ANNALES D'ÉCONOMIE ET DE STATISTIQUE. – N° 62 – 2001

Estimation of Fractionally ARIMA Models for the UK Unemployment

Luis A. GIL-ALANA *

ABSTRACT. – The UK unemployment is examined by means of ARFIMA models using SOWELL's [1992] estimation procedure. A model-selection strategy based on diagnostic tests on the residuals, along with likelihood criteria is adopted to determine the correct model specification. The results suggest that the UK unemployment is well described as an ARFIMA model, with the order of integration fluctuating between 1 and 2. Thus, the standard approach of first differences leads to series with long memory behaviour.

Estimation des modèles fractionnaires ARIMA pour le chômage au Royaume-Uni

RÉSUMÉ. – Nous analysons le chômage au Royaume-Uni à l'aide de modèles ARFIMA, estimés par la méthode de SOWELL [1992]. Les modèles sont sélectionnés en utilisant une approche basée sur les tests de diagnostics sur les résidus et sur des critères liés à la vraisemblance. Les résultats indiquent que le chômage au Royaume-Uni est correctement décrit par un modèle ARFIMA, avec un degré d'intégration compris entre 1 et 2. L'approche classique qui consiste à différencier les séries induit donc une mémoire longue dans le chômage.

---

* L.A. GIL-ALANA: Humboldt Universität zu Berlin, Institut für Statistik und Ökonometrie, Berlin, Germany and University of Navarre, Department of Economics, Pamplona, Spain.
The author gratefully acknowledges the comments of two anonymous referees. Also, financial support from the *European TMR Grant No. ERBFMRX-CT-98-0213* is acknowledged.

1 Introduction and Summary

It is broadly accepted that one feature of macroeconomic time series is that the level of the series evolves or changes with time, although in a rather smooth way. A common practice to explain and model these smooth movements was to assume that the series were I(0) stationary around a deterministic trend, *via* a polynomial and/or a trigonometric function of time, which were fitted by linear regression techniques. A second way came after NELSON and PLOSSER's [1982] influential work, who, following the work and ideas of BOX and JENKINS [1970], argued that these fluctuations in the level were better explained by means of so-called unit roots or I(1) models. In other words, that the change in level was "*stochastic*" rather than "*deterministic*". Both "*schools*" try to model this persistent trend-cycle behaviour of the data although from a different perspective.

However, in recent years, a new approach explaining the source of the non-stationary nature of macroeconomic time series in terms of fractionally integrated processes has appeared. For that purpose, suppose that v_t is an unobservable covariance stationary sequence with spectral density function that is bounded and bounded away from zero at any frequency, and

$$(1) \qquad (1-L)^d \, x_t = v_t \, , \quad t = 1, 2, \ldots \, .$$

The process v_t could itself be a stationary and invertible ARMA sequence, when its auto-covariances decay exponentially. When $d = 0$ in (1), $x_t = v_t$, and thus, x_t is "*weakly auto-correlated*"', also termed "*weakly dependent*". On the other hand, when $d > 0$, x_t is called "*strongly dependent*", so-named because of the strong association between observations widely separated in time, and one series will be more persistent than another if its order of integration is higher. If $0 < d < 0.5$, x_t is still stationary, but its lag-j auto-covariance γ_j decreases very slowly, like the power law j^{2d-1} as $j \to \infty$ and so the $|\gamma_j|$ are non-summable. We say then that x_t has long memory given that its spectral density $f(\lambda)$ is unbounded at the origin. As d in (1) increases beyond 0.5 and through 1 (the unit root case), x_t can be viewed as becoming "*more non-stationary*" in the sense, for example, that the variance of the partial sums increases in magnitude. This is also true for $d > 1$, so a large class of non-stationary processes may be described by (1) (or (2) below) with $d \geqslant 0.5$. The distinction between I(d) processes with different values of d is also important from an economic viewpoint: if a variable is I(d) with $d \in [0.5,1)$, it will be covariance non-stationary but mean reverting since an innovation will have no permanent effect on its value. This is in sharp contrast to an I(d) process with $d \geqslant 1$, which will be both covariance non-stationary and not mean reverting, in which case the effect of an innovation will persist forever. Thus, d plays a crucial role in explaining the degree of persistence of the series.

Processes like (1) with positive non-integer d are called fractionally integrated and when v_t is ARMA(p,q), x_t has been called a fractionally ARIMA (ARFIMA(p,d,q)) process. Thus the model becomes

$$(2) \qquad \phi(L)(1-L)^d\, x_t = \theta(L)\varepsilon_t, \quad t = 1, 2, \ldots,$$

where $\phi(L)$ and $\theta(L)$ are polynomials of orders p and q respectively, with all zeroes of $\phi(L)$ and $\theta(L)$ outside the unit circle, and ε_t is white noise. These kind of models were introduced by GRANGER and JOYEUX [1980], GRANGER [1980, 1981] and HOSKING [1981], (although earlier work by ADENSTEDT [1974] and TAQQU [1975] shows an awareness of the representation), and were justified theoretically in terms of aggregation by ROBINSON [1978] and GRANGER [1980] and more recently, in terms of the duration of shocks by PARKE [1999]. A complete survey of the literature based on these type of models can be found in BAILLIE [1996], and recent studies using fractional models for assessing the persistence in macroeconomic data are amongst others HASSLER and WOLTERS [1995], GIL-ALANA and ROBINSON [1997, 2001], GIL-ALANA [2000] and HAUSER *et al.* [1998].

In view of the preceding remarks, there is some interest in estimating the fractional differencing parameter *d*, along with the other parameters related to the ARMA representation. SOWELL [1992] analyzed the exact maximum likelihood estimates of the parameters of a fractionally ARIMA model (2) in the time domain, using a recursive procedure that allows quick evaluation of the likelihood function, which is given by

$$(2\pi)^{-T/2}\,|\Sigma|^{-1/2}\exp\left(-\frac{1}{2}X_T'\,\Sigma^{-1}X_T\right),$$

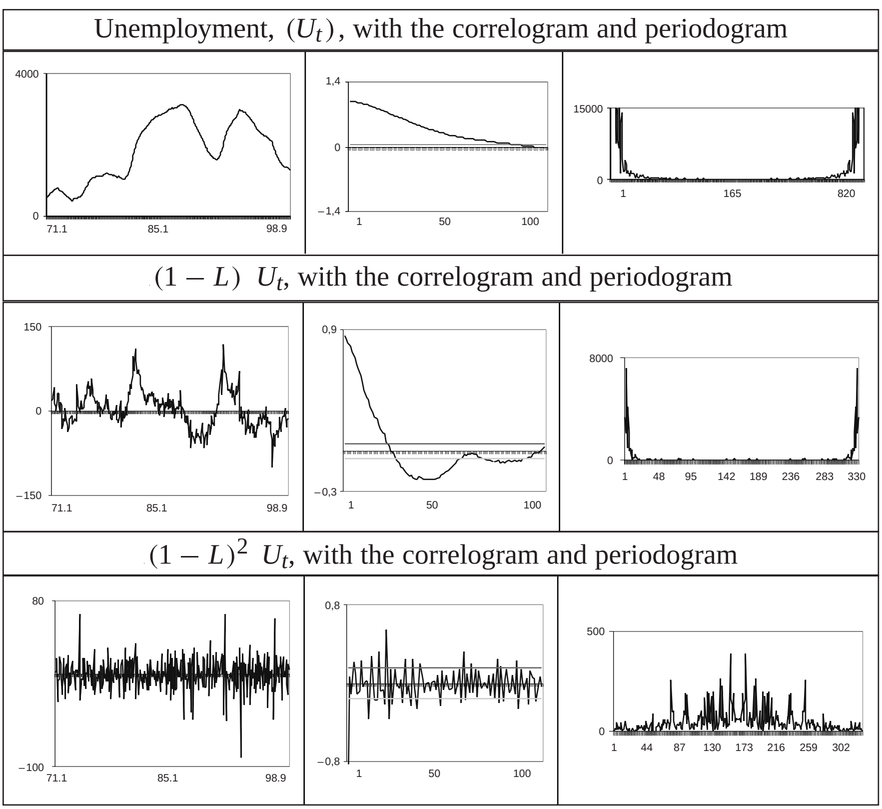
where $X_T = (x_1, x_2, \ldots, x_T)$' and $X_T \sim N(0, \Sigma)$. Other parametric methods of estimating *d* based on the frequency domain were proposed amongst other by FOX and TAQQU [1986] and DAHLHAUS [1989]. Small sample properties of these and other estimates were examined in SMITH *et al.* [1997] and HAUSER [1999]. In the first of these articles, they compare several semi-parametric procedures with the maximum likelihood estimation method of SOWELL [1992], finding that the SOWELL's [1992] procedure outperforms the semi-parametric ones in terms of the bias and the mean square errors. HAUSER [1999] also compares the SOWELL's [1992] procedure with others based on the exact and the *Whittle* likelihood function in the time and in the frequency domain and shows that the SOWELL's [1992] procedure dominates to the others in case of fractionally integrated models. Furthermore, we use, the SOWELL's [1992] estimation method in the empirical application in Section 2 because of the simplicity in the calculations that it affords through Ox (see, DOORNIK and OOMS [1999]).

In this article, we claim that the UK unemployment may be well described by means of fractionally ARIMA models and show that the classical trend-stationary I(0) and unit roots I(1) representations may be too restrictive with respect to the low-frequency dynamics of the series. In the following section, we describe ARFIMA models for different measures of the UK unemployment, while Section 3 contains some concluding remarks.

2 Application to the UK Unemployment

In this section, we analyze the UK unemployment from January 1971 to September 1998 by means of fractionally integrated ARMA models. A measure of unemployment, based on administrative sources, is the number of people claiming unemployment benefit. This measure is known as the *Claimant Count* (CC) and is available monthly, and though it does not provide a measure corresponding to the *International Labour Organization* (ILO) definition, it moves approximately in the same way as other measures such as the one obtained from the *Labor Force Survey* (LFS). Calling this variable U_t, we also look at its logarithmic transformation, log U_t. Another measure, related with the unemployment rate, is the CC as a percentage of

FIGURE 1
Claimant Count Series



the workforce. We also carry the analysis on this variable, (u_t), along with its logistic transformation, u_t^*, where

$$u_t^* = \log\left(\frac{u_t}{1-u_t}\right).^1$$

Figure 1 shows the CC series (U_t), its first and second differences, with their corresponding correlograms and periodograms. Very similar pictures were obtained for the remaining measures of unemployment and they can be found in GIL-ALANA [1999a]. The slow decay observed in the correlogram and the large peak around the zero frequency in the periodogram of the original series give us an indication of the possibly fractionally integrated structure for unemployment. The correlogram of the first differenced series still present a slow decay and/or oscillation, with significant autocorrelations even at large lags, which may indicate that the series still has a degree of long memory. Finally, in the second differenced series, we observe large negative values in the correlogram at lag 1, and also, the periodogram shows values close to 0 at the zero frequency, which may both suggest that the series is now overdifferenced.

We estimate for each of the measures of unemployment different ARFIMA(*p*,*d*,*q*) models with *p* and *q* smaller than or equal to three,[2] using the SOWELL's [1992] procedure of estimating by maximum likelihood in the time domain. This procedure consistently estimates *d* and the other parameters in the model when $d < 0.5$. Thus, in order to assure stationarity, we estimate the models in second differences, adding then two to the resulting values to obtain estimates of *d*.

Table 1 summarizes the estimated *d*'s for the different ARMA representations based on the second differenced series.[3] We observe that the estimates vary substantially depending on the series and on the orders of the AR and MA components. They lie between 0.76 for u_t when the disturbances are ARMA(3,3) and 2.07 for U_t with ARMA(3,1) v_t. In general, we observe higher estimates for U_t than for the remaining series. In fact, we see in this series that in 8 out of the 16 estimates of *d*, the values are greater than 2. For log U_t. apart from the ARMA(1,3) model, all the estimates are constrained between 1.63 and 1.93. Looking at u_t, four estimates are below 1.49, one is 1.50 and the remaining are between 1.70 and 1.99. Similarly, for u_t^*, only three estimates are below 1.50 while the others lie between 1.64 and 1.89.

All these estimated values of *d* were obtained by maximum likelihood. A crucial fact when estimating with parametric approaches is that the model must be correctly specified; if it is misspecified, the estimates of *d* are liable to be inconsistent. In fact, misspecification of the short run components of the series can invalidate the estimation of its long run behaviour.

In order to analyse which might be the best model specification for each of the series we proceed as follows: firstly we select for each series only those

1. See WALLIS [1987] for a justification based on the logistic transformation being defined between $\pm\infty$ so that standard distribution apply.
2. Higher orders of *p* and *q* were also performed but most of the models were inappropriate in view of the large number of parameters used to describe the short-run dynamics of the series.
3. If the true *d* is larger than 0.5, then by its construction, the exact maximum likelihood procedure at most estimates a value of 0.5.

TABLE 1

Maximum Likelihood Estimates of d in ARFIMA(p,d,q) Models for the UK Unemployment

ARMA (p, q)	U_t	LOG U_t	u_t	u_t^*
(0, 0)	1.66	1.63	1.50	1.49
(1, 0)	1.83	1.71	1.70	1.64
(0, 1)	2.00	1.75	1.89	1.85
(1, 1)	2.01	1.86	1.97	1.85
(2, 0)	1.92	1.74	1.88	1.71
(0, 2)	2.01	1.85	1.99	–
(2, 1)	2.02	1.86	1.97	1.85
(1, 2)	1.36	–	1.43	–
(2, 2)	1.33	1.86	1.97	1.09
(3, 0)	2.05	1.84	1.92	1.81
(0, 3)	2.00	1.86	–	1.85
(3, 1)	2.07	1.93	1.27	1.89
(3, 2)	1.89	1.77	0.90	1.66
(1, 3)	1.39	0.86	–	1.87
(2, 3)	1.18	1.81	1.99	1.84
(3, 3)	2.03	–	0.76	1.12

–: *The model failed to achieve convergence after 240 iterations. All the estimates are based on the second differences.*

TABLE 2

Best Model Specification for Each of the UK Unemployment Series According to the "Model Selection" and "Likelihood" Criterions

Series	ARFIMA (p, d, q)	t – tests'			AR parameters			MA parameters		
		$t_{d=0}$	$t_{d=1}$	$t_{d=2}$	ϕ_1	ϕ_2	ϕ_3	θ_1	θ_2	θ_3
U_t	(3, 2.05, 0) (0.08)	25.6	13.1	0.62'	-0.56 (0.09)	-0.29 (0.09)	-0.16 (0.07)	–	–	–
	(2, 1.33, 2) (0.30)	4.43	1.10'	-2.23	1.65 (0.17)	-0.69 (0.13)	– –	-1.52 (0.18)	0.68 (0.18)	–
Log U_t	(3, 1.77, 2) (0.06)	29.5	12.8	-3.83	0.65 (0.08)	-0.84 (0.06)	-0.17 (0.08)	-0.87 (0.01)	1.00 (0.02)	–
u_t	(2, 1.88, 0) (0.06)	31.3	14.6	2.00	-0.61 (0.07)	-0.26 (0.06)	–	–	–	–
	(1, 1.43, 2) (0.08)	4.93	1.48'	-1.96	0.89 (0.08)	–	–	-1.10 (0.22)	0.36 (0.14)	–
u_t^*	(1, 1.64, 0) (0.05)	32.8	12.8	-7.20	-0.25 (0.07)	–	–	–	–	–
	(2, 1.09, 2) (0.13)	8.38	0.69'	-7.00	1.87 (0.11)	-0.89 (0.10)	–	-1.65 (0.11)	0.71 (0.09)	–

*: *The first row of each series corresponds to the best model specification according to the 'model selection' criterion based on diagnostic tests on the residuals and LR tests. The second row indicates the best model according to the AIC and BIC.*

': *Non-rejection values at the 95 % significance level. Standard errors in parenthesis.*

models which pass several diagnostic tests on the residuals. In particular, we apply tests of normality, heteroscedasticity, autoregressive conditional heteroscedasticity (ARCH) and Lung & Box tests. Then, we concentrate on these selected models and analyse them carefully through a model-selection criterion based on LR tests on nested parametric hypotheses, along with the Akaike (AIC) and Bayesian (BIC) information criteria. (It should be borne in mind however that the AIC and BIC may not necessarily be the best criteria for applications involving fractional differences).[4] A more detailed description of how these procedures were implemented on the four measures of unemployment can be found in GIL-ALANA [1999a].

Table 2 summarizes the selected models for each of the series according to both the selection criterion based on the diagnostic and LR tests, and the one based on the likelihood criteria. We see that the only series for which both model selection procedures lead to the same specification is the log of the CC series (log U_t), which produces an ARFIMA(3, 1.77, 2) model. One may interpret this as evidence for an appropriate choice of the prior data transformation in comparison with the remaining series, for example, by taking logs, eliminating a changing variance which may influence the MLE in case of the U_t.[5] It may also result surprising that both information criteria (the AIC and the BIC) lead to the same specification for each series. In general, the BIC selects a more parsimonious specification. However, the fact that both criteria choose a highly parameterized model for each series (with significance coefficients, in view of the low standard errors) may give some support for these model specifications. We observe, in this table, that all the orders of integration are greater than 1. In fact, the null hypothesis of I(0) stationary ARMA representations is rejected for all the series and all the models, and the I(1) or unit root hypothesis is rejected in 4 out of the 7 models presented.[6] The null hypothesis of two unit roots is also rejected in all but one of the models. We see that for the CC series (U_t), our "*model-selection*" criterion chooses an ARFIMA(3, 2.05, 0) model, clearly rejecting the I(0) and I(1) hypotheses. However, according to the AIC and BIC, the best model specification for U_t seems to be an ARFIMA(2, 1.33, 2), and in this case, the unit root null hypothesis cannot be rejected. Looking at the log of the CC series, (log U_t), both criteria lead to the same specification: an ARFIMA(3, 1.77, 2), and in this case, the I(0) and I(1) hypotheses are clearly rejected. That means that the growth rate of the number of unemployees has a strong component of long memory, with shocks affecting the series, taking a long time to return to its original level. For the series corresponding to the CC as a percentage of the workforce (u_t), we find two plausible models: an ARFIMA(2, 1.88, 0), using the "*model-selection*" procedure, and an ARFIMA(1, 1.43, 2) with the "*likelihood-based*" procedures. According to the first of these two models, both the

4. These criteria concentrate on the short-term forecasting ability of the fitted model and may not give sufficient attention to the long run properties of the ARFIMA models. (see, *eg*, HOSKING [1981, 1984]). Another recent paper about model selection in the presence of long and short memory processes is BERAN *et al.* [1998]. They propose versions of the AIC, BIC and the HQ (HANNAN and QUINN [1998]) in case of fractional autoregressions but do not consider MA components.

5. In addition, this model has the lowest standard error for the estimated *d*. Also, the standard errors of the AR and MA coefficients are relatively low.

6. Note that the SOWELL's [1992] estimation procedure is based on maximum likelihood and thus, conventional tests based on the statistic $(d - \hat{d})/SE(\hat{d})$ can be performed.

TABLE 3

Impulse Response Functions for the First Differences of the UK Unemployment

Series:	U_t	U_t	Log U_t	u_t	u_t	u_t^*	u_t^*
ARFIMA model	(3, 2.05, 0)	(2, 1.33, 2)	(3, 1.77, 2)	(2, 1.88, 0)	(1, 1.43, 2)	(1, 1.64, 0)	(2, 1.09, 2)
0	1.000	1.000	1.000	1.000	1.000	1.000	1.000
1	.490	.460	.550	.270	.220	.390	.310
2	.511	.466	.529	.402	.390	.427	.300
3	.505	.514	.518	.478	.412	.354	.302
4	.597	.558	.526	.373	.417	.331	.303
5	.555	.589	.495	.399	.415	.307	.300
6	.562	.606	.437	.395	.410	.289	.293
7	.564	.610	.396	.378	.402	.275	.282
8	.575	.602	.403	.379	.392	.263	.267
9	.573	.587	.434	.373	.382	.252	.249
10	.577	.565	.439	.368	.371	.244	.229
11	.579	.538	.403	.365	.359	.236	.207
12	.582	.510	.358	.361	.348	.229	.184
13	.583	.480	.347	.358	.337	.223	.160
14	.585	.450	.376	.355	.326	.217	.136
15	.581	.420	.401	.353	.316	.212	.112
16	.589	.392	.389	.350	.305	.207	.089
17	.591	.365	.346	.348	.296	.203	.067
18	.592	.341	.319	.345	.286	.199	.047
19	.594	.318	.332	.343	.277	.195	.028
20	.595	.297	.365	.341	.268	.191	.011
40	.615	.138	.266	.315	.162	.150	.005
60	.627	.102	.286	.300	.120	.130	.015
80	.636	.082	.253	.290	.099	.117	.002
100	.643	.071	.226	.283	.086	.108	.005
200	.649	.061	.243	.277	.077	.101	.003

*: *The first column for each series corresponds to the best model according to the 'model selection' criterion based on diagnostic tests on the residuals, and the second column corresponds to the best model according to the AIC and BIC.*

I(1) and the I(2) hypotheses are rejected, however, using the second specification, the I(1) null is not rejected and the I(2) is slightly rejected. Finally, looking at the logistic transformation of u_t, *ie*, u_t^*, a higher order of integration is again observed when using the "*model-selection*" procedure, choosing the ARFIMA(1, 1.64, 0). In this case, once more the I(1) and I(2) hypotheses are rejected, but using the AIC and BIC, the selected model is an ARFIMA(2, 1.09, 2)[7] model and the unit root null hypothesis cannot then be rejected.

Table 3 resumes the first twentieth (and the 40^{th}, 60^{th}, 80^{th}, 100^{th} and 120^{th}) impulse responses for the first differenced series. We see that for U_t, the conclusions will be very different depending on the model chosen. Thus, if U_t is modelled as an ARFIMA(3, 2.05, 0), the effect of a shock affecting the

7. In this case, SOWELL's [1992] procedure was also implemented in first differences and the result was practically the same as with second differences ($d = 1.089$).

differenced series will be permanent. On the contrary, if we take Ut as an ARFIMA(2, 1.33, 2), shocks on $(1-L)\,U_t$ will tend to disappear in the long run, though we observe that even 20 periods after the shock, almost 30 % of its effect still remains on the series. Similarly, for log U_t and u_t, the effect of the shocks on the differenced series are highly persistent: 25.3 % of the initial shock for the growth rate of the CC series and 29 % and 9.9 %, (depending on the selected model), for $(1-L)u_t$, still remain after 80 periods. Finally, for $(1-L)u_t^*$, the effect of the shocks are smaller, though still persistent: 15 % when modelling u_t^* as an ARFIMA(1, 1.64, 0) and 0.5 % with u_t^* as an ARFIMA(2, 1.09, 2) after 40 periods, though in this latter model, 11.2 % of the shock still persists after 15 periods.

3 Concluding Comments

We have showed, in this paper, that the UK unemployment may be well described as a fractionally integrated ARMA (ARFIMA) model, with the order of integration varying substantially depending on the model chosen for the short-run components of the series. This order of integration seems to fluctuate between 1 and 2, and this is observed for the four different measures of unemployment used. Thus, we show that the standard approach of taking first differences to get I(0) stationary series when modelling the UK unemployment may be too restrictive, leading to series with a strong component of long memory behaviour.

These results are completely in line with those obtained in GIL-ALANA [1999b, c]. In these two papers, the same four measures of unemployment for the same period of time were examined, in the former paper, using a parametric testing procedure of fractionally integrated hypotheses proposed by ROBINSON [1994a], and, in the latter, using several semi-parametric techniques for estimating *d* that were recently developed by ROBINSON in a number of papers (ROBINSON [1994b, 1995a, b]). In both cases, he obtained strong evidence in favour of a large degree of integration for the UK unemployment, corroborating the finding obtained here that the UK unemployment is a very highly persistent variable.

Also FUNKE [1999] and TSCHERNIG and ZIMMERMANN [1992], for the US and German unemployment, respectively, found evidence of fractional models, though their orders of integration were smaller than those reported here for the UK case. In this respect, HARVEY and CHUNG [1999] found that the CC series may be modeled in terms of a local linear trend model, which corresponds itself to an ARIMA model with an order of integration of 2, and thus, showing as well a large degree of persistence in its behaviour. Furthermore, FUNKE [1999] used a semi-parametric procedure due to GEWEKE and PORTER-HUDAK [1983] and this method was severely criticized by SOWELL [1990] for its sensitivity to short-run components which cannot be handled by the GEWEKE and PORTER-HUDAK's [1983] method.

Finally, the fact that both model selection procedures (the one based on diagnostic tests on the residuals and LR tests, and the one based on information criteria) lead to the same specification only for the log of the CC series suggests that this series may be the appropriate transformation of the data when using ARFIMA models. ■

• References

ADENSTEDT R.K. (1974). – « On Large Sample Estimation for the Mean of a Stationary Random Sequence », *Annals of Statistics*, 2, pp. 1095-1107.

BAILLIE R. T. (1996). – « Long Memory and Fractional Integration in Econometrics », *Journal of Econometrics*, 73, pp. 5-59.

BERAN J., BHANSALI R.J., OCKER D. (1998). – « On Unified Model Selection for Stationary and Non-stationary Short- and Long-Memory Autoregressive Processes », *Biometrika*, 85, pp. 921-934.

BOX G.E.P., JENKINS G.M. (1970). – *Time Series Analysis, Forecasting and Control*, San Francisco, Holden Day.

DAHLHAUS R. (1989). – « Efficient Parameter Estimation for Self-Similar Processes », *Annals of Statistics*, 17, pp. 1749-1766.

DOORNIK J. A., OOMS M. (1999). – « A Package for Estimating, Forecasting and Simulating ARFIMA Models: Arfima Package 1.0 for Ox », Oxford: *www.nuff.ox.ac.uk/Users/Doornik*.

FOX R., TAQQU M. (1986). – « Large Sample Properties of Parameter Estimates for Strongly Dependent Stationary Gaussian Time Series », *Annals of Statistics*, 14, pp. 517-532.

FUNKE M. (1999). – « The Long Memory Property of the US Unemployment Rate », *Mimeo*.

GEWEKE J., PORTER-HUDAK S. (1983). – « The Estimation and Application of Long Memory Time Series Models », *Journal of Time Series Analysis*, 4, pp. 221-228.

GIL-ALANA L.A. (1999a). – « Estimation of Fractional ARIMA Models for the UK Unemployment. Center for Economic Forecasting », London Business School, *Discussion Paper* No. 05-99.

GIL-ALANA L.A. (1999b). – « Testing the Order of Integration in the UK Unemployment », Center for Economic Forecasting, London Business School, *Discussion Paper* No. 06-99.

GIL-ALANA L.A. (1999c). – « Semi-parametric Estimation of the Fractional Differencing Parameter in the UK Unemployment », Center for Economic Forecasting, London Business School, *Discussion Paper*, forthcoming.

GIL-ALANA L.A. (2000). – « Mean Reversion in the Real Exchange Rates », *Economics Letters*, 69, pp. 285-288.

GIL-ALANA L.A., ROBINSON P.M. (1997). – « Testing of Unit Root and Other Non-stationary Hypotheses in Macroeconomic Time Series », *Journal of Econometrics*, 80, pp. 241-268.

GIL-ALANA L.A., ROBINSON P.M. (2001). – « Seasonal Fractional Integration in the UK and Japanese Consumption and Income », *Journal of Applied Econometrics*, 16, pp. 95-114.

GRANGER C.W.J. (1980). – « Long Memory Relationships and the Aggregation of Dynamic Models », *Journal of Econometrics*, 14, pp. 227-238.

GRANGER C.W.J. (1981). – « Some Properties of Time Series Data and their Use in Econometric Model Specification », *Journal of Econometrics*, 16, pp. 121-130.

GRANGER C.W.J., JOYEUX R. (1980). – « An Introduction to Long Memory Time Series and Fractional Differencing », *Journal of Time Series Analysis*, 1, pp. 15-29.

HANNAN E.J., QUINN B.G. (1979). – « The Determination of the Order of an Autoregression », *Journal of the Royal Statistical Society,* Series B. 41, pp. 190-195.

HASSLER U., WOLTERS J. (1995). – « Long Memory in Inflation Rates: International Evidence », *Journal of Business and Economic Statistics,* 19, pp. 37-45.

HARVEY A.C., CHUNG C.-H. (1999). – « Estimating the Underlying Change in UK Unemployment », *Mimeo*.

HAUSER M.A. (1999). – « Maximum Likelihood Estimators for ARFIMA Models: A Monte Carlo Study », *Journal of Statistical Planning and Inference,* forthcoming.

HAUSER M.A., PÖTSCHER B.M., RESCHENHOFER E. (1998). – « Measuring Persistence in Aggregate Output: ARMA Models, Fractionally Integrated ARMA Models and Non-Parametric Procedures », *Empirical Economics,* 24, pp. 243-269.

HOSKING J.R.M. (1981). – « Fractional Differencing », *Biometrika*, 68, pp. 165-176.

HOSKING J.R.M. (1984). – « Modelling Persistence in Hydrological Time Series Using Fractional Differencing » *Water Resources Research*, 20, pp. 1898-1908.

NELSON C.R., PLOSSER C.I. (1982). – « Trends and Random Walks in Macroeconomic Time Series », *Journal of Monetary Economics*, 10, pp. 139-162.

PARKE W.R. (1999). – « What is Fractional Integration? », *Review of Economics and Statistics*, 8, pp. 632-638.

ROBINSON P.M. (1978). – « Statistical Inference for a Random Coefficient Autoregressive Model », *Scandinavian Journal of Statistics*, 5, pp. 163-168.

ROBINSON P.M. (1994A). – « Efficient Tests of Non-Stationary Hypotheses », *Journal of the American Statistical Association*, 89, pp. 1420-1437.

ROBINSON P.M. (1994b). – « Semi-Parametric Analysis of Long Memory Time Series », *Annals of Statistic*, 22, pp. 515-539.

ROBINSON P.M. (1995a). – « Log-Periodogram Regression of Time Series with Long-Range Dependence », *Annals of Statistics*, 23, pp. 1048-1072.

ROBINSON P.M. (1995b). – « Gaussian Semi-Parametric Estimation of Long-Range Dependence », *Annals of Statistics*, 23, pp. 1630-1661.

SMITH J., TAYLOR N., YADAV S. (1997). – « Comparing the Bias and Misspecification in ARFIMA Models », *Journal of Time Series Analysis*, 18, pp. 507-527.

SOWELL F. (1992). – « Maximum Likelihood Estimation of Stationary Univariate Fractionally Integrated Time Series Models », *Journal of Econometrics*, 53, pp. 165-188.

TAQQU M.S. (1975). – « Weak Convergence to Fractional Brownian Motion and to the Rosenblatt Process », *Zeitschift. Wahrscheinlichkeitstheorie verw*., 31, pp. 287-302.

TSCHERNIG R., ZIMMERMANN K.F. (1992). – « Illusive Persistence in German Unemployment », *Recherches Économiques de Louvain*, 58, pp. 441-453.

WALLIS K. (1987). – « Time Series Analysis of Bounded Economic Variables », *Journal of Time Series Analysis*, 8, pp. 116-132.