Timing of Orders, Orders Aggressiveness and the Order Book at the Paris Bourse

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ABSTRACT. – We offer a statistical model of the order flow and estimate it using high frequency data from the Paris Bourse. Our model jointly explains the duration between two consecutive orders and the relative aggressiveness of the orders, depending upon the past orders and the state of the book. Our results offer evidence of information and liquidity effects, as put forward by market microstructure theories.

Temps d'arrivée, agressivité des ordres et état du carnet d'ordre à la Bourse de Paris

RÉSUMÉ. – Dans cet article, nous construisons et estimons un modèle du flux des ordres boursiers. Pour ce faire, nous utilisons des données à haute fréquence décrivant les ordres émis par les participants du marché automatisé de la Bourse de Paris. Notre modèle explique conjointement la durée séparant deux ordres et l'agressivité des ordres émis, en fonction des ordres passés et de l'état du carnet d'ordre. Les résultats confortent un certain nombre d'implications empiriques de la théorie de la microstructure des marchés financiers.

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1 Introduction

In this paper, we use high frequency data from the Paris Bourse to study the dynamics of the order flow, and assess the empirical validity of some implications of market microstructure theories related to order placement strategies.

We study the flow of orders to buy and to sell Alcatel shares at the Paris Bourse. We are interested in two aspects of this flow. The first one is its dynamics behavior. In this respect, we aim at explaining the duration between successive orders during a trading day. The second one is related to the sequences of orders. In this respect, we try to understand the chaining of buy and sell orders when they are differentiated according to their aggressiveness.

With this aim in view, we build a multi-spell and multi-state transition model (see, for instance, LANCASTER [1990], and KAMIONKA [1992]). Within our model, a spell is the period of time between two successive orders. The state of an order is its type, defined as its direction (buy or sell) and its degree of aggressiveness (ranked from 1 to 6). Such a model jointly explains the time of arrival of the orders and the chaining process of the different types of orders, depending upon a set of predetermined variables.

Considering a particular order and its time of arrival, our model gives the distribution of the waiting time for the next order, and the distribution of the type of this next order, depending upon the state of the market when the former order arrived. This state is described by three groups of variables. First, the timestamp of the order, which enables us to capture the intraday seasonality of the times of arrival. Second, the type of this order, which is used to understand the chaining of the order types. Third, a set of variables related to the state of the order book (the spread and the quantities available at the best bid and ask prices), which enables us to put forward how this state influences the order flow.

Our results are consistent with market microstructure theories of order placement strategies. Consistent with information effects we find imitation in order submissions, a relation between the aggressiveness of an order and the average waiting time for the next order, and a specific pattern of consecutive orders leading to a shift in the bid-ask spread.

We also observe patterns of orders reflecting competition for the supply of liquidity and consumption of liquidity when it is offered at a low price.

Our work is closely related to a study proposed by BIAIS, HILLION and SPATT [1995] – hereafter BHS [1995]. One of their goals was to study the interaction between the order book and the order flow, in order to shed light on various market microstructure theories. Moreover, we use the same classification for the orders.

The main difference between our approach and the one developed in BHS [1995] is that we propose a single and fully coherent statistical model of the order flow, rather than studying separately its various aspects. By modeling both the time interval between two consecutive orders and the type of the orders, we provide a new statistical approach of the order flow.

This approach proves useful. First, using our estimation of the transition model, we are able to compute the risk functions for the transitions between two successive orders. Then, the shape of these functions can be related to the way investors react to new information. Second, while BHS [1995] found evidence of undercutting behavior of investors supported by a strong diagonal effect between orders within the best quotes, we do not find similar evidence. Though this difference could stem from the use of a difference sample, it most likely stems from the use of a single conditional model that allows one to take into account the state of the book. The model shows that the succession of buy or sell orders within the best quotes is rather an indirect effect of the bid-ask spread.

Though more general, our statistical model can be related to the ACD (*Autoregressive Conditional Duration*) models. ENGLE and RUSSEL [1998] introduced this class of models specifically designed for irregularly spaced data. In a model of this class, only the conditional expectation of duration between two consecutive events depends upon the predetermined variables, and in particular upon the past realizations of this duration. Using an ACD model, ENGLE and RUSSEL [1998] study the transaction flow for IBM on the New York Stock Exchange. ENGLE and RUSSEL [1997] study Dollar/Deutschmark quotes on the foreign exchange market. Among others, GHYSELS and JASIAK [1996] and ENGLE (2000) extended ACD models in order to jointly explain time intervals and returns volatility. Usually, the dynamics of the volatility is modeled by an ARCH model in discrete time. This kind of model assumes regularly spaced data, which is not the case with high frequency data. As ENGLE and RUSSEL [1998], they apply their model to IBM transactions. GOURIÉROUX, JASIAK and LE FOL [1996] study the time interval between two consecutive transactions on Alcatel at the Paris Bourse. They analyze the way this distribution evolves during the day, and also consider the traded volumes.

Our approach is different from the one developed in ENGEL [2000]. The central feature of an ACD model is that an exogenous variable can influence the duration distribution only through the expectation of this duration. In contrast, in our approach, an exogenous variable can have a different impact on the mean and on the variance of the distribution.

The paper is organized as follows. In the second section, we briefly present the Paris Bourse, our dataset and how orders are differentiated according to their relative aggressiveness. In the third section, we rely on market microstructure theories to formulate a set of empirical implications. The statistical model of the order flow and the corresponding estimation procedure are exposed in the fourth section. Results are presented in the fifth section and the last section concludes.

2 The Order Flow

The orders we consider are posted by investors on the French stock market. The Paris Bourse is a computerized limit order market. Orders to buy and to sell a particular stock directly compete among themselves in an electronic order book. Matching and corresponding trades are computed according to price and time priority. In the continuous-time market considered in this paper,

orders are posted from 10 a.m. to 5 p.m. and executed as soon as possible. Therefore, the order book can only contain limit orders, such as the best available purchase price is lower than the best sale price. When a market order (*i.e.* an order to buy or to sell at the best currently available price in the book) occurs, it is matched with the best orders on the opposite side of the book and one or more trades occur. The order book is updated in real time. In addition to limit and market orders, investors can also place "*at best orders*". Such orders have no price limit, and will match best prices until executed in full.
Information on the state of the book (in practice the five best prices on the buy side and the five best prices on the sell side, with the corresponding cumulated volumes) is made available to investors in real time through computer screens.

In this paper, we use the dataset known as Historical Market Data, edited by the Paris Bourse. This dataset contains the history of the order flow and order book for each stock traded in Paris, since 1995. For each order, the dataset reports the code of the stock, the date and time of arrival of the order, its direction (buy or sell), the quantity demanded or offered, the type of the order (*i.e.* limit, market, or at best order), and, in the case of a limit order, the limit price. The state of the book is described by the best bid price and the best ask price (the bid-ask quote), and by the quantities available at these prices. This information is recorded and time-stamped each time it changes.

Like Gouriéroux, Jasiak and Le Fol [1996], we study Alcatel, one of the most heavily traded shares at the Paris Bourse.[1] Alcatel is traded on the *"Règlement Mensuel"* market, and is included in the CAC40 index. The period we consider extends from October 25 to December 23, 1996, which covers 40 trading days.

Following BHS [1995], we classified the buy and sell orders according to their aggressiveness. Large buy orders (type 1) and large sell orders (type 7) are the most aggressive orders. They specify a greater quantity than that available at the best price on the opposite side of the book, and are executed in full at different prices. Market buy orders (type 2) and market sell orders (type 8) are less aggressive because they can only hit the current best price. The remaining quantity is converted into a limit order at this price. Small buy orders (type 3) and small sell orders (type 9) are executed in full, but at a single price, because they specify a lower quantity than that available at the best price. The other order types do not immediately trigger a transaction. Buy orders within the best quotes (type 4) and sell orders within the best quotes (type 10) are limit orders with a limit price above the best bid price and below the best ask price. Such orders do not demand immediate execution but obtain the price priority. Buy orders at the best quote (type 5) and sell orders at the best quote (type 11) are limit orders with a limit price equal to the best currently available price. They also get price priority, but, because of time priority, will be executed after those posted earlier at the same price. Finally, the less aggressive types are buy orders below the best quote (type 6) and sell orders above the best quote (type 12), which have neither price nor time priority.

1. Over the year 1996, we computed a daily average of 975 transactions and more than 345 000 shares exchanged. Some other statistics are given in Appendix A.

The numbering of these twelve types and the relative frequency of each type in our dataset are reported in Table 1.[2] Since in our model we must clearly identify the timing of orders, groups of orders with the same time stamp cannot be handled. We decided to select only one order in each group, by randomly picking it according to the empirical probabilities of transition between the twelve different types, computed over our dataset.[3] This operation reduces our population from (roughly) 64 000 orders to 61 000.

Our data is different from the one used in BHS [1995] in three aspects. First, we do not consider cancellations. We made this choice because our dataset only reports cancellations of orders at the best quotes. Second, in order to keep a reasonable size for our statistical model, we also do not consider *"applications"*. Applications are prearranged trades that can be executed between or at the best quotes. Since they are negotiated outside the system, we postulate that they are less closely embedded into the order flow than the other types. Moreover, since they are prearranged, it is very difficult to rank them according to their relative aggressiveness. Finally, they are rather rare. Third, we consider all the orders to buy below the quotes or to sell above the quotes, whereas BHS [1995] can only take into account orders of these types occurring within the five best bid and ask prices.

TABLE 1
The Twelve Types of Order

n.	*Type*	*freq. in %*
Buy side:		41.51
1	large buy orders	2.79
2	market buy orders	3.14
3	small buy orders	10.21
4	buy orders within the best quotes	6.49
5	buy orders at the best quote	4.80
6	buy orders below the best quote	14.08
Sell side:		58.49
7	large sell orders	3.09
8	market sell orders	3.68
9	small sell orders	23.92
10	sell orders within the best quotes	6.28
11	sell orders at the best quote	4.12
12	sell orders above the best quote	17.41

2. One can see that sell orders are more frequent than buy orders. This is consistent with the fact that the average size of a sell order (694 in our dataset) is lower than the average size of a buy order (1050).

3. These probabilities are reported in Appendix B.

3 Empirical Implications

In this section, we set-up some empirical implications of microstructure theories for the arrival of orders and its relation with the state of the market. The theory usually puts forward two motives for order placement: liquidity and information.

3.1 Liquidity Effect

Here, in the spirit of Demsetz [1968], we consider a market for an asset such as no one has any private information about its value. Investors buy and sell for uninformational reasons. In this context, an investor posting a limit order provides liquidity to the market, whereas an investor placing a market order consumes liquidity. Because he makes an immediate execution of the order possible, the former is rewarded by the latter to an amount related to the size of the spread.

The basic feature in this kind of model, as in Biais [1993], Foucault [1995], Ho and Stoll [1983] or Kyle [1985], is a price competition for the supply of liquidity.

In a simple model of Bertrand competition, the investors placing limit orders are competing among themselves for price priority. Posting a better limit price means winning the current price priority but also lowering the reward for supplying liquidity to the market. In this context, the arrival of a limit order within the best quotes means that the competition is running. Therefore, it should favor the arrival of an order of the same type. Also, since an order within the best quotes is the most significant type among the limit orders from the point of view of price competition, the reaction of the market after this type of limit order should be the shortest one.

We can summarize this as follows:

Empirical implications 1 (Competition for price priority): (a) An order to buy (resp. to sell) within the best quotes (types 4, resp. 10) tends to be followed by an order of the same type. (b) The market reacts more quickly after a buy (resp. sell) order within the best quotes than after a limit order at or outside the best quotes (types 5 and 6, resp. 11 and 12).

If the quantity at the best price is very small, it is reasonable to place a limit order at this price rather than a limit order with a better price. Of course, a limit order at the best quote will not get time priority, but it may expect to be executed at a relatively good price after a relatively short delay. In addition, an order within the quotes means, in average and *ceteris paribus*, that the length of the queue at the best price is now smaller. This will give incentives to other liquidity traders to place limit orders at the new best quote rather than inside the best quotes.

This analysis leads to the following implications:

Empirical implications 2 (Choice of queuing): (a) A large quantity demanded (resp. offered) a the best price favors the arrival of a buy (resp. a sell) order within the best quotes (type 4, resp. 10) and unfavors the arrival of a buy (resp. a sell) order at and outside the best quotes (types 5 and 6, resp.11 and 12). (b) A limit order within the best quotes favors the arrival of an order at the best quotes on the same side of the book, and conversely.

Now, let's consider the dynamics of production and consumption of liquidity. As in FOUCAULT [1993], we can say that a large bid-ask spread favors limit orders within the best quotes because a large spread means that the reward for liquidity supply is large. In the same spirit, we can expect to see some particular sequences of orders. First, if a supplier places an order within the best quotes in order to get price priority, it means that the liquidity becomes cheaper. Potential consumers are therefore induced to accept this liquidity by posting a small order with an opposite direction. Second, if someone places a market order he consumes liquidity but also offers liquidity (the unexecuted part of the order) at a very interesting price. The shift of the bid-ask spread may induce an investor to place a market order with an opposite direction: he consumes the liquidity at a good price, and restores the state of the spread.

Empirical implications 3 (Provision and consumption of liquidity): (a) A large spread favors the arrival of limit orders (types 4 to 6, resp. 10 to 12) and unfavors the arrival of an immediate execution order (types 1 to 3, resp. 7 to 9). (b) A buy (resp. a sell) order within or at the best quotes (types 4 and 5, resp. 10 and 11) favors the arrival of a small sell (resp. buy) immediate execution order (types 9, resp. 3). (c) A market buy (type 2) tends to follow a market sell (type 8) and conversely.

3.2 Information Effect

As in GLOSTEN and MILGROM [1985], we now consider the informational content of the orders in a market where some investors have a private information about the future value of the asset.

The first implication of the information effect is what BHS [1995] call a diagonal effect: orders of the same type tend to occur in sequences. Following BHS [1995], this can be justified by an imitation effect, by strategic order splitting, or by similar but successive reactions of different investors to the same event.

Empirical implications 4 (Diagonal effect): An order of a given type tends to be followed by an order of the same type.

In addition, we may say that the intensity of a signal conveyed by an order is a monotonous function of its aggressiveness: an investor who places, say, a very aggressive buy order, strongly believes that the value of the asset is higher than the current market prices. Since the market will interpret this event in this way, it will quickly react.

Empirical implications 5 (Intensity of the signal): The market reacts more quickly to aggressive orders.

Finally, we can set up a last empirical implication by analyzing how the liquidity suppliers may react to a strong signal. If a large purchase occurs, the best ask price will rise, thus widening the spread by the upper side. For liquidity suppliers on the buy side, this means that the value of the asset is farther from the current best bid price. The reward for liquidity is now higher, and this will tend to induce a new price competition.

Empirical implications 6 (Shift in the bid-ask spread): A large buy order (type 1) (resp. sell, type 7) tends to be followed by a limit buy (resp. sell) within the best quotes (type 4, resp. 10).

4 Modeling

4.1 Orders Arrival Process

We analyze the sequence of orders posted between October 25 and December 23, 1996. For each of the forty days in our sample, we consider the orders recorded from 10 a.m. to 5 p.m. Each order is characterized by its day of arrival, its time of arrival (the number of seconds elapsed since 10 a.m.) and its type (a number between 1 and 12, as defined in table 1). Over each day, orders are numbered according to their rank of arrival.

FIGURE 1
Spell n

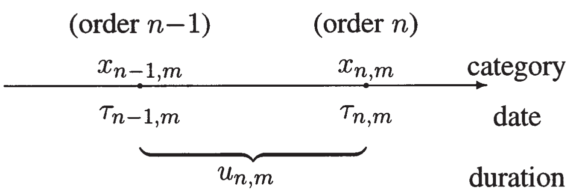
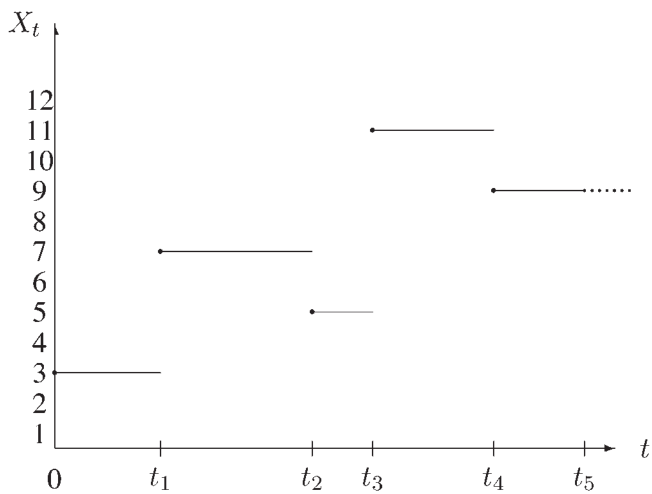


FIGURE 2
A sample path of the process X_t



Let $M = 40$ be the total number of days in our sample and N_m the total number of orders recorded over the day m ($m = 1,\ldots,M$).

Let's consider the n-th order recorded during the day m. We will use the following notations (see Figure 2):

- $\tau_{n,m}$ is its arrival time: $\tau_{n,m} \in \{0,\ldots,25200\}$;
- $x_{n,m}$ is its type: $x_{n,m} \in E$, where $E \equiv \{1,\ldots,12\}$ is the discrete state space of the twelve types;
- $u_{n,m} \equiv \tau_{n,m} - \tau_{n-1,m}$ is the number of seconds elapsed since the arrival of the previous order during the same day.

What we want to modelize is a process such as the realization over a given day, say over day m, is a sequence of types and durations, denoted by $(y_{1,m},\ldots,y_{N_m,m})$, where $y_{n,m} \equiv (x_{n,m},u_{n,m})$. The length of such a sequence may be different for each day: over day m, the pair $y_{n,m}$ is available only for $n = 1,\ldots,N_m$.

In this paper, we consider that these sequences are generated by a single continuous time process $X_t, t \in \mathrm{R}$, in the discrete state space E, where X_t is defined as the type of the last order posted on the market at time t of the trading day.

Therefore, a realization X_t is defined by: $x_t = x_{n-1,m} \forall t \in [\tau_{n-1,m},\tau_{n,m})$.

The process X_t is constant between two consecutive orders and can only change when a new order is posted. A sample path of this process, from 10:00 to 10:01 a.m., is plotted in Figure 2. The first order, posted at 10:00 a.m., is a small buy order (state 3). The process is equal to 3 at time $t = 0$ and is constant until the arrival time of the next order. The next order is a large sell order (state 7).

The forty days of our sample give us forty realizations of the process. For a particular day m, $X_{\tau_{n,m}}$ is the random variable of the type of the order posted at time $\tau_{n,m}$. Hereafter, we will use the notation $X_{n,m} \equiv X_{\tau_{n,m}}$.

We model a sequence of realizations of the process[4], one for each trading day, writing the density of a realization as a product of marginal and conditional densities. We have, then

$$(1) \qquad f(y_{2,m},\ldots,y_{N_m,m} \mid y_{1,m};\theta) = f(y_{2,m} \mid y_{1,m};\theta)\times \\ f(y_{3,m} \mid y_{1,m},y_{2,m};\theta) \times \ldots \times f(y_{N_m,m} \mid y_{1,m},\ldots,y_{N_m-1,m};\theta),$$

where θ is a vector of parameters.

For each spell, we have to write the joint distribution of the ordered pair $y_{n,m} = (x_{n,m},u_{n,m})$. We assume that, conditionally to the past of the process, the distribution of $Y_{n,m}$ depends only on the most recent realization of the process, $y_{n-1,m}$. Moreover, given $Y_{n-1,m} = y_{n-1,m}$, the distribution of $Y_{n,m}$ only depends on the type of the last order, $x_{n-1,m}$.

The duration $U_{n,m}$ ($n = 2,\ldots,N_m$) is positive and continuous. The distribution of this random variable can be characterized by its hazard function, its

4. The likelihood function is written conditionally on the first observation of the day $y_{1,m}$, for $m = 1,\ldots,M$.

density probability function, its cumulative distribution function or its survival function (see, for instance, LO *et alii* [1997]).

Let z be a vector of explanatory variables and Z the corresponding random variable. This vector includes all of the past realizations of the process and, in particular, the type of the last order.

Let $S_j(u \mid z;\theta) = \text{Prob}[u \leqslant U_{n,m} \mid X_{n-1,m}{=}j;\, Z{=}z]$ denote the survivor function. Since each duration is a continuous variable, the survival function is equal to one minus the cumulative distribution function.

The cumulative distribution of the duration is

$$F_j(u \mid z;\theta) = 1 - S_j(u \mid z;\theta). \tag{2}$$

The density probability function can be obtained using the following expressions:

$$\begin{aligned} f_j(u \mid z;\theta) &= \frac{d}{du} F_j(u \mid z;\theta) \\ &= -\frac{d}{du} S_j(u \mid z;\theta). \end{aligned} \tag{3}$$

Let $h_j(u \mid z;\theta)$ be the hazard function of the duration until the next order given that the last one is of type j ($j \in E$). By definition, we have

$$h_j(u \mid z;\theta) = \lim_{\Delta \to 0^+} \frac{\text{Prob}\left[u \leqslant U_{n,m} < u+\Delta \mid U_{n,m} \geqslant u;\, X_{n-1,m}{=}j;\, Z{=}z\right]}{\Delta},$$

so, we obtain

$$h_j(u \mid z;\theta) = \frac{f_j(u \mid z;\theta)}{S_j(u \mid z;\theta)}. \tag{4}$$

The hazard function gives the arrival rate of a new order given that no order has been recorded since u units of time. It is an indicator of market activity. For instance, the conditional mean duration between two orders and the transition probabilities can be calculated using this hazard function (see section 4.3).

Using equation (3) and the fact that $S_j(0 \mid z;\theta) = 1$, one can verify that an expression of the survival function is

$$S_j(u \mid z;\theta) = \exp\left(-\int_0^u h_j(s \mid z;\theta)ds\right). \tag{5}$$

Several specifications can be retained for the hazard functions. For instance, if the hazard functions are constant with the duration u, then the durations are exponentially distributed. In this case, the waiting time until the next order, given that an order of type j has been posted, is equal to $h_j(u \mid z;\theta)^{-1}$. Therefore, when the hazard function is high, the mean duration between two successive orders is tight. Generally, when the hazard functions are not constant with respect to u, the distribution of the duration that we have to wait until the next order, given the type of the last order and all the past of the process, is not exponentially distributed. For instance, if we assume that the elapsed time between two successive orders is distributed as a Weibull random variable (*cf.* GOURIÉROUX [2000]), the conditional hazard functions can be increasing or decreasing with respect to the duration u according to the values of the estimated parameters.

Here, we have to model jointly the elapsed time between two successive orders posted on the market and the type of the next order given the past of the process. In order to do so, we assume that when an order of a given type has been posted on the market, twelve durations are simultaneously and independently drawn. These durations are the times we may have to wait to observe an order of type k ($k = 1,\ldots,12$) posted on the market. We assume that the next observed order is the one with the smallest duration. Therefore, only one of these twelve durations are observed and the eleven other ones are considered as right censored. Among these twelve latent durations, the one we observe will contribute to the likelihood function through its density probability function. The other eleven durations are censored, so we know that they are greater than the observed one. Thus, these eleven censored durations will contribute to the likelihood function *via* the probability that the corresponding durations are greater than the observed one.

In this modeling, several states (the types of the orders) are considered as well as several spells (the period of time between two successive orders). Since our model is multi-spell and multi-state, we use a competing risk model for each ordered pair whose components are the duration and the type of the next order. In this kind of model, the destinations are different categories of orders. From a given date, the placement of an order of a given type will censor the durations we have to wait to observe the placement of orders belonging to the other categories. When we consider the type of the next order, the realization of a given category precludes the realization of all the other ones.

The observed duration, given that the last order is of type j, is the minimum of the realizations of twelve independent random variables. The distributions of these twelve random variables can be characterized by their conditional hazard functions $h_{j,k}(u \mid z; \theta_k)$, for $k = 1,\ldots,12$. The function $h_{j,k}$ can be interpreted as the arrival rate of an order belonging to category k given that the category of the last order is of type j and that no order belonging to category k was posted since u units of time.

Under these assumptions, the survival function of the duration given that the previous order is of type j is

$$(6) \qquad S_j(u \mid z; \theta) = \exp\left(- \int_0^u \sum_{\ell=1}^{12} h_{j,\ell}(s \mid z; \theta_\ell)\, ds \right).$$

The probability density function of the ordered pair whose components are the duration between two orders, u, and the type of the next order, k, given that the type of the last order is j, is

$$(7) \qquad f_j(u,k \mid z; \theta) = h_{j,k}(u \mid z; \theta_k) \exp\left(- \int_0^u \sum_{\ell=1}^{12} h_{j,\ell}(s \mid z; \theta_\ell)\, ds \right),$$

where $\theta = (\theta_1,\ldots,\theta_{12})$.

This probability density function can be written as the product of

• the probability that the next order is of type k, given that the last order is of type j and the duration between the two successive orders is u;

• the probability density function of the duration between the two successive orders given the type of the first one.

Indeed, we have

$$f_j(u,k \mid z;\theta) = \frac{h_{j,k}(u \mid z;\theta_k)}{h_j(u \mid z;\theta_k)}\, f_j(u \mid z;\theta) \tag{8}$$
$$= \pi_{jk}(u \mid z;\theta)\, f_j(u \mid z;\theta),$$

where $\pi_{jk}(u;z;\theta)$ is the probability that an order of type k is posted after u units of time given that the last order is of type j.

So, the probability density function of the duration between order j and the next order (whatever is the type of this order) is

$$f_j(u \mid z;\theta) = \sum_{\ell=1}^{12} h_{j,\ell}(u \mid z;\theta_\ell)\exp\left(-\int_0^u \sum_{\ell=1}^{12} h_{j,\ell}(s \mid z;\theta_\ell)\,ds\right). \tag{9}$$

The hazard function associated to this duration is

$$h_j(u \mid z;\theta_\ell) = \sum_{\ell=1}^{12} h_{j,\ell}(u \mid z;\theta_\ell). \tag{10}$$

Hereafter, we assume that the latent duration we have to wait in order to observe an order of type k, given that the last order is of type j ($j,k \in E$), has a distribution with hazard function

$$h_{j,k}(u \mid z;\theta_k) = \exp(\bar{z}'_{jk}\alpha_k)\,\frac{\gamma_k\, u^{\gamma_k-1}}{(1+\beta_k u^{\gamma_k})}, \tag{11}$$

where $\theta_k = (\alpha'_k,\beta_k,\gamma_k)'$, $k = 1,\ldots,12$, and $\bar{z}_{jk} = \psi_{jk}(z)$. The functions $\psi_{jk}(z)$ give the vector of explanatory variables. These functions are specific to the transition.[5]

Therefore, the latent duration for the arrival of an order of type k ($k \in E$) is distributed as a Singh-Maddala random variable with parameters α_k, $\beta_k > 0$ and $\gamma_k > 0$. This family includes several usual distributions (exponential, Weibull, log-logistic). When $0 < \gamma_k < 1$, the hazard function $h_{j,k}(u \mid z;\theta_k)$ is strictly decreasing with the duration u. When $\gamma_k > 1$, the hazard function is first increasing then decreasing. If β_k is different from 0, then the hazard function is not the one associated to a Weibull distribution.[6]

The likelihood function will be the product of $M = 40$ contributions. Each contribution is written conditionally on the first order executed after 10:00 a.m. Each one is the conditional density function of the realization of the process for a given trading day. The contribution of the realization of the process for a given trading day is a product of conditional density functions because we consider here a semi-markovian process.

---

5. In particular, if we have $\gamma_k = \gamma$ and $\beta_k = \beta$, for all $k \in E$, then $\pi_{jk}(u;z;\theta) = \exp(z'_{jk}\alpha_k)/\sum_{\ell/=j}\exp(z'_{j\ell}\alpha_\ell)$.
6. If $\beta_k = 0$, the hazard function is strictly increasing if $\gamma_k > 1$ and strictly decreasing if $0 < \gamma_k < 1$. When $\gamma_k = 1$ and $\beta_k = 0$, the hazard function is constant and the latent duration is exponentially distributed.

4.2 The Likelihood Function

Given the assumptions on the joint distribution of the durations between successive orders and the type of these orders, the expression of the likelihood function is

$$
\begin{aligned}
L(\theta) &= \prod_{m=1}^{M} f(y_{2,m},\ldots,y_{N_m,m} \mid y_{1,m}; z_m,\theta) \\
&= \prod_{m=1}^{M}\prod_{n=2}^{N_m} f_{x_{n-1,m}}(u_{n,m},x_{n,m} \mid z_{n,m};\theta) \qquad (12)\\
&= \prod_{m=1}^{M}\prod_{n=2}^{N_m} h_{x_{n-1,m},x_{n,m}}(u_{n,m}|z_{n,m};\theta_{x_{n,m}})\, S_{x_{n-1,m}}(u_{n,m}|z_{n,m};\theta).
\end{aligned}
$$

Let $S_{j,k}(u \mid z;\theta_k) = \exp(-\int_0^u h_{j,k}(s \mid z;\theta_k)\,ds)$ be the survival function associated to the latent duration we have to wait in order to observe an order of type k given that the last order is of type j ($j,k \in E$). We assume that $\delta^k_{n,m} = 1$ if at the time $\tau_{n,m}$ of the trading day m an order belonging to category k is posted on the market ($\delta^k_{n,m} = 0$ otherwise).

The likelihood function can be written as

$$
\begin{aligned}
L(\theta) &= \prod_{k=1}^{12}\prod_{m=1}^{M}\prod_{n=2}^{N_m} h_{x_{n-1,m},k}(u_{n,m} \mid z_{n,m};\theta_k)^{\delta^k_{n,m}}\, S_{x_{n-1,m},k}(u_{n,m} \mid z_{n,m};\theta_k) \qquad (13)\\
&= \prod_{k=1}^{12} L_k(\theta_k)
\end{aligned}
$$

where

$$
L_k(\theta_k) = \prod_{m=1}^{M}\prod_{n=2}^{N_m} h_{x_{n-1,m},k}(u_{n,m} \mid z_{n,m};\theta_k)^{\delta^k_{n,m}}\, S_{x_{n-1,m},k}(u_{n,m} \mid z_{n,m};\theta_k).
$$

Therefore, the likelihood function is the product of twelve functions $L_k(\theta_k)$, $k = 1,\ldots,12$. The function L_k depends on the vector of parameters θ_k. For all $k \in E$, a parameter θ_k only appears in L_k. In order to obtain the maximum likelihood estimation of the parameters θ, we can then maximize independently the functions $L_k(\theta_k)$ with respect to θ_k ($k = 1,\ldots,12$).

4.3 Functions of Interest

Using the estimates of the parameters, it is possible to calculate some functions which are useful for the analysis of the order arrival process. For instance, the mean waiting time until an order is posted given that the last one is of type j ($j \in E$) is obtained using the conditional density function of the duration (see equation 9). The expression of this conditional density is

$$(14) \quad \mathrm{E}(u_{n,m} \mid X_{n-1,m} = j; z) = \int_0^{+\infty} u \sum_{k=1}^{12} h_{j,k}(u \mid z; \theta_k)\, S_j(u \mid z; \theta)\, du.$$

Given that an order of type k will follow an order of type j ($j,k \in E$), the mean duration between two successive orders is given by the expression

$$(15) \quad \mathrm{E}(u_{n,m} | X_{n-1,m} = j, X_{n,m} = k; z) = \int_0^{+\infty} u\, \frac{h_{j,k}(u | z; \theta_k)}{\pi_{jk}(z; \theta)}\, S_j(u | z; \theta)\, du,$$

where $\pi_{jk}(z; \theta)$ is the probability that an order of type k is placed on the market given the last order is of type j ($j,k \in E$). This conditional probability is obtained by integration of the joint density (7) with respect to the duration u:

$$(16) \qquad \pi_{jk}(z; \theta) = \int_0^{+\infty} h_{j,k}(u \mid z; \theta_k)\, S_j(u \mid z; \theta)\, du.$$

5 Results

We maximize the functions $L_k(\theta_k)$, $k \in E$, with respect to the vectors of parameters θ_k, $k \in E$. The estimation results are presented in Appendix C. These twelve maximizations generate the ML estimates of the parameters of the process (see equation 13). Each column corresponds to a given type of orders and each row of the table is associated to a given parameter of the process. The hazard function (see equation 11) are indexed by the vectors of parameters θ_k, $k = 1,\ldots,12$, where $\theta_k = (\beta_k, \gamma_k, \alpha'_k, \lambda_{1,k}, \lambda_{2,k})'$.

For a given type of orders k, α_k is the vector of parameters associated respectively to:

- a constant;
- an indicator that the last order is of type j ($j = 1,\ldots,12, j \neq k$);
- the depth, in percentage, at the best ask quote;
- the depth, in percentage, at the best bid quote;
- the spread times 100;
- the date, in hours, of the last executed order;
- the square of this date.

5.1 Hazard Functions

The estimation results are given in Appendix C. These results show that the estimates of the parameters β_k and γ_k are significantly different from zero for all the types. The estimate $\hat{\beta}_k$ is strictly positive and $\hat{\gamma}_k$ is always greater than one.

Since $\hat{\beta}_k$ is positive, the latent distribution of the duration between successive orders is not a Weibull distribution. The exponential distribution is a Weibull distribution, thus the latent distributions of the elapsed time between successive orders are not exponential. This shows the interest for using the Singh-Maddala distribution.

For a given last order j and a fixed next order k, the hazard function h_{jk} depends on the duration, the time of the last order in the trading day, the spread, the depth at best ask quote and the depth at the best bid quote. The parameterization of the distribution of the process is such that the hazard function of a given latent duration is shifted upward or downward when a given exogenous variable increases. The level and the direction of this shift depend on the value of the corresponding estimated parameter. In order to study the hazard functions with respect to duration and time of the day, we fix the spread and the quantities at the best bid-ask quotes to their respective mean values calculated over the sample.[7]

Since $\hat{\gamma}_k$ is greater than one for all k, each hazard function is first increasing then decreasing with respect to the duration u. For instance, the hazard function for a large buy order when the last order posted on the market is a large buy order is plotted in Figure 3.

This function is drawn assuming that the last order has been placed at 10:00 a.m.

The shape of the hazard function shows how the most aggressive buyers react to the arrival of an aggressive bid. The corresponding hazard function increases during the first five seconds after the record of the order. This period of time corresponds to the technical delays of transmission of information on the market and to the mean number of seconds necessary for the investors to react. The hazard function is maximum about five seconds after the recording of the last order. After this first period of time, this information has less and less influence on the behavior of the investors. Indeed, traders who wanted to react to the arrival and characteristics of the last order have already placed orders of different types. Hence, the decrease of the hazard function shows that the prices do not incorporate immediately all the available information because traders react by placing orders with delays.

In order to examine the shape of the hazard function with respect to the time t of the last order, the duration u is set equal to its mean value given that an order of type j will be followed by an order of type j.

For j and k corresponding to a large buy order (type 1), this empirical mean is equal to 7.69 seconds and the hazard function is plotted in Figure 4.

This U-shaped pattern is obtained whatever the category of the last order, j, and the category of the next order, k. This is a characteristic of the intraday activity of the market. Indeed, it is often underlined that activity is large at the beginning and at the end of the trading day, and tight in the middle of the day. In order to study the effect of the type of the next order, we compare the last figure with the one obtained when the next order is a new bid below the best bid quote (type 6). Figure 5 shows that the corresponding hazard function is higher in the morning (10:00 a.m.) than at the end of the trading day whereas

7. The empirical mean value of the spread is 0.13 %, the mean quantity at the best ask quote is equal to 738.09 and the mean quantity at the best bid quote is 778.46.

FIGURE 3

Hazard Function For a Large Buy Order after a Large Buy Order

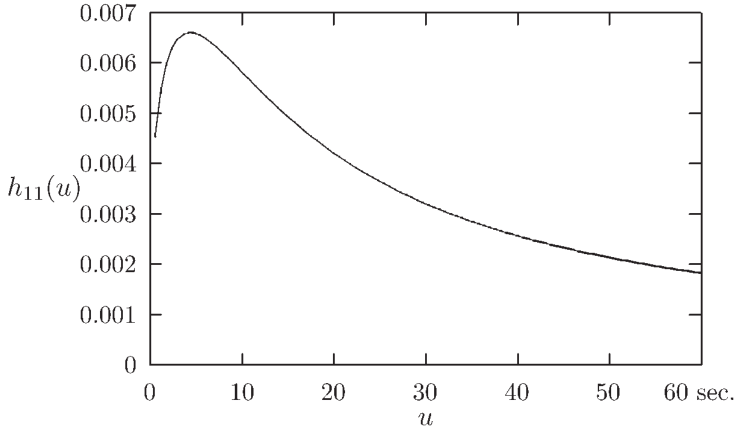


FIGURE 4

Intraday Activity (h_{11}),

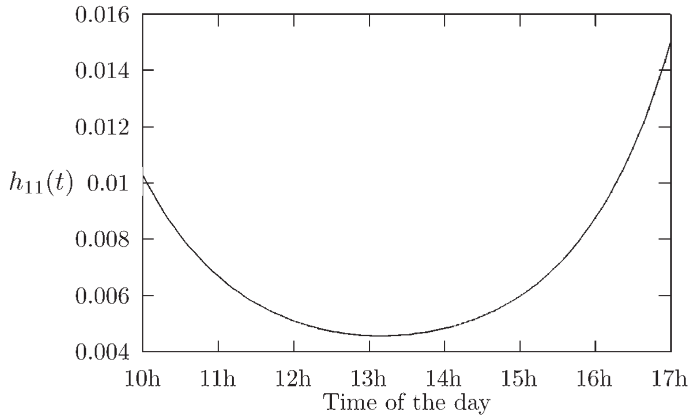
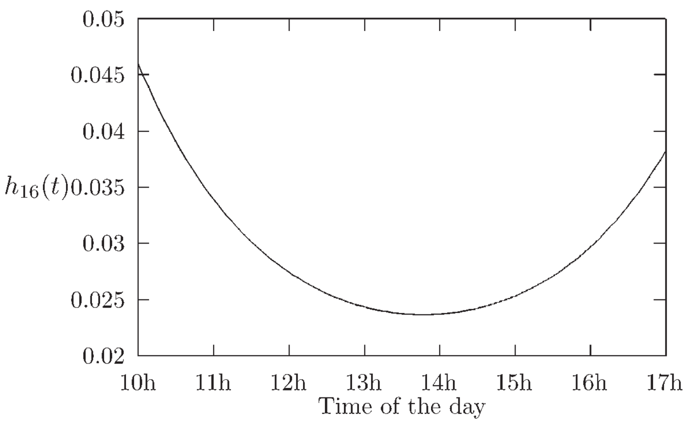


FIGURE 5

Intraday Activity (h_{16}),



the hazard function h_{11} is higher at the end of the day. This analysis is valid whatever the category of the last order.

These differences in the activity relative to the types of orders can be considered as a consequence of the mechanism contributing to the price discovery during a trading day. In the morning, buyers and sellers avoid to post immediate execution orders, because they think that the market has not yet found the "*fair*" price. They prefer, rather, to place limit orders. Later in the day, when the investors think that the market price is fair, aggressive orders are preferred. Moreover, this phenomenon is strengthened by the fact that the less aggressive orders have a decreasing probability to be executed when closing time approaches.

5.2 Transition Probabilities

Using the estimated values of the parameters of the process we can obtain an estimation of the transition probabilities between categories of orders given the time of the last order and the variables associated to the spread. These variables are set equal to their empirical mean values.

We assume that the last order was placed at 10 o'clock. The transition probabilities are given in table 2 for transitions to buy orders and in table 3 for transitions to sell orders.[8]

The transition probabilities are in percentage. A given row of the two tables (a type of the last order) adds up to 100 %. The three largest numbers of each column (type of the next order) are in bold face type. For a given type of the next order (k), if the transition probability for a given type of the last order (j) is in bold face type, the corresponding probability is greater than the unconditional probability that the next order placed by the investors is of type k.

The undercutting or outbidding behavior of investors is not observed in the estimates of the transition probabilities. Indeed, the diagonal effect in the case of bid (or ask) orders within the best bid (or ask) quote (empirical implication 1.a) does not appear. Of course, the corresponding figures (resp. 5.41 and 4.64) are among the six largest numbers of their column but are not among the three largest ones.

We suggest two explanations for this result. First, the weakness of transitions (4,4) and (10,10) can be related to the success of the empirical implication 2.b. Indeed, probabilities corresponding to transitions (4,5), (5,4), (11,10) and (10,11) are among the three largest figures in the same column. The traders arbitrate between competing for price priority and queuing at the best quote.

Second, several traders can react simultaneously to a new limit order within the quotes. Each one believes that he is posting an order within the quotes. However, these orders will not be classified as type 4 or type 10. If these orders are placed at the same price, the first will be recorded in the data set within the quotes but the others will be recorded at the best quotes. If the last orders are placed with a smaller (greater) price, these orders are recorded as bids (asks) below (above) the best bid (ask) quote. For bid orders, a large probability in

8. Standard-errors for these probabilities are given in Appendix D. Transition probabilities when the last order is executed at 01:30 am and 05:00 p.m. are given in appendix E and F. The relations observed between transition probabilities are constant with the time of the last order.

TABLE 2
Conditional Probabilities of Buy Orders

	Type of Buy Orders					
Event	Large buy	Market buy	Small buy	Buy within	Buy at	Buy below
Large buy	**4.34**	**4.61**	10.83	**9.63**	4.72	14.70
Market buy	**2.73**	**4.44**	**12.96**	**5.97**	**6.07**	**20.91**
Small buy	2.51	**2.99**	**15.39**	5.46	3.44	13.45
Buy Within	**2.86**	2.92	10.65	5.41	**5.75**	**19.70**
Buy at	2.28	2.67	9.78	**8.35**	**5.99**	15.86
Buy below	1.79	1.98	10.34	5.14	4.28	**21.25**
Large sell	1.68	1.34	7.96	4.29	3.34	13.18
Market sell	1.47	2.05	7.97	3.32	3.44	13.64
Small sell	1.30	1.71	10.34	4.50	3.76	13.38
Sell within	1.78	1.47	10.71	4.58	3.12	14.14
Sell at	1.39	2.07	**11.88**	4.60	3.86	12.78
Sell above	1.89	1.83	9.73	5.42	3.30	13.11

TABLE 3
Conditional Probabilities of Sell Orders

	Type of Sell Orders					
Event	Large sell	Market sell	Small sell	Sell within	Sell at	Sell above
Large buy	0.99	1.32	21.98	3.10	3.29	20.73
Market buy	1.88	1.98	19.59	2.71	3.43	17.56
Small buy	1.47	1.39	**27.06**	3.92	3.68	19.46
Buy Within	1.59	1.22	21.75	4.40	3.56	20.40
Buy at	1.28	1.66	24.66	4.36	3.52	19.77
Buy below	1.80	1.91	23.95	4.36	3.18	20.21
Large sell	**3.74**	**3.14**	**25.79**	**8.53**	3.88	23.37
Market sell	**2.58**	**3.55**	25.35	**5.11**	**5.07**	**26.66**
Small sell	1.96	**2.46**	**32.69**	4.85	3.75	19.49
Sell within	**2.87**	2.43	24.96	4.64	**5.49**	**24.02**
Sell at	2.46	2.35	24.98	**5.57**	**5.46**	22.80
Sell above	1.92	1.92	24.54	4.10	3.28	**29.17**

(4,4) will be shifted to (4,5) and (4,6). Moreover, probabilities (5,5), (4,6), (6,4) and (6,6) should be large too. This is verified for all these probabilities except for the one associated to transition (6,4). We find the same type of results for ask orders.

The scenario stated by empirical implication 3.b (supply and consumption of liquidity) is confirmed by the results. Small trades on one side of the market are more frequent after new orders within and at the best quote on the other side of the market. This scenario is verified when the last order is an order to sell (types 10 and 11) but not when it is an order to buy (types 4 and 5). Moreover, market orders to buy (resp. to sell) should be more frequent after market orders to sell (resp. to buy). This consequence of the scenario associated to the empirical implication 3.c is confirmed by the results. Indeed, the probabilities in (8,2) and (2,8) are among the largest of their semi-column.

The diagonal effect stated by empirical implication 4 is more important for the most aggressive orders. In each sub-group of immediate execution orders (1 to 3 and 7 to 9) the largest transition probabilities are located on the main diagonal, and close to the main diagonal (this is particularly true for buy orders). This result is consistent with the definition of the categories according to the direction and aggressiveness of orders. The only exceptions in the diagonal effect correspond to transitions (4,4) and (10,10).

Empirical implication 6 states that the spread can be shifted downward or upward after the arrival of information conveyed by large sales (or purchases). The large probabilities in (1,4) and (7,10) reflect this information effect. New orders within the best quotes are more frequent after a very aggressive order in the same direction.

5.3 State of the Book

Tables 4 and 5 report the signs of the estimated parameters of the three variables related to the order book (*i.e*. the spread S computed in percentages, the quantity available at the best price on the sell side, Q_A, and on the buy side, Q_B).

These results must be related to empirical implications 2.a (queuing is preferable when quantity at the best price is small) and 3.a (posting a better price is interesting when the spread is wide).

In table 4 (buy orders), columns 4 to 6, the signs of the parameters of Q_B are consistent with implication 2.a. In table 5 (sell orders), columns 4 to 6, the signs for Q_A are not at odds with this implication, however not significant in the case of type 11.

In both tables, all the signs in the line labeled with S are consistent with implication 3.a.

The other signs in these two tables are not related to any of our implications. *A priori*, we do not expect a buy order to depend upon Q_A, the quantity available on the sell side. Table 4 shows that this is the case for limit orders (the parameters are not significant for types 4 to 6), but not for the three first types. This may be a classification bias: for a given quantity specified by an order to buy (type 1, 2 or 3), the type computed for this order will depend upon Q_A. This is consistent with the signs reported in table 4 for buy orders, and also in table 5 for sell orders.

We have no interpretation to propose for the positive signs of Q_B in the case of an order to sell at the best quote (type 11), for the positive signs of Q_B in case of types 1 to 3, and for the signs of Q_A in case of type 7 and 8.

TABLE 4
Impact of the State of the Book on the Arrival of Buy Orders

	Type of Buy Orders					
State of the book	Large buy	Market buy	Small buy	Buy within	Buy at	Buy below
Depth (ask side)	–	–	+	n.s.	n.s.	n.s.
Size of the spread	–	–	–	+	+	+
Depth (bid side)	+	+	+	+	–	–

TABLE 5
Impact of the State of the Book on the Arrival of Sell Orders

	Type of Sell Orders					
State of the book	Large sell	Market sell	Small sell	Sell within	Sell at	Sell above
Depth (ask side)	+	+	n.s.	+	n.s.	–
Size of the spread	–	–	–	+	+	+
Depth (bid side)	–	–	+	n.s.	+	n.s.

TABLE 6
Average Waiting Time (in sec.) after an Order of a Given Type

	Large buy	Market buy	Small buy	Buy within	Buy at	Buy below
Waiting time	8.04	8.92	9.50	9.14	10.72	9.60
	Large sell	Market sell	Small sell	Sell within	Sell at	Sell above
Waiting time	8.01	8.63	10.16	9.56	10.26	9.86

5.4 Waiting Times

The average waiting time for a new order after an order of a given type, computed at 10 a.m. and with the mean values of the variables related to the book, are reported for each type in table 6.[9]

For buy and sell orders, the average waiting time is lower after an order within the best quotes than after an order at or outside the best quotes. This is consistent with empirical implication 1.b: from the point of view of price competition, an order within the best quotes is more significant.

The negative relation between waiting time and aggressiveness, as postulated by implication 5, is confirmed within each group of immediate execution orders (types 1 to 3 and 7 to 9). The waiting time then decreases for buy and sell orders within the best quotes (4 and 10), and then increases for an order at the best quote (types 5 and 11) and finally decreases for an order outside the best quote (types 6 and 12). The first decrease may be related to the bias pointed during the discussion of competition in price. The decrease between small trades and limit orders within the quotes shows that the market reacts very quickly to two events: the arrival of a very aggressive order, which conveys a clear information about the true value of the asset, and the arrival of an order within the best quotes, which points out to the market that the competition for liquidity supply is running hot.

6 Conclusion

Our results are consistent with those of BHS [1995]: the facts seem to be compatible with the existence of investors whose strategies of order placement are based on information or liquidity related reasons.

The main difference is that we do not find a strong diagonal effect between orders within the best quotes, related to a possible Bertrand competition for price priority. BHS [1995] put forward this diagonal effect by computing the empirical probabilities of transition between the different types of orders. In our statistical model, the arrival of an order is explained by the type of the last order, but, jointly, we also take into account the state of the order book. By doing so, we show that the succession of buy or sell orders within the best quotes is rather an indirect effect of the bid-ask spread: the arrival of an order within the best quotes is mainly explained by a wide spread.

In addition, we show that the hazard function computed from the joint distribution of the duration between two successive orders and the types of the orders is increasing and then decreasing. This finding excludes that the distribution is of the Weibull class, and, consequently, of the exponential class too. We propose an interpretation for this shape in terms of reaction delays of the investors to the information broadcasted by the electronic quotation system.

9. We performed the same calculations at 1:30 and 5 p.m. (see Appendix G and Appendix H). As for transition probabilities, the hierarchy of waiting times are very stable during the day.

Our work may be extended in many ways. In particular, and in order to better understand the dynamics of the order placement strategies, some variables related to the past of a spell could be added to the model. When explaining the arrival of an order, the previous duration will enable to capture the positive autocovariance between durations, related to the clustering in order placements. Moreover, the type of the first order of this chain and the state of the book when it occurs will show the reaction of the investors not only to the state of the book but also to the dynamics of this book. ■

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APPENDIX

A The Alcatel Share

Summary Statistics (December 31, 1996)

Price in FF	416.8
Number of shares on the market	161 788 874
Market Capitalization in FF bio	67.5

Daily Average Statistics over 1996 (250 Trading Days)

Transactions	975.2
Number of shares traded	345 884
Number of applications	13.6
Shares traded by application	18.5 %
Turnover	0.22 %

B Empirical Probabilities of Transition

Figures are in percentage. For a given row, the figures over the two tables add up to 100.

	To					
From	Large buy	Market buy	Small buy	Buy within	Buy at	Buy below
Large buy	6.65	5.00	13.34	8.88	6.11	12.55
Market buy	5.47	8.30	13.77	7.59	5.66	12.36
Small buy	2.86	3.69	16.49	6.39	4.36	12.81
Buy Within	6.28	2.90	10.14	7.28	8.20	14.00
Buy at	3.09	4.42	8.53	9.61	6.49	15.43
Buy below	2.62	4.15	8.71	8.44	4.91	19.89
Large sell	1.39	3.31	7.25	5.57	1.87	13.44
Market sell	1.53	3.83	7.09	3.75	2.54	12.74
Small sell	2.14	2.27	10.44	5.09	4.60	12.60
Sell within	2.17	1.54	8.64	7.01	4.77	14.19
Sell at	2.37	2.59	9.75	6.29	4.68	12.09
Sell above	2.40	2.42	8.70	5.84	4.33	12.96

From	To Large sell	Market sell	Small sell	Sell within	Sell at	Sell above
Large buy	2.18	3.51	16.32	5.79	2.02	17.65
Market buy	1.37	4.53	19.43	3.82	2.78	14.91
Small buy	2.14	2.64	22.55	5.87	4.72	15.49
Buy Within	2.56	2.01	19.25	6.21	3.61	17.58
Buy at	2.63	2.97	22.60	5.01	4.11	15.12
Buy below	2.64	3.16	20.90	5.80	3.52	15.26
Large sell	6.09	5.61	23.94	9.21	5.28	17.03
Market sell	5.24	7.66	28.21	7.01	4.59	15.80
Small sell	2.80	3.43	30.26	5.83	3.84	16.69
Sell within	6.54	3.35	23.67	7.18	5.29	15.64
Sell at	3.20	4.68	23.78	9.03	6.12	15.43
Sell above	3.13	4.33	21.16	6.76	4.13	23.84

C Results of the Estimation

	Outcome (buy orders)					
Parameter (student)	Large buy	Market buy	Small buy	Buy within	Buy at	Buy below
β	0.0456	0.0344	0.0261	0.0293	0.0191	0.0478
	(9.46)	(9.60)	(13.37)	(12.91)	(8.78)	(24.06)
γ	1.3345	1.2865	1.1633	1.2813	1.1906	1.4601
	(24.00)	(26.78)	(52.55)	(40.68)	(35.27)	(53.38)
Constant	-4.3618	-4.1669	-4.2522	-6.1190	-5.5163	-4.3252
	(-28.68)	(-31.08)	(-65.64)	(-65.82)	(-47.56)	(-75.65)
Large buy	–	0.1248	-0.2057	0.6893	0.0215	-0.2267
	–	(0.96)	(-2.41)	(7.70)	(0.17)	(-3.15)
Market buy	-0.5462	–	-0.1160	0.1197	0.1789	0.0422
	(-3.71)	–	(-1.59)	(1.07)	(1.56)	(0.72)
Small buy	-0.6843	-0.4531	–	-0.0264	-0.4493	-0.4521
	(-5.74)	(-4.47)	–	(-0.34)	(-4.63)	(-10.30)
Buy Within	-0.5197	-0.4409	-0.3327	–	0.1032	-0.0376
	(-4.07)	(-3.81)	(-5.47)	–	(1.09)	(-0.85)
Buy at	-0.8742	-0.6613	-0.5533	0.2977	–	-0.3787
	(-6.20)	(-5.25)	(-7.96)	(3.59)	–	(-6.86)
Buy below	-1.0270	-0.8712	-0.4048	-0.0928	-0.2365	–
	(-8.58)	(-8.35)	(-8.38)	(-1.27)	(-2.72)	–
Large sell	-0.9489	-1.1105	-0.5108	-0.1173	-0.3199	-0.3339
	(-5.00)	(-5.40)	(-5.35)	(-1.01)	(-2.33)	(-4.65)
Market sell	-1.1386	-0.7456	-0.5749	-0.4380	-0.3600	-0.3601
	(-6.79)	(-5.55)	(-6.77)	(-3.36)	(-2.76)	(-5.53)
Small sell	-1.3925	-1.0664	-0.4553	-0.2768	-0.4205	-0.5106
	(-12.09)	(-11.08)	(-10.79)	(-3.95)	(-5.09)	(-15.20)
Sell within	-1.0332	-1.1669	-0.3669	-0.2052	-0.5525	-0.4059
	(-7.35)	(-8.35)	(-5.86)	(-2.33)	(-4.96)	(-7.92)
Sell at	-1.3338	-0.8830	-0.3231	-0.2628	-0.4018	-0.5620
	(-6.52)	(-5.52)	(-4.81)	(-2.57)	(-3.39)	(-8.87)
Sell above	-0.9969	-0.9707	-0.4885	-0.0634	-0.5229	-0.5051
	(-8.75)	(-9.68)	(-10.64)	(-0.89)	(-5.98)	(-13.82)
Depth (ask side)	-0.1298	-0.0634	0.0136	0.0002	0.0021	0.0017
	(-18.25)	(-12.91)	(24.29)	(0.10)	(1.17)	(1.57)
Depth (bid side)	0.0138	0.0112	0.0098	0.0106	-0.0209	-0.0060
	(7.66)	(6.30)	(10.26)	(8.12)	(-7.25)	(-4.27)
Spread	-2.3117	-7.8104	-1.2943	4.3539	1.8375	0.8554
	(-7.07)	(-19.49)	(-8.18)	(45.03)	(10.20)	(7.68)
λ_1	-0.5138	-0.4051	-0.5460	-0.3267	-0.3179	-0.3510
	(-10.42)	(-8.68)	(-21.43)	(-10.77)	(-8.61)	(-16.76)
λ_2	0.0808	0.0609	0.0819	0.0506	0.0486	0.0465
	(12.10)	(9.66)	(23.59)	(11.99)	(9.53)	(15.80)

Parameter (student)	*Outcome (sell orders)* Large sell	Market sell	Small sell	Sell within	Sell at	Sell above
β	0.0413	0.0380	0.0192	0.0296	0.0320	0.0373
	(9.86)	(10.25)	(20.41)	(12.84)	(10.62)	(22.60)
γ	1.3353	1.2685	1.1656	1.2794	1.2710	1.2910
	(25.19)	(28.62)	(86.99)	(40.59)	(31.95)	(62.70)
Constant	-4.2768	-4.3499	-3.6513	-6.3464	-5.8016	-3.7950
	(-29.03)	(-34.83)	(-92.65)	(-65.34)	(-45.92)	(-79.55)
Large buy	-1.3296	-0.9321	-0.1838	-0.2518	-0.2984	-0.1721
	(-5.71)	(-4.89)	(-3.13)	(-2.02)	(-1.94)	(-2.63)
Market buy	-0.7754	-0.6133	-0.3930	-0.4787	-0.3449	-0.4240
	(-4.89)	(-4.57)	(-6.94)	(-3.26)	(-2.30)	(-6.51)
Small buy	-1.0734	-1.0210	-0.1288	-0.1651	-0.3326	-0.3765
	(-8.30)	(-9.11)	(-4.34)	(-1.98)	(-3.16)	(-9.84)
Buy Within	-0.9607	-1.1151	-0.3104	-0.0147	-0.3296	-0.2948
	(-6.92)	(-8.40)	(-7.82)	(-0.17)	(-2.84)	(-6.57)
Buy at	-1.3084	-0.9367	-0.3272	-0.1600	-0.4731	-0.4563
	(-6.54)	(-6.03)	(-7.90)	(-1.63)	(-3.76)	(-8.95)
Buy below	-0.8761	-0.7073	-0.2581	-0.0644	-0.4828	-0.3442
	(-7.55)	(-7.32)	(-9.38)	(-0.85)	(-4.76)	(-10.34)
Large sell	–	-0.0602	-0.0206	0.7628	-0.1299	-0.0493
	–	(-0.47)	(-0.39)	(8.45)	(-0.92)	(-0.83)
Market sell	-0.4316	–	-0.1063	0.1843	0.0721	0.0193
	(-3.11)	–	(-2.26)	(1.70)	(0.57)	(0.38)
Small sell	-0.8408	-0.5042	–	-0.0101	-0.3688	-0.4304
	(-7.78)	(-5.84)	–	(-0.14)	(-3.97)	(-15.02)
Sell within	-0.4089	-0.4641	-0.2143	–	0.0639	-0.1700
	(-3.28)	(-4.16)	(-5.70)	–	(0.60)	(-4.01)
Sell at	-0.6197	-0.5553	-0.2767	0.1219	–	-0.2795
	(-4.36)	(-4.38)	(-6.19)	(1.30)	–	(-5.36)
Sell above	-0.8379	-0.7227	-0.2583	-0.1512	-0.4756	–
	(-7.46)	(-7.75)	(-10.32)	(-2.02)	(-4.87)	–
Depth (ask side)	0.0088	0.0076	0.0012	0.0111	-0.0046	-0.0054
	(5.53)	(4.85)	(1.50)	(10.19)	(-1.92)	(-4.58)
Depth (bid side)	-0.1631	-0.1129	0.0138	0.0000	0.0061	0.0057
	(-21.99)	(-19.94)	(24.88)	(0.02)	(3.28)	(6.03)
Spread	-1.6480	-4.1678	-0.6500	4.6762	2.1608	0.0924
	(-5.46)	(-13.16)	(-6.63)	(47.40)	(12.03)	(0.87)
λ_1	-0.5325	-0.2749	-0.4568	-0.2574	-0.3510	-0.4772
	(-11.35)	(-6.28)	(-28.10)	(-8.25)	(-8.94)	(-25.48)
λ_2	0.0795	0.0440	0.0630	0.0418	0.0530	0.0619
	(12.23)	(7.33)	(27.94)	(9.62)	(9.76)	(23.43)

D Standard-Errors of the Transition Probabilities

The standard-errors are computed at 10:00 a.m. using the mean values of the variables related to the state of the book ($Q_A = 738.09$, $Q_B = 778.46$ and $S = 0.0013$).

	To					
From	Large buy	Market buy	Small buy	Buy within	Buy at	Buy below
Large buy	0.52	0.54	0.87	0.73	0.53	0.99
Market buy	0.35	0.47	0.88	0.59	0.59	1.09
Small buy	0.24	0.26	0.65	0.34	0.27	0.56
Buy Within	0.31	0.30	0.61	0.38	0.44	0.80
Buy at	0.28	0.31	0.65	0.56	0.51	0.81
Buy below	0.17	0.18	0.47	0.29	0.28	0.66
Large sell	0.30	0.27	0.73	0.45	0.42	0.89
Market sell	0.23	0.26	0.65	0.40	0.40	0.84
Small sell	0.12	0.14	0.41	0.24	0.23	0.44
Sell within	0.22	0.19	0.63	0.35	0.30	0.69
Sell at	0.26	0.31	0.75	0.42	0.40	0.77
Sell above	0.17	0.16	0.42	0.28	0.22	0.46

	To					
From	Large sell	Market sell	Small sell	Sell within	Sell at	Sell above
Large buy	0.22	0.24	1.16	0.36	0.45	1.20
Market buy	0.27	0.26	1.02	0.37	0.45	1.04
Small buy	0.16	0.14	0.75	0.27	0.30	0.69
Buy Within	0.19	0.15	0.81	0.33	0.34	0.83
Buy at	0.24	0.24	0.94	0.37	0.37	0.91
Buy below	0.17	0.16	0.64	0.26	0.24	0.62
Large sell	0.44	0.37	1.18	0.65	0.48	1.20
Market sell	0.31	0.36	1.07	0.49	0.53	1.15
Small sell	0.16	0.18	0.65	0.25	0.24	0.53
Sell within	0.30	0.25	0.86	0.35	0.45	0.90
Sell at	0.30	0.28	1.01	0.45	0.52	1.04
Sell above	0.17	0.16	0.60	0.23	0.23	0.69

E Conditional Probabilities at 1:30

($Q_A = 738.09$, $Q_B = 778.46$ and $S = 0.0013$).

	Type of buy orders					
Event	Large buy	Market buy	Small buy	Buy within	Buy at	Buy below
Large buy	**3.80**	**4.84**	9.25	**12.02**	6.27	14.83
Market buy	**2.38**	**4.64**	**11.08**	**7.42**	**8.09**	**20.93**
Small buy	2.21	**3.16**	**13.38**	6.89	4.67	13.59
Buy Within	**2.49**	3.05	9.12	6.73	**7.68**	**19.69**
Buy at	1.95	2.75	8.29	**10.22**	**7.92**	15.54
Buy below	1.56	2.07	8.89	6.41	5.75	**21.24**
Large sell	1.46	1.39	6.75	5.31	4.41	13.21
Market sell	1.30	2.17	6.88	4.18	4.63	13.81
Small sell	1.13	1.79	8.88	5.60	5.04	13.32
Sell within	1.56	1.55	9.29	5.77	4.22	14.24
Sell at	1.21	2.16	**10.23**	5.72	5.18	12.73
Sell above	1.67	1.95	8.54	6.89	4.52	13.31

	Type of sell orders					
Event	Large sell	Market sell	Small sell	Sell within	Sell at	Sell above
Large buy	0.81	1.74	21.29	4.42	3.83	16.87
Market buy	1.53	2.60	19.05	3.85	3.98	14.25
Small buy	1.22	1.85	26.80	5.65	4.32	15.98
Buy Within	1.30	1.61	21.19	6.25	4.13	16.55
Buy at	1.03	2.15	23.80	6.09	4.02	15.77
Buy below	1.47	2.52	23.45	6.21	3.70	16.42
Large sell	**3.05**	**4.13**	**24.79**	**12.07**	4.49	18.91
Market sell	**2.13**	**4.73**	**24.90**	**7.34**	**5.94**	**21.86**
Small sell	1.60	**3.24**	**32.01**	6.89	4.35	15.79
Sell within	**2.37**	**3.24**	24.67	6.67	**6.43**	**19.67**
Sell at	2.01	3.10	24.50	**7.92**	**6.34**	18.48
Sell above	1.59	2.59	24.58	5.95	3.89	**24.12**

F Conditional Probabilities at 5:00

($Q_A = 738.09, \quad Q_B = 778.46$ and $S = 0.0013$).

	Type of buy orders					
Event	Large buy	Market buy	Small buy	Buy within	Buy at	Buy below
Large buy	**6.28**	**5.39**	13.17	**11.76**	5.56	12.37
Market buy	**3.97**	**5.22**	**15.83**	**7.32**	**7.20**	**17.68**
Small buy	3.66	**3.52**	**18.87**	6.72	4.09	11.40
Buy Within	**4.21**	3.48	13.21	6.73	**6.93**	**16.87**
Buy at	3.33	3.16	12.04	**10.31**	**7.16**	13.49
Buy below	2.67	2.39	12.99	6.49	5.23	**18.42**
Large sell	2.47	1.59	9.87	5.34	4.02	11.29
Market sell	2.21	2.49	10.08	4.22	4.23	11.91
Small sell	1.93	2.05	12.92	5.65	4.57	11.54
Sell within	2.64	1.76	13.39	5.75	3.78	12.20
Sell at	2.04	2.46	**14.70**	5.71	4.64	10.92
Sell above	2.85	2.24	12.40	6.94	4.09	11.51

	Type of sell orders					
Event	Large sell	Market sell	Small sell	Sell within	Sell at	Sell above
Large buy	1.18	1.67	19.76	3.99	3.81	15.31
Market buy	2.25	2.52	17.70	3.50	4.00	13.04
Small buy	1.77	1.77	**24.54**	5.09	4.30	14.49
Buy Within	1.93	1.58	19.96	5.77	4.21	15.35
Buy at	1.54	2.13	22.46	5.67	4.13	14.77
Buy below	2.21	2.50	22.29	5.80	3.81	15.41
Large sell	**4.53**	**4.06**	**23.67**	**11.20**	4.59	17.60
Market sell	**3.19**	**4.68**	**23.77**	**6.84**	**6.11**	**20.47**
Small sell	2.40	**3.20**	**30.26**	6.42	4.47	14.78
Sell within	**3.51**	3.16	23.11	6.14	**6.54**	**18.22**
Sell at	2.98	3.03	22.87	**7.29**	**6.44**	17.12
Sell above	2.38	2.55	23.19	5.52	3.98	**22.54**

G Waiting Times (in sec) at 1:30

($Q_A = 738.09, \quad Q_B = 778.46$ and $S = 0.0013$).

	Large buy	Market buy	Small buy	Buy within	Buy at	Buy below
Waiting time	21.60	25.01	27.73	25.85	31.29	27.82
	Large sell	Market sell	Small sell	Sell within	Sell at	Sell above
Waiting time	21.24	24.21	29.70	28.00	30.31	29.72

H Waiting Times (in sec) at 5:00

($Q_A = 738.09, \quad Q_B = 778.46$ and $S = 0.0013$).

	Large buy	Market buy	Small buy	Buy within	Buy at	Buy below
Waiting time	8.09	9.03	9.65	9.40	10.95	10.04
	Large sell	Market sell	Small sell	Sell within	Sell at	Sell above
Waiting time	8.25	9.10	10.57	9.95	10.55	10.50