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Optimal Licensing in a Spatial Model

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ABSTRACT. – We analyze a multi-stage non-cooperative game involving an outside patent-holder, who seeks to licence a process innovation, and two price-setting firms located on a circumference. Three licensing policies are studied: the auction, the fixed fee and the per unit output royalty. The main finding is that, contrary to standard results, royalties yield higher payoffs to the patent-holder than do an auction policy or a fixed fee policy regardless of the size of the innovation. Besides, a conflict between private and social interests arises since consumers are better off when the technology is licensed via fees.

Contrats optimaux de licences dans un modèle d'économie spatiale

RÉSUMÉ. – Nous analysons un jeu non-coopératif à plusieurs étapes où un titulaire d'un brevet d'invention cherche à vendre des licences d'exploitation d'une innovation technologique à deux firmes localisées autour d'un cercle. Trois politiques de licence d'exploitation sont étudiées : une vente aux enchères, un prix fixe et des *royalties* (c'est-à-dire des droits par unité de production). Nous montrons que, contrairement aux résultats standards de la littérature, des *royalties* procurent un bénéfice plus élevé au titulaire du brevet qu'une vente aux enchères ou qu'un prix fixe quelque soit l'ampleur de l'innovation. En outre, il y a un conflit entre les intérêts privés et sociaux : pour les consommateurs, il est préférable que la licence d'exploitation soit vendue au moyen d'un prix fixe.

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1 Introduction

The incentives for licensing play an important role in the *R&D* literature. The protection of intellectual property rights by patent laws provides the owner of an invention, when it is itself a non-producer, with *ex post* incentives to licensing technology and to recover part of its incurred *R&D* expenses. The owner of a cost-reducing process innovation may use several licensing policies. This paper explores three of them: 1) the auction, where a limited number of licences is auctioned off and this number is common knowledge before the sealed bid auction takes place, 2) the fixed fee, by which the licensee pays a lump-sum fee and, 3) the royalty, where the licensee pays a royalty payment per unit of production. The analysis is conducted in a spatial duopoly where firms compete in prices. In contrast with the existing literature on licensing, we show that licensing by means of a royalty yields higher payoffs to the patent holder than do the other two licensing policies.

The scarce existing evidence on patent licensing practices reveals that licensing by means of a fixed fee is less frequently used than royalties or a combination of them.[1] However, the theoretical literature with the assumption of complete information has shown that fixed fee licensing is superior to royalty licensing for both the patent-holder and consumers. Theoretical justifications for using royalties can be found by introducing uncertainty or asymmetric information. In face of uncertainty concerning the success of the new technology, royalties permit to share the risk between the patent-holder and the licensee. Also, when one of the parties enjoys some informational advantage over the other, royalties may signal good innovations or allow for the separation of good applicants from bad ones. Notable examples include GALLINI and WRIGHT [1990], MACHO-STADLER and PÉREZ-CASTRILLO [1991] and BEGG [1992].

Our paper is a contribution to explaining the superiority of royalty licensing to auction and fixed fee licensing policies, without assuming uncertainty nor alluding to informational asymmetries. This is obtained as part of the subgame perfect equilibrium (SPE) of a multi-stage non-cooperative game involving an outside patent-holder and two price-setting firms located on a circumference. This is in contrast with MUTO [1993], who studies the same problem by employing the representative consumer approach to product differentiation. MUTO [1993] finds that royalty licensing may be superior to the other two policies when the size of the innovation is small. We show that royalty licensing is the most preferred policy to the patent-holder regardless of the size of the innovation. This result is robust to assuming a patent holder which is itself a producer and also, to the possibility of licensing to a potential entrant.

1. ROSTOKER [1984] reports, for a sample of 37 US firms, that royalty alone was used in 39 % of the time, fixed fee alone in 13 %, and royalty plus fixed fee licensing was employed 46 % of the time. For a sample of Spanish firms, MACHO-STADLER *et al.* [1996] find that about 60 % of the contracts are based exclusively on royalty payments. In addition, CAVES *et al.* [1983] documented that restrictive terms such as market restriction, production location restriction and technology flow-back requirements were also commonly included in licensing contracts.

Spatial or address models of economic behaviour are particularly interesting for analyzing product differentiation. An important difference with the representative consumer approach is that commodities are described by their addresses in an attribute space and preferences are defined over this space, that is, consumers' preferences are heterogeneous. Also, it permits utility maximization over discrete alternatives allowing the consumers to buy from a single variety.[2] These theoretical features match with some real world examples such as the banking industry. Suppose a software company that develops an improvement related with ATM performance. It is sold to branches located in geographical space and consumers may visit one or another branch to use their ATM machines.

Formal analysis on licensing was initiated by ARROW [1962] and MCGEE [1966]. These authors consider licensing of an invention by means of a royalty and introduced the concept of a derived demand for a licence, respectively, and suggested that licences might be auctioned. Thus, the three licensing policies examined in this paper are the ones that have mostly received the attention of researchers in the field. KAMIEN [1992] contains an excellent survey of the literature and concentrates on these licensing policies in a game-theoretic framework. The theoretical work on licensing has normally been done in the context of homogeneous goods —*Cournot* competition— and where the patent holder is outside the industry. Representative papers include KAMIEN and TAUMAN [1986], KATZ and SHAPIRO [1986] and KAMIEN, OREN and TAUMAN [1992]. These papers show that royalties are dominated by other modes of licensing both in terms of the patent holder's payoffs and consumer surplus.

The papers by KAMIEN and TAUMAN [1986] and by KAMIEN, OREN and TAUMAN [1992] include an analysis of *Bertrand* competition. There is an equivalence among policies which follows naturally from the assumption of product homogeneity. As already mentioned, MUTO [1993] studies the licensing of a cost-reducing innovation to a *Bertrand* differentiated duopoly. Most of the literature on licensing has been devoted to the case of process innovations. Little has been done concerning product innovations, as far as we know. A couple of examples are KAMIEN, TAUMAN and ZANG [1988] and CABALLERO, MONER-COLONQUES and SEMPERE-MONERRIS [1995], in a non-spatial and a spatial setting, respectively. Both obtain that licensing by means of fixed fees is the preferred licensing policy.

The remainder of the paper is organized as follows. First, we present the model and section 3 describes the product market equilibria. Sections 4, 5 and 6 examine the fixed fee licensing policy, the royalty licensing policy and the auction licensing policy, respectively. The main results are provided in section 7, robustness in section 8 while other licensing policies are discussed in section 9. Brief concluding remarks close the paper.

2. A third approach is that of the discrete choice models that assumes that consumers' choice is viewed as a probabilistic process. The connections and differences among these three approaches are remarkably established in ANDERSON *et al.* [1992].

2 The Model

We posit an industry consisting of two identical firms (firm A and firm B) located on a circumference of two units length. The distance between them is one, this meaning that the good produced by firm A is maximally differentiated from the good produced by firm B. We assume that entry costs are so high that the structure of the market is fixed. Variables p_j and x_j denote, respectively, firm $j's$ price and output level, $j = A, B$. There is a continuum of consumers, indexed by i, located on the circumference. Each consumer i maximizes the following utility function,

(1) $$u_i = (v - p_j - at_{ij}^2)h_i$$

where v is the consumers' valuation of the good, a stands for the consumers' transportation cost parameter, t_{ij} is the distance between consumer i and firm j, and finally h_i is the amount of the good bought by consumer i, which is assumed to belong to the $\{0,1\}$ set. For the sake of the exposition, symmetry allows us to consider only half of the circumference in the remainder of the analysis.[3] From the consumers maximizing behaviour and assuming that the whole market is covered,[4] we obtain that firm j's demand, $x_j(p_j, p_l)$, is:

(2) $$\max\left[0,\ \min\left(\frac{p_l - p_j + a}{2a},\ 1\right)\right] \text{ j}, \ell = \text{A, B, and j} \neq \ell$$

We further assume that firms' cost functions are linear and identical, $C_j = cx_j$, where c, $0 < c < v$, is the constant marginal cost of production. Firms maximize profits by choosing its price simultaneously and independently, where firm $j's$ profits is:

(3) $$\Pi_j = (p_j - c)x_j \text{ for j = A, B}$$

There is also a patent holder with an innovation which seeks to license the patent to both firms or one so as to maximize his payoffs. We assume that the

3. We assume quadratic transportation costs and a circumference space to ensure that the firms location would be optimal if they were given the option to choose their location strategically in an early stage. Furthermore, the same qualitative results will follow under linear transportation costs.

4. This assumption is typical of spatial models. Technically, this ensures that reaction functions intersect on the increasing linear portion. The assumption is satisfied for v big enough. As indicated in the introduction, the banking industry fits well with the specified spatial model. Take, *e.g.*, the market for deposits. The buy/not buy decision can be interpreted as customers choosing one bank for their deposits. Total market coverage means that all customers, we believe, keep their savings in banks. Other industries that may meet the assumptions are the precision instruments industry and, in general, any industry where the good sold by firms located in space is a necessary input for other firms (the consumers in our model).

patent holder is an independent research laboratory and cannot enter the market of the final good directly.[5] The technology supplied by the patent holder consists of an improvement in the production process of the downstream firms that reduces their unitary costs of production in some amount ε, where $0 < \varepsilon < c,$ the same for any firm.

To place the model and compare our results with the received literature on licensing, we assume that the patent holder can use one of three different licensing policies: a) either a fixed fee policy, where a lump-sum licence fee, $F, \ F \in \mathbb{R}^{+},$ is charged to any firm willing to get the technology, or b) a royalty policy, where a constant payment per unit of output or royalty, R, $0 < R \leqslant \varepsilon,$ is charged to any firm willing to obtain the technology, or c) an auction policy, where a given number of licences, $k, \ k \in \{1,2\},$ is auctioned off through a sealed-bid first price auction with zero reservation price.[6] Licences are sold to the highest k bidders at their bid price and in the event of a tie, licensees are arbitrarily chosen. Resale of licences is ruled out.

The interaction between the patent holder and the duopolists is characterized by a three-stage noncooperative game.[7] The game is played only once, there is no uncertainty, and all relevant information is common knowledge to all the players. In the first stage, the patent holder announces the licensing policy along either with the lump-sum licence fee, or the constant payment per unit output or the number of licences to be auctioned off. In the second stage, duopolists simultaneously and independently either decide whether to buy the licence if the fixed fee policy or the royalty policy was announced in the first stage, or how much to bid in the case of the auction policy. Then, commonly knowing which firm holds a licence, both duopolists decide simultaneously and independently their price levels. The game is solved in the standard backward way.

All agents in the game maximize their payoffs, the patent holder's payoffs are the rents extracted with the licensing policy, and the duopolists' payoffs are their profits net of licence expenses. The subgame perfect equilibrium of this game is analyzed in the following sections which will specify the licensing policy, number of licensees and prices in the market.

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5. This is not an unrealistic assumption. After the second world war a class of specialized process design and engineering firms have appeared in several industries which play an important role in the development and diffusion of process innovations, see ARORA [1997], for the case of the chemical industry. The case when the patent holder is itself a producer is discussed in section 7 below and does not alter the results.

6. Note that when the patent holder announces to auction off a number of licences which coincides with the number of potential buyers, the potential licensees will bid nothing provided they will get one licence anyway. The patent holder must set a minimum bid (reservation price) to avoid such an event. We will consider only a type of auction such that the patent holder cannot set the minimum bid in an arbitrary way; it must coincide with the continuation value of the game for the patent holder, which is assumed zero. Arbitrary minimum bids raise credibility problems (see SEMPERE-MONERRIS and VANNETELBOSCH [2001]).

7. Note that in order to focus on the trading of the technology, we are not presenting a fourth stage in which consumers decide whether to buy a unit of output and from which duopolist. This is in fact captured by equations (1) and (2).

3 Description of the Product Market Equilibria

We analyze in this section the duopolists' decisions on prices taking into account that, at this stage, the production technology that each duopolist is employing is common knowledge. Then, as a result of the agents' decisions on earlier stages of this game we may end up with two qualitatively different equilibria: either a symmetric equilibrium in which both firms hold the same marginal cost, or an asymmetric equilibrium. In what follows we denote by $p_j(c_j, c_l)$, $x_j(c_j, c_l)$ and $\Pi_j(c_j, c_l)$ $j, l = A, B, j \neq l$, the equilibrium prices, outputs and profits for firm j when it produces with marginal cost c_j, while its rival produces with c_l.

Symmetric Equilibrium

In this case, both duopolists produce with the same technology, either with $c_A = c_B = c - \delta$, where $\delta \in \{\varepsilon, \varepsilon - R\}$, if both have purchased a licence, or with $c_A = c_B = c$, if neither of them has purchased a licence. The equilibrium prices, outputs and profits are:

$$\begin{aligned}
&p_A(c,c) = p_B(c,c) = a + c && \text{if none buys}\\
&p_A(c-\delta, c-\delta) = p_B(c-\delta, c-\delta) = a + c - \delta && \text{if both buy}\\
&x_A(c,c) = x_B(c,c) = x_A(c-\delta, c-\delta) = x_B(c-\delta, c-\delta) = \frac{1}{2}\\
&\Pi_A(c,c) = \Pi_B(c,c) = \Pi_A(c-\delta, c-\delta) = \Pi_B(c-\delta, c-\delta) = \frac{a}{2}
\end{aligned} \tag{4}$$

Asymmetric Equilibrium

In this case only one firm, say firm A, has purchased a licence and therefore $c_A = c - \delta$, whereas firm B has not, $c_B = c$.

$$\begin{aligned}
p_A(c-\delta, c) &= a + c - \frac{2\delta}{3} & p_B(c, c-\delta) &= a + c - \frac{\delta}{3}\\
x_A(c-\delta, c) &= \frac{1}{2} + \frac{\delta}{6a} & x_B(c, c-\delta) &= \frac{1}{2} - \frac{\delta}{6a}\\
\Pi_A(c-\delta, c) &= \frac{a}{2}\left(1 + \frac{\delta}{3a}\right)^2 & \Pi_B(c, c-\delta) &= \frac{a}{2}\left(1 - \frac{\delta}{3a}\right)^2
\end{aligned} \tag{5}$$

Note that

$$\begin{aligned}
\Pi_A(c-\delta, c) > \Pi_A(c,c) = \Pi_B(c,c) = \Pi_A(c-\delta, c-\delta)\\
= \Pi_B(c-\delta, c-\delta) > \Pi_B(c, c-\delta),
\end{aligned}$$

that is, symmetry implies that both duopolists obtain the same profits and they are independent of the technology used. Additionally, an exclusive licensee obtains greater profits than a nonexclusive licensee which also obtains greater profits than a nonlicensee with one licence in the market.

It is important to mention that depending on the magnitude of the cost asymmetry we may find situations in which the most inefficient duopolist produces zero, *i.e.*, it is expelled from the market. This happens when $\delta \geqslant 3a$. Therefore, under this condition the equilibrium variables become,

$$(6)\quad \begin{aligned} p_A^d(c-\delta,c) &= c+\frac{a-\delta}{2} & p_B^d(c,c-\delta) &= c \\ x_A^d(c-\delta,c) &= 1 & x_B^d(c,c-\delta) &= 0 \\ \Pi_A^d(c-\delta,c) &= \frac{a+\delta}{2} & \Pi_B^d(c,c-\delta) &= 0 \end{aligned}$$

where superscript d stands for "drastic". Although one firm has been expelled from the market, it does not mean that the most efficient firm can behave as a local monopolist.[8] The best it can do is to set a limit price such that the rival's best response is not to be active. The lowest possible price by the most inefficient firm is c. Then, firm A's best response price is $p_A(p_B = c) = c + (a-\delta)/2,$ and substituting back that price pair into firm A's demand function we obtain $x_A = min((a+\delta)/4a, 1).$ It turns out that the latter term is the smallest one as long as $\delta \geqslant 3a$. This is the only sensible definition of a drastic innovation in this spatial setting. The firm that becomes a licensee serves the whole market.

4 The Fixed Fee Licensing Policy

In this section, we analyze the case when the patent holder announces in the first stage of the game that a fixed fee policy is selected and that the lump-sum license fee charged to any firm willing to get the technology is F, $F \in \mathbb{R}^+$. After F is announced, firms decide simultaneously and independently whether to buy a licence. The strategy of firm j is a pair (d_j, p_j). Firm $j's$ decision about purchasing the licence is denoted by $d_j(F)$, where $d_j(F) = 1$ means buying a licence for a given F, and $d_j(F) = 0$, means not buying. The second element of the pair, p_j, is a function of the fee, F, and of s, where s denotes, the subset of licensees. The payoffs are defined by,

$$\Gamma_j^F(F,(d_A,p_A),(d_B,p_B)) = \begin{cases} (p_j - c + \varepsilon)\, x_j(p_A,p_B) - F & \text{for} \quad j \in s \\ (p_j - c)\, x_j(p_A,p_B) & \text{for} \quad j \notin s \end{cases}$$

$$\Gamma_{PH}^F(F,(d_A,p_A),(d_B,p_B)) = Fk$$

8. In our spatial model the equilibrium monopoly price is either $\frac{2v+c-\delta}{3}$ for $0 < \delta < 3a - (v-c)$ or $v - a$ for $\delta \geqslant 3a - (v-c)$. The relevant one is the latter since $\delta \geqslant 3a$. However, under this case and noting that the market is completely covered, the equilibrium monopoly price is always greater than c. Hence, a positive output would be sold by the inefficient firm. We conclude that the definition of a drastic innovation in the *Arrow* sense does never occur in this model.

where Γ_j^F stands for the firms' payoffs (profits net of licence expenses) and Γ_{PH}^F are the patent holder's payoffs for the case of a fixed fee licensing policy. Finally, k denotes the number of licences sold.

The maximum amount the patent holder can extract from a firm is its willingness to pay to become a licensee. Apply $\delta = \varepsilon$ in (4), (5) and (6) to obtain firms' profits. Suppose that the process innovation is nondrastic, $0 < \varepsilon < 3a$. Then, if the patent holder sells one licence, the fixed fee, F, must satisfy the following:

(7) $$F \leqslant \Pi_A(c-\varepsilon,\ c) - \Pi_A(c, c) = F_1$$

while the fixed fee when two licences are sold must satisfy:

(8) $$F \leqslant \Pi_B(c-\varepsilon,\ c-\varepsilon) - \Pi_B(c, c-\varepsilon) = F_2$$

where $F_1 > F_2$ as long as ε and a are positive. In words, a firm accepts the licence as long as the opportunity cost of not being a licensee plus the fixed fee do not exceed the profits of buying the licence.

Suppose that, on the contrary, the process innovation is drastic. If one licence is sold then the fixed fee F_1^d will equal $\Pi_A^d(c-\varepsilon,\ c) - \Pi_A(c, c)$; F_2^d will equal $\Pi_B(c-\varepsilon,\ c\ -\varepsilon) - \Pi_B^d(c, c-\varepsilon)$ in the case where the patent holder sells two licences.

The patent holder's optimal choice under the fixed fee licensing policy is summarized in the next lemma.

LEMMA 1. Under the fixed fee licensing policy,

a) the subgame perfect equilibrium for the case of a nondrastic innovation $(0 < \varepsilon < 3a)$ is:

$$F(\varepsilon) = \begin{cases} F_2 = \frac{\varepsilon(6a-\varepsilon)}{18a} & \text{if } 0 < \varepsilon \leqslant 2a \\ F_1 = \frac{\varepsilon(6a+\varepsilon)}{18a} & \text{if } 2a < \varepsilon < 3a \end{cases}$$

$$\Gamma_{PH}^F(\varepsilon) = \begin{cases} \frac{\varepsilon(6a-\varepsilon)}{9a} & \text{if } 0 < \varepsilon \leqslant 2a \\ \frac{\varepsilon(6a+\varepsilon)}{18a} & \text{if } 2a < \varepsilon < 3a \end{cases}$$

$$(d_A^F, d_B^F) = \begin{cases} (1,1) & \text{if } F(\varepsilon) = F_2 \\ (1,0) \text{ or } (0,1) & \text{if } F(\varepsilon) = F_1 \end{cases}$$

$$\Gamma_A^F(\varepsilon) = \Gamma_B^F(\varepsilon) = \frac{a}{2}\left(1 - \frac{\varepsilon}{3a}\right)^2 \quad \text{if } 0 < \varepsilon \leqslant 2a$$

$$\Gamma_A^F(\varepsilon) = \frac{a}{2},\ \Gamma_B^F(\varepsilon) = \frac{a}{2}\left(1 - \frac{\varepsilon}{3a}\right)^2 \quad \text{if } 2a < \varepsilon < 3a \text{ and } (d_A^F, d_B^F) = (1,0)$$

b) the subgame perfect equilibrium for the case of a drastic innovation $(3a \leqslant \varepsilon < c)$ is:

$$F(\varepsilon) = F_1^d = \frac{\varepsilon}{2},\quad \Gamma_{PH}^F(\varepsilon) = \frac{\varepsilon}{2}$$

$$(d_A^F, d_B^F) = (1,0) \text{ or } (0,1)$$

$$\Gamma_A^F(\varepsilon) = \frac{a}{2},\ \Gamma_B^F(\varepsilon) = 0 \text{ if } (d_A^F, d_B^F) = (1,0)$$

Proof. See the Appendix.

On the one hand, there is an effective reduction in the marginal cost of production of licensees because the payment in exchange for the licence is a fixed amount. This leads to a reduction in the post-invention equilibrium prices. On the other, firms are never better off than before the process innovation is sold. A nonlicensee always obtains lower payoffs whereas a licensee ends up with lower payoffs unless it becomes an exclusive licensee.

Note that in characterizing the above result, we have derived the demand function for licences: there is an inverse relation between price (the fixed fee) and the number of licences sold. The bigger the size of the innovation the more likely it is that only one licence be sold. In that event, the new technology allows the exclusive licensee to capture more consumers. Since total demand is given, this is at the expense of the rival's market share. Further note that with a fixed fee licensing policy the complete diffusion of the new technology is not guaranteed.

5 The Royalty Licensing Policy

We turn now to analyze the case when the patent holder announces in the first stage of the game that a royalty licensing policy is selected and that the constant payment per unit of output or royalty charged to any firm willing to get the technology is $R,\ 0 < R \leqslant \varepsilon$. Once R has been announced, firms decide simultaneously and independently whether to buy a licence. Firm $j's$ strategy is a pair (d_j, p_j). We follow a similar notation as above. When firm j buys a licence, this is denoted by $d_j(R) = 1$ whereas not buying appears as $d_j(R) = 0$. The price set by firm j, p_j, is a function of R and of s, that is, of the royalty and the subset of licensees. The payoffs are

$$\Gamma_j^R(R,(d_A,p_A),(d_B,p_B)) = \begin{cases} \left(p_j - c - R + \varepsilon\right) x_j(p_A,p_B) & \text{for } \ j \in s \\ \left(p_j - c\right) x_j(p_A,p_B) & \text{for } \ j \notin s \end{cases}$$

$$\Gamma_{PH}^R(R,(d_A,p_A),(d_B,p_B)) = R\sum_{j\in s} x_j$$

where Γ_j^R stands for the firms' payoffs and Γ_{PH}^R are the patent holder's payoffs for the case of a royalty licensing policy.

Note that when royalties are used, the reaction function of the purchasing firms is affected by the difference between ε and R. By taking $\delta = \varepsilon - R$ in (4), (5) and (6) we obtain the downstream game equilibrium variables. Also note that if $R > \varepsilon$, then it will imply that a licensee is worse off than a rival that does not hold a licence. Consequently, no licence would be sold. Suppose now that $R < \varepsilon$. In that case both firms are willing to buy a licence since, irrespectively of the rival's choice, any firm is better off buying the licence, that is, $\Pi_j(c-\varepsilon+R,c-\varepsilon+R) > \Pi_j(c,c-\varepsilon+R)$ and $\Pi_j(c-\varepsilon+R,c) > \Pi_j(c,c)$ for $j = A,B$.

Finally, note that if R equals ε, a firm is indifferent between purchasing and not since marginal costs are equal to c. Nevertheless in a subgame perfect equilibrium with $R = \varepsilon$, it is apparent that $k = 2$. The patent holder earns more by selling two licences rather than one; when two licences are sold, the whole market is served by using the new technology whereas in case one licence is sold only a share of it is served with the new technology.[9] It follows that,

LEMMA 2. Under the royalty licensing policy the subgame perfect equilibrium is:

$$R(\varepsilon) = \varepsilon, \qquad \Gamma^R_{PH}(\varepsilon) = \varepsilon$$
$$(d^R_A, d^R_B) = (1,1), \quad \Gamma^R_A(\varepsilon) = \Gamma^R_B(\varepsilon) = \frac{a}{2}$$

The proof is straightforward from the above discussion. Under a royalty licensing policy, since at equilibrium $R = \varepsilon$ and two licences are sold, the current marginal cost of production is unaffected and consequently no benefit reverts to consumers given that equilibrium prices do not change. Licensees earn the same as before the innovation. In contrast with the fixed fee licensing policy, the innovation is always completely diffused.

6 The Auction Licensing Policy

In this section we study the case where the patent holder announces in the first stage that the auction licensing policy has been selected and that the number of licences to be auctioned off is k, $k \in \{1,2\}$. We focus here on the case of nonarbitrary reservation price auctions, that is, the patent holder cannot credibly choose a reservation price different from the continuation value of the game for him, z, under which he will not sell a licence. Assume $z = 0$. In the second stage, the sealed-bid first price auction with nonarbitrary reservation price is played. Both firms decide independently and simultaneously how much to bid for a licence knowing k. Remark that, in contrast with the previous licensing policies, the price paid for a licence is determined not by the patent holder but rather by buyers' competition. Licences are sold to the $k-$highest bidders at their bid price and, in the event of a tie, licensees are arbitrarily chosen. Let $b = (b_A(k), b_B(k))$ denote the bids submitted by the duopolists and let $s = s(k)$ be the set of the k licensees. Also, denote by p_j the price set by firm j which is a function of the bids, b, and of s. The payoffs are defined by,

9. In the case of a drastic innovation, we know that $\delta \geqslant 3a$, which in the case of a royalty policy becomes $R \leqslant \varepsilon - 3a$, that is, a bound on the royalty which is smaller than ε. Therefore, the patent holder is worse off with only one licensee who serves the whole market than with both firms with the new technology serving the whole market.

$$
\Gamma_j^a(b,s) = \begin{cases} \left(p_j - c + \varepsilon\right) x_j(p_A, p_B) - b_j(k) & \text{for} \quad j \in s \\ \left(p_j - c\right) x_j(p_A, p_B) & \text{for} \quad j \notin s \end{cases}
$$

$$
\Gamma_{PH}^a(b,s) = \sum_{j \in s} b_j(k)
$$

where Γ_j^a stands for the firms' payoffs and Γ_{PH}^a are the patent holder's payoffs for the case of an auction licensing policy. The discussion that follows is, in fact, the proof of the next proposition. Suppose that the patent holder announced $k = 2$ in the first stage. Since we are considering only auctions with nonarbitrary reservation price, both firms will make a bid equal to $z = 0$ and will obtain the licence. Consequently, the patent holder will never auction two licences. Note that the presence of a potential entrant would make the choice of $k = 2$ with a reservation price different from z a credible announcement. By having a third option, or the threat of a third option, both incumbents would bid according to the entrant's willingness to pay for a licence. They would know that otherwise the patent holder could always try and sell the innovation to a third firm, which would enter the market and affect incumbents' profits. But the possibility of entry would only partially solve the problem since now the patent holder could consider to auction three or more licences.

Suppose now that the patent holder selects $k = 1,$ then the highest bid any firm will submit is its willingness to pay to become a licensee. But here, in contrast to the fixed fee licensing policy, a firm will accept the licence as long as the opportunity cost of not being the licensee and the *rival getting the licence* plus the bid do not exceed the profits of buying the licence. That is, apply $\delta = \varepsilon$ in (4), (5) and (6) to obtain firms' profits. Then, if the patent holder announces one licence, firm $j's$ bid must satisfy the following:

$$
(9) \qquad \forall j = A, B \;\; b_j(1) \leqslant \Pi_j(c - \varepsilon,\, c) - \Pi_j(c, c - \varepsilon) = b(1)
$$

when the process innovation is nondrastic, $0 < \varepsilon < 3a$. For the drastic case it must satisfy:

$$
(10) \qquad \forall j = A, B \;\; b_j(1) \leqslant \Pi_j^d(c - \varepsilon,\, c) - \Pi_j^d(c, c - \varepsilon) = b^d(1)
$$

Therefore, both firms make the same bid equal either to $b_1(1)$ or to $b_1^d(1)$, depending on the nature of the process innovation. The licensee is arbitrarily chosen by the patent holder. We may state the following result.

LEMMA 3. Under the auction licensing policy,
a) the subgame perfect equilibrium for the nondrastic innovation case is,

$$
k(\varepsilon) = 1, \qquad \Gamma_{PH}^a(\varepsilon) = \frac{2\varepsilon}{3}
$$

$$
(b_A, b_B) = (\frac{2\varepsilon}{3}, \frac{2\varepsilon}{3}), \quad \Gamma_A^a(\varepsilon) = \Gamma_B^a(\varepsilon) = \frac{a}{2}\left(1 - \frac{\varepsilon}{3a}\right)^2
$$

b) the subgame perfect equilibrium for the drastic innovation case is,

$$k(\varepsilon) = 1, \qquad \Gamma^a_{PH}(\varepsilon) = \frac{a+\varepsilon}{2}$$
$$(b_A, b_B) = \left(\frac{a+\varepsilon}{2}, \frac{a+\varepsilon}{2}\right), \quad \Gamma^a_A(\varepsilon) = \Gamma^a_B(\varepsilon) = 0$$

As in the case of fixed fee licensing, post-invention equilibrium prices are lower since the bid is a fixed payment. Besides, the announcement of $k = 1$ introduces a threat on the potential licensee in the sense that "*either I purchase the licence or my competitor will*". Irrespective of the size of the innovation, the licensee is worse than before the innovation is auctioned off. Furthermore, the exclusive licensee is willing to bid a higher amount in the event of a drastic innovation. This is so because it will be driven out of the market in case it is not the buyer. Under this policy, the technology is never completely diffused.

Consider for a moment auctions with arbitrary reservation prices. The patent holder would choose a reservation price slightly below the benefit to a firm if all were licensed, that is a reservation price equal to $\Pi_i(c-\varepsilon, c-\varepsilon) - \Pi_j(c, c-\varepsilon) > z$, both firms would bid that reservation price and obtain the licence. Hence, for $k = 2$ an arbitrary reservation price auction licensing policy is equivalent to a fixed fee policy when both firms buy the licence.

7 Optimal Licensing Policy and Welfare Considerations

This is the central Section to our paper. We now proceed to examine which of the three licensing policies is optimal for the patent holder. It amounts to comparing the patent holder's payoffs under each policy. Then, we establish which of these licensing policies is the most preferred one from the consumers' viewpoint.

PROPOSITION 1. Licensing by means of a royalty policy generates the highest payoffs to the patent holder. Two licences are sold at a royalty that equals the size of the innovation. Market price and licensees' payoffs coincide with the pre-invention ones. Also, the auction licensing policy generates higher payoffs to the patent holder than the fixed fee licensing policy.

Proof. See the Appendix.

Several reasons may explain this result which, to the best of our knowledge, is the first theoretical finding stating the superiority of a royalty licensing policy *regardless* of the character of the innovation (whether drastic or not). To see the intuition behind this result, it is useful to note the nature of the third-stage strategic variables: prices are strategic complements, that is, duopolists' reaction functions are upward sloping. If the patent holder licenses a process innovation to both firms, then both reaction functions will shift inwards. In such an event, the patent holder evaluates the role of the licensing policies at hand in controlling for competition intensity since prices would fall. The patent holder's problem now resembles that of an owner of a firm instructing its manager's behaviour. In this sense, the owner would like the manager to behave less/more aggressively in the product market when the variables are strategic complements/substitutes – see *e.g.* FERSHTMAN and JUDD [1987]. In our setting, the patent holder may reach a more collusive outcome by means of a royalty licensing policy. In particular, by setting a royalty equal to the size of the innovation the duopolists' reaction functions do not change. The fixed fee and the auction licensing policies cannot undo the aforementioned inward shift associated with the sale of the process innovation, since they involve the payment of a fixed amount independent of licensees' output.

Therefore, with royalty licensing the patent holder does influence equilibrium prices in such a way that they do not fall below the pre-invention level. Thus, the patent holder can realize the total difference between the pre-invention and post-invention marginal cost of production levels. On the other hand, the fixed fee and the auction licensing policies lead to lower prices and this is certainly positive for consumers. However, prices below the pre-invention level do not imply a higher demand since, in this spatial set up, the market is completely covered. Therefore, regarding rent extraction the price effect becomes the sole effect since the output effect is null. These arguments explain the prevalence of a royalty licensing policy over a fixed fee and an auction licensing policies.

We have also shown that the patent holder prefers an auction licensing policy to a fixed fee licensing policy. The difference between both policies, and the reason for such result, stems from the fact that a licensee's rejection decision reduces the number of licences by one under a fixed fee policy, whereas in an auction policy the number of licences is predetermined. There is a strategic element at play and the patent holder may take a better advantage of such strategic interaction by employing an auction licensing policy.[10]

We proceed now to consider the desirability of optimal licensing policies when a public agency cares for consumers' welfare or alternatively for social welfare. The following proposition summarizes the result:

PROPOSITION 2. A fixed fee licensing policy weakly dominates an auction licensing policy, which in turn strongly dominates a royalty licensing policy in consumer surplus and in social welfare terms.

Proof. See the Appendix.

---

10. Other licensing mechanisms can be studied which may include a sort of bargaining game between the patent holder and the licensees. This has been done by SEMPERE-MONERRIS and VANNETELBOSCH [1997]. The patent holder prefers a take-it or leave-it bargaining policy to the fixed fee licensing policy, to the alternating bids bargaining policy and to the simultaneous bids bargaining policy. If we only considered auctions with nonarbitrary reservation prices, then it would no longer be true that an auction licensing policy dominates a fixed fee policy.

Our last result highlights that, no matter we consider consumer surplus only or total welfare, a royalty licensing policy should not be viewed as the desirable licensing policy. However, this is the preferred policy for the patent holder. Such a conflict of interests has not been found on earlier work on this field. Also note that the complete diffusion of the technology is not necessarily a good indication of a welfare improvement, since it might be associated with greater prices.

8 Robustness

We examine the robustness of our findings in light of two changes in the basic assumptions. Firstly, one wonders what would happen was the patent holder itself a producer in the market. The above effect induced by royalties in controlling for competition intensity is reinforced. In general, a royalty reduces the patent holder's incentives to use the licensed technology because it releases royalty revenues. However, the royalty alters the marginal cost of production on which the licensee bases its decisions. With royalty licensing, the patent holder can still extract licensee's rents and compete with a lower marginal cost than the rival firm. Such an effect does not arise with the other two licensing policies. It can be proven that royalty licensing remains optimal for the patent holder in a differentiated address duopoly. Our findings qualitatively coincide with those obtained by earlier work, yet the equilibrium royalty is always set below the size of the innovation (see the Appendix). ROCKETT [1990], for the homogeneous products case, considers two-part tariff contracts to show that, without imitation, the royalty allows the patent holder to extract the entire benefit of the licenses, no fixed fee is charged, and the newest technology is licensed. WANG and YANG [1999] also find a strategic rationale for royalty licensing under *Bertrand* competition in a differentiated non-address duopoly.

Secondly, we discuss next that the royalty licensing policy remains optimal under the possibility of licensing to a potential entrant. Let us suppose that the patent holder considers to license the process innovation to the incumbents, A and B, and to an entrant, E, *i.e.*, allow for a change in the preceding two-firm market structure. To elaborate on the above analysis, several assumptions must be made. The potential entrant bears a fixed entry cost such that with the old technology it does not enter the market. Further assume that relocation costs are prohibitive. Thus, if entry takes place, then firm E will be located halfway between firms A and B, and we need now consider the two-unit length circumference, because when entry occurs, we end up with a spatially differentiated triopoly which is asymmetric in locations.

The possibility of entry affects the three licensing policies as follows. Concerning the royalty licensing policy, note that by setting $R = \varepsilon$, the payoffs to the patent holder are 2ε when the process innovation is licensed to both the incumbents. The patent holder cannot improve on that by selling the innovation to all firms. The reason is that although all the market is served by the three firms and the incumbents pay $R = \varepsilon$, the highest per unit of output royalty that the entrant is willing to pay is smaller than ε, since the entrant has to recover entry costs. The question is: *can the patent holder do better with any of the other two licensing policies?*

Under the fixed fee licensing policy, there are three further options, in addition to the ones analyzed above. The patent holder can license the technology to only the entrant, to an incumbent and the entrant, or to all three firms. Naturally, the patent holder must be able to price discriminate and, most importantly, the threshold to have a drastic innovation is redefined, and now it depends both on how many licences are issued and who obtains the licence. It should be noted that the entrant's willingness to pay, compared to that of an incumbent, is greater because the opportunity cost of not being a licensee implies no entry, whereas it is lower since it incurs entry costs. Whether the entrant's or the incumbent's willingness to pay is larger will depend on the size of the innovation. It can be proven that, regardless of the patent holder's optimal choice under fixed fee licensing it cannot improve on 2ε. Finally, the auction licensing policy is affected in the sense that the above referred credibility problem for $k = 2$ disappears. That problem now applies to $k = 3$. The patent holder will auction either one or two licences. Suppose that $k = 1$. The threat of entry would increase the incumbents' highest possible bid. On the other hand, an incumbent's actual bid must take into account that neither the entrant nor the rival incumbent gets the licence. It turns out that the incumbents bid according to those in Lemma 3 which are higher than the entrant's bid. A similar argument applies for $k = 2$. In fact, when the number of licences coincides with the number of incumbents, the winning bids are dictated by the maximum amount the entrant is willing to bid. Hence, the entrant will never obtain a licence. It can be shown that royalty licensing remains optimal for the patent holder under the possibility of entry.[11]

9 Other Licensing Policies

As mentioned in the introduction, it is commonly observed that many licensing contracts include both a fixed and a variable part, *i.e.*, a two-part tariff licensing policy. We noted that two-part tariffs can be justified by alluding to informational asymmetries between the patent holder and the licensees. Assume two-part tariffs were allowed in the model. The patent holder would maximize $kF + Rx$ subject to two constraints, one is that $R \leqslant \varepsilon$ and the other is the participation constraint by a licensee. Now, the fee is a function of the royalty and this optimization problem yields a corner solution with $R = \varepsilon$ and a zero fee. Therefore, our central result is robust to this variation in the set of contracts.

Most relationships between upstream and downstream firms generally include a number of more or less complex contracting arrangements. These contractual provisions, broadly named vertical restraints, not only specify the terms of payments but also comprise clauses that alter one or the other party's behaviour. Typical examples are exclusive territories, resale price mainte-

11. The interested reader may obtain these computations from the authors upon request.

nance and exclusive dealing clauses. In this sense, technology transfers through licensing contracts are not an exception (see CAVES, [1983], for evidence of this kind of clauses in licensing contracts). A spatial model is particularly convenient to investigate the effects of including an exclusive territorial clause along with either a fixed fee or a royalty. Note that only symmetric equilibria are meaningful. It is the case that both licensing policies (a fixed fee licensing policy with a territorial restriction and a royalty licensing policy with a territorial restriction) yield the same profits to the patent holder.[12] The intuition for this result follows from the literature on vertical restraints: two instruments overcome the two externalities, the vertical one between the patent holder and the licensee and the horizontal one between licensees: intrabrand competition is eliminated (see MATHEWSON and WINTER, [1984]).[13] The same qualitative result is obtained when a resale price maintenance clause is considered. Just note that this case requires some more algebra since the possibility of an asymmetric equilibrium is now open.

Finally, suppose that the patent holder may practise licensing discrimination in the following sense: it offers one fixed fee licence and sells the other licence through a royalty contract. It can be shown that the patent holder will set a royalty equal to the size of the innovation and a positive fee equal to F_1. Nevertheless, its payoffs are lower than under a royalty licensing policy. Therefore, we conclude that our central result is not affected by the possibility of licensing discrimination.

10 Concluding Remarks

The diffusion of technological improvements, either product or process innovations, may take place through licensing contracts, cooperative research agreements or imitation. This paper has investigated three commonly observed licensing policies for the owner of a process innovation in a spatial model.

Several types of considerations have been used by the existing literature to justify the use of royalties. One of them is that royalties serve as a risk-sharing instrument between the patent-holder and the licensee. Another one is to resort to incomplete information scenarios where royalties play the role of signalling devices, or work as elements which induce the self-selection of licensees who underestimate the value of the patent. In this paper, we have added an argument without introducing uncertainty nor asymmetric information. Instead, price competition and demand elasticity considerations become relevant.

12. These computations can be found in the CORE *Discussion Paper* version of the paper.

13. See, for more details, CABALLERO-SANZ and REY [1996] and DOBSON and WATERSON [1996] who have surveyed the literature on vertical restraints and tried to develop a practical framework for their analysis. Some of the ideas expressed by these authors are incorporated in the *Green Paper on Vertical Restraints in EC Competition Policy, Communication* adopted by the *Commission on 22.1.1997*. Only recently has a follow-up to the *Green Paper* been published (*Communication* adopted *on 30.9.1998*).

We have shown that royalties are superior to a fixed fee policy and an auction policy, and this result is robust to a number of changes in some of the basic assumptions. This theoretical prediction is sounder with empirical observations than earlier work on licensing, since royalties are commonly used in licensing contracts. A second finding, different from previous work, is that private and social interests do not coincide. The spatial nature of the model makes it particularly interesting for further theoretical work. Possible extensions include the possibility of allowing for the market not being completely covered, or to consider an elastic demand. ■

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Appendix

A *Proofs*

Proof of Lemma 1.

For part a), we check when $2F_2 \geqslant F_1$, that is,

$$2\left[\frac{a}{2}-\frac{a}{2}\left(1-\frac{\varepsilon}{3a}\right)^2\right] \geqslant \frac{a}{2}\left(1+\frac{\varepsilon}{3a}\right)^2-\frac{a}{2}$$

which yields the condition for selling two licences to be an equilibrium, that is, $0 < \varepsilon \leqslant 2a$.

For part b), it is easy to verify that whenever $\varepsilon \geqslant 2a$, then $2F_2^d < F_1^d$ is always satisfied.

Proof of Proposition 1.

By lemmas 1, 2 and 3, the patent holder's equilibrium payoffs under the three licensing policies are:

$$\Gamma^F_{PH}(\varepsilon) = \begin{cases} \frac{\varepsilon(6a-\varepsilon)}{9a} & \text{if } 0 < \varepsilon \leqslant 2a \\ \frac{\varepsilon(6a+\varepsilon)}{18a} & \text{if } 2a < \varepsilon < 3a \\ \frac{\varepsilon}{2} & \text{if } 3a \leqslant \varepsilon < c \end{cases}$$

$$\Gamma^R_{PH}(\varepsilon) = \varepsilon$$

$$\Gamma^a_{PH}(\varepsilon) = \begin{cases} \frac{2\varepsilon}{3} & \text{if } 0 < \varepsilon < 3a \\ \frac{a+\varepsilon}{2} & \text{if } 3a \leqslant \varepsilon < c \end{cases}$$

Note that ε is always greater than $\Gamma^a_{PH}(\varepsilon)$ since for the drastic innovation case $\varepsilon \geqslant 3a$. Similarly, $\Gamma^a_{PH}(\varepsilon) > \Gamma^F_{PH}(\varepsilon)$ since $\frac{2\varepsilon}{3} > \frac{\varepsilon(6a-\varepsilon)}{9a}$ for all ε, and $\frac{2\varepsilon}{3} > \frac{\varepsilon(6a+\varepsilon)}{18a}$ for all $\varepsilon < 6a$ when ε is restricted to belong to the $(2a, 3a)$ interval. For the drastic case it is even more obvious that $\Gamma^a_{PH}(\varepsilon) > \Gamma^F_{PH}(\varepsilon)$.

Proof of Proposition 2.

Under a royalty licensing policy, we have that consumer surplus is:

$$CS_2^R = 2\int_0^{\frac{1}{2}} (v-c-a-at^2)dt = v-c-\frac{13}{12}a \quad for \quad 0 < \varepsilon < c$$

where subscripts denote the number of licences issued, and superscripts stand for the means of licensing.

With a fixed fee licensing policy, we have that consumer surplus is:

$$CS_2^F = 2\int_0^{\frac{1}{2}}(v - c - a + \varepsilon - at^2)dt,$$
$$CS_1^F = \int_0^{\left(\frac{1}{2}+\frac{\varepsilon}{6a}\right)}(v - c - a + \frac{2\varepsilon}{3} - at^2)dt$$
$$+\int_0^{\left(\frac{1}{2}-\frac{\varepsilon}{6a}\right)}(v - c - a + \frac{\varepsilon}{3} - at^2)d$$

and $CS_1^{Fd} = \int_0^1 (v - c - (\frac{a-\varepsilon}{2}) - at^2)dt,$ which correspond to:

$$\begin{aligned} CS_2^F &= v - c - \frac{13}{12}a + \varepsilon & \text{if } 0 < \varepsilon < 2a \\ CS_1^F &= v - c - \frac{13}{12}a + \frac{\varepsilon}{2} + \frac{\varepsilon^2}{36a} & \text{if } 2a \leqslant \varepsilon < 3a \\ CS_1^{Fd} &= v - c + \frac{\varepsilon}{2} - \frac{5}{6}a & \text{if } 3a \leqslant \varepsilon < c \end{aligned}$$

Finally, with an auction licensing policy the consumer surplus reads:

$$\begin{aligned} CS_1^a &= v - c - \frac{13}{12}a + \frac{\varepsilon}{2} + \frac{\varepsilon^2}{36a} & \text{if } 0 < \varepsilon < 3a \\ CS_1^{ad} &= v - c + \frac{\varepsilon}{2} - \frac{5}{6}a & \text{if } 3a \leqslant \varepsilon < c \end{aligned}$$

A fixed fee licensing policy weakly dominates an auction licensing policy since $CS_2^F > CS_1^a$ if $0 < \varepsilon < 2a$, $CS_1^F = CS_1^a$ if $2a \leqslant \varepsilon < 3a$ and $CS_1^{Fd} = CS_1^{ad}$ if $3a \leqslant \varepsilon < c$. An auction licensing policy strongly dominates a royalty licensing policy since $CS_1^a > CS_2^R$ if $0 < \varepsilon < 3a$ and $CS_1^{ad} > CS_2^R$ if $3a < \varepsilon < c$. Consequently, a fixed fee policy strongly dominates a royalty policy in consumer surplus terms.

Under a royalty licensing policy, we have that social welfare is:

$$SW_2^R = CS_2^R + \Gamma_{PH}^R(\varepsilon) + \Gamma_A^R(\varepsilon) + \Gamma_B^R(\varepsilon) = v - c - \frac{a}{2} + \varepsilon \quad for \quad 0 < \varepsilon < c$$

With a fixed fee licensing policy we have that social welfare is:

$$\begin{aligned} SW_2^F &= v - c - \frac{1}{12}a + \varepsilon & \text{if } 0 < \varepsilon < 2a \\ SW_1^F &= v - c - \frac{1}{12}a + \frac{1}{2}\varepsilon + \frac{5}{36a}\varepsilon^2 & \text{if } 2a \leqslant \varepsilon < 3a \\ SW_1^{Fd} &= v - c - \frac{1}{3}a + \varepsilon & \text{if } 3a \leqslant \varepsilon < c \end{aligned}$$

Finally, with an auction licensing policy social welfare reads:

$$
\begin{aligned}
SW_1^a &= SW_1^F \quad \text{if } 0 < \varepsilon < 3a \\
SW_1^{ad} &= SW_1^{Fd} \quad \text{if } 3a \leqslant \varepsilon < c
\end{aligned}
$$

A fixed fee licensing policy weakly dominates an auction licensing policy since $SW_2^F > SW_1^a$ if $0 < \varepsilon < 2a$, being equivalent otherwise. An auction licensing policy strongly dominates a royalty licensing policy since $SW_1^a > SW_2^R$ if $0 < \varepsilon < 3a$ and $SW_1^{ad} > SW_2^R$ if $3a \leqslant \varepsilon < c$. Consequently, a fixed fee policy strongly dominates a royalty licensing policy in social welfare terms.

Optimal Licensing Policy when the Patent Holder is One of the Two Producers

Assume that the patent holder is one of the two producers, say A. We next prove that it remains optimal for the patent holder to sell one licence to its competitor by means of a per unit of output royalty. The total payoffs accruing to the patent holder come from two sources, first its market profits and second those coming from the sale of the innovation to the rival.

— Suppose that a fixed fee licensing policy is chosen. Since both firms in the market share the same technology and marginal costs, both obtain $\Pi_A(c-\varepsilon, c-\varepsilon) = \Pi_B(c-\varepsilon, c-\varepsilon) = \frac{a}{2}$ in the downstream market. However, firm B pays a fixed fee to firm A which is equal to its willingness to pay to become a licensee, $F_B = \Pi_B(c-\varepsilon, c-\varepsilon) - \Pi_B(c, c-\varepsilon)$, and therefore, total payoffs to the patent holder are equal to $\Gamma_A^F = \Pi_A(c-\varepsilon, c-\varepsilon) + \Pi_B(c-\varepsilon, c-\varepsilon) - \Pi_B(c, c-\varepsilon)$.

— Suppose on the contrary, that a royalty licensing policy is chosen. In this case, as long as R is not zero, the downstream industry is asymmetric in marginal costs, being R the difference between c_A and c_B. Each firm obtains in the market, $\Pi_A(c-\varepsilon, c-\varepsilon+R)$ and $\Pi_B(c-\varepsilon+R, c-\varepsilon)$, respectively. Once again the patent holder extracts at most the licensee's willingness to pay for the licence. In addition to the usual constraint $0 < R \leqslant \varepsilon$, the royalty, R, must be such that the following constraint is satisfied: $\Pi_B(c-\varepsilon+R, c-\varepsilon) - Rx_B(c-\varepsilon+R, c-\varepsilon) \geqslant \Pi_B(c, c-\varepsilon)$. Since $R = \varepsilon$ violates the latter constraint, it is precisely the binding constraint and we conclude that the royalty chosen by the patent holder must be smaller than ε. Therefore, we know that for any R, the patent holder receives a total payment from the licensee equal to $Rx_B(c-\varepsilon+R, c-\varepsilon) = \Pi_B(c-\varepsilon+R, c-\varepsilon) - \Pi_B(c, c-\varepsilon)$. This means that the total payoffs to the patent holder are equal to $\Gamma_A^R = \Pi_A(c-\varepsilon, c-\varepsilon+R) + \Pi_B(c-\varepsilon+R, c-\varepsilon) - \Pi_B(c, c-\varepsilon)$. By comparing Γ_A^R with Γ_A^F the royalty licensing policy yields higher payoffs to the patent holder as long as $\Pi_A(c-\varepsilon, c-\varepsilon+R) + \Pi_B(c-\varepsilon+R, c-\varepsilon) - 2\Pi_A(c-\varepsilon, c-\varepsilon)$ be positive, which is the case for all $R > 0$. Although, the amount of the royalty set by the patent holder is smaller than the *size* of the innovation, through a royalty policy the patent holder is a more efficient competitor than its rival. In this particular case, the auction and the fixed fee licensing policy are equivalent. Finally, in the case of a drastic innovation, $\varepsilon > 3a$, the question for the patent holder is whether to license its technology. It is easy to prove that a drastic innovation is never licensed.