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Optimal Employment Subsidies To Heterogeneous Workers:

Unemployment-Trap, Job-Additionality and Tax Rates

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ABSTRACT. – Unemployment and welfare benefits generate unemployment traps. In this paper, we design the optimal employment subsidies that allow governments to reduce unemployment traps under constant budget deficit and without diminishing workers' welfare. We explore the effects of work incentives on the shape and on the properties of employment subsidies. We finally address the issues of *self-help* effect and job-additionality.

Subventions optimales à l'emploi : trappe à chômage, effet d'aubaine et taux effectifs de taxation

RÉSUMÉ. – La présence d'allocation de chômage entraîne la création d'une trappe à chômage pour les travailleurs de faible productivité. Dans cet article, nous analysons une subvention à l'emploi qui permet la réduction de cette trappe à chômage sans détérioration du bien-être des travailleurs et du budget du gouvernement. Nous étudions les questions d'additionalité et de taux effectifs de taxation ainsi que l'impact des incitations au travail sur la forme des subventions.

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1 Introduction

The presence of unemployment and welfare benefits have been disputed intensely for their disincentive properties on the unemployed and welfare recipients. These individuals indeed loose their entitlement to unemployment and welfare programs when they choose to work. As they naturally prefer to remain unemployed, they are said to fall into the "*unemployment trap*". This problem has been well documented by authors like ATKINSON [1995] and VAN PARIJS [1995] who argue for a global reform of the tax-benefit system. They propose a basic-income-linear-tax system in which a constant marginal tax rate would restore work incentives and in which redistribution would be performed through a lump sum transfer.

Yet, less dramatic remedies for the work discouraging effects of unemployment and welfare benefits have been proposed by economists and policy makers. Employment subsidies, tax credits or reductions in social contributions for low income workers offer appropriate theoretical responses to this issue (see DRÈZE and SNEESSENS [1994], PHELPS [1994] and SNOWER [1994]). In practice, such remedies have already been used with some success. The Earned Income Tax Credit (EITC) that offers tax reductions to US low income earners is recognized for its significant effect on labor supply (HOFFMAN and SEIDMAN [1990], EISSA and LIEBMAN [1995] and KATZ [1996]). Tax credit, reductions in social contributions and employment subsidies have been introduced in many European countries (*ie,* Belgium, France, Germany, Netherlands, ...) during last years.

Work incentives constitute the central problem of the theory of optimal income taxation (MIRRLEES [1971]). However, this theory hardly reconciles the redistribution objectives involved in taxation with low unemployment rates. A strong income redistribution is consistent with large unemployment traps. In their simulations of (highly) redistributive tax systems, KANBUR and TUOMALA [1996] find optimal unemployment rates that rise up to 19 % (*Table 3, p. 280*). Low productivity individuals indeed choose to be unemployed not only because of their low marginal rate of substitution between consumption and effort or working time (SEADE [1977]), but mainly because incentive problems imply that they should face high marginal tax rates (TUOMALA [1990], PIKETTY [1997]). Highly redistributive societies should thus accept large unemployment traps.

Still, in highly redistributive nations such as in the European Union, there exists a strong popular concern about unemployment, a concern that is relieved by political institutions and by governments and inter-governmental institutions.[1] Indeed, the role of employment in the satisfaction of social needs of individuals is extensively discussed in many fields of social sciences and is relayed into the political debate. Moreover, recent economic literature on employment subsidies and tax credits (KATZ [1996], BLUNDELL and MACCURDY [1999]), and on unemployment traps focuses less on redistribu-

1. See Title 1, Article 2 of the consolidated version of the *Treaty of the European Union*. See also, the *Employability objective* of the *Luxembourg Job Summit Agreement 1997*.

tion issues than on relieving work disincentive effects of welfare programs, redistribution being already achieved by the latter.

The current paper departs from the traditional welfare analysis used in public economics. We study the situation in which the government designs employment subsidies in order to reduce the unemployment trap under plausible institutional constraints. Governments have indeed limited control over economic and institutional variables. In many European countries, unemployment benefits are determined by institutions gathering employees, employers and government representatives (*eg*, UNEDIC in France). In the US, the main welfare benefit programs are ruled by local states, not by the federal government that proposes the EITC. It is reasonable to assume that in the short term, a government has no control over the unemployment and welfare benefits and that it cannot alter the tax system over the whole population.

We assume that the government's agenda is to reduce the disincentive effects of unemployment and welfare programs, *ie*, to diminish the unemployment trap. This set-up has three advantages. First, as argued above and by KANBUR, KEEN and TUOMALA [1994], it is closely related to the "*normal terms of practical policy formulation and evaluation*". Second, it yields analytical results and insights that could hardly be obtained under traditional welfare objectives. The present model can then be viewed as a benchmark on which the traditional discussion on redistribution can be added. Finally it allows us to concentrate on the incentive issues of employment subsidies, on their shapes and on their properties, which are the focus of the paper.

Work incentives are directly related to the asymmetry of information between workers and government. In this paper we discuss several aspects of this relation. We first explore the effects of work incentives on employment subsidies and the possibility of eliminating the unemployment trap. In relation to this, we address the issue of the *self-help* effect introduced by KANBUR, KEEN and TUOMALA [1994] in poverty alleviation policies. Anti-unemployment policies may induce such a *self-help* effect by setting negative marginal tax rates on subsidized workers. Negative marginal tax rates induce individuals to work more than the efficient work (or effort) level and thereby permit a reduction in the amount of subsidy. The welfare of low ability workers may however worsen under such policies. We also qualify the problem of *job additionality* (FOLEY [1992], MEAGER and EVANS [1998]) according to which many workers receive a subsidy although they would have been hired without it. We finally give some theoretical basis for the shape of the employment subsidies or tax credits. That is, we relate the bell-shaped EITC to the possible accumulation (mass) of workers at low skill levels.

Of course, anti-unemployment policies should not target only the supply side of the labor market, as proposed in this paper. DRÈZE and SNEESSENS [1994]), SNOWER [1994], PHELPS [1994] and many others also suggest policies that apply on the demand side of the labor market. The government can also support the employment in private firms through employment subsidies to firms. In this case, the efficacy of the policy is reduced by the asymmetric information between the government and the private firms. Private firms indeed seek subsidies for jobs that would have been hired without a subsidy (see PICARD [2001]). In this paper, we leave aside the issue of policy design on the supply side of the labor market and we focus on the problems of unemployment trap generated on the supply side of the labor market.

The paper is structured in the following way. Section 2 presents the model, Section 3 studies the benchmark case of the optimal employment subsidy under complete information. Section 4 proposes the design of employment subsidies under incomplete information. Job additionality, *self-help* effects, subsidy shapes and the possibility of removing the unemployment trap are discussed. Section 5 concludes. Proofs are relegated to the appendix.

2 The Model

Workers

Workers have heterogeneous skills or abilities. Ability $x \in [\underline{x}, \bar{x}]$ is distributed according to the c.d.f. $N(x)$ and the p.d.f $n(x) > 0$. Work productivity or effort h accounts for *efficiency units* that workers provide at work. Working generates a disutility $d(h,x) \geqslant 0$ with $d(0,x) = 0$. Since larger productivity levels are increasingly costly for workers, it must be that $d_h > 0$ and $d_{hh} > 0$. Moreover, as more able workers suffer from less disutility, work disutility and marginal work disutility decrease with ability, *ie,* $d_x \leqslant 0$ and $d_{hx} < 0$.[2]

We assume additively separable utility functions, $U = c - d(h,x)$, where c stands for the worker's consumption. Workers consume their whole revenues which are composed of net earnings and subsidies $c = \theta wh$ where $\theta = 1 - \tau$ and where τ is a linear tax rate. The total tax on each worker is therefore $T = \tau wh$. The wage w is the competitive *wage per efficiency unit*.[3] The worker's utility is therefore

$$U(h,x) = \theta wh - d(h,x).$$

The assumption of additive separable utility function prohibits income effects. Lump sum subsidies to workers do not change their choice of productivity level. On the one hand, as in ATKINSON and STIGLITZ [1980], ATKINSON [1995] and in DIAMOND [1998], this assumption is made for tractability purpose. It allows to isolate information rents and to work out the government budget expression. Utility functions that generate income effects necessitate to resort to numerical simulations at the cost of significant loss of insight (see KANBUR, KEEN and TUOMALA [1994]). On the other hand, this assumption remains consistent with the empirical fact of the low income effect attributable to low income workers (see BLUNDELL [1992]). The labor supply indeed indicates strong elasticities of worked-hours to wage-per-hour for low earnings workers. The labor supply becomes backward bending, indicating

2. One can interpret h as worked hours as in MIRRLEES [1971] or as education in ATKINSON and STIGLITZ [1980].

3. The wage is endogenized in a earlier version of the paper (available upon request to the author).

the presence of a significant income effect, only for high earnings workers, to whom employment subsidies do not apply. Finally, many empirical studies confirm the fact that the substitution effect dominates the income effect for female labor supply and that they largely dominates for mothers with dependent children. The above assumption is thus a valid approximation for the study of employment subsidies targeted at such groups as the EITC.

In the initial tax-benefit system, we assume no subsidies, $s = 0$. For simplicity, we assume a linear tax τ on earnings[4] and an unemployment or welfare benefit b. We assume that the government does not control the size of such benefits. Indeed, in many European countries, unemployment benefits are determined by institutions consisting of employees, employers and government representatives. In the US, the major assistance program to low income families, Aid to Families with Dependent Children, is ruled by states, whereas the tax level and the EITC falls under the juridiction of the federal government. We also assume that the government can not reform the level of tax τ on the whole population.

Optimal levels of productivity and utility levels in the initial tax-benefit system are denoted by a hat. The worker's utility is then

$$\widehat{U}(x,w) = \max_h [b, \theta wh - d(h,x)].$$

We denote as the *marginal worker*, the worker who is indifferent between work and unemployment. In the initial tax-benefit system, the marginal worker has the ability x_0 such that

$$\theta w \widehat{h}(x_0) - d(\widehat{h}(x_0), x_0) = b.$$

Workers with ability $x < x_0$ choose unemployment ($\widehat{h}(x) = 0$). Workers with ability $x > x_0$ exert a productivity level $\widehat{h}(x)$ such that $d_h(\widehat{h}, x) = \theta w$. The productivity level of employed workers increases as x and θw rise while their utility $\widehat{U}$ increases in x and θw.

The presence of a benefit in the initial tax-benefit system creates an unemployment trap. The total tax T on each worker is τwh if he works ($\widehat{h} > 0$) and $-b$ if he does not work ($\widehat{h} = 0$). The marginal tax rate

$$T' \equiv \frac{\partial T}{\partial (wh)}$$

measures the increase in tax involved by a marginal rise of earnings. On the one hand, employed workers face a constant marginal tax rate equal to τ. At this rate, the worker provides an inefficient level of productivity, the efficient level of productivity being attained when the marginal tax rate is zero *ie,* when $d_h(h,x) = w$. On the other hand, unemployed workers face a 100 % marginal tax rate since they loose their entitlement to the benefit as soon as they choose to work. The unemployed worker has no incentive to work.

4. Real tax systems involve tax "*brackets*" on which marginal tax rates are constant. Subsidized workers usually belong to the lowest tax bracket. If not, the model can easily be adapted, but without any gain in insight.

In the initial tax-benefit system, the government's budget deficit includes unemployment benefits minus tax revenues

$$B(x_0) = \int_{\underline{x}}^{x_0} bn(x)dx + \int_{x_0}^{\bar{x}} -\tau w\widehat{h}(x)n(x)dx,$$

while the employment level is simply

$$E(x_0) = \int_{x_0}^{\bar{x}} n(x)dx.$$

Government

The government proposes a non-linear subsidy that is a function of the earnings from work $s(wh)$. This is equivalent to choosing the employment contracts $(h(\cdot), s(\cdot))$ that maximize its objective. Subsidies never hurt the workers: $s(\cdot) \geqslant 0$. In contrast to the literature in public economics, we assume that the government follows a non-welfarist agenda. It seeks to reduce the unemployment trap (*ie*, to increase employment E) at some cost to the budget deficit B.[5]

We assume that the government limits its intervention to a category of workers or to a segment of the labor market. For instance, the Targeted Job Tax Credit in the US (see KATZ [1996]) and the Earned Income Tax Credit target working mothers with dependent children. Also, wage subsidies supported by the European Union's Social Funds target under-developed districts or regions. The government makes the following cost-benefit analysis:

$$\max_{\{h(\cdot), s(\cdot)\}} [\lambda' E - \lambda B] \text{ s.t. } s(\cdot) \geqslant 0$$

where the parameters λ' and λ are the government's valuation of employment and the budget deficit. As in LAYARD and GLAISTER [1994], these parameters are assumed to be exogenous since the subsidy program has little impact on the total employment, on the total budget deficit and thus on the global distortion introduced by the tax and subsidy system. Without loss of generality, λ' can be normalized to 1. The ratio $1/\lambda$ can be interpreted as the government's monetary value of a marginal job.

5. According to ATKINSON [1995, p. 16], "*it is important to extend the range of objective functions to include non-welfarist goals: ie, those which are not based solely on consideration of individual welfare, as conventionally understood.*" For this author (p. 75), "*the notion of [work participation] is related to that of 'social exclusion', which is receiving a great deal of attention in the European Union [...]*". Also, LINDBECK [1988] argues that *individual freedom* is hindered when an individual is trapped in a certain income bracket with little possibility to change his economic situation by his own effort. This pleads for intervention toward low earning workers. Many labor economists share the view that taxes in the labor market should be revised in light of their of employment impact (see, for instance, PISSARIDES [1998] and SORENSENS [1999]). Finally, one may claim that the objective of unemployment trap is justified on the ground that government intervention is expected to apply only to a small fringe of the population, *ie,* the group of subsidized workers. Redistributive issues within this group may be disregarded.

When the government offers employment subsidies to a large set of the population, the incidence of subsidies on the overall economy may be significant. Non-categorical employment subsidies fall in this category. As shown in section 4.8, the parameter λ must then be endogenized. In the other sections the parameter λ is exogenous.

3 Complete Information

Although unrealistic, the analysis of employment subsidies under complete information provides some insights about the properties of optimal employment subsidies. Suppose that the government is informed on the true value of the ability x of unemployed workers ($x \leqslant x_0$). The government can offer a lump sum subsidy $s(x)$ that is tailored to each worker's ability x. The government can also specify the effort level $h(x)$ and thus the earnings level $\theta w h(x)$ that each subsidized worker must provide in order to get the subsidy $s(x)$. If a worker does not accept the effort level prescribed by the government, he is not entitled to subsidy. In this complete information setting, we show that the unemployment trap is eliminated by the application of optimal employment subsidies.

The employment and budget levels depend on the *marginal worker* with ability x_1, who is indifferent between work and unemployment. Total employment is

$$E(x_1) = \int_{x_1}^{\bar{x}} n(x)dx$$

while budget deficit accounts for unemployment benefits and subsidies minus tax receipts:

$$B(x_1,s(\cdot),h(\cdot)) = \int_{\underline{x}}^{x_1} bn(x)dx + \int_{x_1}^{x_0} (s(x) - \tau w h(x))n(x)dx + \int_{x_0}^{\bar{x}} -\tau w h(x)n(x)dx.$$

Since the government's instrument is restricted to the use of subsidies, workers' utility should not drop because of the subsidies. This implies the following participation constraint:

$$\text{(PC)} \qquad \theta w h(x) - d(h(x),x) + s(x) \geqslant \widehat{U}(x).$$

The government selects the ability of marginal worker x_1, the subsidy scheme $s(\cdot)$ and the productivity schedule $h(\cdot)$ that maximize its objective under the above participation constraint:

$$\max_{\{x_1,s(\cdot),h(\cdot)\}} E(x_1) - \lambda B(x_1,s(\cdot),h(\cdot))\text{s.t.}(PC).$$

Under complete information, the government grants subsidies only to unemployed workers ($x \leqslant x_0$). All subsidized workers are thus *additional:* they would not have been hired without subsidies. Their reservation utility is $\widehat{U}(x) = b$. The government has no benefit from increasing the subsidy $s(x)$ above the value that makes the participation constraint binding. The optimal subsidy must therefore be

$$s^*(x) = b - \theta w h(x) + d(h(x), x). \tag{1}$$

The subsidy compensates for the difference between each worker's utility from subsidized job and his utility level in unemployment.

Since employment is a *head-count* measure in which infra-marginal workers $x \in]x_1, x_0]$ do not have any marginal impact, only the marginal worker x_1 is relevant to employment. Infra-marginal workers must then be accounted for the budget deficit they cause. Thus, *the government must minimize the budget deficit on subsidized workers in order to concentrate the maximal subsidy on the marginal worker* x_1.[6] The head-count nature of employment allows the following decomposition of the government's problem:

$$\max_{x_1} E(x_1) - \lambda B^*(x_1) \tag{2}$$

where $\quad B^*(x_1) = \int_{\underline{x}}^{x_1} bn(x)dx + \bar{B}^*(x_1) + \int_{x_0}^{\bar{x}} -\tau w \widehat{h}(x) n(x) dx$

and $\quad \bar{B}^*(x_1) = \min_{\{s(\cdot), h(\cdot)\}} \int_{x_1}^{x_0} [s^*(x) - \tau w h(x)] n(x) dx.$

Using (1), the subsidy schedule $s^*(x)$ can be eliminated such that

$$\bar{B}^*(x_1) = \min_{h(\cdot)} \int_{x_1}^{x_0} [d(h(x), x) - w h(x) + b] n(x) dx.$$

Given that this expression has a strictly convex integrand, it yields the unique solution $h^*(\cdot)$ given by

$$d_h(h^*, x) = w.$$

Under complete information, there exists no tax distortion for subsidized workers and the latter exert the efficient productivity level.

Moreover, reductions in the unemployment trap are compatible with decreases in budget deficit. Equation (1) can indeed be written as

$$s^*(x) - b - \tau w h^*(x) = -\big(w h^*(x) - d(h^*(x), x)\big) \leqslant 0. \tag{3}$$

6. The same idea is developed in ATKINSON [1993] and BOURGUIGNON and FIELDS [1990] who describe the properties of head-count measures in poverty alleviation frameworks and under complete information. In particular, BOURGUIGNON and FIELDS denote optimal policies resulting from head-count objectives as "*Type-r*" (Type-rich) because money had to be allocated to the richest of the poor.

The inequality follows from the facts that utilities are non negative in the absence of taxation. For each worker, the employment subsidy is covered by the unemployment benefits and by the tax receipts recovered on behalf of his earnings. Hence, any decrease in x_1 enhances the employment while it reduces the budget deficit. The marginal change in the government's objective is given by

$$\text{(4)} \qquad \frac{d}{dx_1}\left[E(x_1) - \lambda B^*(x_1)\right] = \lambda n(x_1)\left[s^*(x_1^*) - b - \tau w h^*(x_1^*) - \frac{1}{\lambda}\right]$$

By (3), the square bracketed term in (4) is negative. Hence, decreases in x_1 always enhance the objective and the unemployment trap can obviously be eliminated ($x_1^* = \underline{x}$).

This result is generated by the assumption of complete information but not from the absence of income effect in the worker's decision. Indeed, under complete information, the government obliges subsidized workers to remain on the same indifference curve. It thus completely controls for the income effect. However, complete information on workers generates significant tax gains with respect to the initial-tax system since the government is now able to tax additional workers without any distortion on work incentives.

In practice, the government can not be informed about workers' abilities. It has an incomplete information on workers and the above employment subsidies are not incentive compatible. Workers with high abilities are indeed enticed to mimic workers with lower abilities. Incentive compatibility is a relevant issue not only between groups of subsidized and unsubsidized workers but also within the group of subsidized workers, as shown in the next section.

4 Incomplete Information

Under incomplete information, the government is not able to observe and verify the true value of the workers' ability. Still, the government can observe and verify the worker's earning wh. It designs employment subsidies as contracts $(wh(\cdot), s(\cdot))$ that respect incentive compatibility and participation constraints.

This section relies on JULLIEN [1997] and DIAMOND [1998]. The use of additively separable utility function allow us to express optimal employment subsidies in terms of information rents. We first present and decompose the problem, using the head-count nature of employment and using the properties of incentive compatibility and participation constraints. We establish the link between worker utility and information rents, and we then characterize the optimal employment contracts for infra-marginal workers. We discuss the issue of job additionality, we study the likelihood of a *self-help* effect and we analyze the optimal shape of the subsidies. We finally concentrate on the optimal size of the unemployment trap.

4.1 Problem Decomposition

Under incomplete information, employment subsidies must be incentive compatible in the sense that no worker with ability x should be enticed to mimic workers with $x' \neq x$. Incentive compatible subsidies can be described as the contracts $(wh(\tilde{x}),\ s(\tilde{x}))$ in which each worker reveals his ability $\tilde{x}$ to the government. By the revelation principle, incentive compatibility can be replaced by direct truth revelation. That is,

(IC) $$x = arg \max_{\tilde{x}} \theta wh(\tilde{x}) - d(h(\tilde{x}), x) + s(\tilde{x}).$$

Applying standard techniques used in mechanism design (see Appendix 1), this condition reduces to the following first and second order conditions:

(IC1) $$U' = -d_x(h(x), x),$$

(IC2) $wh(\cdot)$ is monotonically increasing.

From the first condition, utility increases at a rate that prevents each worker from mimicking other workers of higher and/or lower abilities. The second condition requires that the government can infer the worker's ability from the observation of his earnings wh. When wh is a strictly increasing function of x, employment subsidies are *separating* contracts: workers with different abilities choose different contracts. When wh is constant over some interval, workers *bunch* at a particular level of productivity.

Since workers accept only positive subsidies, their participation constraints can be written as

(PC) $$\theta wh(x) - d(h(x), x) + s(x) \geqslant \widehat{U}(x)\ \forall x.$$

Again let x_1 be the ability of the *marginal worker* who is indifferent between subsidized work and unemployment. Workers with $x > x_1$ who choose to work with a subsidy are called *infra-marginal workers*. The employment level and the budget deficit are

$$E(x_1) = \int_{x_1}^{\bar{x}} n(x)dx,$$

$$B(x_1, s(\cdot), h(\cdot)) = \int_{\underline{x}}^{x_1} bn(x)dx + \int_{x_1}^{\bar{x}} [s(x) - \tau wh(x)]n(x)dx.$$

The government's problem is

$$\max_{\{x_1, s(\cdot), h(\cdot)\}} E(x_1) - \lambda B(s(\cdot), h(\cdot), x_1) \text{ s.t.} (IC1),\ (IC2) \text{ and } (PC).$$

Due to the head-count nature of the employment objective, the problem decomposes into the choice of the marginal worker x_1 and the choice of the productivity level of infra-marginal workers to whom only a financial trade-off is applied. As in section 3, the government *minimizes transfers to infra-marginal subsidized workers in order to allocate the maximum subsidy* *to the marginal worker*. But, in contrast to the case of complete information, incentive compatibility hinders government from pulling the utility levels of subsidized workers to the levels that they obtained in the initial tax-benefit system.

In this paper, we restrict our attention to the situation in which subsidies are granted to a part of the working population. Hence, there exists a worker with ability x_2 who is indifferent between subsidized and unsubsidized work ($x_1 \leqslant x_2 < \bar{x}$). Subsidies are granted to workers with abilities $x \in [x_1, x_2]$. Since unsubsidized workers $x \in]x_2, \overline{x}]$ face a linear tax, they do not mimic each other. Unsubsidized workers may nevertheless want to imitate subsidized workers and incentive compatibility between groups of subsidized and unsubsidized workers must be checked. This amounts to checking incentive compatibility for the worker with ability x_2.

LEMMA 1: $s(x_2) = s_x(x_2) = 0, U(x_2) = \widehat{U}(x_2)$ and $h(x_2) = \widehat{h}(x_2)$.

Proof. See Appendix 1. ■

Incentive compatible employment contracts never provide the subsidized worker with ability x_2 with a strictly positive subsidy, since, otherwise, his (continuous) right-hand neighbor $x_2 + \varepsilon$ (with infinitely small $\varepsilon > 0$) will be enticed to mimic him. The same argument applies to the left-hand neighbor $x_2 - \varepsilon$. As consequence, the subsidy should be zero and should have zero slope at x_2. Continuity of utility U at x_2 is proved in the same way. Continuity of productivity levels h is obtained from the first order conditions.

By Lemma 1, the participation constraint (PC) can be transformed as

(PC1) $$U(x) \geqslant b,$$

(PC2) $$U(x_2) = \widehat{U}(x_2),$$

(PC3) $$h(x_2) = \widehat{h}(x_2).$$

We denote the optimal productivity levels and subsidies by $h^*(\cdot)$ and $s^*(\cdot)$, and the budget deficit induced by the optimal subsidy $s^*(\cdot)$ by $B^*(x_1, x_2)$. Then, the government's program becomes

$$\max_{\{x_1,x_2\}} E(x_1) - \lambda B^*(x_1,x_2) \text{ s.t.}(PC1) \text{ and } (PC2)$$

where

(5) $$B^*(x_1,x_2) = \int_{\underline{x}}^{x_1} bn(x)dx + \bar{B}^*(x_1,x_2) + \int_{x_2}^{\bar{x}} -\tau w \widehat{h}(x) n(x) dx$$

and where

$$\bar{B}^*(x_1,x_2) = \min_{s(\cdot),h(\cdot)} \int_{x_1}^{x_2} [s(\cdot) - \tau w h(\cdot)] n(x) dx$$

$$\text{s.t.}(IC1),(IC2) \text{ and } (PC3).$$

The problem is solved firstly for control variables $(s(\cdot),h(\cdot))$ within the interval $[x_1,x_2]$ and then for x_1 and x_2.

4.2 Utility and Information Rents

The relationship between utility levels and information rent is easily expressed for additively separable utility functions. The incentive compatibility condition $(IC1)$ implies that utility levels increase within the interval $[x_1,x_2]$. The participation constraint $(PC1)$ binds at x_1 since the government does not benefit from increasing the utility of worker x_1 above its reservation value b. Thus, one gets after integration

$$U(x) = b + \int_{x_1}^{x} -d_x(h(x),x)dx. \tag{6}$$

Whereas subsidized workers obtain a utility level equal to b under complete information, they get that level plus an *information rent* under incomplete information. By (6), a subsidized worker with ability x extracts an information rent that decreases as the productivity of subsidized workers with ability lower than x drops.

At the ability x_2, the utility obtained with the information rent is no longer larger than the utility of an unsubsidized work. By $(PC2)$, we have

$$\int_{x_1}^{x_2} -d_x(h^*(x),x)dx + b = \theta w\widehat{h}(x_2) - d(\widehat{h}(x_2),x_2). \tag{7}$$

4.3 Optimal Employment Contracts

Necessary and sufficient conditions for optimal subsidies and productivities of infra-marginal workers $x \in [x_1,x_2]$ can now be derived. Let first the inverse hazard rate be defined as

$$H(x,\Gamma) \equiv \frac{\Gamma - N(x)}{n(x)}$$

and its derivative with respect to x be denoted as H'. Then, holding the employment level constant (and thus x_1), the interior solutions $(s^*(\cdot),h^*(\cdot))$ of the government's program are given in the following proposition.

PROPOSITION 1: If $H' \leqslant 0$ for Γ defined below and if $d_{xhh} \leqslant 0 \leqslant d_{hxx}$, then the optimal employment subsidy exists and is unique. It satisfies

$$w - d_h(h^*,x) = -H(x,\Gamma)\, d_{hx}(h^*,x), \tag{8}$$

$$\Gamma \equiv N(x_2) + \tau w \frac{n(x_2)}{-d_{hx}(\widehat{h}(x_2),x_2)} > N(x_2). \tag{9}$$

Proof. See Appendix 1.

The condition (8) expresses the trade-off between efficient productivity levels and information rent. The left hand side corresponds to the marginal utility from work. Under complete information, this is zero and subsidized workers provide the efficient levels of productivity (see expression (4)). However, under incomplete information, the positive right hand side appears and expresses the workers' marginal utility of hiding information to the government. To diminish information rents, the government asks for productivity levels that are lower than those obtained under complete information.

Finally, the subsidized worker with ability x_2 faces a marginal tax rate equal to τ . Indeed, at x_2, the condition (8) reduces to $d_h(x_2, h^*(x_2)) = \theta w$, which implies that $h^*(x_2) = \widehat{h}(x_2)$. The term Γ captures the continuity requirements on marginal tax rates and utility levels imposed by the presence of unsubsidized workers.[7] This property distinguishes the present model from the standard optimal taxation models in which the most able worker is not taxed at the margin (Γ should be equal to 1).

The Figure 1 illustrates the productivity and utility levels for a particular set of parameters. By offering employment subsidies, the government is able to smooth away the discontinuity in the workers effort choice around x_0 and to reduce the convexity of the worker's utility profile.

In Proposition 1, the condition $d_{hhx} \geqslant 0$ is the sufficient condition for the existence of globally optimal contracts whereas the set of conditions $H' \leqslant 0$ and $d_{xhh} \leqslant 0 \leqslant d_{hxx}$ determines the sufficient conditions for employment subsidies to be implemented as separating contracts. Those conditions deserve two comments. On the one hand, those are sufficient conditions. The existence and the separating nature of employment subsidies occur under less demanding conditions. On the other hand, the conditions on the disutility function have relevant economic interpretations. In conjunction with $d_{hx} < 0$, the condition $0 \leqslant d_{hxx}$ implies that the work disutility decreases with skills but that it decreases less at higher skill levels. The advantage of high skills thus vanishes with abundance of skills. The condition $d_{xhh} \leqslant 0$ implies that disutility functions are less convex in effort h at higher skills. This property can hardly be disputed since the reverse condition yields the unattractive property that the disutility level of the high skill workers must become larger than the disutility level of low skill workers as soon as effort level reaches some positive threshold.

The monotone hazard rate condition $H' \leqslant 0$ in Proposition 1 guarantees that employment subsidies are implemented as separating contracts. This condition may not be fulfilled. There indeed exists some presumption that the density of the skill distribution presents a peak around some very low skills. As developed in SNOWER [1994], long-term unemployed workers loose their skills after long unemployment spells. By this process, low skill workers accumulate and the skill distribution may become very dense for very low skills. As consequence, the optimal productivity level $h^*(x)$ is no longer monotonic. Proposition 1 does not apply and some *bunching* occurs. That is,

7. We have here an extended monotone hazard rate property. Models in contract theory imply that either $\frac{N(x)}{n(x)}$ or $\frac{1-N(x)}{n(x)}$ be monotonously increasing (see JULLIEN [1997]). Here, the presence of non-constant participation constraint imposes further restriction on this condition.

the government provides the same lump sum subsidy S^B to the group of low skill workers $x \in [\underline{x}, x_1']$ where x_1' ($\underline{x} \leqslant x_1 \leqslant x_1'$) is the ability of the worker who is indifferent between the subsidies S^B and $s^*(\cdot)$. The subsidy S^B is granted only to workers with earnings wh^B.

Bunching at low skill levels can be explained as follows. Suppose, for the purpose of this exposition, that the peak in the skill distribution occurs for $x \in [\underline{x}, x']$, and that the group of workers included in this peak is negligible compared to the whole working population $(N(x')/(1 - N(x')) \simeq 0)$. Therefore, workers whose abilities lie above the peak ($x > x'$) are likely to exert the same productivity levels and to accept the subsidies defined in Proposition 1. Suppose also that the marginal workers belong to the dense part of the peak ($x_1 < x'$). Since the density of marginal workers is non negligible, subsidies to marginal workers have a strong impact on employment. The government should therefore target these workers with large subsidies. To minimize the cost of subsidies, the government should also increase the earnings of marginal workers by asking them large productivity levels. The incentive problem is then exacerbated because large productivity requirements for the marginal workers ($x_1 < x'$) are incompatible with moderate productivity requirements for subsidized workers with abilities lying out to the peak ($x > x'$). The government is no longer able to extract information from these workers. It can only propose the same subsidy to this group.

More formally, incentive compatibility imposes that optimal subsidies and productivity levels are continuous at x_1': $S^B = s^*(x_1')$ and $h^B = h^*(x_1')$. Within the interval $]x_1, x_2]$, the government faces a financial problem, that is,

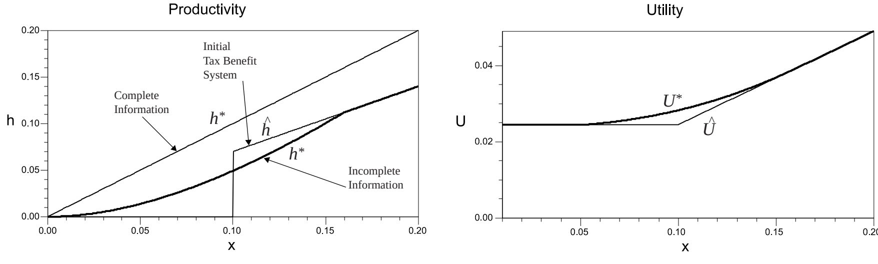
$$\min_{x_1'} \int_{x_1}^{x_1'} [S^B - \tau w h^B)] n(x) dx + \int_{x_1'}^{x_2} [s^*(x) - \tau w h^*(x)] n(x) dx,$$

where $s^*(\cdot)$ and $h^*(\cdot)$ are given in Proposition 1. The first order condition is

$$\int_{x_1}^{x_1'} \frac{\partial}{\partial x_1'} [s^*(x_1') - \tau w h^*(x_1')] n(x) dx = 0.$$

FIGURE 1

Productivity and Utility Levels with/without Employment Subsidies (Parameters: $d(x,h) = h^2/2x$, $\tau = 0.30$, $w = 1$, $N(x)$ = Uniform distribution over $[0,1]$, $x_0 = .1$, $x_1 = 0.052$ and $x_2 = 0.16$.)



After some transformation, this yields

$$
\begin{aligned}
(N(x_1') - N(x_1))\frac{\partial}{\partial x_1'}[s^*(x_1') - \tau w h^*(x_1')] &= 0, \\
(N(x_1') - N(x_1))\frac{\partial}{\partial x_1'}[U^*(x_1') - w h^*(x_1') + d(h^*(x_1'),x_1')] &= 0, \\
-(N(x_1') - N(x_1))[w - d_h(h^*(x_1'),x_1')]h_x^*(x_1') &= 0.
\end{aligned}
$$

Hence, suppose that the set of bunching workers $[x_1,x_1']$ is not empty. Since condition (8) implies that $w - d_h(h^*(x_1'),x_1') < 0$, we can readily derive the following proposition:

> PROPOSITION 2: When the density of the skill distribution has a peak around some very low skills, the government may propose bunching contract (S^B, wh^B) to the group of workers with ability $x \in [\underline{x}, \widehat{x}']$ where $h_x^*(x_1') = 0$ and $h^B = h^*(x_1')$.

Workers who accept the subsidy S^B offer the same level of productivity h^B. Their utility decreases as their ability drops. Finally, the ultimate worker who accepts work with the subsidy S^B is the marginal worker with ability x_1.

4.4 Job-Additionality

A popular belief among economists and policy makers is that incentive problems emanate from the presence of non-additional workers (see FOLEY [1992] and MEAGER and EVANS [1998]). *Non-additional workers* $x \in [x_0, x_2]$ are subsidized workers who would nevertheless be hired under the initial tax-benefit system. *Additional workers* $x \in [x_1, x_0[$ would not have been hired without the subsidy. The distinction between additional and non-additional workers is also instructive in explaining Proposition 1.

The trade-off between efficient productivity and information rent applies to additional workers as well as to non-additional workers. Suppose first that the government is informed on the workers' additionality. It can then exclude non-additional workers from the subsidy scheme and focus only on the additional workers $x \in]x_1, x_0]$. In this case, the standard result of optimal non-linear taxation applies (MIRRLEES [1971]): the government pays an information rent to the most productive worker x_0 but imposes a zero marginal tax on the latter since no other worker can mimic him ("*efficiency at the top*"). Because of this incentive problem, the government is also obliged to reduce the subsidies to workers with lower ability; the least able subsidized worker gets no rent.

Of course, the government does not observe the additional status of workers. The presence of non-additional workers introduces a second incentive problem: non-additional workers seek to get the subsidy proposed to additional workers. In response to this problem, the government is obliged to reduce further the net earnings of additional workers by diminishing the size of their subsidy and their productivity levels.

From this analysis, it is obvious that incentive problems come from the presence of *both* additional and non-additional workers. Both types of workers do not respond “*honestly*” to the government.

4.5 Poverty, *Self-Help* Effect and Marginal Tax Rates

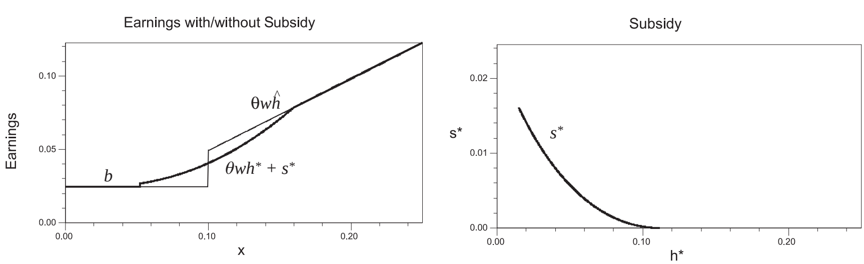
KANBUR, KEEN and TUOMALA [1994] show the existence of a *self-help* effect under the poverty alleviation objective. A negative marginal tax rate on the poorest workers indeed constitutes an optimal poverty alleviation policy since these workers are enticed to “*help themselves*” out of poverty by working more than their efficient level of effort. By focussing on poverty measures, such policies have the pernicious property that workers may end up with lower utility levels compared to the no-policy situation. In this section, we first study the impact of employment subsidy on earnings, then we check the presence of the *self-help* effect though the analysis of the marginal tax rates.

In our model, the total earnings of workers (*ie,* the sum of benefits, net earnings from work and subsidies) rise or drop according to whether workers are additional or not. This is depicted in the first panel of Figure 2. It presents the total earnings for all types of workers with and without subsidies. Non-additional workers nevertheless compensate their loss in earnings by relaxing their effort levels. Their work disutility is smaller and their utility levels are larger with a subsidy (see again Figure 1). In contrast to KANBUR, KEEN and TUOMALA [1994] the least able workers do not get poorer in the presence of employment subsidies. Only non-additional workers becomes poorer, but still those workers do increase their utility levels with a subsidy.

It nevertheless remains unclear whether the *self-help* effect occurs under the objective of reducing the unemployment trap. We now show that the *self-help* effect is unlikely unless the skill distribution includes a peak for low abilities.

The worker's utility can be written as $wh - d(h,x) - T(wh)$ where $T(wh)$ is the total tax to the government. The worker's optimal decision on productivity h requires that $w - d_h = wT'$. Then, by (8), we can make the following proposition.

FIGURE 2
Earnings and Subsidies



PROPOSITION 3: If the conditions of Proposition 1 hold, the marginal tax rate is

$$T' = -\frac{1}{w} H(x,\Gamma) \, d_{hx}(h^*,x) > 0. \tag{10}$$

Marginal tax rates are always positive and the self-help effect does not appear.

No worker is induced to work more than the efficient productivity level in order to alleviate the unemployment trap. This result originates from the head-count nature of the employment objective and the separating property of employment subsidies. Since the government's trade-off with respect to infra-marginal workers is only a financial one, there is no point in increasing productivity levels beyond the efficient levels since such levels provide the largest tax receipts. Incentive problems force the government to ask for lower productivity levels, which leads to larger marginal taxes. The absence of the *self-help* effect is true for any other head-count objective.

The *self-help* effect may nevertheless occur when the skill distribution has a peak around some very low skills. Indeed, negative marginal taxes upon these workers would induce them to exert larger productivity levels. Such marginal tax rates would increase the earnings of these workers and would require lower subsidies. However, since incentive problems limit the size of such earnings and productivity levels, marginal tax rates on the low ability subsidized workers are downward constrained. The possibility of the *self-help* effect depends on the bunching productivity level $h^B = h^*(x_1')$ and on the ability of marginal worker, x_1. Indeed, if the latter has a sufficiently low ability, the productivity level h^B that he will exert will be larger than his efficient level. However, in contrast to KANBUR, KEEN and TUOMALA [1994], workers never end up with lower utility levels compared to those obtained in the initial tax-benefit system. Participation constraints here insure that workers always benefit from the subsidies. Thus, the *self-help* effect may appear but it is not accompanied by the exploitation of the least able individual.

4.6 Subsidy Shape

Little rationale has been presented to explain the optimal shape of employment subsidies. DRÈZE and SNEESSENS [1994], who argue for an employment subsidy that decreases with earnings, are not explicit about the shape of the subsidy. In practice, employment subsidies may have various shapes. The UK Family Credit (FC) is a subsidy scheme that decreases with earned income whereas the US Earned Income Tax Credit (EITC) presents a bell shape (see KATZ [1996], BLUNDELL and MACCURDY [1999]): it grants zero tax credit for very low earnings, increasing tax credits above a first threshold, constant tax credit above a second threshold, diminishing tax credit after a third threshold and finally zero credit for large earnings. In this section, we provide some theoretical foundation on this issue.

The shape of optimal employment subsidies is related to the marginal tax rate in the following way. By (6), we have

$$\theta w h^*(x) - d(h^*(x),x) + s^*(x) = \int_{x_1}^{x} -d_x(h^*(z),z)dz + b.$$

By differentiating this expression by x and by using (9) and (10), we successively get

$$\begin{aligned} s^{*\prime}(x) &= -(\theta w - d_h(h^*(x),x))h_x^*(x), \\ &= (\tau w + \frac{1}{w}H(x,\Gamma)\, d_{hx}(h^*,x))h_x^*(x), \\ &= (\tau - T')h_x^*(x). \end{aligned}$$

Since earnings $wh^*(x)$ are positively related to abilities (see (IC2)), employment subsidies decrease with earnings if, and only if, the marginal tax rate lies above τ. While the marginal tax rate is equal to τ at x_2, it may lie above or below this value for any other subsidized workers with $x < x_2$. Differentiating (10) yields

$$\frac{dT'}{dx} = -\frac{H}{w}\Big(\frac{H'}{H}d_{hx}(h^*,x) + d_{hxx} + d_{hhx}h_x^*\Big).$$

This expression is unsigned. Indeed, under the conditions in Proposition 1, we have that $H' < 0$, $d_{hhx} \leqslant 0 \leqslant d_{hxx}$. The two first terms in brackets are positive while the last term is negative. Therefore, marginal tax rates diminish with larger earnings if and only if the two first terms dominate the last one. If they diminish with larger earnings, marginal tax rates surely lie above τ and employment subsidies decrease when earnings grow. This would give a theoretical basis for the decreasing shape of the FC. However, if marginal tax rates increase with larger earnings, they may lie below τ for some interval of very low abilities. Employment subsidies may then increase with earnings for very low earnings, which would then give a basis for the bell shaped EITC.

It is nevertheless difficult to find simple examples of utility functions and smooth skill distributions such as uniform and Log-Normal distributions that yield bell shaped subsidies. (For example, the subsidy shape in Figure 2 is a decreasing function).[8] Moreover, recent literature on optimal non-linear taxation strongly contributes to the argument for diminishing marginal tax rates (regressive taxation) for low earning workers. PIKETTY [1997] stresses that when governments must raise large amounts of taxes, they need to set a high average tax on average income earners. Reducing the earnings of low skill workers through high marginal taxes stimulates work incentives for average income earners and thus provides larger tax proceeds to redistribute. Also, DIAMOND [1998] argues that the distribution of ability $N(x)$ plays a crucial role in determining the exact shape of the tax schedule. In the present setup, diminishing marginal tax rates prevail if the ratio H'/H is sufficiently negative. This generally means that the skill distribution is not too dense for low

8. PICARD [1998] shows that multiplicatively separable disutility functions $d(h,x) = B(h)A(x)$ yield diminishing marginal tax rates if $H'/H < -A'/A + A''/A'$. This condition imposes no restriction on $B(h)$. As soon as separating contracts can be implemented ($H' \leqslant 0$), the condition is satisfied for power or exponential forms of the function $A(x)$.

skills. In such a case, employment subsidies should not be bell-shaped but monotonically decreasing.

As mentioned in previous sections, there is a presumption that the skill distribution is very dense for low skills. The government must then propose the bunching contracts (S^B, wh^B) to a group of workers with low ability $x \in [\underline{x}, x_1']$. On the one hand, earnings wh^B acts as a lower bound for workers' eligibility to employment subsidies. On the other hand, the government also sets very low marginal tax rates for the least productive of workers who select the subsidy $s^*(\cdot)$. Indeed, when the skill distribution density $n(x)$ is large and when the cumulative distribution $N(x)$ gets smaller and smaller, marginal tax rates in expression (10) tend to very small values. There must then exist a group of subsidized workers with ability $x \geqslant x_1'$ who face marginal tax rates lower than τ. For this group of workers, the subsidy $s^*(\cdot)$ increases in ability x and in earning wh^*. Accumulation of workers at very low skills is therefore a rationale for bell shaped subsidies such as the EITC.

4.7 Optimal Size of Unemployment Trap

While the unemployment trap can be eliminated under complete information ($x_1 = \underline{x}$), incentive problems may thwart this outcome under incomplete information. In this section, we derive and comment on the necessary condition for the optimal size of unemployment trap. We also provide a condition on workers' behavior that guarantees that optimal employment subsidies simultaneously bring more employment and lower budget deficits.

An interior solution in x_1 to the government's problem should solve

$$\frac{d}{dx_1}\left[E(x_1) - \lambda B^*(x_1)\right] = 0.$$

This condition can be transformed into a simple formula when employment subsidies are implemented as separating contracts (no peak for low abilities).

$$\frac{d}{dx_1}\left[E(x_1) - \lambda B^*(x_1)\right]$$
$$= \lambda n(x_1)\left[s^*(x_1) - b - \tau w h^*(x_1,\Gamma) - \frac{\Gamma - N(x_2)}{n(x_1)} d_x(h^*(x_1,\Gamma), x_1) - \frac{1}{\lambda}\right]$$

PROPOSITION 4: Suppose that employment subsidies are implemented as separating contracts and that the unemployment trap is not eliminated. Then, the government should set the employment level such that

$$(11) \quad s^*(x_1) - b - \tau w h^*(x_1,\Gamma) - \frac{\Gamma - N(x_2)}{n(x_1)} d_x(h^*(x_1,\Gamma), x_1) - \frac{1}{\lambda} = 0.$$

Proof. See Appendix 2. ■

Condition (11) is similar to the interior solution of condition (4). The optimal subsidy to the marginal worker should be equal to the sum of recovered tax, unemployment benefit and the marginal social value of employment $1/\lambda$, minus the information cost due to the shirking of subsidized workers.

Under incomplete information, some shirking occurs. At the margin, the impact of incomplete information is financial. First, incentive problems lower the tax recovered on the marginal worker. By Proposition 3, marginal tax rates are larger under incomplete information than under complete information. The productivity level of the marginal worker x_1 and the tax recovered on behalf of that worker are thus smaller under incomplete information. Hence, the third term of the left hand side of (11) is smaller under incomplete information. Second, a rise in the subsidy not only increases employment but also the incentive problems among infra-marginal workers. In response to this, the government asks lower productivity levels from all subsidized workers, which reduces tax proceeds. This effect is expressed through the positive fourth term in (11) which incorporates the cumulative effect of subsidized workers through the term $(\Gamma - N(x_2))/n(x_1)$.

When the skill distribution is dense for low skill workers, expression (11) no longer determines the optimal level of employment. A more intricate formula is required since the government must consider the subsidy and the tax recovered on behalf of workers who take a bunching subsidy S^B. As this case does not add much to the current discussion, we do not present it in this text.

In this paper the workers' shirking behavior is the unique force that limits the benefits of an employment subsidy. A larger sensitivity of the work disutility to skill levels (larger $|d_x|$) induces higher information rents and smaller subsidies. Condition (11) can be transformed to provide some indication on whether an employment subsidy can be applied. In the initial situation we have a marginal worker with ability x_0 and productivity level $h_0 \equiv \widehat{h}(x_0)$. If we remain close to the initial situation, $x_1 \simeq x_0 \simeq x_2$, $s^*(x_1) \simeq s^*(x_0) \simeq s^*(x_2) \simeq 0$ and $h^*(x_1) \simeq h^*(x_0) \simeq h^*(x_2) \simeq h_0$. Using these equalities and (9), we have that

$$\left[\frac{dB^*(x_1)}{dx_1}\right]_{x_1 \simeq x_0 \simeq x_2} = -n(x_0)\left[-b - \tau w h_0 - \frac{\Gamma - N(x_0)}{n(x_0)} d_x(h_0,x_0)\right] = n(x_0)\left[\tau w h_0 + b - \tau w \frac{d_x(h_0,x_0)}{d_{hx}(h_0,x_0)}\right].$$

When this expression is positive, the budget deficit decreases when x_1 decreases slightly below x_0. Rearranging this expression, we can state that optimal employment subsidies simultaneously increase employment and decrease budget deficit if

(12) $$\left[\frac{\partial \log |d_x(h,x))|}{\partial \log h}\right]_{h_0,x_0} \geqslant \frac{1}{1 + \frac{b}{\tau w h_0}}.$$

Small employment subsidies are thus beneficial if the marginal disutility from work decreases rather rapidly as ability increases above x_0. In this case, more able workers do not have much incentive to mimic workers with lower ability.

The relevance of this condition is nevertheless better explained by the following short example: suppose that $d(h,x) = B(h)A(x)$, with $B(h) \equiv h^\alpha$ $(\alpha > 1)$. Then one can compute that the elasticity of labor supply $\varepsilon \equiv (\partial h/\partial w)(w/h)$ is equal to $1/(\alpha - 1)$ (> 0) and that the left hand side of (12) is equal to α, that is, $(1+\varepsilon)/\varepsilon \geqslant 1$. In this example, the condition

(12) is always satisfied even if labor supply is very elastic ($\varepsilon = \infty$). In general, employment subsidies will reduce simultaneously the unemployment trap and the budget deficit when labor supply is not too elastic and/or when the tax-benefit ratio $b/\tau w h_0$ is large enough.

4.8 Eliminating the Unemployment Trap

When the left hand side of (11) is always negative, the government benefits from reducing x_1, until $x_1 = \underline{x}$. In spite of incentive problems, the unemployment trap could thus be eliminated as under complete information. Employment subsidies might even rule out the unemployment trap without worsening the budget deficit. Indeed, suppose that the government seeks to reduce the unemployment trap without diminishing the utility of any worker and that it does not allow to inflate the budget deficit above its initial value $B(x_0)$. The government solves the problem

$$\max_{\{h(\cdot),s(\cdot)\}} E \text{ s.t. } s(\cdot) \geqslant 0 \text{ and} B \leqslant B(x_0)[\lambda]$$

where $\lambda \geqslant 0$ is the Lagrange multiplier of the budget constraint. This problem has the same structure as in the previous sections except that λ is now an endogenous variable. A larger initial budget deficit $B(x_0)$ relaxes the government budget constraint and decreases λ.

As with many contributions in optimal non-linear taxation, the properties of tax or subsidy schemes under fixed budget constraint cannot be discussed without resorting to numerical simulations. Whereas they are no proof, numerical exercises are suggestive on the size and the impact of the forces in play. In the present exercise, we focus on the issue of unemployment trap by assuming that all workers work when the economy offers no tax and no benefit.[9] That is, $d_h(0,\underline{x}) < w$. This assumption is satisfied for power and exponential forms of disutility functions that we have tested: $d(h,x) \equiv B(h)A(x)$ with $B(h) = h^{\alpha}$ ($\alpha > 1$) and $A(x) > 0\ \forall x \in]\underline{x},\bar{x}[$ or with $B(h) = e^{\beta h}$ ($\beta > 0$) and $0 < A(\underline{x})\beta < w$. In our simulations, we have associated those disutility functions with the usual benchmark skill distributions. We used Log-Normal distributions as in Mirrlees [1971] and Kanbur and Tuomala [1996] as well as the Uniform distributions. We tested a large variety of parameters for these functions. We did not find any example in which the optimal employment subsidies did not rule out the unemployment trap while diminishing the budget deficit.

In this paper, we give only a sample of those simulations. Table 1 reports the savings in budget deficit $B(x_0) - B^*(\underline{x})$ and the proportion of subsidized workers $N(x_2)$ obtained from applying optimal employment subsidies. Plausible values for the linear tax τ and the initial unemployment rate u are presented. Unemployment benefit in the initial tax-benefit system is endogenously determined as $b = b(\tau,u)$. In order to allow a comparison between the various economies, budget savings are divided by the value of the unemployment benefit prevailing in each initial tax-benefit system. Table 1 therefore presents the values of the government's gain in terms of the amount of unemployment benefit recovered through the subsidies.

9. The marginal rate of substitution between effort and consumption is always low enough so that the least able worker chooses to work.

TABLE 1
Simulation: Budget Savings and Subsidized Population.
Standard Log-Normal Distribution and $d = h^2/2x$

$(\frac{B(x_0) - B^*(\underline{x})}{b}), N(\check{x}))$	$\tau = .30$	$\tau = .40$	$\tau = .50$
$u = .10$	(.049, .18)	(.057, .18)	(.068, .18)
$u = .15$	(.073, .26)	(.085, .26)	(.102, .27)
$u = .20$	(.095, .34)	(.112, .35)	(.134, .35)

According to Table 1, optimal employment subsidies would cover 18 % of the population when they are applied in an initial economy with 10 % unemployment rate and with a 30 % linear tax rate. The budget deficit would be reduced by an amount equivalent to the unemployment budget allocated to 4,9 % of the population.

Budget savings are important although there is incomplete information. In fact, the cost of subsidies accruing to non-additional workers is always recovered by the savings in unemployment benefits and in recovered tax revenues on behalf of additional workers. Recovered tax revenues are important when the initial tax rate τ is large. For a 50 % linear tax rate, recovered tax on the first additional worker (at x_0) can become as large as the recovered unemployment benefit. Moreover, recovered taxes on behalf of most productive additional workers are much more important than taxes recovered on other workers. In these simulations, the least productive subsidized workers face higher marginal tax rates. They do not contribute much to production and to tax proceeds. Therefore, the most productive additional workers bear the cost of non-additional workers.

When the skill distribution presents a peak for very low skill workers, the above result is strengthened. The presence of the peak concentrates more information on the low ability group; it thus provides more discriminatory power to the government.[10] If the employment content of the peak is negligible compared to the whole working population, the presence of the peak will have no impact on workers with abilities largely above the peak. But, the increase in information with respect to low ability workers is beneficial to the government.

5 Conclusion

This paper focuses on the issue of the unemployment trap and its remedies in a constrained tax-benefit environment. Basically, the effect of employment subsidies is to smooth marginal tax rates by moving the large marginal tax

10. The reader will convince himself by verifying that a discrete information set of few elements with equal probabilities provides more discriminatory power than the continuous information set with uniform probability distribution that contains these few elements.

rates imposed on low ability workers (100 % when there are no subsidies) to more able workers. Our analysis suggests that incentive problems generated by the asymmetry of information are not important enough to impede a simultaneous gain in employment and government budget. Optimal employment subsidies therefore reach the same work incentive objective as the radical tax reforms proposed by ATKINSON [1995] and VAN PARIJS [1995]. But, as subsidies are here confined to a small group of workers, they do however not present the political difficulties of large reforms.

The paper also provides insights into the issue of job-additionality, the shape of employment subsidies and the existence of the *self-help* effect as studied in KANBUR, KEEN and TUOMALA [1994]. In particular, we show that a bell shape of optimal employment subsidies (such as that of the US Earned Income Tax Credit) and the existence of a *self-help* effect must be related to the accumulation of low skill workers, such as that created by the skill deterioration of the long-term unemployed. ■

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APPENDIX

Appendix 1

The Government's Problem

A subsidized worker has the utility $U = \theta wh - d(h,x) + s$. The government designs contracts $(wh(\tilde{x}), s(\tilde{x}))$ where the worker reveals his ability as $\tilde{x} = \tilde{x}(x)$. By the revelation principle, incentive compatibility is equivalent to direct truth revelation. That is,

$$\text{(IC)} \qquad x = arg \max_{\tilde{x}} \theta wh(\tilde{x}) - d(h(\tilde{x}),x) + s(\tilde{x}).$$

Applying the envelope theorem and truth revelation ($\tilde{x}(x) = x$), it is easy to show that $dU/dx = -d_x(h,x)$ which yields the first incentive compatibility condition $(IC1)$.

To obtain the second order condition, it is convenient to denote $\theta wh - d(h,x)$ by V. Then, the condition $\partial^2 U/\partial^2 \tilde{x} \leqslant 0$ is equivalent to the inequality $V_{hh}(h_x)^2 + V_h h_{xx} + s_{xx} \leqslant 0$. Differentiating totally the first order condition and using the truth revelation $\tilde{x} = x$ yields the condition $V_{hh}(h_x)^2 + V_{hx}h_x + V_h h_{xx} + s_{xx} = 0$. Thus, the previous inequality is equivalent to $V_{hx}h_x \geqslant 0$. Given that $V_{hx} = -d_{hx} > 0$, the second order condition reduces to $h_x \geqslant 0$ which is $(IC2)$.

In the paper, we denoted by x_1, the skill of the marginal worker who is indifferent between unemployment and subsidized work. The workers' reservation utility was $\widehat{U}(x) = \max_h [b, \theta wh - d(h,x)]$. The government's problem is therefore

$$\text{(PC)} \qquad \max_{\{x_1, s(\cdot), h(\cdot)\}} E(x_1) = \int_{x_1}^{\bar{x}} n(x)dx - \lambda\Big[\int_{\underline{x}}^{x_1} bn(x)dx + \int_{x_1}^{\bar{x}} [s(x) - \tau wh(x)]n(x)dx\Big]$$

$$\text{s.t. } (IC1),\ (IC2) \text{ and } U(x) \geqslant \widehat{U}(x)$$

This problem can be decomposed into two parts: first, the design of the optimal subsidy on the interval $[x_1, x_2]$ and second, the optimal choice of x_1 and x_2 where x_2 is the highest skill of a subsidized worker $(x_1 \leqslant x_2 < \bar{x})$. Let us denote the optimal contract on the interval $[x_1, x_2]$ by $(h^*(\cdot), s^*(\cdot))$. The utility of a subsidized worker is denoted by U^*.

Incentive compatibility applies to the worker x_2 as Lemma 1 shows.

LEMMA 1: $s(x_2) = 0$, $U^*(x_2) = \widehat{U}(x_2)$, $h^*(x_2) = \widehat{h}(x_2)$, $s_x(x_2) = 0$ and $U^{*\prime}(x_2) = \widehat{U}'(x_2)$.

The lemma tells us that the optimal value of the state variable $U(x)$must end on the curve $\widehat{U}(x)$ at x_2 with continuous slope.

PROOF: It is straightforward to show that incentive compatibility at x_2 requires that $s(x_2) = 0$ and $U^*(x_2) = \widehat{U}(x_2)$. Indeed, unsubsidized workers have abilities $x \in]x_2, \bar{x}]$. If $s(x_2) > 0$, some unsubsidized workers with $x = x_2 + \varepsilon$ (for small $\varepsilon > 0$) would take the subsidy $s(x_2)$which is a contradiction. If $s(x_2) < 0$, some subsidized workers with $x = x_2 - \varepsilon$ would not take the subsidy, which is also a contradiction. A similar argument show that $U^*(x_2) < \widehat{U}(x_2)$ and $U^*(x_2) > \widehat{U}(x_2)$ are contradictions.

Given that $s(x_2) = 0$ and $U^*(x_2) = \widehat{U}(x_2)$, it is trivial to show that $h^*(x_2) = \widehat{h}(x_2)$.
To show that $s_x(x_2) = 0$, we write the first order conditions at $x' = x_2 - \varepsilon$ and at $x" = x_2 + \varepsilon$ (small $\varepsilon > 0$):

$$h_x(x')[\theta w - d_h(h(x'), x')] + s_x(x') = 0,$$

$$\theta w - d_h(\widehat{h}(x''), x'') = 0.$$

When $\varepsilon \to 0$, $x' \to x"$ and $h(x') \to \widehat{h}(x'')$. Hence, $s_x(x_2) = 0$.
Finally, let us note that $U^{*\prime} = -d_x(h^*(x), x)$ and $\widehat{U}' = -d_x(\widehat{h}(x), x)$. Since $h^*(x_2) = \widehat{h}(x_2)$, it must be that $U^{*\prime}(x_2) = \widehat{U}'(x_2)$.

Optimal Subsidy and Productivity

Since $U = \theta wh - d(h,x) + s(x)$, the term $[s(x) - \tau wh(x)]$ can be replaced by $[U - wh + d(h,x)]$. Suppose then that the constraint $(IC2)$ is slack (this will be checked *ex post*). In this case, the program can be transformed to the optimal control problem

$$\min_{U(\cdot),h(\cdot)} \int_{x_1}^{x_2} [U - wh + d(h,x)]n(x)dx \text{ s.t. } \dot{U} = -d_x(h,x) \quad [\mu]$$

with the particular end-point conditions $U(x_2) = \widehat{U}(x_2)$ and $h(x_2) = \widehat{h}(x_2)$. In this optimal control problem, h is the control variable, U is a state variable whereas the variable $\mu = \mu(x)$ is the co-state variable of the motion equation. The Hamiltonian function is

$$[U - wh + d(h,x)]n(x) + \mu(x)[-d_x(h,x)].$$

An interior maximum is characterized by the following necessary conditions:

- The equation of motion for the co-state variable $\mu : \dot{\mu} = -n(x)$. Therefore,

(13) $$\mu^* = \Gamma - N(x)$$

with Γconstant.

• The first order condition for an interior maximum must have that the derivative of the Hamiltonian function with respect to h is zero. So,

$$(-w + d_h(h^*,x))n(x) - \mu^*(x)d_{hx}(h^*,x) = 0, \text{ or} \tag{14}$$

$$(-w + d_h(h^*,x))n(x) - (\Gamma - N(x))d_{hx}(h^*,x) = 0.$$

Therefore $h^* = h^*(x,\Gamma)$.

• The transversality conditions on the terminal point x_2 implies that $\widehat{h}(x_2) = h^*(x_2)$ where $\widehat{h}(x_2)$ satisfies $d_h(\widehat{h}(x_2),x_2) = \theta w = (1-\tau)w$. In (14), this yields

$$\begin{aligned}(-w + d_h(\widehat{h}(x_2),x_2))n(x_2) &= (\Gamma - N(x_2))d_{hx}(\widehat{h}(x_2),x_2), \text{ or}\\ -\tau w n(x_2) &= (\Gamma - N(x_2)d_{hx}(\widehat{h}(x_2),x_2).\end{aligned}$$

That is,

$$\Gamma = N(x_2) - \tau w \frac{n(x_2)}{d_{hx}(\widehat{h}(x_2),x_2)} > N(x_2). \tag{15}$$

By the MANGASARIAN'S [1966] Sufficient Condition Theorem, two conditions must be fulfilled in order to have a unique solution in the above optimal control program. First, the co-state variable $\mu^*(x)$ should be strictly positive, which is true by (13) and (15). Second, the Hamiltonian function should be weakly jointly convex in (U,h). This is true if $d_{hhx} \leqslant 0$.

Finally, we have supposed that the incentive compatibility condition $(IC2)$ was satisfied. This condition discards the possibility of bunching. By (14), one can compute

$$h_x^* = -\frac{d_{hx} - \frac{\Gamma - N(x)}{n(x)}d_{hxx} - (\frac{\Gamma - N(x)}{n(x)})' d_{hx}}{d_{hh} - \frac{\Gamma - N(x)}{n(x)}d_{hhx}}$$

The productivity level h^* is strictly increasing in x if $(\frac{\Gamma - N(x)}{n(x)})' \leqslant 0$ and $d_{hxx} \geqslant 0 \geqslant d_{hhx}$. This proves the proposition 1.

Appendix 2

Let us denote the productivity levels of subsidized and unsubsidized workers as $h^* = h^*(x,\Gamma)$ and $\widehat{h} = \widehat{h}(x)$. Let us denote the budget B^* as

$$B^*(x_1,x_2,\Gamma) = \int_{\underline{x}}^{x_1} bn(x)dx + \int_{x_1}^{x_2} [s^* - \tau wh^*]n(x)dx + \int_{x_2}^{\bar{x}} -\tau w\widehat{h}n(x)dx.$$

It is readily observed from this expression that, since $s^*(x_2) = 0$ and $h^*(x_2,\Gamma) = \widehat{h}(x_2)$, $B^*_{x_2} = 0$. A marginal change in the skill of the worker x_2 does not alter the size of subsidy s^* and the tax receipts τwh^*. Marginal changes in the government objective are then equal to

$$dB^* = B^*_{x_1}dx_1 + B^*_{\Gamma}d\Gamma.$$

The relationship between x_1 and Γ is found by differentiating totally (7). After simplification, this yields

$$(dx_1)\, d_x(h^*(x_1,\Gamma),x_1) - (d\Gamma) \int_{x_1}^{x_2} d_{hx}(h^*(x,\Gamma),x)h^*_{\Gamma}(x,\Gamma)dx = (dx_2)\left\{d_x(h^*(x_2,\Gamma),x_2) - d_x(\widehat{h}(x_2),x_2)\right\}$$

By Lemma 1, the right hand side of this equality is zero. Hence,

$$(16) \quad (dx_1)\, d_x(h^*(x_1,\Gamma),x_1) - (d\Gamma) \int_{x_1}^{x_2} d_{hx}(h^*(x,\Gamma),(x)h^*_{\Gamma}(x,\Gamma)dx = 0.$$

Variations in employment and budget deficit with respect to $x_1$are given by

$$E^*_{x_1} = -n(x_1),$$

$$B^*_{x_1} = -n(x_1)\left(s^*(x_1) - \tau wh^*(x_1,\Gamma) - b\right).$$

Variations in budget deficit with respect to Γ, B^*_{Γ}, are more difficult to compute. Using expression (7), we get

$$\int_{x_1}^{x_2} [s^* - \tau wh^*]n(x)dx$$

$$= \int_{x_1}^{x_2} [U^* - wh^*(x,\Gamma) + d(h^*(x,\Gamma),x)]n(x)dx$$

$$= \int_{x_1}^{x_2} [b + \int_{x_1}^{x} -d_x(h^*(z,\Gamma),z)dz - wh^* + d(h^*(x,\Gamma),x)]n(x)dx.$$

Integration by part of the double integral term yields

$$\int_{x_1}^{x_2}\int_{x_1}^{x} -d_x(h^*(z,\Gamma),z)dz\, n(x)dx$$
$$= \left[N(x)\int_{x_1}^{x} -d_x(h^*(z,\Gamma),z)dz\right]_{x_1}^{x_2} - \int_{x_1}^{x_2} -d_x(h^*(x,\Gamma),x)\; N(x)dx$$
$$= N(x_2)\int_{x_1}^{x_2} -d_x(h^*(z,\Gamma),z)dz - N(x_1)\,0 + \int_{x_1}^{x_2} d_x(h^*(x,\Gamma),x)\; N(x)dx$$
$$= \int_{x_1}^{x_2} -d_x(h^*(x,\Gamma),x)\;\left(N(x_2)-N(x)\right)dx.$$

Therefore,

$$\begin{aligned}B^*(x_1,x_2,\Gamma) &\\ &= \int_{\underline{x}}^{x_1} bn(x)dx \\ &+\int_{x_1}^{x_2}[(b-wh^*+d(h^*,x)-\frac{N(x_2)-N(x)}{n(x)}d_x(h^*,x))]n(x)dx \\ &+\int_{x_2}^{\bar{x}} -\tau w\widehat{h}n(x)dx.\end{aligned}$$

Finally, we get

$$\begin{aligned}B^*_\Gamma &= \frac{\partial}{\partial\Gamma}B^*(x_1,x_2,\Gamma) \\ &= \int_{x_1}^{x_2}\frac{\partial}{\partial\Gamma}[(b-wh^*+d(h^*,x)-\frac{N(x_2)-N(x)}{n(x)}d_x(h^*,x))]n(x)dx \\ &= \int_{x_1}^{x_2}[-w+d_h(h^*,x)-\frac{N(x_2)-N(x)}{n(x)}d_{hx}(h^*,x))]h^*_\Gamma n(x)dx \\ &= [\Gamma-N(x_2)]\int_{x_1}^{x_2} d_{hx}(h^*,x)h^*_\Gamma dx.\end{aligned}$$

where we use (8) for the last equality. From this expression and from (16), marginal increases in the objective function are given by

$$\begin{aligned}dE^*-\lambda dB^* &= dx_1\left[-n(x_1)-\lambda\left(B^*_{x_1}+B^*_\Gamma\tfrac{d\Gamma}{dx_1}\right)\right] \\ &= dx_1\left[-n(x_1)-\lambda\left(-n(x_1)\right)\left(s^*(x_1)-\tau wh^*(x_1,\Gamma)-b\right)\right. \\ &\qquad\left.-\lambda\left(\Gamma-N(x_2)\right)d_x(h^*(x_1,\Gamma),x_1)\right] \\ &= \lambda n(x_1)dx_1\left[-\frac{1}{\lambda}+s^*(x_1)-\tau wh^*(x_1,\Gamma)-b\right. \\ &\qquad\left.-\frac{\Gamma-N(x_2)}{n(x_1)}d_x(h^*(x_1,\Gamma),x_1)\right]\end{aligned}$$

which proves Proposition 4.