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Median-Unbiased Estimation in Fixed-Effects Dynamic Panels

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ABSTRACT. – This paper extends ANDREWS' [1993] median-unbiased estimation for autoregressive/unit root time series to panel data dynamic fixed effects models. It is shown that median-unbiased estimation applies straightforwardly to models that include linear time trends as well as to those including more general time specific effects. Using Monte Carlo simulations, median-unbiased LSDV estimators are computed and found to be robust to groupwise heteroskedastic and cross-sectionally correlated disturbances. The behavior of these estimators in the presence of exogenous regressors as well as AR parameter heterogeneity is also evaluated in this paper. As an application, these estimators are used to evaluate conditional convergence in the cases of 48 USA states, 13 OECD countries, and two wider samples from SUMMERS and HESTON's Penn World Tables, with 57 and 100 countries. It is found that median-unbiased estimates support conditional convergence only among USA states and OECD countries.

Estimation sans biais de médiane dans les modèles dynamiques de panel avec effets fixes

RÉSUMÉ. – Cet article étend la méthode d'estimation sans biais de médiane des séries temporelles autorégressives et/ou avec racine unitaire proposée par ANDREWS [1993], aux modèles dynamiques de panel avec effets fixes. On montre que l'estimation sans biais de médiane s'applique aisément autant aux modèles contenant des tendances temporelles linéaires, qu'à ceux incluant des effets temporels spécifiques plus généraux. Par des simulations de Monte Carlo, les estimateurs à effets fixes sans biais de médiane sont calculés et se révèlent robustes à l'hétéroscédasticité spécifique de groupe et à la corrélation des résidus entre coupes transversales. Le comportement de ces estimateurs en présence de régresseurs exogènes ainsi que d'hétérogénéité du paramètre autorégressif est aussi évalué dans cet article. Les estimateurs sont utilisés dans une application dont l'objectif est d'étudier la convergence conditionnelle du produit par tête de 48 Etats des U.S.A., 13 pays membres de l'OCDE et deux échantillons plus importants provenant des Penn World Tables de SUMMERS et HESTON, avec 57 et 100 pays. Les résultats d'estimation sans biais de médiane sont en faveur de la convergence conditionnelle uniquement pour les Etats des U.S.A. et les pays membres de l'OCDE.

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1 Introduction

The bias problem in dynamic panel data models estimated from finite samples has been well documented. NICKELL [1981], SEVESTRE and TROGNON [1985], HSIAO [1986] have shown that for a small time dimension (*T*) the magnitude of the asymptotic bias of the least squares dummy variable (LSDV) estimator is appreciable. BEGGS and NERLOVE [1988] show that the bias becomes larger if the cross-sectional dimension of the panel (*N*) is also small. The use ofthe LSDV estimator in typical panels (small *T* and large *N*) is, therefore, not recommended. Instead, estimators with consistency properties relying on the cross-sectional dimension of the panel being large have been proposed. This is the case of IV (ANDERSON and HSIAO [1981, 1982], HSIAO [1986]) and GMM (ARELLANO and BOND [1991], ARELLANO and BOVER [1995], AHN and SCHMIDT [1995]) methods, among others.

The increasing interest of researchers for applying panel data techniques to problems involving cross-country information is creating new problems since those samples generally have larger time dimensions but much shorter cross-sectional dimensions than typical panels and, more important, they maybe highly trended as well. In several cases most of the above methods cannot be implemented because of *T* being large relative to *N*. This is also the case of 2SLS methods as in KEANE and RUNKLE [1992]. More seriously, in contexts where the AR parameter is high, say 0.95, most estimators may become biased and imprecise. Under these circumstances, most bias formulae given in the papers mentioned before, may not be accurate either.

This paper extends ANDREWS' [1993] median-unbiased estimation for autoregressive/unit root time series to dynamic fixed effects models. Median-unbiased estimation seems to be a reliable method to deal with the bias and efficiency problems in the aforementioned context. This approach is sample specific and is based on the distribution of the LSDV estimator, which is well behaved and has a relatively small variance even in the unit root case. The justification for using median-unbiased estimation in dynamic panel data models is similar to the one for KIVIET's [1995] LSDVc estimator.The method exploits the fact that even though the LSDV is inconsistent and biased in finite samples it is however relatively efficient. KIVIET [1995] derives a formula to estimate the bias of the LSDV estimator for finite *N* and *T*, which is then subtracted from the original estimate. He shows that the bias corrected LSDV method performs well in a number of experimental designs. CERMEÑO [1997] finds that in contexts with no exogenous regressors, KIVIET's approximation formula to the bias works quite well for an AR parameter value such as 0.5. However, for higher values, say 0.85, 0.95 or 0.99, KIVIET's bias correction method produces quite biased and imprecise results. In contrast, median-unbiased estimation can be implemented for a range of AR parameter values on the interval [– 1, 1]. Thus, highly persistent AR processes do not pose any problem for the method. In addition, the method can be applied to samples of any finite dimension. Obviously, this method only applies to dynamic panel data models with no exogenous regressors.

The rest of the paper is organized as follows. Section 2 extends ANDREWS' [1993] median unbiased estimation to a panel data context and explains its implementation. Section 3 explores the robustness of median-unbiased estimators to groupwise heteroskedasticity and cross-sectional correlation. The behavior of these estimators in the presence of exogenous regressors and AR parameter heterogeneity is also explored in this section. Section 4 presents an empirical application of the method to the controversial issue of conditional convergence. Finally, Section 5 concludes.

2 Median-Unbiased Estimation in Dynamic Panel Data

This section extends ANDREWS' [1993] median-unbiased estimation of first order autoregressive/unit root (AR/UR) time series models to panel data.This paper considers a two way dynamic error-components model or a dynamic-panel data model as described in the panel data literature, *e.g.* HSIAO [1986], or BALTAGI [1995]. Specifically, the case with fixed individual and time effects and no exogenous regressors is considered. Extension of ANDREWS' approach from time series to a dynamic panel of N cross-sections over $T+1$ periods is straightforward. It will suffice to show that the LSDV estimator (which is the OLS analogue in panel data models with fixed effects) is invariant to the individual specific effects, time specific effects (or time trend coefficients) and the variance of the innovations. It seems worthwhile to define the method itself and its implementation. A median-unbiased estimate is the value of the AR parameter for which the median of the distribution of the LSDV estimator (for the same sample dimensions) equals the actual LSDV estimate. This implies subtracting the median-bias from the actual LSDV estimate. For example, suppose that for a sample of dimensions $N = 57,\ T+1 = 41$, using model (3) given below, a LSDV estimate equal to 0.9172 is obtained. To obtain a median-unbiased estimate, this value is matched to the median (0.5^{th} quantile) of the distribution of the LSDV 1 (see Table A.1 in the Appendix), in order to find the AR parameter value whose median equals the actual LSDV estimate. A median of 0.9172 corresponds exactly to an AR = 0.99. Thus, the median-unbiased estimate equals 0.99 in this case. [1] In most cases, median-unbiased estimates are computed by linear extrapolation. Implementation of median-unbiased estimation requires, for a given sample size: (i) Computing the median (and other quantiles as well), of the distribution of the LSDV estimator for a grid of AR values. This will give a (unique) mapping between true AR values and medians of the LSDV estimator. (ii) Using the previous mapping to recover true AR (median-unbiased) values from actual LSDV estimates, as explained in the example given before.

1. ANDREWS [1993], p. 140 and MADDALA and KIM [1998], pp. 141-144 give similar examples for time series models.

Following ANDREWS' definition of models, consider the latent variable model

(1) $$y^*_{it} = \beta y^*_{it-1} + \nu_{it}, \qquad i = 1,\ldots,N\ , \ t = 1,\ldots,T,$$

where $\nu_{it} \sim iid(0,\sigma^2)$, $y^*_{i0} \sim (0,\sigma^2/(1-\beta^2))$ if $\beta \in]-1,1[$, and y^*_{io} is some constant or random variable if $\beta = 1$. [2] Thus, for each cross-section, the process given by (1) is strictly stationary with mean zero in the former case, and a random walk with arbitrary initial condition in the latter case. Define the following model for y_{it}, the observable variable:

(2) $$y_{it} = \mu_i + \lambda_t + y^*_{it} \qquad i = 1,\ldots,N\ , \ t = 1,\ldots,T$$

where μ_i and λ_t are individual and time specific effects respectively, which are assumed fixed. From (1) and (2) it can be obtained:

(3) $$y_{it} = \tilde{\mu}_i + \tilde{\lambda}_t + \beta y_{it-1} + \nu_{it}$$

where $\tilde{\mu}_i = \mu_i(1-\beta)$ and $\tilde{\lambda}_t = \lambda_t - \beta\lambda_{t-1}$. In the case where $\beta \in]-1,1[$, for each cross-section $(y_{it} - \lambda_t)$ will be a strictly stationary process with mean μ_i. In this case, $(y_{i0} - \lambda_0) \sim (\mu_i,\sigma^2/(1-\beta^2))$. For the case $\beta = 1$, $(y_{it} - \lambda_t)$ is a random walk process with arbitrary initial condition. Performing the Within transformation (WALLACE and HUSSAIN [1969]) on model (3) gives:

(4) $$\tilde{y}_{it} = \beta\tilde{y}_{it-1} + \tilde{\nu}_{it},$$

where $\tilde{y}_{it} = (y_{it} - \bar{y}_{i.} - \bar{y}_{.t} + \bar{y}_{..})$, $\tilde{y}_{it-1} = (y_{it-1} - \bar{y}_{i.,-1} - \bar{y}_{.t-1} + \bar{y}_{..,-1})$, $\tilde{\nu}_{it} = (\nu_{it} - \bar{\nu}_{i.} - \bar{\nu}_{.t} + \bar{\nu}_{..})$. For each transformed variable, the second, third and fourth terms are the individual (over time for each cross-section), cross-sectional (over cross-sections at a given time), and overall (over both cross-sections and time) means of the corresponding original variables. The OLS estimator applied to the transformed model (4) is known as Within group or LSDV estimator (LSDV1 in this paper). This is given by:

(5) $$\beta_{LSDV1} = \left(\sum_{i=1}^{N}\sum_{t=1}^{T}\tilde{y}_{it}\tilde{y}_{it-1}\right) / \left(\sum_{i=1}^{N}\sum_{t=1}^{T}\left(\tilde{y}_{it-1}\right)^2\right).$$

Since the Within transformation sweeps out both individual and time specific effects, β_{LSDV1} is independent of these effects, and so is its distribution. It can be also shown that β_{LSDV1} is invariant with respect to σ^2.

Consider the particular case in which the time specific effects take the form of a simple linear time trend, that is $\lambda_t = \theta t$. In this case, model (3) becomes

(6) $$y_{it} = \tilde{\mu}_i + \tilde{\theta}t + \beta y_{it-1} + \nu_{it},$$

where $\tilde{\mu}_i = \mu_i(1-\beta) + \theta\beta$, and $\tilde{\theta} = \theta\,(1-\beta)$. In the case $\beta \in]-1,1[$, for each cross-section, y_{it} is a strictly stationary process around a linear time trend with intercept μ_i and slope θ. The initial condition for each cross-

2. The normality assumption is not made since IMHOF's [1961] algorithm is not used in this paper. Instead, Monte Carlo simulations will be used.

section will be $y_{i0} \sim \left(\mu_i, \sigma^2 / \left(1-\beta^2\right)\right)$. For $\beta = 1, y_{it}$ is a random walk process with drift θ for each cross-section and with arbitrary initial condition. Subtracting individual means from (6) gives the Within transformation

(7) $$(y_{it} - \bar{y}_{i.}) = \tilde{\theta}\left(t - \bar{t}\right) + \beta\left(y_{it} - \bar{y}_{i.}\right) + \left(\nu_{it} - \bar{\nu}_{i.}\right),$$

where $\bar{t} = T\,(T+1)\,/2$. The OLS estimator of β in (7), called β_{LSDV2}, will also be evaluated in this paper.

A few remarks are in order here. First, since the Within transformation wipes out individual and time specific effects the previous results hold if these effects were random, as long as they are independent from each other and from y_{i0} and ν_{it}. Second, the invariance results also hold in the cases in which the time specific effects take the form of higher order time trends (*i.e.* quadratic), or if individual time effects or time trends are allowed for. Finally, the previous results apply in the case of one-way error-components models that exclude time specific effects. In this case $y_{it} = \mu_i + y^*_{it}$, and all previous invariance results are obvious. Only models given by (3) and (6) will be considered.

Quantiles of the LSDV1 and LSDV2 for a few relevant AR parameter values and sample sizes are shown in Table A.1 in the Appendix. They have been tabulated using Monte Carlo simulations with 10001 replications. Most sample sizes correspond to those of actual panels of per capita income that will be used in the empirical application later in Section 4. Monte Carlo simulations have been chosen instead of IMHOF's [1961] algorithm for practical reasons. The computational work has been made using GAUSS programs.

The previous results show, numerically, a monotonically increasing relationship between true AR parameter values and the median (and other quantiles as well) of the distribution of the LSDV estimators. Also, they show that the downward biases are sizable. In particular, for a given sample size, the bias becomes larger as the true AR coefficient approaches one. In the same way, the median (and mean) biases become larger and the 90 % confidence intervals become wider the shorter is the time dimension of the samples. For the sample dimensions considered, using LSDV estimators is likely to produce downward biased point estimates of the AR parameter. Moreover, those estimates may be consistent with stationary processes when in fact they are non-stationary.

3 Robustness of Median-Unbiased Estimators

This section explores the robustness of median-unbiased estimators to groupwise heteroskedasticity and cross-sectional correlation. This exercise seems to be necessary because these problems are likely to be present in practice. In addition, some simulation experiments are implemented in order to determine the behavior of median-unbiased estimators in the presence of exogenous regressors and AR parameter heterogeneity.

3.1. **Groupwise Heteroskedasticity and Cross-Sectional Correlation**

Table 1 describes the relevant design. The variance levels of the error term for each cross-section were obtained from the covariance matrix (diagonal elements) of actual samples of output per capita in logarithms, after removing individual and time specific effects. Notice that the highest ratio of maximum to minimum variances is quite large (124 times approximately). The values for the minimum and maximum off-diagonal elements of the correlation matrix of disturbances were obtained in the same way, but they were truncated to values between – 0.1 to + 0.1 approximately. Their average absolute value is in the range of 0.05 to 0.06.

In general, the results show that median-unbiased estimators are quite robust to the simultaneous presence of groupwise heteroskedaticity and cross-sectional correlation. Table A.2 in the Appendix, shows the relevant quantiles. In all cases, the median quantiles are practically unaffected. The 90 % intervals, though, are widened moderately. In particular, they become wider the shorter the time dimension of the samples. It should be noticed that ANDREWS [1993] median-unbiased estimators in time series are also found to be quite robust to non-*iid* and non-normal error structures.

TABLE 1
Design of Heteroskedasticity and Cross-Sectional Correlation

Cross-Sectional Dimensions	Ratio Max/Min Variance	Min/Max Cross Correlation
N = 13	9.02	– 0.10/+ 0.10
N = 48	37.83	– 0.11/+ 0.11
N = 57	106.31	– 0.11/+ 0.11
N = 100	123.77	– 0.11/+ 0.11

3.2. **Exogenous Regressors**

This section examines the behavior of the LSDV estimators in the presence of exogenous regressors. The potential over-correction of the bias that may arise by using median-unbiased estimation omitting exogenous regressors is also examined. The data is generated using models (3) and (6) including an exogenous regressor with a coefficient of one. This regressor has been generated from the uniform distribution on the interval (0,1) and is kept fixed across replications. The AR values considered are 0.5, 0.7, 0.9, and 0.99. Quantiles of the distribution of the LSDV estimators in the presence of exogenous regressors are reported on Table A.3 in the Appendix. The results are in line with those in SEVESTRE and TROGNON [1985] who, contrary to what NICKELL [1981] asserted, find that the bias of the LSDV estimator becomes less important when the model includes exogenous regressors. In general the biases are smaller than in the cases with no exogenous regressors (given in Table A.1), particularly when the AR parameter is high. However, for relatively small time dimensions, the biases are still sizable. Of course, their specific magnitudes will depend on the particular values of the exogenous regressors.

Table A.4 in the Appendix reports quantiles of the LSDV estimators when the exogenous regressor is omitted. In general, the median quantiles are lower compared to those obtained with the correctly specified model (Table A.3). However, they are much higher than the median quantiles obtained from models with no exogenous regressors given in Table A.1. Ignoring the presence of exogenous regressors may, therefore, result in an over correction of the bias, leading to an over estimation of the persistence of the dynamic process. For example, consider that the true model includes an exogenous regressor as designed in this study. For the case $N = 100$, $T + 1 = 31$ and the LSDV1 estimator, if the exogenous regressor is omitted, an estimate of 0.9 corresponds to a median-unbiased estimate of approximately 0.95 (Table A.4). However, ignoring the exogenous regressor and using Table A.1 to correct for the bias, will result in a median unbiased estimate of 1.

3.3. AR Parameter Heterogeneity

This section explores the behavior of median-unbiased estimators when AR parameter heterogeneity is allowed for. Instead of having a single parameter value common to all cross-sections, different values for each cross section are considered such that their mean equals a predetermined value. The AR values are drawn from a normal distribution and kept constant across replications. Then the biases (respect to the mean value of the AR parameters) of the LSDV estimators implemented under the assumption of AR parameter homogeneity are computed. Table 2 presents a brief description of the heterogeneity of AR values used in the simulations. Only the mean AR values 0.5, 0.7, and 0.9 are considered.

TABLE 2
Design of Heterogeneity of AR Parameter Values

Cross-Sectional Dimensions	Mean	Max	Min	Std. Dev.
N = 13	0.5	0.7071	0.3566	0.1246
	0.7	0.8186	0.5808	0.0764
	0.9	0.9674	0.8360	0.0416
N = 48	0.5	0.6572	0.2270	0.1056
	0.7	0.8520	0.4791	0.0837
	0.9	0.9988	0.7783	0.0420
N = 57	0.5	0.7910	0.2986	0.0934
	0.7	0.9644	0.4050	0.1088
	0.9	0.9905	0.7823	0.0413
N = 100	0.5	0.8690	0.2948	0.1009
	0.7	0.9302	0.4653	0.0928
	0.9	0.9697	0.8158	0.0312

Table A.5 in the Appendix shows the simulation results. For the cases of AR parameter heterogeneity considered here, the LSDV estimators implemented under the assumption of identical AR coefficients for all cross-sections, approximate reasonably well the mean value of these coefficients when $T + 1 = 120$. Similar results are obtained for $T + 1 = 63$, except when AR = 0.9 where the LSDV1 estimator is upward biased. In all other cases, the LSDV estimators are downward biased (respect to the mean of the AR coefficients). As in the case of AR parameter homogeneity, the absolute value of the bias is larger, the smaller the magnitude of *T* and the larger the (true) mean of the AR coefficients.

4 An Application to Conditional Convergence

The empirical work on convergence is controversial. The cross-section regression approach (as in BAUMOL [1986], BARRO [1991], BARRO and SALA-I-MARTIN [1992], MANKIW, ROMER and WEIL [1992], among others) has been criticized on several aspects such as the wasting of useful information across time by averaging growth rates and its underlying assumption of homogeneity across countries. More seriously, the uniform finding that economies converge at the rate of about 2 percent per year almost every where can be shown as evidence that this statistical apparatus is flawed (QUAH [1993a, 1993b]). EVANS [1996, 1997] shows formally that the cross-section regression approach in fact produces biased results.

Several alternative approaches have been proposed since then. EVANS [1994, 1996, 1997, 1998] and EVANS and KARRAS [1996a, 1996b] have proposed several methods that exploit the cross-sectional and/or time series dimensions of the data to test endogenous against exogenous growth and to estimate growth regressions consistently. In general, they find evidence supporting exogenous growth theories in samples such as the USA states, 13 OECD countries and a 48-country sample from the Penn World Tables. They find conditional convergence, in the sense of economies having parallel balanced growth paths. Absolute convergence does not hold even in the case of the USA states.

On the other hand, studies of wider and more heterogeneous samples of economies have produced mixed results. LEE, PESARAN and SMITH [1995] using a unit root approach cannot reject the hypothesis of non-stationary output processes in 3 samples of countries. This result is in spite of their high time series estimates of convergence rates (about 20 % per year), which they attribute to highly upward biases in their point estimates. In contrast, ISLAM [1995] using the same 98-country sample as in MANKIW, ROMER and WEIL [1992] finds much faster conditional converge rates if a dynamic panel data model is used instead of a cross-section or pooled regression model. He uses the Minimum Distance (MD) and Least Squares Dummy Variable (LSDV) estimators obtaining similar results. His results, though, might be significantly biased in favor of high convergence rates since he uses data spaced five years apart, which results in a very short time dimension of the sample (only 5 points in time). On the other hand, LEE, PESARAN and SMITH [1998] and MADDALA and WU [1997] have pointed out that imposing the restriction that economies have identical autoregressive and time trend parameters, can produce very misleading results on convergence.

This section uses median-unbiased estimators in panel data to evaluate conditional convergence in the sense of economies having parallel balanced growth paths. It is found here, that even when the assumption of equal autoregressive coefficients and common trends is imposed *a priori*, convergence in the sense given previously is likely to happen only in samples of relatively homogeneous countries once the downward biases are corrected. Four panels of yearly per capita income are studied. They include 48 USA states for the period 1929-1991, 13 OECD countries during 1870-1989, 57 countries over the period 1950-1990 and 100 countries, including the previous 57 countries, during 1960-1990. The last two samples are taken from SUMMERS and HESTON's Penn World Tables (PWT), version 5.6. Additional information on these samples is provided in the Appendix.

Models (3) and (6) given before are used. These models are useful to characterize whether deviations of per capita output of economies around a common trend are stationary or not. In the first case, the dynamics of output per capita of economies will be consistent with conditional convergence. Even though conditional convergence can be taken as evidence in favor of exogenous growth models, it should be pointed out a similar output dynamics would be consistent with a technological imitation mechanism as in endogenous growth models. Interestingly, an endogenous growth model that explicitly takes into account the interdependence among economies will also predict conditional convergence, *i.e.* see HOWITT and AGHION [1998]. Thus, the distinction among exogenous and endogenous growth models could not be made on the basis of convergence results only.

The results are shown in Table 3. The estimates of the AR coefficient labeled LSDV1 and LSDV2 correspond to models given by (3) and (6) respectively. Uncorrected estimates are obtained using the actual data samples. The corresponding median-unbiased estimates have been obtained by correcting the actual LSDV for the median-bias, as explained in section 2.They are computed using the quantiles under groupwise heteroskedasticity and cross-sectional correlation since these problems are presumed to be present in the actual data. Also, median-unbiased estimates of the AR coefficient from quantiles under *iid* errors are reported for comparison. The quantiles used for the median-bias correction are not reported but they are available upon request. They have been computed for a more concentrated grid of parameter values (between 0.81 and 1) using 20001 replications.

The implied rates of convergence are reported in both cases. They are approximately equal to one minus the AR parameter value. The rates reported in parenthesis in the last column are obtained using the 0.05^{th} and 0.95^{th} quantiles, and give the lower and upper values of the corresponding 90 % confidence intervals. In most cases the median-unbiased point estimates and confidence intervals have been computed by linear interpolation. It can be seen that the uncorrected LSDV1 and LSDV2 estimates are consistent with conditional convergence in all samples. However, after the median-bias correction, conditional convergence holds only in the cases of the USA states and OECD countries, and takes place at much slower rates than the ones implied by the uncorrected estimates. In the case of the PWT samples of countries, the median-unbiased estimates do not support conditional convergence.

TABLE 3
Uncorrected and Median-Unbiased Estimates of AR Coefficients

Sample (N,T+1)	Uncorrected		Median-Unbiased	
Estimator	Estimates	Implied Rate of Converg.	Estimates	Implied Rate of Converg.
US States (48,63)				
LSDV 1	0.9109	9.3	0.9511	5.0 (2.6-7.5)
	(0.0117)		0.9505	5.1 (3.3-6.8)
LSDV 2	0.8811	12.7	0.9188	8.5 (5.9-11.3)
	(0.0131)		0.9177	8.9 (7.3-9.4)
OECD (13,120)				
LSDV 1	0.9576	4.3	0.9792	2.1 (0.0-4.2)
	(0.0112)		0.9788	2.1 (1.0-3.0)
LSDV2	0.9686	3.2	0.9938	0.6 (0.0-2.4)
	(0.0111)		0.9938	0.6 (0.1-1.0)
PWT-1 (57,41)				
LSDV 1	0.9569	4.4	1.0000	0.0
	(0.0098)		1.0000	0.0
LSDV 2	0.9555	4.6	1.0000	0.0
	(0.0099)		1.0000	0.0
PWT-2 (100,31)				
LSDV 1	0.9537	4.7	1.0000	0.0
	(0.0088)		1.0000	0.0
LSDV 2	0.9533	4.8	1.0000	0.0
	(0.0088)		1.0000	0.0

* In the last two columns and for each estimator, the values in the first row are computed using quantiles under groupwise heteroskedasticity and cross-sectional correlation. The values in the second row correspond to quantiles under *iid* errors. Numbers in parenthesis below uncorrected estimates are autocorrelation and heteroskedasticity consistent standard errors.

5 Conclusion

This paper has implemented median-unbiased estimation in dynamic panel data models. First, it has been shown that ANDREWS' [1993] approach applies straightforwardly to LSDV estimators in dynamic fixed effects models with no exogenous regressors. Second, median-unbiased LSDV estimators have been found to be quite robust to groupwise heteroskedastic and cross-sectional correlated innovations. Relative to alternative estimation or bias-correction techniques, the method has the advantage that it can be applied to panels of any finite cross-sectional and time dimensions and with highly persistent dynamics as would be the case of most cross-country panels.

Concerning the possible omission of exogenous regressors, it has been found that application of median-unbiased estimation may over-correct the downward bias, leading to an over-estimation of the AR parameter value. Also this study has found that (at least for the cases considered) ignoring AR parameter heterogeneity leads to downward biased (respect to the mean of individual AR values) LSDV point estimators, except in those cases where $T + 1$ is relatively high (63 and 120).

The empirical application has found that unadjusted LSDV estimators would be consistent with conditional convergence in the four samples studied. Median-unbiased estimates, however, support conditional convergence, only for the USA states and OECD samples, and at much slower rates than unadjusted LSDV estimates would imply.

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APPENDIX

About the Data

The USA states sample includes real per capita personal income of 48 contiguous states over the period 1929-1991. The data source is The U.S. Department of Commerce. See EVANS and KARRAS [1996a] for details.

The OECD sample includes per capita real gross domestic product during the period 1870-1989 for Australia, Austria, Belgium, Canada, Denmark, Finland, France, Italy, Norway, Sweden, United Kingdom, United States and West Germany. The Data source is Angus MADDISON [1991]. See EVANS [1996, 1998] for details.

The other two samples include per capita gross domestic product in constant international prices (RGDPCH) from the Penn World Tables 5.6 by SUMMERS and HESTON [1991, 1995]. Countries with complete information over the periods 1950-1990 and 1960-1990, except oil and centrally planned countries were selected. The PWT-1 sample includes 57 countries. They are Egypt, Kenya, Mauritius, Morocco, Nigeria, South Africa, Uganda, Canada, Costa Rica, Dominican Republic, El Salvador, Guatemala, Honduras, Mexico, Nicaragua, Panama, Trinidad and Tobago, The United States, Argentina, Bolivia, Brazil, Chile, Colombia, Ecuador, Guyana, Paraguay, Peru, Uruguay, Venezuela, India, Japan, Pakistan, Philippines, Sri-Lank, Thailand, Austria, Belgium, Cyprus, Denmark, Finland, France, West Germany, Greece, Iceland, Ireland, Italy, Luxembourg, Netherlands, Norway, Portugal, Spain, Sweden, Switzerland, Turkey, United Kingdom, Australia, New Zealand.

The PWT-2 sample has 100 countries including (in addition to the previous 57 countries): Algeria, Benin, Burkina Faso, Burundi, Cameroon, Cape Verde, Central, Chad, Comoros, Congo, Gabon, Gambia, Ghana, Guinea, Guinea Bissau, Ivory Coast, Lesotho, Madagascar, Malawi, Mali, Mauritania, Mozambique, Namibia, Rwanda, Senegal, Seychelles, Togo, Tunisia, Zambia, Zimbabwe, Jamaica, Bangladesh, Hong Kong, Indonesia, Israel, Jordan, Korea, Malaysia, Singapore, Syria, Taiwan, Fiji, Papua New Guinea.

TABLE A.1
Quantiles of LSDV Estimators of the AR Coefficient

Estimator	LSDV 1			LSDV 2		
AR/Quantile	0.05	0.5	0.95	0.05	0.5	0.95
N = 13, T + 1 = 120						
0.50	0.4476	0.4872	0.5242	0.4480	0.4861	0.5214
0.70	0.6514	0.6845	0.7159	0.6513	0.6836	0.7138
0.90	0.8582	0.8817	0.9022	0.8578	0.8807	0.9004
0.99	0.9503	0.9660	0.9784	0.9493	0.9648	0.9771
1.00	0.9590	0.9740	0.9849	0.9578	0.9727	0.9840
N = 48, T + 1 = 63						
0.50	0.4475	0.4751	0.5019	0.4471	0.4747	0.5010
0.70	0.6476	0.6711	0.6940	0.6472	0.6704	0.6930
0.90	0.8471	0.8646	0.8804	0.8463	0.8640	0.8797
0.99	0.9301	0.9436	0.9554	0.9295	0.9430	0.9548
1.00	0.9384	0.9512	0.9625	0.9374	0.9504	0.9617
N = 57, T + 1 = 41						
0.50	0.4286	0.4608	0.4919	0.4284	0.4601	0.4911
0.70	0.6258	0.6540	0.6811	0.6254	0.6533	0.6800
0.90	0.8202	0.8422	0.8624	0.8194	0.8413	0.8615
0.99	0.8988	0.9172	0.9334	0.8978	0.9164	0.9326
1.00	0.9063	0.9245	0.9400	0.9059	0.9237	0.9396
N = 100, T + 1 = 31						
0.50	0.4199	0.4479	0.4756	0.4195	0.4474	0.4749
0.70	0.6136	0.6382	0.6617	0.6130	0.6375	0.6611
0.90	0.8000	0.8206	0.8397	0.7994	0.8200	0.8391
0.99	0.8747	0.8925	0.9086	0.8741	0.8919	0.9080
1.00	0.8825	0.8997	0.9152	0.8819	0.8992	0.9148
N = 100, T + 1 = 21						
0.50	0.3843	0.4197	0.4545	0.3837	0.4190	0.4533
0.70	0.5705	0.6035	0.6347	0.5702	0.6027	0.6338
0.90	0.7482	0.7760	0.8018	0.7474	0.7751	0.8009
0.99	0.8187	0.8431	0.8654	0.8169	0.8423	0.8646
1.00	0.8244	0.8497	0.8724	0.8236	0.8489	0.8711
N = 100, T + 1 = 11						
0.50	0.2790	0.3313	0.3803	0.2776	0.3300	0.3791
0.70	0.4431	0.4929	0.5395	0.4416	0.4916	0.5381
0.90	0.5911	0.6379	0.6810	0.5897	0.6364	0.6793
0.99	0.6510	0.6954	0.7362	0.6492	0.6941	0.7346
1.00	0.6580	0.7014	0.7426	0.6566	0.7001	0.7404

TABLE A.2

Quantiles under Heteroskedasticity and Cross-Correlation

Estimator	LSDV 1			LSDV 2		
AR/Quantile	0.05	0.5	0.95	0.05	0.5	0.95
N = 13, T + 1 = 120						
0.50	0.4421	0.4865	0.5291	0.4432	0.4859	0.5265
0.70	0.6480	0.6849	0.7203	0.6481	0.6840	0.7186
0.90	0.8555	0.8815	0.9052	0.8552	0.8805	0.9036
0.99	0.9487	0.9656	0.9793	0.9477	0.9644	0.9782
1.00	0.9569	0.9735	0.9862	0.9560	0.9725	0.9852
N = 48, T + 1 = 63						
0.50	0.4362	0.4749	0.5124	0.4361	0.4746	0.5117
0.70	0.6373	0.6706	0.7032	0.6368	0.6700	0.7025
0.90	0.8398	0.8638	0.8868	0.8394	0.8633	0.8861
0.99	0.9246	0.9426	0.9594	0.9241	0.9421	0.9588
1.00	0.9332	0.9505	0.9666	0.9327	0.9499	0.9661
N = 57, T + 1 = 41						
0.50	0.4091	0.4602	0.5118	0.4085	0.4597	0.5108
0.70	0.6080	0.6532	0.6974	0.6074	0.6525	0.6966
0.90	0.8067	0.8404	0.8735	0.8062	0.8398	0.8728
0.99	0.8876	0.9156	0.9428	0.8868	0.9149	0.9421
1.00	0.8957	0.9231	0.9491	0.8951	0.9224	0.9483
N = 100, T + 1 = 31						
0.50	0.3969	0.4472	0.4077	0.3966	0.4467	0.4971
0.70	0.5925	0.6365	0.6814	0.5924	0.6360	0.6805
0.90	0.7841	0.8189	0.8538	0.7835	0.8184	0.8531
0.99	0.8610	0.8907	0.9196	0.8604	0.8901	0.9191
1.00	0.8692	0.8979	0.9263	0.8687	0.8973	0.9258
N = 100, T + 1 = 21						
0.50	0.3565	0.4188	0.4814	0.3562	0.4182	0.4805
0.70	0.5461	0.6020	0.6593	0.5455	0.6013	0.6585
0.90	0.7269	0.7737	0.8208	0.7262	0.7729	0.8201
0.99	0.7982	0.8403	0.8830	0.7974	0.8395	0.8822
1.00	0.8062	0.8470	0.8890	0.8053	0.8462	0.8882
N = 100, T + 1 = 11						
0.50	0.2378	0.3293	0.4189	0.2368	0.3281	0.4176
0.70	0.4062	0.4904	0.5766	0.4054	0.4888	0.5755
0.90	0.5568	0.6343	0.7134	0.5556	0.6330	0.7122
0.99	0.6151	0.6908	0.7675	0.6138	0.6895	0.7659
1.00	0.6231	0.6973	0.7700	0.6216	0.6959	0.7689

TABLE A.3
Quantiles of LSDV Estimators with Exogenous Regressors

Estimator	LSDV 1			LSDV 2		
AR/Quantile	0.05	0.5	0.95	0.05	0.5	0.95
N = 13, T + 1 = 120						
0.50	0.4498	0.4881	0.5242	0.4511	0.4873	0.5211
0.70	0.6534	0.6859	0.7162	0.6540	0.6852	0.7142
0.90	0.8620	0.8840	0.9030	0.8655	0.8852	0.9022
0.99	0.9746	0.9822	0.9885	0.9787	0.9843	0.9891
N = 48, T + 1 = 63						
0.50	0.4508	0.4771	0.5026	0.4506	0.4767	0.5017
0.70	0.6512	0.6739	0.6953	0.6518	0.6739	0.6950
0.90	0.8551	0.8708	0.8850	0.8598	0.8740	0.8875
0.99	0.9653	0.9723	0.9787	0.9697	0.9754	0.9807
N = 57, T + 1 = 41						
0.50	0.4337	0.4642	0.4939	0.4335	0.4638	0.4931
0.70	0.6320	0.6585	0.6839	0.6322	0.6583	0.6836
0.90	0.8343	0.8541	0.8719	0.8394	0.8577	0.8743
0.99	0.9466	0.9571	0.9664	0.9524	0.9613	0.9695
N = 100, T + 1 = 31						
0.50	0.4255	0.4521	0.4785	0.4255	0.4520	0.4782
0.70	0.6204	0.6441	0.6664	0.6219	0.6452	0.6670
0.90	0.8184	0.8369	0.8537	0.8286	0.8450	0.8606
0.99	0.9253	0.9367	0.9473	0.9405	0.9492	0.9575
N = 100, T + 1 = 21						
0.50	0.3932	0.4272	0.4604	0.3931	0.4270	0.4600
0.70	0.5850	0.6151	0.6448	0.5871	0.6166	0.6456
0.90	0.7835	0.8074	0.8295	0.7885	0.8115	0.8326
0.99	0.8895	0.9064	0.9219	0.8896	0.9054	0.9198
N = 100, T + 1 = 11						
0.50	0.2973	0.3469	0.3949	0.2982	0.3472	0.3949
0.70	0.4708	0.5180	0.5615	0.4742	0.5209	0.5640
0.90	0.6466	0.6878	0.7269	0.6502	0.6902	0.7277
0.99	0.7295	0.7668	0.8009	0.7257	0.7618	0.7943

TABLE A.4
Quantiles of LSDV Estimators Omitting Exogenous Regressors

Estimator	LSDV 1			LSDV 2		
AR/Quantile	0.05	0.5	0.95	0.05	0.5	0.95
N = 13, T + 1 = 120						
0.50	0.4451	0.4844	0.5213	0.4471	0.4845	0.5196
0.70	0.6492	0.6825	0.7137	0.6503	0.6825	0.7124
0.90	0.8590	0.8819	0.9014	0.8629	0.8832	0.9005
0.99	0.9740	0.9816	0.9879	0.9780	0.9836	0.9884
N = 48, T + 1 = 63						
0.50	0.4501	0.4773	0.5036	0.4495	0.4769	0.5029
0.70	0.6502	0.6739	0.6957	0.6504	0.6736	0.6951
0.90	0.8541	0.8703	0.8850	0.8584	0.8730	0.8865
0.99	0.9646	0.9715	0.9779	0.9690	0.9747	0.9800
N = 57, T + 1 = 41						
0.50	0.4308	0.4623	0.4930	0.4303	0.4615	0.4919
0.70	0.6298	0.6568	0.6834	0.6296	0.6565	0.6827
0.90	0.8322	0.8526	0.8705	0.8373	0.8560	0.8730
0.99	0.9456	0.9562	0.9658	0.9510	0.9601	0.9683
N = 100, T + 1 = 31						
0.50	0.4233	0.4512	0.4781	0.4226	0.4503	0.4772
0.70	0.6188	0.6429	0.6661	0.6195	0.6430	0.6655
0.90	0.8176	0.8363	0.8533	0.8269	0.8436	0.8595
0.99	0.9250	0.9367	0.9474	0.9399	0.9488	0.9570
N = 100, T + 1 = 21						
0.50	0.3925	0.4277	0.4618	0.3918	0.4264	0.4603
0.70	0.5842	0.6153	0.6453	0.5852	0.6156	0.6451
0.90	0.7824	0.8069	0.8291	0.7870	0.8102	0.8317
0.99	0.8883	0.9054	0.9208	0.8885	0.9045	0.9188
N = 100, T + 1 = 11						
0.50	0.2938	0.3451	0.3944	0.2946	0.3447	0.3942
0.70	0.4683	0.5168	0.5619	0.4723	0.5201	0.5646
0.90	0.6450	0.6869	0.7265	0.6492	0.6906	0.7286
0.99	0.7285	0.7667	0.8002	0.7263	0.7629	0.7962

TABLE A.5

Quantiles of LSDV under AR Parameter Heterogeneity

Estimator	LSDV 1			LSDV 2		
Mean AR/Quantile	0.05	0.5	0.95	0.05	0.5	0.95
N = 13, T + 1 = 120						
0.50	0.4648	0.5076	0.5478	0.4649	0.5066	0.5446
0.70	0.6649	0.7001	0.7327	0.6651	0.6990	0.7303
0.90	0.8748	0.9015	0.9263	0.8759	0.8998	0.9216
N = 48, T + 1 = 63						
0.50	0.4596	0.4886	0.5158	0.4580	0.4867	0.5138
0.70	0.6721	0.6981	0.7232	0.6657	0.6916	0.7162
0.90	0.9360	0.9515	0.9640	0.8960	0.9159	0.9338
N = 57, T + 1 = 41						
0.50	0.4385	0.4722	0.5054	0.4377	0.4713	0.5041
0.70	0.6557	0.6915	0.7321	0.6623	0.6985	0.7402
0.90	0.8345	0.8586	0.8810	0.8534	0.8748	0.8954
N = 100, T + 1 = 31						
0.50	0.4431	0.4752	0.5111	0.4320	0.4621	0.4926
0.70	0.6641	0.6954	0.7263	0.6375	0.6655	0.6932
0.90	0.8413	0.8617	0.8797	0.8297	0.8490	0.8667
N = 100, T + 1 = 21						
0.50	0.3938	0.4311	0.4679	0.3962	0.4335	0.4706
0.70	0.5830	0.6177	0.6504	0.5927	0.6274	0.6601
0.90	0.7499	0.7785	0.8045	0.7741	0.8009	0.8250
N = 100, T + 1 = 11						
0.50	0.2967	0.3511	0.4058	0.2862	0.3391	0.3902
0.70	0.4759	0.5289	0.5798	0.4590	0.5103	0.5597
0.9	0.6039	0.6515	0.6944	0.6111	0.6575	0.6995