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Causality between Returns and Traded Volumes

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ABSTRACT. – This paper examines causality between the series of returns and transaction volumes in high frequency data. The dynamics of both series is restricted to transitions between a finite number of states. Depending on the state selection criteria, this approach approximates the dynamics of varying market regimes, or in a broader sense reflects the time varying heterogeneity of traders behavior. Our analysis is based on returns and volumes represented by chains with constant or time varying transition probabilities. The univariate return series is examined to identify varying market regimes and determine the impact of state specification on temporal dependence. In the bivariate framework we investigate co-movements between volumes and transaction prices, and propose tests for Granger causality. The trade size threshold yielding a dichotomous process featuring maximum volume-price causality is proposed as a volume classification criterion. We apply our methods to the Alcatel stock data recorded in real and calendar time, and discuss implications of the sampling frequency.

Causalité entre rendements et volumes de transactions

RÉSUMÉ. – Ce papier s'intéresse à la causalité entre rendements et volumes sur données haute-fréquence. La dynamique de ces séries est restreinte à un nombre fini d'états et les processus sont représentés par des chaînes avec une matrice de transition qui peut être constante ou fonction du temps. Dans le cadre multivarié, nous analysons de facon précise les co-mouvements entre volumes et prix dans le but de mettre en évidence des régimes de transactions reflétant l'hétérogénéité des investisseurs. Nous effectuons, en particulier, une analyse causale complète. L'approche est appliquée aux données de cotations correspondant au titre Alcatel, considérées à la fois en temps de transactions et en temps calendaire, ceci permettant de discuter les effets de la fréquence de transactions.

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1 Introduction

The empirical evidence documenting the relationship between prices and traded volumes is constantly growing. It has significantly progressed since the market breaks of 1987 and 1989 with episodes of high price volatility coupled with large trading volumes. More recently, the implementation of electronic trading systems provided new areas of investigation for applied research due to the accessibility of intraday data. In the literature we now find empirical studies examining the price-volume relationship in data sampled at various frequencies. There exist studies involving daily data (Tauchen, Pitts [1983], Karpoff [1987], Gallant, Rossi, Tauchen [1992], Lamoureux, Lastrapes [1994], Jones, Kaul, Lipson [1994]), hourly or half-hourly data (Mulherin, Gerety [1988], Foster, Viswanathan [1993]), and high frequency data recorded in real time at unequal intervals determined by trade arrivals (see, Jones, Kaul, Lipson [1994], Engle, Russell [1998], Darolles, Gouriéroux, Le Fol [2000]). Although empirical findings reported in these studies coincide to some extent, a formal ground for comparison has not been established yet and inference based on data observed at distinct sampling frequencies appears sometimes hard to reconcile. For this reason, researchers need to develop adequate procedures of temporal aggregation and learn to distinguish for example, the proportional variation of daily volume attributed to volumes of individual trades, from that due to the trade intensity (Engle, Russell [1998], Gouriéroux, Jasiak, Le Fol [1999]). Another difficulty stems from the necessity to aggregate outcomes of distinct individual behaviors, and various strategies on a given trading day. The comparability of empirical results is also impaired by the diversity of markets due to the coexistence of order driven markets equipped with electronic order matching devices (*e.g.* the Paris Bourse, the Toronto Stock Exchange) and markets organized with market-makers, like the New York Stock Exchange. On markets belonging to the last category we find intermediaries, whose task is to ensure minimum liquidity. Their role on the order driven markets is automatically fulfilled by a specific type of order, called the market order.[1] It is executed at the best current market price and hence guarantees sufficient liquidity for orders queued in the middle of the order book. Moreover, there are selected investors who provide liquidity at prices differing substantially from the market price, and benefit in exchange from lower transaction costs. Among existing markets, we also distinguish markets involving single price-clearing auctions which take place at fixed times, studied among others by Tauchen and Pitts [1983] and adopted as a general framework in a majority of theoretical papers, and continuous auction markets.

Let us briefly summarize some "*stylized facts*" concerning the price-volume relationship documented in the literature:

1. Note the difference between the definition of market order on the NYSE, and on an order driven market, like for example the Paris Bourse. In Paris, the "*market order*" is executed at the best current ask or bid price in the order book, up to the volume available for this price. The eventually outstanding volume remains in the queue as a limit order.

i) The expected returns depend on traded volumes. This dependence is interpreted in the literature as a liquidity risk premium, and represents a counterpart of the volatility risk premium. The negative correlation is confirmed by inference from simple linear regressions of returns on volumes, disregarding the volatility effect. Surprisingly, linear regressions involving explanatory variables other than volume produce contradictory evidence. It is hence unclear to what extent the negative sign of the regression coefficient on volume depends on the presence and type of other regressors. Equivalently the sign of the volatility risk premium depends on the information set available to econometricians. It seems that the presence of volume as an explanatory variable does not alter the sign of correlation. Indeed, GALLANT, ROSSI and TAUCHEN [1992] found a positive return-volatility relationship after introducing the lagged volume as a conditioning variable.

ii) The early empirical investigations were focused on the contemporaneous relationship between the absolute values of price changes as a proxy for volatility and volumes, and documented a positive correlation between these variables. (See, *e.g.* TAUCHEN, PITTS [1983], and KARPOFF [1987] for a survey). In particular LAMOUREUX, LASTRAPES [1991] showed that volume contains significant information to improve the prediction of price volatility.

iii) There also exists evidence for converse effects. The prediction of intertrade durations, which partly determine the daily exchanged volume, is improved by conditioning on lagged prices or lagged volatilities. Such causal relationship has been systematically revealed by studies on high frequency data based on ACD-GARCH models, Autoregressive Multinomial models (ENGLE, RUSSELL [1998]) or models involving the birth and death processes (DAROLLES, GOURIÉROUX, LE FOL [2000]).

iv) The dynamic relationship between prices and volumes is highly nonlinear, and cannot be sufficiently explored by studies involving only the first and second order conditional moments. For example, the leverage effect, *i.e.* the asymmetric volatility response to positive and negative changes in prices, becomes substantially attenuated by conditioning on lagged volumes. The nonlinearity justifies the growing interest in nonparametric analysis of price and volume dynamics (GALLANT, ROSSI, TAUCHEN [1992]).

v) At a daily sampling frequency, the linear volatility-volume relationship may even disappear, when the number of transactions is included in the set of conditioning variables (JONES, KAUL, LIPSON [1994]). It is not clear whether this result would hold if nonlinear patterns were accommodated, and data sampled at higher frequencies were considered.

In the literature, there exist various approaches to examining the price-volume relationship which rely on a common assumption of heterogeneous traders, who may be time constrained or unconstrained, liquidity suppliers or consumers, and finally, informed or uninformed. We seek to answer the question whether the dynamics of these heterogeneous groups can explain the price-volume relationship. A major difficulty involved in this task is the nonobservability of traders' identities and characteristics contrary to prices and volumes. Yet, we can reveal, to some extent, the latent heterogeneity of traders by investigating the joint behavior of returns and volumes. This paper proposes a method of identifying different endogenous trading regimes. These regimes are determined by significant downward and upward price movements and small and large traded volumes. The core of the paper constitutes the selection of price and volume thresholds, maximizing the price-volume causation.

The paper is organized as follows: in section 2, we consider an univariate series of stock returns $\Delta \log p_t$ and a two state chain $Z_t(a) = \mathbf{1}_{\Delta \log p_t > a}$ characterizing the return dynamics. We discuss the state selection yielding uncorrelated qualitative return processes. The qualitative bivariate specification provides a convenient framework for inference on nonlinear causal links between prices and volumes. In section 3, we consider jointly the returns $\Delta \log p_t$ and volumes v_t summarized by qualitative indicator variables $Z_t(a) = \mathbf{1}_{\Delta \log p_t > a}$, $Y_t(c) = \mathbf{1}_{v_t > c}$. We focus on the first order nonlinear dynamics of the joint process $[Z_t(a), Y_t(c)]$, and propose various regressions to recover the transition matrix at horizon 1. Next, we analyze the dependence of the causality measures at lag one on selected thresholds. We propose the choice of the threshold c in order to maximize the causal relationship between $Z_t(a)$ and $Y_t(c)$.

The empirical results are presented in section 4, where we analyse high frequency data for the Alcatel stock traded on the Paris Bourse. Two sampling schemes, corresponding to the transaction and calendar time, respectively, are examined. Section 5 concludes the paper. Technical details are presented in the Appendix.

2 Univariate Dynamic Patterns

2.1 Financial Applications

In this section, we investigate the dynamics of a chain (Z_t) making transitions between two states 0 and 1.

The two state framework may be used in various ways to analyze the trading process, *i.e.* stock prices and volumes. The states are distinguished according to some dichotomous qualitative features of the quantitative series of prices and volumes. For example, we can consider:

a) the direction (increase or decrease) of the price (or log-price) evolution:

$$Z_t = \begin{cases} 1, & \text{if} \quad \log(p_t) - \log(p_{t-1}) > 0, \\ 0, & \text{otherwise ;} \end{cases}$$

b) the comparison of price change with a given threshold not necessarily equal to zero:

$$Z_t(a) = \begin{cases} 1, & \text{if} \quad \log(p_t) - \log(p_{t-1}) > a, \\ 0, & \text{otherwise.} \end{cases}$$

This approach can be applied to compare the price evolution with the behavior of the riskfree asset, by choosing $a = \log(1 + r)$, where r is the riskfree rate.

There exist other possibilities of threshold selection which may also be considered for financial analysis. For instance, we may fix a at a very large-value (resp. small value) in an analysis of the dynamics of positive [resp.negative] extreme returns. This state specification can further be adapted to a multistate setup where, for example, several price modifications are distinguished (see *e.g.* HAUSMAN, LO, MACKINLAY [1992], ENGLE, RUSSELL [1998]).

c) The qualitative analysis may also be used to study the dynamics of the trade initiating side of the market by verifying if the observed trading prices correspond to asks or bids (HEDVALL, ROSENQUIST [1997]):

$$Z_t = \begin{cases} 1, & \text{if} \quad p_t = ask_t, \\ 0, & \text{if} \quad p_t = bid_t. \end{cases}$$

d) Some other qualitative features may concern joint trading, for example, involving substitution between an underlying asset and a derivative:

$$Z_t = \begin{cases} 1, & \text{if both asset and option are traded between } t \text{ and } t + \Delta t, \\ 0, & \text{otherwise,} \end{cases}$$

where Δt is a predetermined time interval.

e) Finally, we may also examine some features of the volume series and, for example distinguish trades of small and large sizes:

$$Z_t(c) = \begin{cases} 1, & \text{if} \quad v_t > c, \\ 0, & \text{otherwise.} \end{cases}$$

2.2 The Uncorrelated States Specification

In example b) of qualitative processes representing returns, we defined a family of chains indexed by the threshold a. In this section, we study the dynamics of the chain $(Z_t(a) = 1_{y_t > a})$ and its dependence on the selected a. In particular, we investigate to what extent serial correlation of the underlying series is altered by imposing a qualitative representation. There exists empirical evidence suggesting that, even if returns exhibit some temporal dependence when analyzed as a quantitative process, serial correlation may disappear in a qualitative series of sign change indicators.

In the first step, we specify the relationship between the threshold a and the first order autocorrelation as a function of a. For applied work, we introduce the empirical first order autocorrelation $\hat{\rho}_T(a)$ and define the threshold estimator:

$$\hat{a}_T = \min_a \hat{\rho}_T(a)^2,$$

which approximates the value a providing the lowest absolute value of the first order autocorrelation. Since the estimated first order autocorrelation is a stepwise function of a, the solution $\hat{a}_T$ may not be unique or appear as a solution of the first order conditions. Hence, it may be useful to introduce a smoothed first order autocorrelogram involving the empirical autocorrelations $\hat{\rho}_{T,k}(a)$ of $Z_t(k;\ a) = F\left(\frac{Y_t - a}{k}\right)$, where F is a given c.d.f., for example a standard normal, and k is a bandwidth. The corresponding estimator of minimal correlation threshold is:

$$\hat{a}_{T,k} = \min_a \hat{\rho}^2_{T,k}(a).$$

The following proposition is easily derived under standard regularity conditions.

PROPOSITION 1: Let (Y_t) be a strongly stationary process with a continuous density function and consider the qualitative process defined by $Z_t(a) = 1_{Y_t \geqslant a}$. If the theoretical first order autocorrelation

$$a \to \rho(a) = Corr[Z_t(a), Z_{t-1}(a)],$$

admits a unique zero first order autocorrelation threshold a_1, then for $T \to +\infty$ and $k \to 0$ at an appropriate rate:

i) $\hat{a}_{T,k}$ is a consistent estimator of a_1;

ii) For T sufficiently large, $\hat{\rho}_{T,k}[\hat{a}_{T,k}] = 0$;

iii) $\hat{a}_{T,k}$ is asymptotically normal, with

$$\mathrm{Var}_{asy}[\sqrt{T}(\hat{a}_{T,k} - a_1)] =$$

$$\left\{ f(a_1) \frac{P(Y_{t-1} > a_1 | Y_t = a_1) + P(Y_t > a_1 | Y_{t-1} = a_1) - 2P(Y_t > a_1)}{P(Y_t > a_1)(1 - P(Y_t > a_1))} \right\}^{-2} \eta^2,$$

where $f(.)$ is the marginal density of Y_t and the expression $\eta^2 = V_{as}[\sqrt{T}\hat{\rho}_T(a_1)]$ is given in the appendix.

Proof: The derivation of the asymptotic variance is given in the Appendix.

The second condition can be used to detect the existence of a zero first order autocorrelation threshold a_1. Indeed if the zero autocorrelation threshold does not exist the minimum of ρ^2 is strictly positive and the same holds asymptotically for $\hat{\rho}^2_{T,k}[\hat{a}_{T,k}]$.

This approach may be directly extended to accommodate autocorrelations of higher orders by estimating the thresholds:

$$\hat{a}_{T,k}(h) = \min_a \hat{\rho}_{T,k}(h;\ a)^2,$$

where $\hat{\rho}_{T,k}(h;\ a)$ is the smoothed autocorrelation of order h of $(Z_t(a))$. In the next step, the estimated thresholds $\hat{a}_{T,k}(h)$ may be compared. For example, the existence of a threshold a^* such that $\hat{a}_{T,k}(h) \approx a^*$, $\forall h \geqslant 1$ and $\hat{\rho}_{T,k}(a^*)$ are close to zero for $h \geqslant 1$ is a strong evidence in favor of $(Z_t(a))$ being a white noise.

3 Causality Tests

The chain approach can be extended to a bivariate setup for a joint qualitative analysis of returns and volumes. In this section, we study causal relations between the series and develop various inference methods. We introduce thresholds distinguishing significant and non significant positive price changes, and large and small volumes:

$$Z_t(a) = \begin{cases} 1, & \text{if} \quad \log(p_t) - \log(p_{t-1}) > a, \\ 0, & \text{otherwise,} \end{cases}$$

$$Y_t(c) = \begin{cases} 1, & \text{if} \quad v_t > c, \\ 0, & \text{otherwise,} \end{cases}$$

and consider the qualitative bivariate process $[Z_t(a), Y_t(c)]'$. We restrict our attention to the first order transition probabilities defined by:

$$P(Z_t = i, Y_t = j \,|\, Z_{t-1} = k, Y_{t-1} = l) \; = \; \Pi_{ij|kl}, \quad i,j,k,l = 0,1.$$

3.1 A Lag One SUR Representation of the Chain

The elements of the transition matrix are directly related to the coefficients in the following Seemingly Unrelated Regression (SUR) model (Zellner [1962]):

$$Z_t Y_t = \beta_{11} + \alpha_{11|11} Z_{t-1} Y_{t-1} + \alpha_{11|10} Z_{t-1}(1 - Y_{t-1}) + \alpha_{11|01}(1 - Z_{t-1})Y_{t-1} + u_{11t},$$

$$Z_t(1 - Y_t) = \beta_{10} + \alpha_{10|11} Z_{t-1} Y_{t-1} + \alpha_{10|10} Z_{t-1}(1 - Y_{t-1}) + \alpha_{10|01}(1 - Z_{t-1})Y_{t-1} + u_{10t},$$

$$(1 - Z_t)Y_t = \beta_{01} + \alpha_{01|11} Z_{t-1} Y_{t-1} + \alpha_{01|10} Z_{t-1}(1 - Y_{t-1}) + \alpha_{01|01}(1 - Z_{t-1})Y_{t-1} + u_{01t}.$$

The matrix representation of the system of equations is:

$$\begin{pmatrix} Z_t Y_t - m_{11} \\ Z_t(1 - Y_t) - m_{10} \\ (1 - Z_t)Y_t - m_{01} \end{pmatrix} = A \begin{pmatrix} Z_{t-1} Y_{t-1} - m_{11} \\ Z_{t-1}(1 - Y_{t-1}) - m_{10} \\ (1 - Z_{t-1})Y_{t-1} - m_{01} \end{pmatrix} + u_t, \tag{3.1}$$

where $E(u_t | Z_{t-1}, Y_{t-1}) = 0$. We obtain a new parametrization of the transition matrix in terms of limiting probabilities (3 parameters) and the matrix of speed adjustment coefficients A (9 parameters).

3.2 The Noncausality Hypotheses

Let us now consider the non causality hypotheses at lag one. By definition (GEWEKE [1982], GRANGER [1969]) Y does not cause Z if the conditional density of Z_t given Y_{t-1}, Z_{t-1} is equal to the density of Z_t given Z_{t-1} only. This condition can be transformed into a set of linear constraints involving the parameters of the SUR model. From this specification we deduce:

$$Z_t = \beta_{11} + \beta_{10} + (\alpha_{11|11} + \alpha_{10|11}) Z_{t-1} Y_{t-1} + (\alpha_{11|10} + \alpha_{10|10}) \\ Z_{t-1}(1 - Y_{t-1}) + (\alpha_{11|01} + \alpha_{10|01})(1 - Z_{t-1}) Y_{t-1} + u_{11t} + u_{10t}.$$

When $Y_{t-1} = 1$ the deterministic part of the model is:

$$\beta_{11} + \beta_{10} + (\alpha_{11|01} + \alpha_{10|01}) + (\alpha_{11|11} + \alpha_{10|11} - \alpha_{11|01} - \alpha_{10|01}) Z_{t-1},$$

while for $Y_{t-1} = 0$ the expression simplifies to:

$$\beta_{11} + \beta_{10} + (\alpha_{11|10} + \alpha_{10|10}) Z_{t-1}.$$

The null hypothesis of unidirectional noncausality can be written:

$$H^0_{Y \to Z} : \{\alpha_{11|01} + \alpha_{10|01} = 0,\ \alpha_{11|11} + \alpha_{10|11} - \alpha_{11|10} - \alpha_{10|10} = 0\}.$$

Similarly, other noncausality hypotheses may also be written in terms of regression parameters. Thus, the null hypothesis of noncausality from Z to Y is:

$$H^0_{Z \to Y} : \{\alpha_{11|10} + \alpha_{01|10} = 0,\ \alpha_{11|11} + \alpha_{01|11} - \alpha_{11|01} - \alpha_{01|01} = 0\},$$

while the null hypothesis of instantaneous noncausality between Z and Y is satisfied, when the deterministic part of $Z_t Y_t$ is the product of the deterministic parts of Z_t and Y_t. We deduce four constraints which define this null hypothesis $H^0_{Z \leftrightarrow Y}$:

$$\beta_{11} - \alpha_{11|11} - \alpha_{11|10} - \alpha_{11|01} = \\ \{\beta_{11} + \beta_{01} - (\alpha_{11|11} + \alpha_{01|11}) - (\alpha_{11|10} + \alpha_{01|10}) - (\alpha_{11|01} + \alpha_{01|10})\} \\ \{\beta_{11} + \beta_{10} - (\alpha_{11|11} + \alpha_{10|11}) - (\alpha_{11|10} + \alpha_{10|10}) - (\alpha_{11|01} + \alpha_{10|01})\},$$

$$\alpha_{11|11} = (\alpha_{11|11} + \alpha_{01|11})(\alpha_{11|11} + \alpha_{10|11}),$$

$$\alpha_{11|10} = (\alpha_{11|10} + \alpha_{01|10})(\alpha_{11|10} + \alpha_{10|10}),$$

$$\alpha_{11|01} = (\alpha_{11|01} + \alpha_{01|01})(\alpha_{11|01} + \alpha_{10|01}).$$

3.3 Additional Regressions for Causality Analysis

In the previous subsection, the nonlinear causality analysis at lag one of a bivariate chain was based on regressions involving nonlinear qualitative regressors. We show below supplementary regressions corresponding to each type of causality. This approach greatly simplifies the computation of test statistics.

i) **Unidirectional Causality from *Z* to *Y*.**

Let us consider the regression:

$$Y_t = \beta_{1.} + \alpha_{1.|1.} Y_{t-1} + \alpha_{1.|.1} Z_{t-1} + \alpha_{1.|11} Y_{t-1} Z_{t-1} + u_{1.,t}.$$

The hypothesis of noncausality from Z to Y corresponds to the constraints: $H^0_{Z\to Y} = \{\alpha_{1.|.1} = \alpha_{1.|11} = 0\}$. Interestingly, if the hypothesis is rejected we can find out if this outcome is due to the presence of pure linear dependencies. For this purpose, we can test if $\alpha_{1.|11} = 0$ and $\alpha_{1.|.1} \neq 0$.

ii) **Unidirectional Causality from *Y* to *Z***

The regression to consider is symmetric to the previous one:

$$Z_t = \beta_{.1} + \alpha_{.1|1.} Y_{t-1} + \alpha_{.1|.1} Z_{t-1} + \alpha_{.1|11} Y_{t-1} Z_{t-1} + u_{.1,t},$$

and the null hypothesis of noncausality is:

$$H^0_{Y\to Z} = \{\alpha_{.1|1.} = \alpha_{.1|11} = 0\}.$$

iii) **Instantaneous Causality between *Y* and *Z***

The instantaneous noncausality concerns the absence of influence of the current value of Z_t in the conditional distribution of Y_t given Z_t, Y_{t-1}, Z_{t-1}. The regression corresponding to the conditional distribution contains eight regressors:

$$Y_t = \gamma_1 + \delta_{1..} Z_t + \delta_{.1.} Y_{t-1} + \delta_{..1} Z_{t-1} + \delta_{11.} Z_t Y_{t-1} + \\ + \delta_{1.1} Z_t Z_{t-1} + \delta_{.11} Y_{t-1} Z_{t-1} + \delta_{111} Z_t Y_{t-1} Z_{t-1} + v_{1,t}.$$

The null hypothesis is:

$$H^0_{Z\leftrightarrow Y} = \{\delta_{1..} = \delta_{11.} = \delta_{1.1} = \delta_{111} = 0\}.$$

A rejection may be due either to the lack of a linear relation of Y_t with Z_t, or to interactions of order two, or of order three between the variables.

3.4 Volume State Specification

In the causality analysis presented above, we assumed that the threshold defining the states of the qualitative volume process is fixed and set, for example at the average volume per trade. Intuitively, we might expect that the state specification matters in inference on causality. Then, we can look for the threshold inducing the strongest causality from volume to return at lag 1. More precisely, for each level of c, we compute the chi-square test statistics $\xi(c)$ for testing the hypothesis of noncausality from volume to returns, based on either the SUR model 3.4, or the additional regressions of subsection 3.2. We know that $\xi(c)$ is a measure of the strength of the causal relation, at lag 1 (Gouriéroux, Monfort, Renault [1987]). Then the threshold estimate is defined by:

$$\hat{c}_T = \underset{c}{\text{Argmax}}\, \xi(c).$$

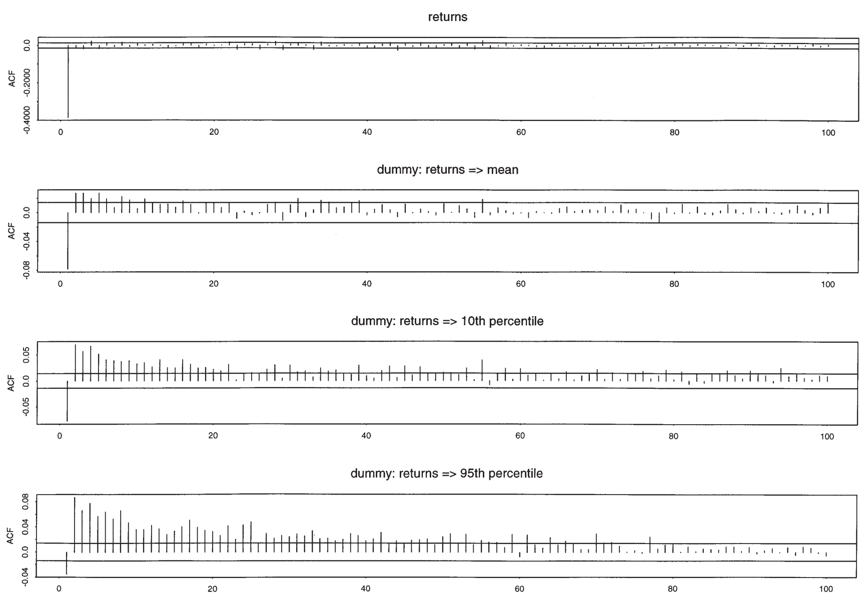
We have now to discuss the choice of the price and volume thresholds with respect to the heterogeneity of traders. Let us assume two groups of investors trading large and small volumes respectively. The microstructure theory suggests some endogenous subperiods during which the small volume traders (resp. large volume traders) trade within their groups. During these periods of homogenous trades, we expect the markets to be efficient and to find a strong relationship between prices and volumes.

The idea of this paper is to characterize the endogenous trading dates of the second group of investors (large volume traders) by a condition $v_t > c$, where c has to be determined. If the price movements are summarized by a qualitative variable $Z_t(a)$ such that $Z_t(a)$ is a weak white noise (which is a type of market efficiency condition), we select the limiting threshold c to maximize the causality from qualitative volume size $Y_t(c)$ to the sign of price changes $Z_t(a)$.

4 Empirical Results

We examine returns and volumes of trades of the Alcatel stock, recorded on the Paris Stock Exchange (Paris Bourse) in July and August 1996. Prior to estimation, the opening trades (the simultaneous trades concealing split orders resp.) were deleted (aggregated resp.). The data set consists of 20405 observations on returns $(\Delta\log(p_t))$ and volume by trade $(v_t = \log(V_t))$. We proceed

FIGURE 1
Real Time: Threshold Effect in Return Persistence



in two steps by investigating separately the data in transaction and calendar times. A unitary increment in transaction time is set by a trade arrival, disregarding the length of the waiting time between transactions. Conventionally, a unitary increment in calendar time corresponds to an integer multiple of one minute and may eventually be arbitrarily selected, depending on the sampling frequency of the data. In our analysis, we use a 1 minute grid which roughly corresponds to an average duration between arrivals of Alcatel trades (52.56 sec). The comparison of results obtained from the calendar time and real time data is expected to reveal the effect of the frequency of trades, which is a liquidity determinant.

4.1 Analysis in Trading Time

(i) **Correlation Analysis and State Selection for Return Series**

It is known that autocorrelograms are very sensitive to the preliminary transformation applied to the series of interest. The panels 1:4 in Figure 1, beginning with the top one, display the autocorrelograms of the quantitative series of returns and three binary series $Z_t(a)$ corresponding to various thresholds equal to the mean, which is close to zero, the 10th and 95th percentiles, respectively. As expected the first order autocorrelation is significantly negative due to the bid-ask bounce. Beyond the bid-ask effect, the higher order autocorrelations in the quantitative series are not significant. This explains the random walk behavior observed in data sampled at lower, constant frequencies. However, we find that the autocorrelation patterns of qualitative and quantitative return processes differ substantially for thresholds different from 0, especially a longer range of persistence is observed when we focus on large returns. The autocorrelograms of the quantitative process and the qualitative process with zero threshold are quite similar.

The autocorrelation size as well as the range of persistence vary when thresholds are modified. Figure 2a displays the autocorrelation of order one in the return series as a function of the threshold varying between the sample minimum and maximum. The behavior of higher order correlations is illustrated in Figure 2b by the statistic $Q^* = \sum_{h=2}^{10} \hat{\rho}_h^2$, estimated again for thresholds varying between the sample minimum and maximum.

We note that zero is a particular threshold for the return series, since the statistics $\hat{\rho}_1^2$ and Q^* are approximately even functions of c. Moreover, it is seen that this threshold provides approximately the minimal value of $\hat{\rho}_1^2 + Q^*$, and that this minimal value is close to zero.

Therefore the series $[Z_t(0)]$ is almost a weak white noise. We select this return threshold for the joint causality analysis below.

(ii) **Joint Causality Analysis and State Selection for Volume Series**

There is a strong evidence of linear causality in the quantitative representations of return and volume processes already mentioned in the empirical literature. The null hypothesis of absence of unidirectional causality is

FIGURE 2A
Correlation at Lag 1 for Varying Thresholds

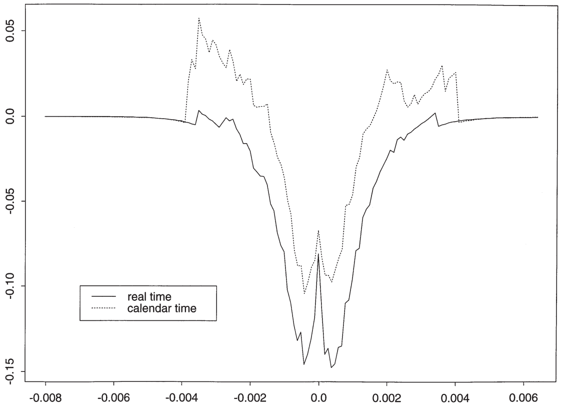


FIGURE 2B
Q*-Statistic for Varying Thresholds Lags 2 to 10

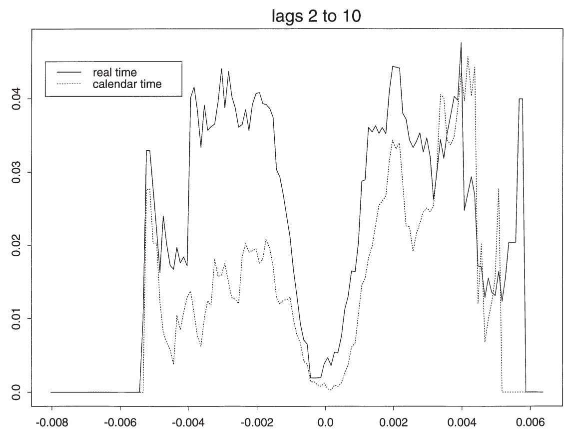
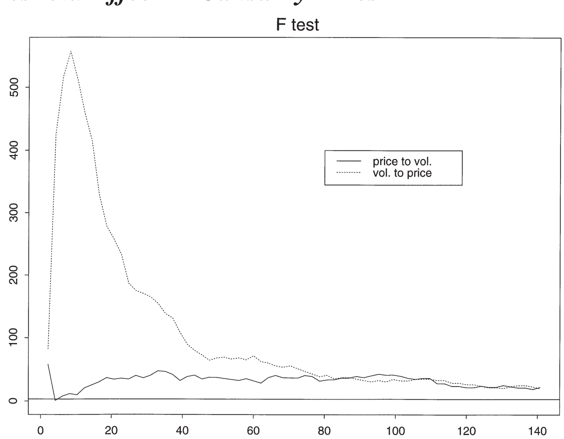


FIGURE 3
Real Time: Threshold Effect in Causality F Test



rejected by standard t-tests of coefficients in linear regressions of returns (volume) on lagged returns and volume. The estimated regressions are:

$$\begin{aligned} r_t &= 0.000013 - \underline{0.3862} r_{t-1} - \underline{0.000004} v_{t-1}, \\ v_t &= \underline{5.8684} - \underline{160.3382} r_{t-1} + \underline{0.1175} v_{t-1}, \\ v_t &= \underline{5.8775} - \underline{672.1791} r_t - \underline{419.9977} r_{t-1} + \underline{0.1148} v_{t-1}, \end{aligned}$$

where all coefficients associated with the unidirectional causality hypotheses are significant (underlined). As well, we reject the null hypothesis of absence of linear instantaneous causality in a regression of volume on returns, past volume and past returns.

However, linear causality analysis depends on the preliminary transformations, which are applied to the series of interest. We illustrate this effect by considering the binary series $Z_t(0)$ and $Y_t(c)$, for varying volume threshold c. We consider the F-statistics for testing the unidirectional causality hypothesis between $Z_t(0)$ and $Y_t(c)$, for volume threshold levels varying between the sample minimum and the 99.94th percentile (see Figure 3). The horizontal line at 2.99 corresponds to the critical value of the F-test for the noncausality hypothesis. We find the maximal causality from volume to returns for a discriminating level of approximately 8.28, which exceeds the average volume. We also note that for this value (close to the median of volume but less than the mean) causality from returns to volume is almost eliminated. Therefore this threshold is a good candidate for discriminating among two categories of traders (see, section 3.4).

4.2 Analysis in Calendar Time

The literature suggests that the dynamic properties in trading and calendar-times may differ due to time deformation (see *e.g.* Ghysels, Gouriéroux, Jasiak [1998]). It has also been observed that correlations in returns are, in general, weaker in trading time than in calendar time. We will see that this decrease in correlations is no longer observed when volumes are also taken into account.

The calendar time sample consists of 9,813 observations. There are 6,709 observations on returns in state 1 and 3,103 on returns in state 0.

(i) **Correlation Analysis and State Selection for Return Series**

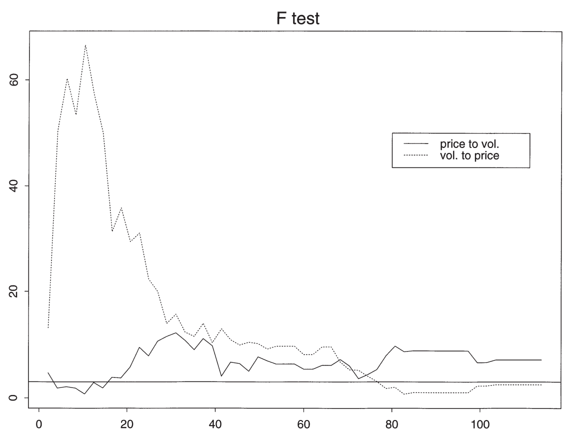
The resampling at equal intervals modifies significantly the serial correlation in the data. There remains a significant negative first order autocorrelation (-0.23) in the quantitative return process, while the persistence range of volume is reduced to two lags. However, the first order autocorrelation function of binary returns computed for varying thresholds can exhibit positive values (see Figure 2a). We still find an even function, with an optimum around the zero threshold. For this threshold the absolute value of the first order autocorrelation is lower in calendar time than in real time.

The Q^* statistic computed for varying thresholds in Figure 2b still displays a minimum at 0.

TABLE 1
Calendar Time: Causality

UNIDIRECTIONAL					
PARAMETER	EST	S.E.	PARAMETER	EST	S.E.
$\beta_{1.}$	0.2733	0.0099	$\beta_{.1}$	0.7755	0.0101
$\alpha_{1.\|1.}$	0.0573	0.0171	$\alpha_{.1\|1.}$	– 0.1371	0.0175
$\alpha_{1.\|.1}$	– 0.0056	0.0117	$\alpha_{.1\|.1}$	– 0.1066	0.0121
$\alpha_{1.\|11}$	– 0.0150	0.0212	$\alpha_{.1\|11}$	0.1126	0.0218
F-test: 1.59, p-value = 0.2024			F-test: 64.35, p-value = 0.000		
INSTANTANEOUS					
PARAMETER	EST	S.E.	PARAMETER	EST	S.E.
γ_1	0.3995	0.0208	$\delta_{11.}$	0.0350	0.0373
$\delta_{1..}$	– 0.1626	0.0236	$\delta_{1.1}$	0.0834	0.0272
$\delta_{.1.}$	0.0126	0.0310	$\delta_{.11}$	– 0.0551	0.0375
$\delta_{..1}$	– 0.0787	0.0235	δ_{111}	0.0935	0.0455
F-test: 106.27, p-value = 0.0000					

FIGURE 4
Calendar Time: Threshold Effect in Causality F Test



(ii) **Causality Analysis and State Selection for Volume Series**

Compared to real time data, the quantitative processes of volume and returns are less correlated. There are significant cross-correlations between lagged volume and returns at lags 0, 1 and 8, while only at lag 0 lagged returns are correlated with volume. We repeat the unidirectional one step linear causality test based on regressions of volume (returns) on past volume and returns. Since the relevant coefficients are not significant, the null hypothesis of unidirectional noncausality cannot be rejected. There is however strong evidence in favor of instantaneous causality. The regressions for linear causality are:[2]

$$
\begin{aligned}
r_t &= 0.000003 - \underline{0.2372} r_{t-1} - 0.000001 v_{t-1} \\
v_t &= \underline{5.9789} - 4.2148 r_{t-1} + \underline{0.0720} v_{t-1} \\
v_t &= \underline{5.9805} - \underline{476.6650} r_t - 117.2836 r_{t-1} + \underline{0.0716} v_{t-1}
\end{aligned}
$$

Inference on qualitative data with thresholds zero and the sample average for returns and volumes, respectively, is still based on the regression method for unidirectional relationships. We do not reject the null hypothesis of noncausality from returns to volume, but we strongly reject the absence of causality from volume to returns as well as the lack of instantaneous causality. Therefore there exists a nonlinear causality from volume to returns (see Table 1).

Figure 4 displays the relationship between the causality test statistics and the selected volume thresholds. Amazingly, the F test statistics admit lower values than in real time. Similarly to the real time results, there exists a threshold maximizing the unidirectional causality from volume to returns, and for which the symmetric unidirectional causality is negligible. Moreover, this particular threshold is approximately the same as in real time. We find a substantially larger range of values eliminating causality from returns to volume, including the volume average.

5 Conclusions

This paper examined the dynamics of two-state representations of stock returns and volumes by qualitative processes featuring some interesting properties. We showed that the range of temporal dependence differs in the quantitative and qualitative data and illustrated the autocorrelation behavior with respect to the state specification. In particular, we found that serially correlated return processes can be transformed into qualitative white noises by selecting an appropriate threshold separating the two states.

The qualitative processes of returns and volumes also display different interactions compared to the quantitative data. In the bivariate setup, we investigated the volume-return relationship and proposed various tests of noncausality hypotheses. Our empirical results indicate that causality directions vary in time and depend on the sampling scheme, such as the real or calendar time scales. We also illustrated the variation of the noncausality test statistic due to adjustments of the state separating threshold. Using the value of test statistic as a measure of strength for a causal relation, we proposed a volume classification based on thresholds maximizing the volume-return causality. ■

2. Statistically significant coefficients are underlined.

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APPENDIX

Asymptotic Distribution of $\hat{\rho}_{T,k}$

For T sufficiently large, the estimator satisfies the first order condition:

$$\hat{\rho}_{T,k}(\hat{a}_{T,k}) = 0.$$

By considering a first order expansion in a neighborhood of the limiting value a_1, we have:

$$\begin{aligned}
\sqrt{T}\hat{\rho}_{T,k}(a_1) &= -\frac{d\hat{\rho}_{T,k}(a_1)}{da}\sqrt{T}(\hat{a}_{T,k} - a_1) + o_p(1) \\
&= -\frac{d\rho(a_1)}{da}\sqrt{T}(\hat{a}_{T,k} - a_1) + o_p(1),
\end{aligned}$$

or equivalently:

$$\sqrt{T}(\hat{a}_{T,k} - a_1) \approx \left[\frac{d\rho(a_1)}{da}\right]^{-1} \sqrt{T}\hat{\rho}_{T,k}(a_1) + o_p(1).$$

If the bandwidth k_T tends to zero sufficiently fast, we get:

$$\sqrt{T}(\hat{a}_{T,k} - a_1) = \left[\frac{d\rho(a_1)}{da}\right]^{-1} \sqrt{T}\hat{\rho}_T(a_1) + o_p(1).$$

For the value a_1, we have $\rho(a_1) = 0$ and it is known that:

$$\sqrt{T}\hat{\rho}_T(a_1) \xrightarrow{d} N(0,\eta^2).$$

where: $\eta^2 = [P(Y_t > a_1)(1 - P[Y_t > a_1])]^{-2}$

$$\sum_{h=-\infty}^{+\infty} E\{(\mathbf{1}_{y_t > a_1} - P[Y_t > a_1])(\mathbf{1}_{y_{t-1} > a_1} - P[Y_t > a_1])$$

$$(\mathbf{1}_{y_{t-h} > a_1} - P[y_1 > a_1])(\mathbf{1}_{y_{t-h-1} > a_1} - P[Y_t > a_1])\}.$$

Moreover, we have:

$$\begin{aligned}\frac{d}{da}[\rho(a_1)] &= \frac{1}{\gamma_0(a_1)}\frac{d\gamma_1(a_1)}{da} - \rho^2(a_1)\frac{d\gamma_0(a_1)}{da}\\ &= \frac{1}{\gamma_0(a_1)}\frac{d\gamma_1(a_1)}{da},\end{aligned}$$

$$\begin{aligned}\left[\frac{d\gamma_1(a)}{da}\right]_{a=a_1} &= \left[\frac{d}{da}E(1_{Y_t>a}1_{Y_{t-1}>a})\right]_{a=a_1} - \left(\frac{d}{da}\left[E(1_{Y_t>a})\right]^2\right)_{a=a_1}\\ &= \left[\frac{d}{da}\int_a^\infty\int_a^\infty f(y_t,y_{t-1})dy_t dy_{t-1}\right]_{a=a_1}\\ &\qquad +2P[Y_t>a_1]f(a_1)\\ &= -\int_{a_1}^\infty f(a_1,y_{t-1})dy_{t-1} - \int_{a_1}^\infty f(y_t,a_1)dy_t\\ &\qquad +2f(a_1)P[Y_t>a_1]\\ &= -f(a_1)[P[Y_{t-1}>a_1|Y_t=a_1]\\ &\qquad +P[Y_t>a_1|Y_{t-1}=a_1]-2P[Y_t>a_1]].\end{aligned}$$