ANNALES D'ÉCONOMIE ET DE STATISTIQUE. – N° 55-56 – 1999

Moment Conditions for Dynamic Panel Data Models with Multiplicative Individual Effects in the Conditional Variance

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ABSTRACT. – Moment conditions are derived for dynamic linear panel data models with linear individual specific effects in the mean and multiplicative individual effects in the conditional ARCH type variance function. The relation and correlation between the linear and multiplicative effects are unrestrained. Moment conditions are derived for non-autocorrelated error processes, *MA*(*q*) processes, and for models that allow for time varying parameters on both the linear mean effects and multiplicative variance effects. The small sample performance of a GMM estimator is investigated in a Monte Carlo simulation study.

Conditions de moments pour des modèles de panel dynamiques avec effets individuels multiplicatifs dans la variance conditionnelle

RÉSUMÉ. – Les conditions de moment sont obtenues pour des modèles de panel dynamiques avec des effets individuels linéaires dans la moyenne et multiplicatifs dans la fonction de variance conditionnelle de type ARCH. La relation et la corrélation entre les effets linéaires et multiplicatifs sont libres. Les conditions de moment sont obtenues dans le cas d'erreurs non-autocorrélées, d'erreurs satisfaisant une représentation *MA*(*q*), et pour des modèles qui autorisent des paramètres fonction du temps dans les effets linéaires dans la moyenne et multiplicatifs dans la variance. Les propriétés de petits échantillons d'un estimateur de la méthode des Moments Généralisée sont étudiées dans un exercice de simulation de Monte-Carlo.

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This research is part of the program of the ESRC Centre for the Micro-Economic Analysis of Fiscal Policy at IFS. The financial support of the ESRC is gratefully acknowledged.

1 Introduction

The estimation of dynamic processes using panel data with a large number of individuals but with a fixed number of time periods, and allowing for linear unobserved individual effects in the mean, are commonplace nowadays. The Generalized Method of Moments (GMM) estimator, see HANSEN [1982], has been developed for dynamic panel data models by HOLTZ-EAKIN, NEWEY and ROSEN [1988], ARELLANO and BOND [1991], ARELLANO and BOVER [1995], AHN and SCHMIDT [1995], BLUNDELL and BOND [1998] and others.

In many applications it is important to model the higher order moments of the dynamic process, like the variance. An example is a model for income dynamics and uncertainty where it is likely that persons at different levels of the income distribution face a different variance of their time-income profile. As with the mean, it is further likely that unobserved individual attributes are important factors for the determination of this variance. One way of modelling this is to specify the dynamic variance process as an ARCH type variance with multiplicative individual effects. ARELLANO [1995] considered such processes, but restricted the multiplicative effects in the variance to be the square of the linear individual effects in the mean.

In this paper we relax this assumption and allow the multiplicative variance effects to be different from the linear mean effects. We derive conditional moment conditions for the parameters in the variance function for estimation by GMM.

In section 2 the basic dynamic autoregressive model with conditional heteroscedasticity is presented, and the moment conditions for the variance parameters are derived. Section 3 extends the analysis for a model with an MA error process, and section 4 considers the case where the individual mean effects are interacted with time effects, as in HOLTZ-EAKIN, NEWEY and ROSEN [1982]. In section 5 we present some Monte Carlo simulation results. Section 6 concludes.

2 Model and Moment Conditions

The panel consists of N individuals (denoted by i) and T time periods (denoted by t). The number of time periods is fixed and consistent estimation relies on a large cross sectional dimension N.

Consider the panel data AR(1) model with individual effects f_i

$$y_{it} = \alpha y_{it-1} + v_{it} \tag{2.1}$$

$$v_{it} = f_i + u_{it}. \tag{2.2}$$

The error process u_{it} has conditional mean zero, but its variance is conditionally heteroscedastic, dependent on the past values of the dependent variable and individual specific effects m_i,

$$E(u_{it} \mid f_i, m_i, y_i^{t-s}) = 0 \tag{2.3}$$

$$E(u_{it}^2 \mid f_i, m_i, y_i^{t-1}) = \sigma_t^2(y_i^{t-1}, \gamma) m_i \tag{2.4}$$

$$E(u_{it} u_{it-s} \mid f_i, m_i, y_i^{t-s-1}) = 0, \tag{2.5}$$

for $s > 0$, and where $y_i^{t-1} = (y_{i1}, \ldots, y_{it-1})$ and $m_i > 0$.

Since T is fixed, no consistent estimates of f_i or m_i can be obtained. To estimate α and γ consistently we can derive suitable orthogonality conditions. Such conditions for the estimation of α have been derived by HOLTZ-EAKIN, NEWEY and ROSEN [1988], ARELLANO and BOND [1991] and AHN and SCHMIDT [1995] who apply HANSEN's [1982] Generalised Method of Moments (GMM) estimator. ARELLANO [1995] has shown how to estimate the coefficients γ under the assumption that $m_i = f_i^2$.

In this and the next two sections, we will focus on the derivation of moment conditions for the estimation of γ, assuming that α is known. Joint estimation of α and γ is considered in the Monte Carlo experiments as presented in section 5.

To derive the moment conditions for the estimation of γ, consider the differenced disturbance $v_{it+1} - v_{it} = u_{it+1} - u_{it}$, which is independent of the linear effects f_i. The correlation of v_{it} with $v_{it+1} - v_{it}$ is given by

$$\begin{aligned} E\left[v_{it}(v_{it+1} - v_{it}) \mid f_i, m_i, y_i^{t-1}\right] &= E\left[(f_i + u_{it})(u_{it+1} - u_{it}) \mid f_i, m_i, y_i^{t-1}\right] \\ &= -E\left[u_{it}^2 \mid f_i, m_i, y_i^{t-1}\right] \\ &= -\sigma_t^2(y_i^{t-1}, \gamma) m_i, \end{aligned} \tag{2.6}$$

utilising the assumptions (2.3), (2.4) and (2.5). Let

$$r_{it}(\gamma) = \frac{v_{it}(v_{it+1} - v_{it})}{\sigma_t^2(y_i^{t-1}, \gamma)},$$

then it follows from (2.6) that

$$E\left[r_{it}(\gamma) \mid f_i, m_i, y_i^{t-1}\right] = -m_i.$$

Next, define $r_{it-1}(\gamma)$ as

$$r_{it-1}(\gamma) = \frac{v_{it-1}(v_{it} - v_{it-1})}{\sigma_{t-1}^2(y_i^{t-2}, \gamma)},$$

and consider the conditional expectation of the first difference $r_{it}(\gamma) - r_{it-1}(\gamma)$ conditional on information up to time $t-2$,

$$E\left[\left(r_{it}(\gamma) - r_{it-1}(\gamma)\right)|y_i^{t-2}\right] =$$

$$E_{f,m}\left[E\left[\left(r_{it}(\gamma) - r_{it-1}(\gamma)\right)|\, f_i, m_i, y_i^{t-2}\right]\right] \tag{2.7}$$

using the law of iterated expectations, and further

$$\begin{aligned} & E\left[\left(r_{it}(\gamma) - r_{it-1}(\gamma)\right)|\, f_i, m_i, y_i^{t-2}\right] \\ =& E\left[\left(E[r_{it}(\gamma)|\, f_i, m_i, y_i^{t-1}] - r_{it-1}(\gamma)\right)|\, f_i, m_i, y_i^{t-2}\right] \\ =& -m_i + m_i = 0, \end{aligned} \tag{2.8}$$

using the fact that $y_i^{t-2} \subset y_i^{t-1}$.

The combination of (2.7) and (2.8) gives the desired moment conditions, which are summarised in the following lemma:

LEMMA 2.1: In the model defined by (2.1) and (2.2) with assumptions (2.3), (2.4) and (2.5), conditional moment restrictions for the estimation of γ are given by

$$E\left[\left(\frac{v_{it}(v_{it+1} - v_{it})}{\sigma_t^2(y_i^{t-1}, \gamma)} - \frac{v_{it-1}(v_{it} - v_{it-1})}{\sigma_{t-1}^2(y_i^{t-2}, \gamma)}\right)|y_i^{t-2}\right] = 0. \tag{2.9}$$

Using these moment conditions, the coefficients γ can be estimated by GMM as in HANSEN [1982]. In principle, all moment conditions of the type

$$E\left[\left(\frac{v_{it}(v_{it+s} - v_{it})}{\sigma_t^2(y_i^{t-1}, \gamma)} - \frac{v_{it-1}(v_{it-1+l} - v_{it-1})}{\sigma_{t-1}^2(y_i^{t-2}, \gamma)}\right)|y_i^{t-2}\right] = 0$$

are valid, for $s = 1, \ldots, T - t$; $l = 1, \ldots, T - t + 1$. However, the extra moment conditions are not informative for γ and are implied by

$$E\left[v_{it}\left(v_{it-1} - v_{it-2}\right)\right] = 0,$$

which together with the moment conditions

$$E\left[y_{is}\left(v_{it} - v_{it-1}\right)\right] = 0 \;\; ; \;\; s = 1, \ldots, t-2,$$

form the set of moment conditions for the estimation of α under assumptions (2.3) and (2.5), see AHN and SCHMIDT [1995].

From (2.9), when multiplied through by $\sigma_{t-1}^2(y_i^{t-2}, \gamma)$, it follows that the following moment conditions are also valid

$$E\left[\left(v_{it}(v_{it+1} - v_{it})\frac{\sigma_{t-1}^2(y_i^{t-2}, \gamma)}{\sigma_t^2(y_i^{t-1}, \gamma)} - v_{it-1}(v_{it} - v_{it-1})\right)|y_i^{t-2}\right] = 0, \tag{2.10}$$

and (2.9) and (2.10) are similar in spirit to the moment conditions derived for models with predetermined variables and multiplicative fixed effects in the mean by WOOLDRIDGE [1997] and CHAMBERLAIN [1992] respectively.

When $m_i = f_i^2$, the second moment of v_{it} is given by

$$E\left[v_{it}^2 | f_i, y_i^{t-1}\right] = \left(1 + \sigma_t^2(y_i^{t-1}, \gamma)\right) f_i^2,$$

and the moment conditions for the estimation of γ are

$$E\left[\left(\frac{v_{it}^2}{\left(1 + \sigma_t^2(y_i^{t-1}, \gamma)\right)} - \frac{v_{it-1}^2}{\left(1 + \sigma_{t-1}^2(y_i^{t-2}, \gamma)\right)}\right) | y_i^{t-2}\right] = 0,$$

see ARELLANO [1995].

There are various specifications possible for the variance function. For example, exponential ARCH type specifications with asymmetric response can be specified as

$$\sigma_{it}^2 = exp\left(\gamma_0 + \gamma_1 v_{it-1} + \gamma_2 v_{it-1}^2\right).$$

3 MA Errors

In the preceding section, the error process u_{it} was not correlated over time. In this section we derive moment conditions for the estimation of γ when the error process has an MA error structure. [1]

The $MA(q)$ error process is given by

$$v_{it} = f_i + u_{it} + \theta_1 u_{it-1} + \theta_2 u_{it-2} + \ldots + \theta_q u_{it-q}, \tag{3.1}$$

where the u_{it} satisfy the conditions (2.3), (2.4) and (2.5). As

$$E\left[v_{it}\left(v_{it+q+1} - v_{it+q}\right)\right] = -\theta_q E\left[u_{it}^2\right],$$

Lemma 2.1 is easily modified to deal with $MA(q)$ errors. The result is stated in the next lemma.

LEMMA 3.1: In the model defined by (2.1) and (3.1) with assumptions (2.3), (2.4) and (2.5), conditional moment restrictions for the estimation of γ are given by

$$E\left[\left(\frac{v_{it}(v_{it+q+1} - v_{it+q})}{\sigma_t^2(y_i^{t-1}, \gamma)} - \frac{v_{it-1}(v_{it+q} - v_{it+q-1})}{\sigma_{t-1}^2(y_i^{t-2}, \gamma)}\right) | y_i^{t-2}\right] = 0.$$

1. The case of autocorrelated errors can be more easily dealt with by adding lags of the dependent variable.

4 Individual Effects interacted with Time Effects

HOLTZ-EAKIN, NEWEY and ROSEN [1982] specified a dynamic model where the individual specific linear effects in the mean were allowed to have time specific coefficients. The model is specified as

$$y_{it} = \alpha y_{it-1} + \phi_t f_i + u_{it}, \tag{4.1}$$

and a set of moment conditions for the estimation of α are given by

$$E\left[\left(v_{it} - \frac{\phi_t}{\phi_{t-1}} v_{it-1}\right) | y_i^{t-2}\right] = 0.$$

If the variance process is the same as specified in section 1, *i.e.* the individual specific multiplicative effects are constant over time, then the moment conditions (2.9) can be adjusted straightforwardly to allow for the time varying mean effects. As

$$E\left[v_{it}\left(\frac{\phi_t}{\phi_{t+1}} v_{it+1} - v_{it}\right)\right] = -E\left[u_{it}^2\right],$$

the moment conditions for γ are:

LEMMA 4.1: In the model defined by (4.1) with assumptions (2.3), (2.4) and (2.5), conditional moment restrictions for the estimation of γ are given by

$$E\left[\left(\frac{v_{it}\left(\frac{\phi_t}{\phi_{t+1}} v_{it+1} - v_{it}\right)}{\sigma_t^2(y_i^{t-1}, \gamma)} - \frac{v_{it-1}\left(\frac{\phi_{t-1}}{\phi_t} v_{it} - v_{it-1}\right)}{\sigma_{t-1}^2(y_i^{t-2}, \gamma)}\right) | y_i^{t-2}\right] = 0.$$

When the multiplicative variance effects are further also allowed to vary over time, and the conditional variance function is specified as

$$E(u_{it}^2 | f_i, m_i, y_i^{t-1}) = \sigma_t^2(y_i^{t-1}, \gamma) m_i \delta_t^2, \tag{4.2}$$

then the moment conditions are:

LEMMA 4.2: In the model defined by (4.1) with assumptions (2.3), (4.2) and (2.5), conditional moment restrictions for the estimation of γ are given by

$$E\left[\left(\frac{v_{it}\left(\frac{\phi_t}{\phi_{t+1}} v_{it+1} - v_{it}\right)}{\sigma_t^2(y_i^{t-1}, \gamma)} - \frac{\delta_t^2}{\delta_{t-1}^2} \frac{v_{it-1}\left(\frac{\phi_{t-1}}{\phi_t} v_{it} - v_{it-1}\right)}{\sigma_{t-1}^2(y_i^{t-2}, \gamma)}\right) | y_i^{t-2}\right] = 0.$$

5 Estimation and Monte Carlo

The parameters α and γ in the model as defined by (2.1)-(2.5) can be jointly estimated by (non-linear) GMM, combining the moment conditions

$$E\left[y_{is}\left(v_{it}-v_{it-1}\right)\right]=0 \quad ; \quad t=3,\ldots,T;\ s=1,\ldots,t-2 \tag{5.1}$$

$$E\left[v_{it}\left(v_{it-1}-v_{it-2}\right)\right]=0 \quad ; \quad t=4,\ldots,T \tag{5.2}$$

$$E\left[y_{il}\left(\frac{v_{it}(v_{it+1}-v_{it})}{\sigma_t^2(y_i^{t-1},\gamma)}-\frac{v_{it-1}(v_{it}-v_{it-1})}{\sigma_{t-1}^2(y_i^{t-2},\gamma)}\right)\right]=0 \quad ; \quad t=3,\ldots,T-1;\ l=1,\ldots,t-2 \tag{5.3}$$

which form a total of $(T-1)(T-2)/2+(T-3)+(T-2)(T-3)/2$ moment conditions. [2]

Define

$$s_i=\begin{pmatrix} v_{i3}-v_{i2} \\ \vdots \\ v_{iT}-v_{iT-1} \\ v_{i4}\left(v_{i3}-v_{i2}\right) \\ \vdots \\ v_{iT}\left(v_{iT-1}-v_{iT-2}\right) \\ \dfrac{v_{i3}(v_{i4}-v_{i3})}{\sigma_3^2(y_i^2,\gamma)}-\dfrac{v_{i2}(v_{i3}-v_{i2})}{\sigma_2^2(y_i^1,\gamma)} \\ \vdots \\ \dfrac{v_{iT-1}(v_{iT}-v_{iT-1})}{\sigma_{T-1}^2(y_i^{T-2},\gamma)}-\dfrac{v_{iT-2}(v_{iT-1}-v_{iT-2})}{\sigma_{T-2}^2(y_i^{T-3},\gamma)} \end{pmatrix}$$

2. The instruments for the moment conditions (5.3) can be any transformation of the y_{il}, see the discussion in WOOLDRIDGE [1997].

and

$Z_i =$

$$\begin{bmatrix} y_{i1} & 0 & 0 & \cdots & 0 & \cdots & 0 & 0 & 0 & 0 & 0 & \cdots & 0 & \cdots & 0 \\ 0 & y_{i1} & y_{i2} & \cdots & 0 & \cdots & 0 & 0 & 0 & 0 & 0 & \cdots & 0 & \cdots & 0 \\ 0 & 0 & 0 & \cdots & 0 & \cdots & 0 & 0 & 0 & 0 & 0 & \cdots & 0 & \cdots & 0 \\ 0 & 0 & 0 & \cdots & y_{i1} & \cdots & y_{iT-2} & 0 & 0 & 0 & 0 & \cdots & 0 & \cdots & 0 \\ 0 & 0 & 0 & \cdots & 0 & \cdots & 0 & I_{T-3} & 0 & 0 & 0 & \cdots & 0 & \cdots & 0 \\ 0 & 0 & 0 & \cdots & 0 & \cdots & 0 & 0 & y_{i1} & 0 & 0 & \cdots & 0 & \cdots & 0 \\ 0 & 0 & 0 & \cdots & 0 & \cdots & 0 & 0 & 0 & y_{i1} & y_{i2} & \cdots & 0 & \cdots & 0 \\ 0 & 0 & 0 & \cdots & 0 & \cdots & 0 & 0 & 0 & 0 & 0 & \cdots & 0 & \cdots & 0 \\ 0 & 0 & 0 & \cdots & 0 & \cdots & 0 & 0 & 0 & 0 & 0 & \cdots & y_{i1} & \cdots & y_{iT-3} \end{bmatrix}$$

where I_{T-3} is the identity matrix of order $T-3$.[3] Further, let $\theta = (\alpha, \gamma)'$ and

$$f_i(\theta) = Z_i' s_i.$$

The GMM estimator $\widehat{\theta}$ for θ minimises

$$\left[\frac{1}{N}\sum_{i=1}^{N} f_i(\theta)\right]' W_N \left[\frac{1}{N}\sum_{i=1}^{N} f_i(\theta)\right],$$

with respect to θ; where W_N is a positive semidefinite weight matrix which satisfies $\operatorname{plim}_{N\to\infty} W_N = W$, with W a positive definite matrix. Regularity conditions are in place such that $\lim_{N\to\infty} \frac{1}{N}\sum_{i=1}^{N} f_i(\theta) = E\left(f(\theta)\right)$ and $\frac{1}{\sqrt{N}} f_i(\theta) \to N(0, \Psi)$.

Let $F(\theta) = E(\partial f_i(\theta)/\partial\theta)$, then $\sqrt{N}\left(\widehat{\theta} - \theta\right)$ has a limiting normal distribution, $\sqrt{N}\left(\widehat{\theta} - \theta\right) \to N(0, V_W)$, where

$$V_W = \left(F'WF\right)^{-1} F'W\Psi WF \left(F'WF\right)^{-1}.$$

In order to investigate the performance of this GMM estimator, we consider the following data generating process:

$$\begin{aligned} y_{it} &= \alpha y_{it-1} + f_i + u_{it} \\ f_i &\sim N(0, \sigma_f^2) \\ \left(u_{it} | y_{it-1}\right) &\sim N\left(0, \exp\left(\gamma_0 + \gamma y_{it-1} + m_i\right)\right) \\ m_i &\sim N\left(0, \sigma_m^2\right). \end{aligned}$$

3. In the Monte Carlo study reported below, $(T-3)$ time specific constants are added to the instrument set for γ. Although the efficiency gain is small, the estimated standard errors are better behaved when these dummies are included.

Data is generated for $T = 5$ and 10, and $N = 100$, 500, and 1000. The process is started at $y_{i0} = 0$, then four periods are generated before the sample is generated. The value of the fixed effect variances are $\sigma_f^2 = \sigma_m^2 = 0.25$. The values for α are 0.5, 0.7 and 0.9, with values for γ equal to 0 and 0.2. The value of γ_0 is set in such a way that the sample variance of u_{it} is approximately equal to $3\sigma_f^2$. A one-step GMM estimator for θ, $\widetilde{\theta}$, is obtained by using the weight matrix $W_{N1} = \left(\frac{1}{N}\sum_i Z_i' Z_i\right)^{-1}$.

TABLE 1A
Monte Carlo Results for $T = 5$

	$\alpha = 0.5$			$\alpha = 0.7$			$\alpha = 0.9$		
	mean	se	est se	mean	se	est se	mean	se	est se
$\gamma = 0$									
$N = 100$									
α_1	0.4897	0.1969	0.1643	0.7168	0.2178	0.1793	0.9313	0.1538	0.1356
α_2	0.5015	0.1457	0.0841	0.7265	0.1743	0.0946	0.9292	0.1425	0.0759
γ_1	0.0037	0.2326	0.3092	– 0.0007	0.1969	0.2793	– 0.0022	0.1367	0.1958
γ_2	0.0073	0.2426	0.2018	– 0.0012	0.2084	0.1840	– 0.0051	0.1488	0.1279
$N = 500$									
α_1	0.5105	0.1030	0.0899	0.7357	0.1521	0.1353	0.9436	0.1281	0.1218
α_2	0.5019	0.0567	0.0486	0.7279	0.1048	0.0698	0.9364	0.1072	0.0677
γ_1	– 0.0047	0.1523	0.2039	– 0.0010	0.1554	0.2188	0.0087	0.1133	0.1676
γ_2	– 0.0055	0.1448	0.1535	– 0.0066	0.1542	0.1651	0.0041	0.1187	0.1263
$N = 1000$									
α_1	0.5040	0.0637	0.0613	0.7259	0.1227	0.1075	0.9472	0.1082	0.1072
α_2	0.5029	0.0387	0.0355	0.7134	0.0734	0.0527	0.9350	0.0912	0.0570
γ_1	0.0030	0.1168	0.1526	0.0007	0.1345	0.1994	– 0.0007	0.1008	0.1596
γ_2	0.0032	0.1102	0.1214	0.0022	0.1306	0.1475	0.0005	0.1046	0.1255
$\gamma = 0.2$									
$N = 100$									
α_1	0.4918	0.2040	0.1599	0.7159	0.2129	0.1773	0.9531	0.1488	0.1242
α_2	0.5032	0.1576	0.0833	0.7236	0.1806	0.0937	0.9566	0.1360	0.0709
γ_1	0.2165	0.2346	0.3052	0.2038	0.2023	0.2675	0.2147	0.1473	0.2107
γ_2	0.1909	0.2476	0.1928	0.1812	0.2085	0.1677	0.1906	0.1616	0.1322
$N = 500$									
α_1	0.5044	0.0992	0.0888	0.7306	0.1461	0.1327	0.9576	0.1188	0.1052
α_2	0.4990	0.0594	0.0493	0.7208	0.1075	0.0711	0.9498	0.1076	0.0665
γ_1	0.1966	0.1482	0.1992	0.2060	0.1500	0.2250	0.2197	0.1178	0.1755
γ_2	0.1824	0.1497	0.1475	0.1895	0.1528	0.1620	0.2007	0.1216	0.1214
$N = 1000$									
α_1	0.5042	0.0646	0.0625	0.7311	0.1237	0.1053	0.9542	0.1125	0.0923
α_2	0.5001	0.0387	0.0366	0.7153	0.0771	0.0563	0.9449	0.0917	0.0593
γ_1	0.1909	0.1270	0.1552	0.2054	0.1318	0.1919	0.2230	0.1036	0.1679
γ_2	0.1834	0.1200	0.1209	0.1917	0.1326	0.1402	0.2071	0.1077	0.1179

α_1, γ_1 are one-step GMM, α_2, γ_2 are two-step GMM. Mean of 1000 replications se: sample standard deviation; est se: mean of estimated standard errors.

Given $\widetilde{\theta}$, the efficient two-step GMM estimator uses as weight matrix $W_{N2} = \left(\frac{1}{N} \sum_i Z_i' s_i \left(\widetilde{\theta} \right) s_i \left(\widetilde{\theta} \right)' Z_i \right)^{-1}$.

TABLE 1B
Monte Carlo Results for $T = 10$

	$\alpha = 0.5$			$\alpha = 0.7$			$\alpha = 0.9$		
	mean	se	est se	mean	se	est se	mean	se	est se
$\gamma = 0$									
$N = 100$									
α_1	0.4442	0.1582	0.0919	0.7078	0.1444	0.0882	0.9564	0.0709	0.0541
α_2	0.4493	0.1505	0.0113	0.7098	0.1385	0.0109	0.9563	0.0698	0.0076
γ_1	0.0033	0.1098	0.1412	0.0018	0.0835	0.1151	– 0.0017	0.0480	0.0714
γ_2	0.0030	0.1087	0.0193	0.0015	0.0820	0.0179	– 0.0007	0.0474	0.0124
$N = 500$									
α_1	0.4820	0.0668	0.0543	0.7269	0.0930	0.0710	0.9538	0.0617	0.0529
α_2	0.4959	0.0307	0.0166	0.7160	0.0508	0.0192	0.9462	0.0581	0.0174
γ_1	– 0.0009	0.0659	0.0903	– 0.0027	0.0545	0.0851	– 0.0002	0.0332	0.0560
γ_2	– 0.0011	0.0552	0.0371	– 0.0009	0.0515	0.0391	0.0002	0.0347	0.0302
$N = 1000$									
α_1	0.4897	0.0432	0.0380	0.7179	0.0733	0.0587	0.9578	0.0587	0.0511
α_2	0.5000	0.0171	0.0130	0.7052	0.0251	0.0153	0.9403	0.0577	0.0153
γ_1	– 0.0024	0.0522	0.0685	0.0013	0.0488	0.0733	– 0.0011	0.0301	0.0518
γ_2	– 0.0006	0.0395	0.0324	0.0023	0.0426	0.0368	– 0.0007	0.0316	0.0318
$\gamma = 0.2$									
$N = 100$									
α_1	0.4493	0.1453	0.0912	0.7224	0.1416	0.0862	0.9765	0.0641	0.0465
α_2	0.4538	0.1382	0.0110	0.7252	0.1371	0.0101	0.9768	0.0631	0.0060
γ_1	0.1912	0.1095	0.1396	0.2064	0.0869	0.1147	0.2249	0.0534	0.0760
γ_2	0.1890	0.1094	0.0180	0.2020	0.0878	0.0161	0.2220	0.0538	0.0106
$N = 500$									
α_1	0.4813	0.0686	0.0541	0.7274	0.0937	0.0694	0.9748	0.0649	0.0463
α_2	0.4952	0.0339	0.0168	0.7138	0.0527	0.0194	0.9715	0.0602	0.0158
γ_1	0.1992	0.0669	0.0899	0.2122	0.0571	0.0846	0.2274	0.0367	0.0577
γ_2	0.1922	0.0568	.0357	0.2012	0.0551	0.0361	0.2145	0.0381	0.0260
$N = 1000$									
α_1	0.4909	0.0439	0.0381	0.7210	0.0732	0.0577	0.9694	0.0685	0.0452
α_2	0.4991	0.0183	0.0134	0.7055	0.0277	0.0159	0.9557	0.0628	0.0159
γ_1	0.1992	0.0522	0.0699	0.2087	0.0483	0.0741	0.2270	0.0334	0.0524
γ_2	0.1942	0.0411	0.0314	0.1992	0.0427	0.0345	0.2133	0.0345	0.0275

α_1, γ_1 are one-step GMM, α_2, γ_2 are two-step GMM. Mean of 1000 replications se: sample standard deviation; est se: mean of estimated standard errors

Results for the Monte Carlo experiments are given in Table 1A and 1B for $T = 5$ and $T = 10$ respectively.[4] The estimator for α is upward biased for $\alpha = 0.7$ and $\alpha = 0.9$. There is no systematic bias in the estimator for γ. The

TABLE 2
Sargan Test Results

	$T = 5,\ DoF = 11$			$T = 10,\ DoF = 76$		
	$\alpha = 0.5$	$\alpha = 0.7$	$\alpha = 0.9$	$\alpha = 0.5$	$\alpha = 0.7$	$\alpha = 0.9$
$\gamma = 0$						
$N = 100$						
mean	11.7435	11.6874	11.6853	78.6737	78.5495	78.3087
variance	19.5180	19.1204	18.2304	25.3295	24.8881	24.1394
$p < 0.10$	0.1110	0.1020	0.1040	0.0010	0.0010	0.0000
$p < 0.05$	0.0520	0.0400	0.0480	0.0000	0.0000	0.0000
$p < 0.01$	0.0050	0.0060	0.0040	0.0000	0.0000	0.0000
$N = 500$						
mean	11.3870	11.4920	11.3501	80.1720	81.4084	80.3298
variance	22.1283	22.2584	22.0550	124.2624	152.3446	131.3430
$p < 0.10$	0.1120	0.1220	0.1140	0.1340	0.1800	0.1590
$p < 0.05$	0.0540	0.0580	0.0540	0.0710	0.1030	0.0800
$p < 0.01$	0.0130	0.0100	0.0090	0.0150	0.0210	0.0090
$N = 1000$						
mean	11.1570	11.6251	11.4034	78.4569	79.4141	81.8165
variance	21.5606	26.3616	24.3043	131.3792	183.3694	176.8718
$p < 0.10$	0.1110	0.1220	0.1240	0.1190	0.1470	0.2150
$p < 0.05$	0.0410	0.0720	0.0580	0.0570	0.0850	0.1250
$p < 0.01$	0.0060	0.0210	0.0140	0.0130	0.0240	0.0350
$\gamma = 0.2$						
$N = 100$						
mean	11.7876	11.6715	11.5384	78.4791	78.3221	78.0095
variance	19.4657	20.7590	17.0209	24.9107	25.3591	26.4413
$p < 0.10$	0.1100	0.1200	0.0900	0.0000	0.0000	0.0000
$p < 0.05$	0.0430	0.0570	0.0310	0.0000	0.0000	0.0000
$p < 0.01$	0.0090	0.0060	0.0040	0.0000	0.0000	0.0000
$N = 500$						
mean	11.6560	11.4762	11.1035	79.7600	81.1115	81.7547
variance	23.1530	23.7077	21.4205	126.3555	152.1419	121.1384
$p < 0.10$	0.1260	0.1190	0.1010	0.1340	0.1770	0.1670
$p < 0.05$	0.0690	0.0530	0.0530	0.0540	0.0960	0.0780
$p < 0.01$	0.0140	0.0130	0.0060	0.0110	0.0300	0.0130
$N = 1000$						
mean	11.1499	11.3486	11.4782	78.4394	79.7112	84.4989
variance	22.2237	23.4938	24.5737	127.9988	153.3866	198.1512
$p < 0.10$	0.1120	0.1170	0.1200	0.1140	0.1370	0.2640
$p < 0.05$	0.0470	0.0540	0.0560	0.0530	0.0680	0.1590
$p < 0.01$	0.0120	0.0130	0.0140	0.0090	0.0160	0.0560

4. For the minimisation of the GMM criterion function, we used the MAXLIK 4.0 routine in GAUSS.

standard deviation of γ is relatively large when $T = 5$, but γ is estimated quite precisely when $T = 10$. For the estimator of γ, there is hardly any efficiency gain in using the optimal weight matrix, whereas there is quite a substantial gain in efficiency for the estimator for α. The estimated standard errors for the estimator of α are downward biased, especially for the two-step GMM estimator when $T = 10$. This is a similar result as in ARELLANO and BOND [1991]. The one-step estimated standard errors of the estimator of γ are upward biased, whereas the two-step estimated standard errors are quite close to the true values when $T = 5$, whereas they are downward biased when $T = 10$.

Table 2 presents results for the SARGAN/HANSEN test statistic for the over identifying restrictions. Under the null, the test statistic is χ^2 distributed with 11 and 76 degrees of freedom for $T = 5$ and $T = 10$ respectively. For $T = 5$, the size performance of the test statistic is reasonable. For $T = 10$, however, the test underrejects when $N = 100$. It overrejects for larger N with the size of the test improving with increasing sample size, apart from when $\alpha = 0.9$. The size is reasonable when $\alpha = 0.5$ for large N. The problem of doing inference when there are many over identifying instruments is apparent and due to the estimation of the efficient weight matrix. A possible solution is to select a subset of the instruments, thus improving inference at the cost of efficiency loss. Alternatively, bootstrap methods may be used in order to perform better small sample inference (see HALL and HOROWITZ [1996] and BROWN, NEWEY and MAY [1998]).

TABLE 3
Monte Carlo Results for Estimation of α and γ in two stages, $T = 5$, $\alpha = 0.7$

	$N = 100$		$N = 500$		$N = 1000$	
	mean	se	mean	se	mean	se
$\gamma = 0$						
α_1	0.6172	0.1724	0.6848	0.1100	0.6935	0.0825
α_2	0.6601	0.1572	0.7052	0.0923	0.7017	0.0634
γ_1	0.0036	0.1491	– 0.0029	0.1206	– 0.0031	0.1130
γ_2	0.0039	0.1692	– 0.0033	0.1344	– 0.0008	0.1225
$\gamma = 0.2$						
α_1	0.6017	0.1863	0.6773	0.1141	0.6912	0.0927
α_2	0.6502	0.1632	0.6958	0.0949	0.7004	0.0704
γ_1	0.1847	0.1520	0.1867	0.1162	0.1974	0.1103
γ_2	0.1915	0.1730	0.1940	0.1336	0.2011	0.1197

α_1, γ_1 are one-step GMM, α_2, γ_2 are two-step GMM.
Mean of 1000 replications. se: sample standard deviation γ_1 and γ_2 are estimated conditional on α_2

Although the use of the combined moment conditions for the joint estimation of α and γ is more efficient, in practice it may be a better idea to estimate α and γ in stages. First, α can be estimated utilising moment conditions (5.1) and (5.2). Subsequently, γ can be estimated from (5.3) substituting in the consistent estimate for α. Advantages of this method are that the moment conditions for α and γ can be tested separately, and that the estimate for α will be consistent even if (5.3) is not valid. A slight disadvantage is that the asymptotic standard error for the estimator for γ has to be adjusted for the separate estimation of α, as in NEWEY [1984]. Simulation results for this two-stage estimation procedure are presented in Table 3 for $T = 5$ and $\alpha = 0.7$. The estimator for γ is conditional on the value of the two-step GMM estimator for α. Apart from the estimate for α when $N = 100$, which is downward biased, the two-step procedure performs better than the estimator that combines all moment conditions in terms of bias and mean squared error.

6 Summary

We have derived orthogonality conditions for estimating the coefficients of the conditional variance of a simple linear autoregressive process with unobserved individual effects. The distinguishing characteristic of our model is that we allow for individual effects both in the conditional mean function and the conditional variance function. The relationship between these effects is left unconstrained. Moment conditions are derived for non-autocorrelated error processes, error processes with an $MA(q)$ structure, and for models that allow for time varying individual effects. A Monte Carlo study shows that the estimation of the parameters of the conditional variance function is feasible, but that large sample sizes are needed for precision.

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