

Estimation and Identification of Structural Parameters in the Presence of Multiple Equilibria : A Comment

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1 Introduction

During those past twenty years, Russell W. COOPER has been an important contributor in the domain of strategic complementarity and multiple equilibria models. In this lecture, he shares with us some thoughts on the estimation and identification problems raised by such models. The two questions he asks are (1) *Can models with multiple equilibria be rejected?* and (2) *Can the parameters of a model with multiple equilibria be identified?* A collection of examples is given that illustrate where are the problems and how can they be solved case by case. As COOPER mentions it; “*theorems would be preferred*”, but it is the interest of those lectures to allow for a specialist to present his thoughts and interrogations even before general results have been obtained, therefore allowing the reader to observe the very process of scientific research.

In this comment, I would like to raise two points that might clarify some aspects of COOPER's lecture. The first one is related to the estimation of multiple equilibria models. *Why would those models be more difficult to esti-*

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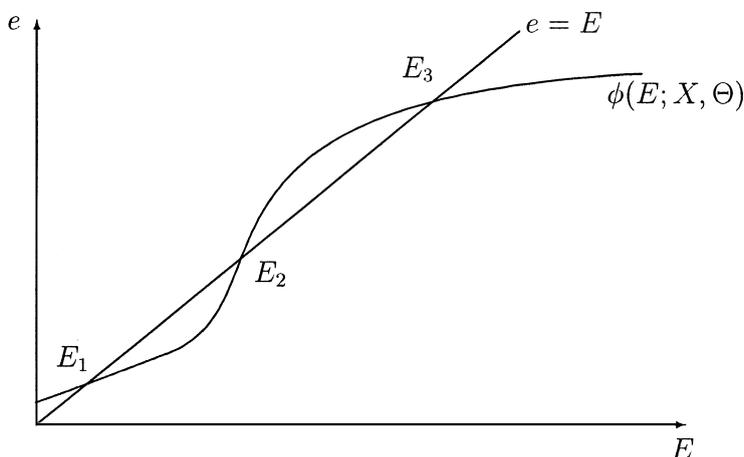
This comment has benefited from discussions with Patrick FÈVE. I would like to take advantage of this comment to express my gratitude to Russ for his help and encouragement during all these years, since I first met him as a PhD student. At that time, he was visiting MAD and Greg LEMON was visiting France with a yellow jersey on his shoulders.

mate than models with unique equilibrium? My view is that, in theory, there would not be more difficult to estimate, had everything been observable by the econometrician. But expectations coordination is crucial in those models, and the mechanisms or variables that drive this coordination are hardly observable. For this reason, both testing for multiplicity and identifying parameters is difficult. My second point is more macro oriented, and is concerned with the relevance of multiplicity for applied macroeconomics. *Which kinds of macroeconomic movements can be the consequence of switches between different equilibria of the same model?* My current opinion is that some cross-sectional variations and some ‘catastrophes’ could be explained by coordination problems between distant equilibria, while this seems less appealing for business cycles fluctuations, since some alternative models with news shocks and expectations revisions are observationally equivalent.

2 The Importance of Expectations Coordination

Here, I use the same notations than COOPER in his section 2. I start from a static game played by a large number of agents, in which $\sigma(e, E; X, \Theta)$ is the payoff of one agent when she plays e and when the average action of the others is E , X being an exogenous variable and Θ a vector of parameters. $e = \phi(E; X, \Theta)$ denotes the best response function of the agent. Assume that the economy displays a configuration in which the set $\xi(X, \Theta)$ has three elements for all possible realization of X , meaning that there exist three Nash equilibria in the model. This is the configuration of Figure 1. A time series

FIGURE 1
A Configuration with Three Equilibria



derived from the model is a sequence $\{E_t\}$ of equilibrium realizations of the static game, for a given sequence $\{X_t\}$.

Assume for a moment that X_t is constant. The theoretical problem raised by this configuration of the model is that any sequence of jumps from one equilibrium to another is an equilibrium sequence. Without a selection device, one cannot make predictions about the future equilibrium path. Once supplemented with an arbitrary selection device, *ie*, an element π in $\Pi(X, \Theta)$, the set of probability distributions over the set of equilibria, one can use the model for prediction. This selection device π can be either a sunspot process, or some backward dynamics in a more complex model (that would need to be fully specified), for example a model without rational expectations but with some learning.

Let us assume that we have such a learning dynamics, and that equilibria E_1 and E_3 are locally stable under the learning dynamics. From the point of view of period t equilibrium of the dynamic model, there is no more multiplicity, although there is still multiplicity of steady states. Let us now assume that X_t is not constant, but observable by the econometrician. For small enough changes in X , the economy will asymptotically stay in the neighborhood of E_1 or E_2 , depending on initial conditions. Assume the economy is in the neighborhood of E_1 . If we were observing such an economy, we would be unable to notice the existence of E_3 , and therefore to realize that other equilibria exist. Assume now that we observe two economies, one being in the neighborhood of E_1 and one in the neighborhood of E_2 . An example of such a configuration is the idea that the American and European labor markets are located on two different equilibria of the same model. Using the cross-section dimension of the data, it will still be difficult to estimate the model and to realize that the two sets of observations are outcomes of the same model. In the case where the model has a simple parametric form, it might be the case that, using the correct parametric form, the function ϕ can be estimated, although such a case is unlikely to occur for real economic phenomena.

Let us now consider a case in which the model is still backward looking, but in which exogenous shocks X_t are large enough to move the economy from the vicinity of E_1 to the vicinity of E_3 and *vice-versa*. In this case, estimating the model using a maximum likelihood procedure and testing for multiplicity is possible. The estimation might be difficult, as the model is non-linear, but it does not raise conceptual problems.

Problems come when we assume that the expectations matter and that there are coordination problems in the economy. Assume now that the selection device is coordination according to a markovian sunspot. At every period t of the dynamic model, the economy can jump from one equilibrium to the other according to the realization of the sunspot. If it is possible for the econometrician to observe both X and the sunspot, then we are back again to a standard problem of estimating a non-linear model. As COOPER mentions it, “*a key issue in the identification is the discrimination between two sources of variations in observable economic variables: sunspots and fundamentals (X_t)*”. My feeling is therefore that if multiplicity raises a problem, it is because it opens the possibility for expectations shocks (sunspots) to play a role in the allocations determination. Because it is difficult to observe them separately from the fundamental shocks, multiplicity raises econometric problems.

3 Which Macroeconomic Fluctuations Do Correspond to Switches Between Equilibria?

Let us again consider the configuration of figure 1. *Which macroeconomic phenomenon is likely to be the consequence of movements from one equilibrium to another?* The first category concerns the cross-sectional dimension of the data. As mentioned previously, the difference between the European and North-American labor market conditions can receive different explanations. The fundamentalist view, for instance BENTOLILA and BERTOLA [1990], would explain those differences by differences in labor market regulation, firing costs, minimum wage, unemployment benefits, etc... Another explanation is that European and American economies are similar, but that agents coordinate on different equilibria in the two countries, as it can happen in theoretical models of the labor market (MANNING [1990 or SAINT-PAUL [1995]). One could be not convinced by the fact that labor market regulations are alike on the two sides of the Atlantic. Politico-economics model can even go further: labor market regulations are not fundamentals but equilibrium outcomes: it is the couple (regulation-unemployment) that can take two equilibrium values in a model in which agents (workers and unemployed) vote for the design of labor markets institutions (SAINT-PAUL [2000]). Politico-economics models also propose to give an explanation for large differences of productivity across countries, that would be created by the *barriers to riches* erected by some group of interest in developing countries (PRESCOT and PARENTE [2000]). Whether or not the cross-sectional variance of the data is large enough to estimate such models is to my knowledge an open question.

A second category of phenomenon is catastrophes, *ie*, brusks, infrequent, large and discontinuous movements of economic activity, that seems unexplained by the movements of current fundamentals. The most obvious example is the Great Depression, and COOPER and CORBAE [2001] show that the 1929-1933 slump can be understood as a switch between two equilibria caused by a change in agents coordination. The Mexican crisis of the early 90s (COLE and KEHOE [2000]) or the recent Asian crisis can also be considered as the results of sunspots phenomena, as no large changes in current fundamentals have been observed. My opinion is that those episodes are indeed the ones for which the multiple equilibria explanation is the most convincing. As I said, those events are infrequent ones, so that there is little hope to precisely test for the multiple equilibria story.

A third category of phenomenon is business cycle. Since Keynes and even earlier before (Pigou or the Stockholm School in the interwar period), expectations, optimism, revisions of believes and animal spirits have been often considered as a major driving force of fluctuations. Here, we do not talk about very large movements of economic activity. Therefore, if there are multiple equilibria, they should be close one to each other. The way the recent literature has tackled this problem is by assuming uniqueness of the steady state but existence of a continuum of equilibrium trajectories around this unique steady state. This indeterminacy, as advocated by BENHABIB and FARMER

[1994] and FARMER [1993] among others, is combined with a sunspot type of selection device, so that the economy fluctuates not only because of shocks to fundamentals, but also because of changes in expectations coordination. Up to now, those models require quite restrictive assumptions about the existence of increasing returns or of large enough externalities, although recent multi-sectoral extensions seems to require less restrictive conditions (NISHIMURA and VENDITTI [2002]). They are also difficult to distinguish from models in which revisions of expectations and news about future fundamentals happen, as there are also models in which current fluctuations are not related to current shocks to fundamentals. Recently, BURNSIDE, EICHENBAUM and REBELO [2001] have proposed a model of the Asian crisis in which the prospect of future deficits, and not a shock on fundamentals or a sunspot, is the cause of the crisis. BEAUDRY and PORTIER [1999] and BEAUDRY and PORTIER [2002] have also build models in which news and expectation revisions are the cause of fluctuations. My opinion is that finding empirical procedures to test between news shocks and animal spirits models is a difficult task. ■

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