive the decision rules analytically, I have used the same method as COOLEY and HANSEN [1995] and computed the decision rules numerically by approximating the original problem with a second order *Taylor* expansion around the constant steady state values in the nominal-growth adjusted economy. As a consequence of this approximation, the method produces linear decision rules (in natural logarithms for K_{t+1}, H_t and $\hat{P}_t$). The algorithm utilized is described in detail in HANSEN and PRESCOTT [1995].

B. *Data Sources and Definitions*

In this appendix, I provide the sources of the data collected in Table B1 below.

TABLE B1
The Data Set

Variables	Sample period	Source
GNP	1960Q1-1997Q4	FRED database, Federal Reserve Bank of St. Louis
GEC	1960Q1-1997Q4	FRED database, Federal Reserve Bank of St. Louis
M1	1959Q1-1997Q4	FRED database, Federal Reserve Bank of St. Louis
POP	1960-1996	OECD Main Economic Indicators
CPI	1959Q1-1997Q4	FRED database, Federal Reserve Bank of St. Louis

Note: All real macroeconomic variables are measured in 1992 billion US dollars. Abbreviations; GNP denotes real (fixed, seasonally adjusted) gross national product; GEC real (chained, seasonally adjusted) government consumption and investment; M1 (not seasonally adjusted) nominal money supply 1; CPI (not seasonally adjusted) consumer price index; POP average US population (for 1997, POP is set equal to average gross growth rate times the value for 1996).

The transformations made to generate the variables used in Table B1 are displayed in Table B2.

TABLE B2
Generation of Composite Quarterly Data Series

Variables	Sample period	Calculation formula
ln Y	1960Q1-1997Q1	ln (GNP/POP)
μ	1960Q1-1997Q4	ln $(\mathrm{M1_t}/\mathrm{M1_{t-4}})$
π	1960Q1-1997Q4	ln $(\mathrm{CPI_t}/\mathrm{CPI_{t-4}})$
ln G	1960Q1-1997Q4	ln (GEC/POP)

Note: To get of $\ln Y - \ln Y^*$,$\ln G$ *and G and* $\pi - \pi^*$, $\ln G$ *and* π *are then subject to Hodrick-Prescott filtering with the smoothness coefficient* λ *set to* $1{,}600$.

To compute measures of the ratio of government expenditures to output and the growth rate in nominal money supply in steady state, $\overline{g}$ and $\overline{\mu}$ respectively, I computed the sums $\frac{1}{152}\sum_{1960Q1}^{1997Q4}(\mathrm{GEC_t}/\mathrm{GNP_t})$ and $\frac{1}{152}\sum_{1960Q1}^{1997Q4}(1+\mu_t)^{\frac{1}{4}} - 1$.