ANNALES D'ÉCONOMIE ET DE STATISTIQUE. – N° 61 – 2001

The Dickey-Fuller Test Family and Changes in the Seasonal Pattern

Antonio MONTAÑÉS, Andreu SANSÓ *

ABSTRACT. – This paper studies the asymptotic behaviour of the Dickey-Fuller family of tests when the variable being considered exhibits a break in the seasonal pattern. We show that the Dickey-Fuller test tends to reject the unit root null hypothesis, except when the break affects all the periods in a similar manner. By contrast, the Dickey-Hasza-Fuller test is biased towards the acceptance of its null hypothesis, the larger the break magnitudes.

Tests de Dickey-Fuller et rupture dans le profil saisonnier

RÉSUMÉ. – Cet article étudie le comportement asymptotique des tests de type Dickey-Fuller lorsque la variable considérée présente une rupture dans son profil saisonnier. Nous montrons que le test de Dickey-Fuller tend à rejeter l'hypothèse nulle de racine unitaire sauf dans la situation où la rupture affecte toutes les fréquences de façon similaire. En revanche, le test de Dickey-Hasza-Fuller est biaisé en faveur de l'acceptation de l'hypothèse nulle, et ce d'autant plus que les ruptures sont grandes.

* A. MONTAÑÉS: Department of Economic Analysis, University of Zaragoza;
A. SANSÓ: Department of Econometrics, Statistics and Spanish Economy, Universitat de Barcelona.
This paper has benefited from the research stay of the first author at Boston University and of the second at the University of Aarhus. The first author acknowledges the financial support of the Spanish Ministry of Education and Culture (DGES PB98-1614) and the second that of the Generalitat de Catalunya and the CICYT (SEC99-0693). The authors also wish to express their thanks for the helpful comments and observations of N. HALDRUP, A. LOPES, A. AGUIRRE and those of an anonymous referee. Obviously, the usual disclaimer applies.

1 Introduction

The debate on the respective merits of the Difference Stationary and Trend Stationary hypotheses has naturally supposed the derivation of a number of methods for determining which of these two is more appropriate. Furthermore, since the seminal works of FULLER [1976] and DICKEY and FULLER [1979], a number of new statistics have been designed in response to these new theoretical developments. For example, testing the I(2) hypothesis has been considered in DICKEY and PANTULA [1987], HALDRUP [1994] or HALDRUP [1998]; the Trend Stationary null hypothesis has been tested in KWIATKOWSKY *et al.* [1992]; whilst PERRON [1989] has tested for unit roots in the presence of structural breaks and DICKEY, HASZA and FULLER [1984] and HYLLEBERG *et al.* [1990] have tested for seasonal unit roots.

The last two issues have a special interest from the point of view of their empirical implications. In this regard, we should bear in mind that most of the series included in time series econometric applications can be affected by at least one break. Similarly, most of the empirical studies do not use yearly data, but rather data with a periodicity of less than one year. Although these two issues have received extensive attention, quite apart from the two above-mentioned papers, there remains only a limited amount of evidence on the behaviour of the unit root tests when these two aspects are considered together. The pioneering works in this field are those of GHYSELS [1994], SMITH and OTERO [1997], FRANSES and PAAP [1997] and FRANSES and VOGELSANG [1998], where the implications of the omission of a structural break in the seasonal pattern on the behaviour of the seasonal/non-seasonal unit roots is either conjectured or analysed by way of Monte Carlo methods. More recently, LOPES and MONTAÑÉS [1998] have derived the asymptotic behaviour of the HEGY family of tests under the presence of such a break. Their results lead us to conclude that these tests are not equally affected by the omission of this break and nor are they always biased towards the acceptance of the unit root hypothesis.

However, this last work did not consider a group of tests that have been presented in the literature. Here, we are thinking in terms of the Dickey-Fuller family of tests, which are based on the analysis of the estimator of the autoregressive parameter for several orders. For example, if we are interested in determining the presence of a unit root in the non-seasonal frequency, we should analyse whether the first order autoregressive parameter is 1. This is precisely the test proposed in DICKEY and FULLER [1979]. Similarly, when analysing quarterly data, we should study the fourth order autoregressive parameter. This test is proposed in DICKEY, HASZA and FULLER [1984]. The particular contribution of this present paper is to derive the asymptotic behaviour of these two above-mentioned tests in those circumstances where the variable being studied exhibits a break in its deterministic seasonal pattern.

To that end, we proceed as follows. Section 2 is devoted to the derivation of the asymptotic behaviour of these tests when the variable is generated by a model that includes seasonal variables, as well as a deterministic trend and a change in the seasonal pattern. We will show that the Dickey-Fuller test is not biased towards the acceptance of the unit root null hypothesis, except when the change in the seasonal pattern affects all the dummy variables in a similar manner. Furthermore, the first order autoregressive parameter can take negative values and, under some combinations of values, can lead the researcher to accept the H_o: $\rho = -1$ hypothesis. For its part, the behaviour of the Dickey-Hasza-Fuller test is different, in that it tends to accept the seasonal unit root null hypothesis when the size of the break magnitudes is greater, independent of the way this break affects the seasonal pattern. Section 3 contains some Monte Carlo exercises, where the validity of our asymptotic results is assessed for finite samples. In Section 4, we present an empirical application that illustrates the importance of the theoretical results obtained in Section 2. Section 5 closes the paper with a review of the main conclusions. The proof of the Theorems reported in Section 2 is relegated to an Appendix.

2 The Asymptotic Behaviour of the Dickey-Fuller Family of Tests in the Presence of a Break in the Seasonal Pattern

This Section is devoted to deriving the asymptotic behaviour of the Dickey-Fuller family of tests when the variable exhibits a break in the seasonal pattern. We will consider the statistics proposed in DICKEY and FULLER [1979] and in DICKEY, HASZA and FULLER [1984]. Both of these are based on the study of an autoregressive parameter: the first order autoregressive parameter in the first case, and the fourth order autoregressive parameter in the second. Consequently, they analyse whether a regular or seasonal differentiation is required in order to achieve the stationarity of the variable. Although these statistics allow for some different specifications of the deterministic elements, we will only consider the most general approach where the variable exhibits a trend. Finally, we will focus exclusively on the quarterly data case, although our results can provide a useful approximation for other periods. Given all these assumptions, in the rest of the paper we will consider that the variable being studied is generated according to the following model:

$$(1) \qquad y_t = \beta t + \sum_{s=1}^{4} d_s D_{st} + \sum_{s=1}^{4} \delta_s D_{st} \left[I_{t \geqslant \tau} \right] + u_t \qquad t = 1,2,T$$

where D_{st} represents the usual dummy variables (s = 1,2,3,4), t is a deterministic trend and $\left[I_{t \geqslant \tau} \right]$ takes the value 1 if $t \geqslant \tau$ and 0 otherwise. Adopting a general approach, we consider that the innovations of the model comply with the conditions reported in Assumption 1 of PERRON [1989], which allows for these innovations to follow any stationary and invertible ARMA model. We also assume that the time of the break can be stated as $\tau = \lambda T$, where $0 < \lambda < 1$. Similarly, we consider that the value of the seasonal break fraction parameter takes a value which guarantees that the break time is an integer number. We further consider that the break can only affect the seasonal components of the same year. Consequently, we do not take into account those cases where, for example, the seasonal pattern is altered in the fourth quarter of year t_o and in the first quarter of year $t_o + 1$.

In what follows, our aim is to derive the asymptotic behaviour of the Dickey-Fuller family of tests when the variable is generated by (1). Given the characteristics of the variable being studied, these statistics can be obtained from the estimation of the following model:

$$y_t = \sum_{s=1}^{4} \mu_s D_{st} + \beta\, t + \rho_k\, y_{t-k} + u_t \quad k = 1,4 \tag{2}$$

with D_{st} $(s = 1,2,3,4)$ being the usual seasonal dummy variables, and the subsequent obtaining of the pseudo t-ratio for testing whether the autoregressive parameter is 1. It is also possible to use the normalised bias, defined as $T\left(\hat{\rho}_k - 1\right)$ $(k = 1,4)$. However, we have chosen to focus our study essentially on the most commonly used pseudo t-ratio. In any event, the asymptotic expressions for the normalised bias can easily be obtained from the limit value of the estimator of the autoregressive parameter.

2.1 The Case of the Dickey-Fuller Statistic

In this section, we study the case of $k = 1$ in (2). The null hypothesis of interest is now $H_o : \rho_1 = 1$; that is to say, we analyse whether the variable exhibits a unit root at frequency zero. As was shown in Dickey, Miller and Bell [1986] and in Ghysels, Lee and Noh [1994], the presence of some dummy variables does not alter the distribution of the pseudo t-ratio when there is an intercept in the regression. Here, it should be appreciated that a set of dummy variables plays the role of an intercept when we test for a unit root at the zero frequency. Consequently, we can employ the critical values that are tabulated in Fuller [1976] or MacKinnon [1991] for comparing the values of the pseudo t-ratio. The asymptotic behaviour of the Dickey-Fuller statistic is reported in the following Theorem.

Theorem 1: Let us assume that the variable y_t is generated by model (1), with the innovations of the model being generated by any stationary and invertible ARMA model, with $\tau = \lambda T$, where $0 < \lambda < 1$. Thus, when we estimate model (2) with $k = 1$, the Dickey-Fuller statistics converge towards the following values:

a) $\hat{\rho}_1 \to \dfrac{\Psi_2 + 16\gamma_1}{\Psi_1 + 16\,\sigma_u^2}$

b) $T^{-1/2}\; t_{\hat{\rho}_1} \to -\sqrt{\dfrac{\left[\Psi_1 - \Psi_2 + 16\left(\sigma_u^2 - \gamma_1\right)\right]^2}{\Psi_1^2 - \Psi_2^2 + 32\left(\Psi_1\sigma_u^2 - \Psi_2\gamma_1\right) + 256\left(\sigma_u^2 - \gamma_1^2\right)}}$

where

$$\Psi_1 = 3\,(2-\lambda)\,\lambda^3 \left(\sum_{i=1}^{4} \delta_i\right)^2 + 4\lambda \sum_{i=1}^{4} \delta_i^2 - \lambda^2 \left[7 \sum_{i=1}^{4} \delta_i^2 + 6 \sum_{i=1}^{3} \delta_i \sum_{j=i+1}^{4} \delta_j\right]$$

$$\Psi_2 = -\kappa \left[3\kappa \left(\sum_{i=1}^{4} \delta_i \right)^2 - 4 \ (\delta_1 + \delta_3) \ (\delta_2 + \delta_4) \right]$$

$$\kappa = \lambda \ (1 - \lambda), \ \sigma_u^2 = \lim_{T \to \infty} T^{-1} \sum_1^T E\left(u_t^2\right)$$

and $\gamma_1 = \lim_{T \to \infty} T^{-1} \sum_1^T E\left(u_t \, u_{t-1}\right)$

PROOF: See Appendix.

The results reported in the above Theorem are valid for the general case. However, it is not easy to draw definitive conclusions for all the possible parameter combinations due to the complexity of the limit values. Nevertheless, some conclusions appear to be clear. First, the shape adopted by the polynomial Ψ_2 suggests that the autoregressive parameter can take negative values. Under these circumstances, the researcher can wrongly conclude that the variable exhibits a negative correlation pattern. Moreover, the value of the autoregressive parameter goes closer to -1 if the innovations of the model (1) show a negative correlation, or when the break magnitude is higher. Thus, the researcher may be induced into thinking that the null hypothesis of interest is $H_o : \rho_1 = -1$ and can eventually accept it. The second conclusion is that the pseudo t-ratio diverges towards $-\infty$ at rate $T^{1/2}$, so that it will tend to reject the unit root null hypothesis when the sample size is larger.

Apart from this general case, it is also of interest to analyse the behaviour of these statistics when the change in the seasonal pattern adopts some particular shapes. Let us consider three specific cases. The first, Case A, studies the situation where the break affects all the seasonal periods in a similar way. Thus, there is no change in the seasonal pattern under this type of break, although the mean of the variable is modified. The second and third cases (Cases B and C, respectively) do allow for a change in the seasonal pattern. However, the difference between them lies in the fact that whilst there is a change in the mean levels of the variables under Case B, this does not take place under Case C. The following Corollary reports the limit values for all three cases.

COROLLARY 1: Assuming the conditions stated in Theorem 1, the Dickey-Fuller statistics asymptotically behave as follows:

A) If $\delta_i = \delta, \ i = 1,...,4$

a) $\hat{\rho}_1 \to \dfrac{\kappa \ (1 - 3\kappa) \ \delta^2 + \gamma_1}{\kappa \ (1 - 3\kappa) \ \delta^2 + \sigma_u^2}$

b) $T^{-1/2} \, t_{\hat{\rho}_1} \to -\sqrt{\dfrac{\left(\sigma_u^2 - \gamma_1\right)}{2\kappa \ (1 - 3\kappa) \ \delta^2 + \sigma_u^2 + \gamma_1}}$

B) If $\delta_i = \delta, \delta_j = 0 \, \forall j \neq i$

a) $\hat{\rho}_1 \to -\dfrac{3\,\kappa^2\,\delta^2 - 16\,\gamma_1}{(4 - 3\kappa)\ \kappa\,\delta^2 + 16\ \sigma_u^2}$

b) $T^{-1/2} t_{\hat{\rho}_1} \to -\sqrt{\dfrac{2\,\kappa\,\delta^2 + 8\left(\sigma_u^2 - \gamma_1\right)}{(2-3\kappa)\,\kappa\,\delta^2 + 8\,\left(\sigma_u^2 + \gamma_1\right)}}$

C) If $\delta_4 = -\sum_{i=1}^{3} \delta_i$

a) $\hat{\rho}_1 \to -\dfrac{\kappa\,(\delta_1 + \delta_3)^2 - 4\,\gamma_1}{2\,\kappa\,\xi_1 + 4\,\sigma_u^2}$

b) $T^{-1/2} t_{\hat{\rho}_1} \to -\sqrt{\dfrac{\left[\kappa\,(\delta_1 + \delta_3)^2 + 2\,\kappa\,\xi_1 + 4\,\left(\sigma_u^2 - \gamma_1\right)\right]^2}{4\,\left(\kappa\,\xi_1 + 2\,\sigma_u^2\right)^2 - \left[\,\kappa\,(\delta_1 + \delta_3)^2 - 4\,\gamma_1\,\right]^2}}$

with $\xi_1 = \sum_{i=1}^{3} \delta_i^2 + \sum_{i=1}^{2} \delta_i \sum_{j=i+1}^{3} \delta_j,$

PROOF: The proof is directly obtained by imposing the above restrictions on the results reported in Theorem 1.

Some important consequences flow from the results of this Corollary. First, we can see that in the presence of a break which simply implies a jump in the mean of the variable y_t, but with no seasonal change, the behaviour of the statistics is different from the case where there is a change in the seasonal pattern. Note that the results reported in Corollary A.a are qualitatively similar to those reported in PERRON [1989] and MONTAÑÉS and REYES [1999] for the case of a change in the intercept model. The estimator of the autoregressive parameter goes closer to 1 the greater the magnitude of the break. Similarly, the pseudo t-ratio diverges. Nevertheless, it is also true that the larger the magnitude of the break, the more difficult it is to reject the null hypothesis.

With respect to Cases B and C, the results are particularly useful in understanding the performance of the Dickey-Fuller statistics in the presence of a break in the seasonal pattern. Let us begin by considering the case where the change in the seasonal pattern only affects one of the dummy variables. The results reported in parts B.a and B.b of Corollary 1 show that the autoregressive parameter can take negative values, generating a spurious negative autocorrelation in the variable. This statistic minimises its value for $\lambda = 0.5$ and, when the break magnitude tends towards ∞, it cannot take values lower than -0.24. Thus, the possibility of accepting the mirror image null hypothesis $H_o : \rho_1 = -1$ is not very high, even in those cases where $\gamma_1 < 0$.

On the other hand, the pseudo t-ratio diverges towards $-\infty$ for any combination of the parameters. Given the range of values of the first order autoregressive parameter, we can conclude that this statistic will tend to reject both the unit root null hypothesis and its mirror image hypothesis.

When the change in the seasonal pattern does not modify the mean of the variable, the results are similar to those described earlier, in that the first order autoregressive parameter can take negative values. This statistic also minimises its value when $\lambda = 0.5$. However, we cannot now appreciate what is the asymptotic limit, in that this depends on the particular shape adopted by the break. For example, when the change in the seasonal level is given by

$\delta_1 = \delta$ and $\delta_i = -\delta/3$, for $i = 2,3,4$, the asymptotic limit of the first order autoregressive parameter is:

$$\hat{\rho}_1 \rightarrow \frac{-\kappa\,\delta^2 + 9\gamma_1}{3\,\kappa\,\delta^2 + 9\,\sigma_u^2} \tag{3}$$

Thus, it is clear that this estimator tends towards $-1/3$ the higher the break magnitude. Therefore, the probability of accepting the mirror image hypothesis is not very high, especially when the innovations show a positive autocorrelation pattern. Something similar occurs when we consider the case where break adopts the shape $\delta_1 = -\delta_3$, $\delta_2 = \delta_4 = 0$. Here, it is evident that the autoregressive estimator now goes to 0, the greater the break magnitude.

However, when we consider the following combination of seasonal break parameters: $\delta_1 = \delta_3 = \delta$, $\delta_2 = \delta_4 = -\delta$, the limit value of the autoregressive parameter is now:

$$\hat{\rho}_1 \rightarrow \frac{-\,\kappa\,\delta^2 + \gamma_1}{\kappa\,\delta^2 + \sigma_u^2} \tag{4}$$

In this case, it is clear that the probability of accepting the $H_o : \rho_1 = -1$ hypothesis is now higher, in that the limit value of this estimator goes closer to -1, the larger the magnitude of the break. We can also note that the limit value reported in (4) is similar to the limit expression obtained in PERRON [1989] under the omission of a break in the intercept of the trend function.

2.2 The Case of the Dickey-Hasza-Fuller Test

Let us now consider the case where the researcher wishes to analyse whether the variable needs to be seasonally differentiated. The Dickey-Fuller family of tests provides us with the opportunity of developing a statistic similar to that analysed earlier, in the form of the DICKEY, HASZA and FULLER [1984] test, hereafter DHF. This is calculated in terms similar to that of the simple Dickey-Fuller test, with the only change being the consideration of $k = 4$ in equation (2). Therefore, the null hypothesis of interest is $H_o : \rho_4 = 1$. Under the assumptions described for the analysis of the Dickey-Fuller statistics, the asymptotic behaviour of the DHF tests under the presence of a change in the seasonal pattern is described in the following Theorem.

THEOREM 2: Let us assume that the variable y_t is generated by model (1), with the innovations of the model being generated by any stationary and invertible ARMA model, with $\tau = \lambda T$, where $0 < \lambda < 1$. Thus, when we estimate model (2) with $k = 4$, the Dickey-Hasza-Fuller statistics converge towards the following values:

a) $\hat{\rho}_4 \rightarrow \dfrac{\Psi_3 + 16\gamma_4}{\Psi_1 + 16\,\sigma_u^2}$

b) $T^{-1/2}\, t_{\hat{\rho}_1} \rightarrow -\sqrt{\dfrac{\left[\Psi_1 - \Psi_3 + 16\,\left(\sigma_u^2 - \gamma_4\right)\right]^2}{\Psi_1^2 - \Psi_3^2 + 32\left(\Psi_1\sigma_u^2 - \Psi_3\gamma_4\right) + 256\left(\sigma_u^2 - \gamma_4^2\right)}}$

where

$$\Psi_1 = 3(2-\lambda)\lambda^3\left(\sum_{i=1}^{4}\delta_i\right)^2 + 4\lambda\sum_{i=1}^{4}\delta_i^2 - \lambda^2\left[7\sum_{i=1}^{4}\delta_i^2 + 6\sum_{i=1}^{3}\delta_i\sum_{j=i+1}^{4}\delta_j\right]$$

$$\Psi_3 = k\left[(4-3\kappa)\sum_{i=1}^{4}\delta_i^2 - 6k\sum_{i=1}^{3}\delta_i\sum_{j=i+1}^{4}\delta_j\right]$$

$$\kappa = \lambda(1-\lambda),$$

$$\sigma_u^2 = \lim_{T\to\infty} T^{-1}\sum_1^T E\left(u_t^2\right) \text{ and } \gamma_1 = \lim_{T\to\infty} T^{-1}\sum_1^T E\left(u_t\, u_{t-1}\right)$$

PROOF: See Appendix.

The behaviour of the DHF statistics is different from that observed in the DF case, with the most important of these being the fact that the autoregressive parameter cannot take negative values. Moreover, this statistic goes to 1, the greater the magnitude of the break. As in the DF case, the pseudo t-ratio diverges towards $-\infty$ for a finite value of the break magnitudes. Thus, the statistics behave correctly, in that they would reject the null hypothesis, at least asymptotically. However, we should also note that the denominator of the pseudo t-ratio in the above expression includes the presence of the break magnitudes. This causes the pseudo t-ratio to move closer to the acceptance zone, the greater the size of these parameters. Consequently, we can expect a serious loss of power in these statistics, especially for the sample sizes that are commonly used in applied studies.

As in the case of the DF statistics, let us particularise the results reported in Theorem 2 to some cases of empirical interest.

COROLLARY 2: If we assume the conditions stated in Theorem 2, the Dickey-Hasza-Fuller statistics asymptotically behave as follows:

A) If $\delta_i = \delta$, $i = 1,\dots,4$

a) $\hat{\rho}_4 \to \dfrac{(1-3\kappa)\,\kappa\,\delta^2 + \gamma_4}{(1-3\kappa)\,\kappa\,\delta^2 + \sigma_u^2}$

b) $T^{-1/2}\, t_{\hat{\rho}_4} \to -\sqrt{\dfrac{\left(\sigma_u^2 - \gamma_4\right)}{2\kappa\,(1-3\kappa)\,\delta^2 + \sigma_u^2 + \gamma_4}}$

B) If $\delta_i = \delta, \delta_j = 0\,\forall j \neq i$

a) $\hat{\rho}_1 \to \dfrac{(4-3\kappa)\,\kappa\,\delta^2 + 16\,\gamma_4}{(4-3\kappa)\,\kappa\,\delta^2 + 16\,\sigma_u^2}$

b) $T^{-1/2} t_{\hat{\rho}_1} \to -\sqrt{\dfrac{8\left(\sigma_u^2 - \gamma_4\right)}{\kappa\,(4-3\kappa)\,\delta^2 + 8\left(\sigma_u^2 + \gamma_4\right)}}$

C) If $\delta_4 = -\sum_{i=1}^{3}\delta_i$

a) $\hat{\rho}_1 \rightarrow \dfrac{\kappa\,\xi_2 + 2\gamma_4}{\kappa\,\xi_2 + 2\,\sigma_u^2}$

b) $T^{-1/2} t_{\hat{\rho}_1} \rightarrow -\sqrt{\dfrac{(\sigma_u^2 - \gamma_4)}{\kappa\,\xi_2 + (\sigma_u^2 + \gamma_4)}}$

$$\xi_2 = \sum_{i=1}^{3} \delta_i^2 + \sum_{i=1}^{2} \delta_i \sum_{j=i+1}^{3} \delta_j$$

PROOF: The proof is obtained directly by imposing the above restrictions on the results reported in Theorem 2.

The results of the above Corollary confirm that the autoregressive parameter goes to 1 the greater the magnitudes of the break. As we can verify, the pseudo t-ratio diverges for a finite value of the break magnitude. However, the greater the value of the parameter δ, the closer the limit value to 0. Thus, the statistic continues to exhibit its power problems when the break is large.

As in the earlier case, we can again see that the asymptotic limit value of the DHF statistics diverges for a finite value of the break magnitude, although a large value of this parameter implies an approximation of this limit to the acceptance zone. The results of these two Corollaries coincide, to some extent, with those obtained in LOPES and MONTAÑÉS [1998]. Thus, we can draw the conclusion that a change in the seasonal pattern which modifies the level of the variable involves a loss of power on the part of the DHF statistic for large values of the parameters which measure the break.

3 A Monte Carlo Study for Finite Samples

In this Section, we present a Monte Carlo study where the finite sample behaviour of the Dickey-Fuller family of tests is analysed in the situation where the variable exhibits a break in the seasonal pattern. The Data Generating Process (DGP) is given by equation (1) with $\beta = 1,\ d_s = 0$ for all $s,\ u_t = \phi u_{t-1} + \varepsilon_t,\ \varepsilon_t \sim iidN(0{,}1)$, and with different values of both ϕ and of the parameters which determine the type of the seasonal break. In this regard, we have considered four sets of values for the seasonal break: a) $\delta_i = \delta\ (i = 1{,}2{,}3{,}4)$, b) $\delta_1 = \delta$ and $\delta_i = 0\ \ (i = 2{,}3{,}4)$, c) $\delta_1 = \delta$ and $\delta_i = -\delta/3\ (i = 2{,}3{,}4)$, and d) $\delta_1 = \delta_3 = \delta, \delta_2 = \delta_4 = -\delta$. Hence, these parameterisations coincide with Cases A, B and C of Corollaries 1 and 2, and Case C with two different sets of values. For each set of values, we have carried out Monte Carlo experiments for the break fractions $\lambda = \{0.3{,}0.5{,}0.7\}$, $\phi = \{-0.9{,}-0.7{,}-0.5{,}-0.3{,}0{,}0.3{,}0.5{,}0.7{,}0.9\}$, for sample sizes $T = \{60{,}120{,}240\}$ and for $\delta = \{1{,}3{,}5{,}7{,}9\}$. Hence, a total of 405 Monte Carlo experiments have been carried out. In order to save space,

TABLE 1

Rejection Frequencies Case 1: $\delta_i = \delta (i = 1,2,3,4)$

ϕ	δ	T	DF	DFI	DHF
0	1	60	1	1	1
		120	1	1	1
		240	1	1	1
	3	60	1	1	1
		120	1	1	1
		240	1	1	1
	5	60	1	1	1
		120	1	1	1
		240	1	1	1
0.5	1	60	0.939	1	1
		120	1	1	1
		240	1	1	1
	3	60	0.9195	1	1
		120	1	1	1
		240	1	1	1
	5	60	0.935	1	1
		120	1	1	1
		240	1	1	1
0.9	1	60	0.091	1	0.992
		120	0.2275	1	0.9995
		240	0.778	1	1
	3	60	0.2115	1	0.9985
		120	0.367	1	1
		240	0.7905	1	1
	5	60	0.4195	1	1
		120	0.554	1	1
		240	0.8445	1	1

TABLE 2

Rejection Frequencies Case 2: $\delta_i = \delta$ and $\delta_i = 0 \quad (i = 2,3,4)$

ϕ	δ	T	DF	DFI	DHF
0	1	60	1	1	1
		120	1	1	1
		240	1	1	1
	3	60	1	1	0.999
		120	1	1	1
		240	1	1	1
	5	60	1	1	0.959
		120	1	1	1
		240	1	1	1
0.5	1	60	0.9625	1	1
		120	1	1	1
		240	1	1	1
	3	60	0.9995	1	0.9995
		120	1	1	1
		240	1	1	1
	5	60	1	1	0.9735
		120	1	1	1
		240	1	1	1
0.9	1	60	0.1045	1	0.989
		120	0.3025	1	1
		240	0.8765	1	1
	3	60	0.4665	1	0.97
		120	0.8375	1	0.9985
		240	1	1	1
	5	60	0.869	1	0.859
		120	0.9945	1	0.975
		240	1	1	0.9995

TABLE 3

Rejection Frequencies Case 3: $\delta_1 = \delta$ and $\delta_i = -\delta/3 \ (i = 2,3,4)$.

ϕ	δ	T	DF	DFI	DHF
0	1	60	1	1	1
		120	1	1	1
		240	1	1	1
	3	60	1	1	0.9935
		120	1	1	1
		240	1	1	1
	5	60	1	1	0.733
		120	1	1	0.977
		240	1	1	1
0.5	1	60	0.969	1	1
		120	1	1	1
		240	1	1	1
	3	60	1	1	0.993
		120	1	1	1
		240	1	1	1
	5	60	1	1	0.8135
		120	1	1	0.992
		240	1	1	1
0.9	1	60	0.1275	1	0.993
		120	0.3755	1	0.9995
		240	0.915	1	1
	3	60	0.714	1	0.9195
		120	0.965	1	0.9915
		240	1	1	1
	5	60	0.981	1	0.608
		120	0.9995	1	0.834
		240	1	1	0.9975

TABLE 4

Rejection Frequencies Case 4:
$\delta_1 = \delta_3 = \delta, \delta_2 = \delta_4 = -\delta$

ϕ	δ	T	DF	DFI	DHF
0	1	60	1	1	1
		120	1	1	1
		240	1	1	1
	3	60	1	0.2445	0.994
		120	1	0.9985	1
		240	1	1	1
	5	60	1	0	0.995
		120	1	0.103	0.9995
		240	1	1	1
0.5	1	60	1	1	0.9995
		120	1	1	1
		240	1	1	1
	3	60	1	0.9405	0.9575
		120	1	1	1
		240	1	1	1
	5	60	1	0.079	0.9515
		120	1	0.9725	0.984
		240	1	1	1
0.9	1	60	0.446	1	0.9755
		120	0.8215	1	0.9995
		240	0.998	1	1
	3	60	0.9995	0.998	0.52
		120	1	1	0.711
		240	1	1	0.9605
	5	60	1	0.888	0.2945
		120	1	1	0.1735
		240	1	1	0.352

Note: δ are the parameters related with the change in the seasonal pattern, λ is the break fraction, T is the sample size and ϕ is the autoregressive parameter; DF means the standard Dickey-Fuller test; DFI means the DF test of the image hypothesis ($\rho = -1$); finally DHF is the Dickey-Hasza-Fuller test. All experiments have been carried out with 2,500 replications and using the 5 % significance level. The DGP is given by expression (1).

we only report those for $\lambda = 0.5$, $\delta = \{1,3,5\}$ and $\phi = \{0,0.5,0.9\}$. The complete list of results will be provided by the authors upon request.

In each experiment, we have computed the standard Dickey-Fuller [DF], the Dickey-Fuller type test of the mirror image hypothesis ($\rho = -1$) [DFI] and the Dickey-Hasza-Fuller [DHF] test. We have used the MacKinnon [1991] response surface to generate the 5 % significance level critical values of the DF and DFI statistics, whilst the critical values of the DHF statistic have been obtained from Table C2 of Ghysels *et al.* [1994]. All experiments have been carried out using the RNDN routine of GAUSS 2.1 and 2,500 replications. Here, it should be noted that the behavior of the DFI statistic for $\phi = \{-0.9,-0.7,-0.5,-0.3\}$ is qualitatively similar to that of DF for $\phi = \{0.3,0.5,0.7,0.9\}$, and that the converse is also true.

Tables 1-4 report the power of the above mentioned statistics. Table 1 shows the results for the case where there is a jump in the mean of the variable which does not have the effect of modifying the seasonal pattern.[1] In this case, we can see that the Dickey-Fuller test decreases its power, the smaller the sample size and the bigger the autoregressive parameter. We can also note that the size of the change favours the power of the test for large values of the autoregressive parameter. The DFI statistic always rejects the null hypothesis, but behaves like the DF statistic for negative values of ϕ. The DHF shows a relatively unimportant lack of power for small samples sizes and a large autoregressive parameter. A further result obtained for this statistic is that it is powerless when the break takes place at the beginning of the sample. Similar results were obtained in Montañés and Reyes [1998].

Table 2 presents the percentage of rejections of the null hypothesis of the tests when the break in the seasonal pattern affects just one of the seasons. In this situation, the DHF statistic suffers from a very small reduction in power. As in the first case, the Dickey-Fuller test shows a decrease in power for large values of the autoregressive parameters and for small sample sizes. The size of the break also favours the power of the test.

Tables 3 and 4 consider Case C of Corollaries 1 and 2. Now, the variable exhibits a break in the seasonal pattern which does not modify the mean of the process. Specifically, Table 3 reports the case where an increase in the mean of a season is balanced by an identical reduction in the mean of the other seasons. Here, the DHF test is clearly affected by this type of break for small sample sizes and large values of ϕ. Furthermore, in this case the size of the break tends to reduce the rejection frequency.

Finally, Table 4 presents the special situation where the change in the mean of a season is compensated by that of the following season. This case mimics the behaviour of the $(1+L)x_t = e_t$ process, so that in some cases we could expect the mirror image hypothesis not to be rejected. As this Table reports, such an occurrence is quite feasible when the change is large and the sample size small. In such a situation, the behaviour of the DHF and DF tests is not seriously affected, except for large values of the autoregressive parameter.

1. We should recall that, given our DGP and for comparative purposes, the Dickey-Fuller power in the case of no break is 1.

4 Empirical Illustration

In order to illustrate the theoretical results that we have reported and discussed in the previous Sections, let us now analyse the time properties of some variables. To that end, we have chosen to study the consumption of Gasoline and Oil Diesel in Spain. These variables are measured in metric Tons and the sample covers the period 1983:1-1999:3. Figures 1 and 2 reflect the evolution of these two variables, showing that the pattern described by these variables changes around 1991, a date that coincides with the "*Gulf War*". Given that the Spanish economy is heavily dependent on oil imports, we could expect that this crisis was capable of modifying the habits of the Spanish agents and, consequently, that the consumption of these two goods would undergo some changes. Thus, the presence of a break in the evolution of these variables seems to be an appropriate hypothesis.

In order to confirm this, we have analysed these variables using of the statistics proposed in BAI and PERRON [1996]. Such statistics allow the researcher to detect several breaks in the evolution of the variables being studied. In our particular case, if we model the consumption of Gasoline and Oil Diesel by assuming that the parameter associated to the deterministic trend is constant, but that of the mean may exhibit some changes, then the Bai-Perron statistics show the presence of a break in 1991:2 for the case of Gasoline consumption, whilst the break appears a little later for Oil Diesel consumption (1991:3). Consequently, if we bear in mind the results of Section 2, we

FIGURE 1
Gasoline Consumption (*in thousands of metric tons*)

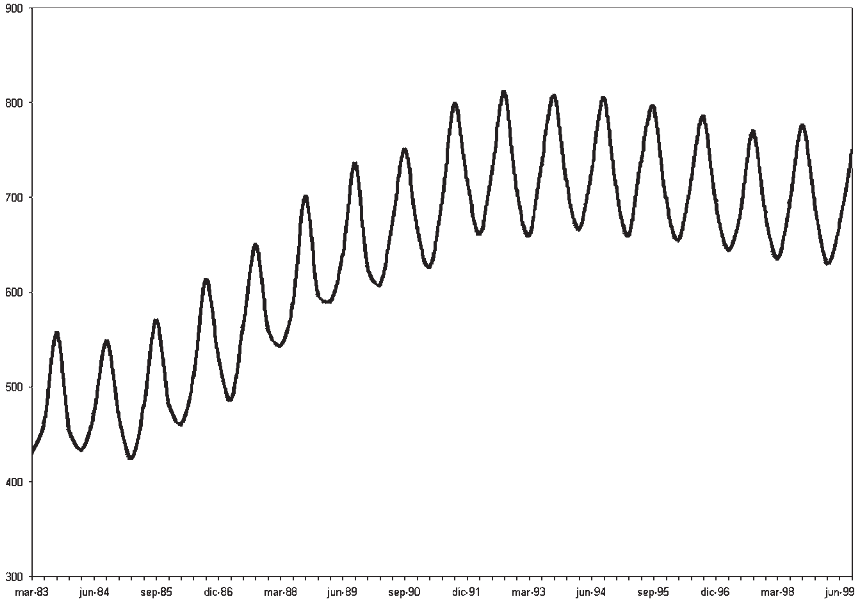

FIGURE 2

Oil Diesel Consumption (*in thousands of metric tons*)

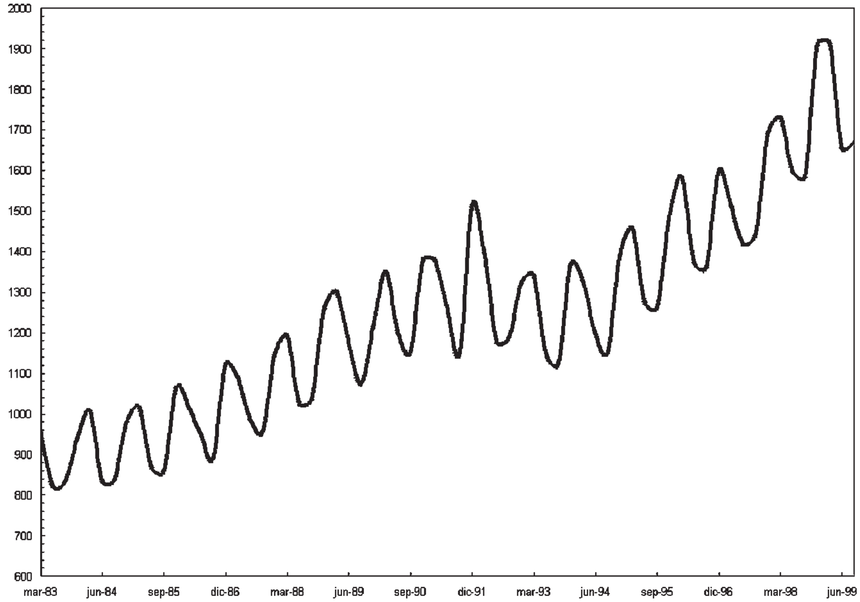

TABLE 5

ADF Results

	$\hat{\alpha}$	$\hat{d}_1$	$\hat{d}_2$	$\hat{d}_3$	$\hat{d}_4$	$\hat{\beta}$	k
			Panel A.				
Gasoline	-0.03	-61.4	14.52	115.8	14.20	0.00	8
	(-0.74)	(-1.91)	(0.44)	(3.55)	(0.40)	(0.00)	
Oil Diesel	-0.22	238.1	103.2	104.4	302.0	2.78	5
	(-1.91)	(2.31)	(0.97)	(1.05)	(3.17)	(2.10)	
			Panel B.				
Gasoline	-0.29	-264.6	-252.2	-139.5	-222.2	2.28	5
	(-3.24)	(-3.73)	(-2.98)	(-1.59)	(-2.95)	(3.18)	
Oil Diesel	-0.90	810.7	662.1	626.4	821.6	12.84	0
	(-4.39)	(4.16)	(3.58)	(4.03)	(5.66)	(4.51)	
			Panel C.				
Gasoline	-0.51	547.6	578.1	640.4	551.4	-1.05	2
	(-3.87)	(4.18)	(4.72)	(5.05)	(4.00)	(-4.80)	
Oil Diesel	-0.34	112.9	-71.0	29.7	263.5	8.2	0
	(-3.47)	(1.44)	(-0.92)	(0.46)	(4.30)	(4.08)	

Note: This Table reports the values of the ADF tests for Gasoline Consumption and Oil Diesel Consumption in Spain. Panel A considers the sample period 1983:1-1999:3. The sample period for Panel B is 1983:1-1991:2, for Gasoline Consumption, and 1983:1-1991:3 for Oil Diesel Consumption. Finally, the sample period for Panel C is 1991:3-1999:3 and 1991:4-1999:3, respectively, for Gasoline and Oil Diesel Consumption. The model estimated is $\Delta y_t = \alpha y_{t-1} + d_1 S_{1t} + d_2 S_{2t} + d_3 S_{3t} + d_4 S_{4t} + \beta t + \sum_{i=1}^{k} \phi_i \Delta y_{t-i} + u_t$, *where t is a deterministic trend and* S_{it} $(i = 1,2,3,4)$ *are four seasonal dummy variables. The value of the lag truncation parameter, k, has been selected using the criteria proposed in* NG *and* PERRON *[1995], using a kmax=8.*

TABLE 6
Dickey-Hasza-Fuller Tests

	$\hat{\alpha}$	$\hat{d}_1$	$\hat{d}_2$	$\hat{d}_3$	$\hat{d}_4$	$\hat{\beta}$	k
				Panel A.			
Gasoline	-0.06	5.98	10.4	16.4	12.2	0.15	5
	(-1.3)	(0.2)	(0.45)	(0.8)	(0.5)	(0.6)	
Oil Diesel	-0.49	448.2	358.5	343.4	450.2	5.77	3
	(-4.4)	(4.4)	(4.3)	(4.3)	(4.5)	(4.64)	
				Panel B.			
Gasoline	-0.44	(4.64)	-323.6	-285.4	-328.4	3.36	3
	(-5.6)	(-4.9)	(-4.8)	(-4.6)	(-4.8)	(5.4)	
				Panel C.			
Gasoline	-0.64	743.1	782.3	831.4	776.7	-1.60	4
	(-5.7)	(6.8)	(6.8)	(6.8)	(6.8)	(-6.2)	

Note: This Table reports the values of the Dickey-Hasza-Fuller tests for Gasoline Consumption and Oil Diesel Consumption in Spain. Panel A considers the sample period 1983:1-1999:3. The sample period for Panel B is 1983:1-1991:2, for Gasoline Consumption and 1983:1-1991:3 for Oil Diesel Consumption. Finally, the sample period for Panel C is 1991:3-1999:3 and 1991:4-1999:3, respectively, for Gasoline and Oil Diesel Consumption. The model estimated is $\Delta^4 y_t = \alpha y_{t-4} + d_1 S_{1t} + d_2 S_{2t} + d_3 S_{3t} + d_4 S_{4t} + \beta t + \sum_{i=1}^{k} \phi_i \Delta^4 y_{t-i} + u_t$, *where t is a deterministic trend,* S_{it} $(i = 1,2,3,4)$ *are four seasonal dummy variables and* $\Delta^4 y_t = y_t - y_{t-4}$. *The value of the lag truncation parameter, k, has been selected using the criteria proposed in* NG *and* PERRON *[1995], using a kmax=8.*

can anticipate that the results of the Dickey-Fuller and the Dickey-Hasza-Fuller tests would lead us to distorted results, given the omission of these breaks.

The values of the Dickey-Fuller and the Dickey-Hasza-Fuller tests are reported in Tables 5 and 6, respectively. Here, we can note that the presence of a unit root is accepted in the regular part for the two variables when the whole sample is considered. By contrast, we can reject the presence of a unit root is rejected in the seasonal part for the consumption of Oil Diesel, but accepted for that of Gasoline.

However, when we split the sample and distinguish between the *pre-* and *post-Gulf War* behaviour, we can observe very different results. Now, the unit root hypothesis can be rejected both for the two variables and for the regular and seasonal part. Therefore, the variables can be better characterised as being stationary around some breaks, rather than as being integrated, which was the result obtained when the sample was not split.

Furthermore, if we focus on the estimation of the seasonal dummy variables, we can see that the estimation of their parameter exhibits significant differences as between the two sub-samples. Thus, we should conclude that the omission of some changes in the seasonal pattern may lead us to an incorrect determination of the time properties of these variables, thereby confirming both the results reported in Section 2 and their utility.

5 Conclusion

In this paper, we have analysed the asymptotic behaviour of the Dickey-Fuller family of tests when the variable being studied exhibits a break in the deterministic seasonal pattern. To be more precise, the two statistics considered are the pseudo t-ratios presented in Dickey and Fuller [1979] and in Dickey, Hasza and Fuller [1984]. Both of these analyse whether the estimator of autoregressive parameter of order k is equal to 1. The latter paper considers the presence of a unit root in the seasonal part ($k = 4$), whilst the former tests for the presence of a unit root in the zero frequency part ($k = 1$).

Our asymptotic results prove that the Dickey-Fuller statistic tends to reject the unit root null hypothesis, regardless of the type of the break. However, our theoretical results show that, when the break affects all the seasonal periods in a similar way, then, the higher the break, the closer the test to the acceptance region. Thus, under these circumstances, we can expect a loss in the power of the ADF statistic as our Monte Carlo simulations have verified. When the break does not modify the seasonal behaviour, the Dickey-Fuller tests are not affected by the presence of this type of break. However, the estimator of the autoregressive parameter may take negative values. Thus, the null hypothesis of interest could now be that of the mirror image. For example, the researcher could be led to accept this hypothesis when the breaks appear under the circumstances whereby the variation of one season is balanced by an inverse modification in the next.

The asymptotic behaviour of the Dickey-Hasza-Fuller statistics is somewhat different, in that they tend to accept the seasonal unit root null hypothesis, the greater the magnitude of the break. Therefore, we can consider this asymptotic performance as being qualitatively similar to that of the Dickey-Fuller tests under the presence of a break in the intercept of the trend function. ■

• References

BAI J., PERRON P. (1996). – « Estimating and Testing Linear Models with Multiple Structural Changes », *Econometrica*, 1998, vol 66, pp. 47-78.

DICKEY D.A., BELL W.R., MILLER R. B. (1986). – « Unit Roots in Time Series Models: Tests and Implications », *The American Statistician*, 40, pp. 12-26.

DICKEY D.A., HASZA D.P., FULLER W. A. (1984). – « Testing for Unit Roots in Seasonal Time Series », *Journal of American Statistical Association*, 79, pp. 355-367.

DICKEY D., FULLER W. (1979). – « Distribution of the Estimator for Autoregressive Time series with a Unit Root », *Journal of the American Statistical Association*, 74, pp. 427-431.

DICKEY D.A., PANTULA S.G. (1987). – « Determining the Order of Differencing in Autoregressive Processes », *Journal of Business and Economic Statistics*, 5, pp. 445-461.

FULLER W. (1976). – *Introduction to Statistical Time Series*. John Wiley and Sons.

FRANSES P.H., VOGELSANG T. (1998). – « On Seasonal Cycles, Unit Roots, and Mean Shifts », *The Review of Economics and Statistics*, 80, pp. 231-40.

GHYSELS E. (1994). – On the Economics and Econometrics of Seasonality, in Sims, C. A. (ed.), *Advances in Econometrics*, Sixth World Congress, vol. I, pp. 257-322, Cambridge University Press.

GHYSELS E., LEE H.S., NOH J. (1994). – « Testing for Unit Roots in Seasonal Time Series », *Journal of Econometrics*, 62, pp. 415-442.

KWIATKOWSKY D., PHILLIPS P.C.B., SCHMIDT P., SHIN Y. (1992). – « Testing the Null Hypothesis of Stationarity against the Alternative of a Unit Root », *Journal of Econometrics*, 54, pp. 159-178.

HALDRUP N. (1994). – « Semiparametric Tests for Double Unit Roots », *Journal of Business and Economic Statistics*, 12, pp. 109-122.

HALDRUP N. (1998). – « A Review of the Econometric Analysis of I(2) Variables », *forthcoming in Journal of Economic Surveys*.

HYLLEBERG S., ENGLE R.F., GRANGER C.W.J., YOO B.S. (1990). – « Seasonal Integration and Cointegration », *Journal of Econometrics*, 49, pp. 215-238.

LOPES A.C.B., MONTAÑÉS A. (1998). – « The Asymptotic and Finite Sample Behaviour of the Seasonal Unit Root Tests Under the Presence of a Break in the Seasonal Pattern », *Working Paper*, University of Zaragoza.

MACKINNON J.G. (1991). – Critical Values for Cointegration Tests in Engle, -R.-F. and CWJ Granger, eds Long-run Economic Relation ships: Reading in Cointegration, *Advanced Tests in Economics*, Oxford University Press, pp. 267-276.

MONTAÑÉS A., REYES M. (1998). – « Effect of a Shift in the Trend Function on Dickey-Fuller Unit Root Tests », *Econometric Theory*, 14, pp. 355-363.

NG S., PERRON P. (1995). – « Unit Root Test in ARMA Models with Data-Dependent Methods for the Selection of the Truncation Lag », *Journal of the American Statistical Association*, vol. 90, pp. 268-281.

PERRON P. (1989). – « The Great Crash, the Oil Shock and the Unit Root Hypothesis », *Econometrica*, 57, pp. 1361-402.

SMITH J., OTERO J. (1997). – « Structural Breaks and Seasonal Integration », *Economics Letters*, 56, pp. 13-19.

APPENDIX

In order to obtain the asymptotic expressions that have been reported in Section 2, we will begin by considering that the variable is generated accordingly to the following model:

$$y_t = \beta t + \sum_{s=1}^{4} d_s D_{st} + \sum_{s=1}^{4} \delta_s D_{st} \left[I_{t \geqslant \tau}\right] + u_t \qquad t = 1,2,T \tag{A.1}$$

where D_{st} represents the usual seasonal dummy variable (s = 1,2,3,4), t is a deterministic trend, $\left[I_{t\geqslant\tau}\right]$ takes the value 1 if $t \geqslant \tau$ and 0 otherwise, and u_t is assumed to follow any stationary and invertible ARMA model.

The Dickey-Fuller family of tests is based on the estimation of the following regression:

$$y_t = \beta t + \sum_{s=1}^{4} d_s D_{st} + \rho_k y_{t-k} + e_t \qquad k = 1,4 \tag{A.2}$$

and the subsequent obtaining of the pseudo t-ratio for testing whether ρ_k is 1. If $k = 1$, we are constructing the Dickey-Fuller test, whilst the Dickey-Hasza-Fuller test is studied when $k = 4$.

An equivalent way to calculate the previous statistics is to estimate the following model:

$$zy_t = \rho_k \, zy_{kt} + e_t \tag{A.3}$$

where zy_t and zy_{kt} ($k = 1,4$) are, respectively, the projections of y_t, y_{t-k} on the space defined by $\{D_{1t}, D_{2t}, D_{3t}, D_{4t}, t\}$.

In order to obtain the asymptotic expressions reported in Theorems 1 and 2, we should first define some different sample moments that are necessary for the calculations. These are reported in the following Lemma.

LEMMA 1: Let us suppose that the variable y_t is generated by model [A.1] and that zy_t and zy_{kt} ($k = 1,4$) are the projections of y_t, y_{t-k} on the space defined by $\{D_{1t}, D_{2t}, D_{3t}, D_{4t}, t,\}$, respectively. Then, if T denotes the available sample size, it is true that:

a) $T^{-1} \sum_{t=1}^{T} zy_{t-k}^2 \to \dfrac{\Psi_1 + 16\,\sigma_u^2}{16} \qquad k = 0, 1, 4$

b) $T^{-1} \sum_{t=1}^{T} zy_t zy_{1t} \to \dfrac{\Psi_2 + 16\,\gamma_1}{16}$

c) $T^{-1} \sum_{t=1}^{T} zy_t zy_{4t} \to \dfrac{\Psi_3 + 16\,\gamma_4}{16}$

with

$$\Psi_1 = 3\,(2-\lambda)\,\lambda^3 \left(\sum_{i=1}^{4} \delta_i\right)^2 + 4\lambda \sum_{i=1}^{4} \delta_i^2 - \lambda^2 \left[7 \sum_{i=1}^{4} \delta_i^2 + 6 \sum_{i=1}^{3} \delta_i \sum_{j=i+1}^{4} \delta_j\right]$$

$$\Psi_2 = -k \left[3k \left(\sum_{i=1}^{4} \delta_i \right)^2 - 4 \ (\delta_1 + \delta_3) \ (\delta_2 + \delta_4) \right]$$

$$\Psi_3 = k \left[(4 - 3\kappa) \sum_{i=1}^{4} \delta_i^2 - 6k \sum_{i=1}^{3} \delta_i \sum_{j=i+1}^{4} \delta_j \right]$$

$$\kappa = \lambda \ (1 - \lambda)$$

The proof of Lemma A.1 is straightforward and, therefore, is not included here. More details can, of course, be obtained upon request.

Once we have the results of Lemma 1, the limit values of the autoregressive parameter reported in Theorems 1 and 2 can easily be obtained by direct substitution of the limit values of Lemma 1 into the OLS estimator of the autoregressive parameter of model (A.3), for $k = 1,4$. The proof for the t-ratios is somewhat more complicated, in that we need to know the asymptotic behaviour of the estimation of the variance. However, it is not difficult to show that, when $k = 1$, it holds that the sum of the squared residuals converges towards the following limit value:

$$\text{(A.4)} \quad T^{-1} SSR_1 \to \frac{\Psi_1^2 - \Psi_2^2 + 32 \left(\Psi_1 \sigma_u^2 - \Psi_2 \gamma_1 \right) + 256 \left(\sigma_u^2 - \gamma_1^2 \right)}{\Psi_1 + 16 \sigma_u^2}$$

and, when $k = 4$, it is true that the sum of the squared residuals goes towards:

$$\text{(A.5)} \quad T^{-1} SSR_4 \to \frac{\Psi_1^2 - \Psi_3^2 + 32 \left(\Psi_1 \sigma_u^2 - \Psi_3 \gamma_k \right) + 256 \left(\sigma_u^2 - \gamma_4^2 \right)}{\Psi_1 + 16 \sigma_u^2}$$

Once we know these two limit values, the asymptotic expressions for the t-ratios reported in Theorems 1 and 2 are easily obtained.