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Inference in codependence: Some Monte Carlo Results and Applications

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ABSTRACT. – In this paper, we investigate through Monte Carlo simulations the behavior of the codependence testing procedure (GOURIÉROUX et PEAUCELLE [1989]) in small samples and in various usual statistical situations. Our results suggest that, except for the pure MA(q) case, important power losses may occur. The simulation results are illustrated by an analysis of OKUN's law conducted for the main OECD countries.

Tester la Codépendance : aspects inférentiels et applications

RÉSUMÉ. – Dans cet article, nous analysons au travers de simulations de Monte Carlo le comportement de la procédure de test de codependence (GOURIÉROUX et PEAUCELLE [1989]) à distance finie et dans differents contextes statistiques couramment rencontrés. Nos résultats suggèrent qu'à l'exception du cas MA(q) pur, d'importantes pertes de puissance surviennent. Nous illustrons l'analyse par une étude de la loi d'OKUN appliquée aux principales économies de l'OCDE.

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1 Introduction

Since the mid eighties, time series econometrics has been subject to quite important theoretical and practical developments. Emphasis has been particularly placed on the statistical treatment of non-stationary variables. This is reflected by the progress in asymptotic theory for integrated variables and in particular by the development of unit root tests and cointegration techniques. This progress towards a more accurate modeling of dynamic processes was associated with numerous economic applications and studies of small samples properties of estimators via Monte Carlo experiments.

Another form of comovements in the short run is codependence. Up to quite recently, in comparison with cointegration, few attention has been devoted to relationships in models involving stationary variables. This lack may be due to the fact that the issue was seen as less crucial. Indeed, unlike for integrated processes, applications of traditional techniques do not provide spurious estimated coefficients. The gap in the dynamic specification for stationary processes has been partially filled by GOURIÉROUX and PEAUCELLE [1989, 1993] with codependence for moving average (MA) models, by TIAO and TSAY [1985, 1989] through scalar component models for general ARMA models, by VELU, REINSEL and WICHERN [1986] or ENGLE and KOZICKI [1993] with the serial correlation common feature approach (a strong form of codependence in which a linear combination yields a white noise process), by AHN and REINSEL [1988] with nested reduced-rank in autoregressive models or by VAHID and ENGLE [1997] with non-synchronous cycles. The common goal of all these approaches is to find linear combinations of time series exhibiting less dynamics that the series analyzed individually.

Taking into account of these constraints yields two main advantages. First, it allows to look for hidden meaningful structures among economic time series. Secondly, the simplification of the whole model by the reduction in the number of estimated parameters leads to an increase in efficiency. Short run comovements analysis is found to be useful when the theory tends to define relationships among stationary time series, like the PHILLIPS curve, the OKUN's law, the relative purchasing power parity, the real interest rates parity. Such an analysis is also appropriate when the time series are cointegrated in level and the analysis is carried out in a VECM form with r cointegrating vectors entering as new stationary variables[1].

The codependence or the common feature approaches have been applied in empirical studies: see inter alia KUGLER and NEUSSER [1993], BEINE and HECQ [1999], ISSLER and VAHID [1993], ROMIJN [1996], JOBERT [1995], VARIYAM [1996], FUNKE and HALL [1995] for some applications.

However, for empirical practice in codependence which often faces small sample sizes, numerous important issues remain unexplored. These problems are quite general, as they concern for instance the choice of the dynamic representation or the robustness of test statistics in the presence of misspecifi-

1. See VAHID and ENGLE [1993], GALLO and KEMPF [1996], ISSLER and VAHID [1993], HECQ, PALM and URBAIN [1997] for integrated studies with both codependence and cointegration.

cation. This paper is devoted to some of the most current problems one may face in practice. Using Monte Carlo simulations, we investigate how the results of the codependence testing procedure are affected in different statistical contexts. While we do not find out important problems of bias in the estimates or problems of size distortions, our results emphasize potential significant power losses, depending on the nature of the misspecification source or the statistical properties of the data.

The paper is organized as follows. The second section reviews the codependence concept, the estimation techniques and the relevant test statistics. The third section is devoted to the simulation exercises and to the inference results. We conduct a small sample Monte Carlo simulation in order to analyze the impact on size, power and estimated coefficients of the choice of the lag length in the MA(q) representation. We also study the consequences when the data generating process (DGP) is an infinite VMA, i.e. a stable VAR, as well as when the specified model ignores an existing long run relationship between the variables in level. We conclude this section by considering the issue of temporal aggregation and of non normal disturbances (via Student's t as well as GARCH errors). The fourth section illustrates the preceding results by analyzing OKUN's law, i.e. the relationship between the variation of unemployment rate and the growth rate of economic activity for the main OECD countries. Section five concludes.

2 Codependence: Technical Background

Since the ENGLE and GRANGER [1987] seminal paper, the use and development of cointegration techniques is a subject of great concern. While cointegration deals with the determination of some long run relationships between non stationary time series, the notion of codependence, introduced by GOURIÉROUX and PEAUCELLE [1989], allows to consider comovements among $I(0)$ variables. In this section, we present the concept of codependence as well as the testing procedure.

2.1. General Considerations about Codependence

To define "a more stable linear relationship", GOURIÉROUX and PEAUCELLE develop an analysis in a moving average framework in terms of dynamic multipliers and persistence. Indeed, consider y_t, a n dimensional vector process, admitting a multivariate MA representation of order q, such that :

(1) $$y_t = m + \varepsilon_t + \Theta_1 \varepsilon_{t-1} + \cdots + \Theta_q \varepsilon_{t-q} \qquad t = 1 \ldots T$$

where m is a $n \times 1$ vector of constants, ε_t is a multivariate Gaussian white noise with Ω_ε its variance-covariance matrix, Θ_j, $j = 1 \ldots q$ are square matrices of size n. Let us assume y_t a multivariate stationary process composed of n $I(0)$ variables [2], a one time transitory shock $\Delta\varepsilon_{t^*}$ on the inno-

2. Notice that unlike the univariate case, a finite VMA(q) may yield a non stationary process like in the presence of exchangeable models (see VAHID and ENGLE (1997), p. 201 for instance).

vation of y_t at time $t = t^*$ takes the values $\Delta\varepsilon_{t^*}$, $\Theta_1 \Delta\varepsilon_{t^*}, \ldots, \Theta_q \Delta\varepsilon_{t^*}, 0, 0$. The impact becomes equal to zero after q periods, which defines the order of persistence of the multivariate moving average process of order q.

Let us consider independent linear combinations u_t, such that:

$$(2) \quad u_t \equiv \alpha' y_t = \alpha' m + \alpha' \varepsilon_t + \alpha' \Theta_1 \varepsilon_{t-1} + \cdots + \alpha' \Theta_q \varepsilon_{t-q} \qquad t = 1 \ldots T$$

where α is a $(n \times s)$ matrix with $1 \leqslant s \leqslant n$ and $\text{rank}(\alpha) = s$. The order of persistence of these combinations is maximum q. However, if that order is less than q and equal to $q - b$, then the time series will be said to be codependent. Codependence holds for the VMA process (1) if there exists a $(n \times s)$ matrix α whose columns span the codependence space. The degree of codependence is the maximum[3] integer b such that $\alpha' \Theta_{q-b} \neq 0$ but $\alpha' \Theta_{q-k} = 0$, $\forall k < b$. In terms of reduced rank of some matrices, this last condition also specifies that $\text{rank}[\Theta_1 : \cdots : \Theta_{q-b}] = n$ but $\text{rank}[\Theta_{q-b+1} : \cdots : \Theta_q] < n$.

Exemple 2.1. Let us consider a simple illustrative example where two variables y_{1t} and y_{2t} exhibit a VMA(2) dynamics such as:

$$\begin{pmatrix} y_{1t} \\ y_{2t} \end{pmatrix} = \begin{pmatrix} \alpha_1 \theta_{1.21} & \alpha_1 \theta_{1.22} \\ \theta_{1.21} & \theta_{1.22} \end{pmatrix} \begin{pmatrix} \varepsilon_{1t-1} \\ \varepsilon_{2t-1} \end{pmatrix} + \begin{pmatrix} \alpha_1 \theta_{2.21} & \alpha_1 \theta_{2.22} \\ \theta_{2.21} & \theta_{2.22} \end{pmatrix} \begin{pmatrix} \varepsilon_{1t-2} \\ \varepsilon_{2t-2} \end{pmatrix} + \begin{pmatrix} \varepsilon_{1t} \\ \varepsilon_{2t} \end{pmatrix}$$

If we ignore that underlying true structure, the specification requires the estimation of eight parameters (plus the variances and the autocovariance). However, recognizing that the first row is a linear combination of the second one, the model may be rewritten in a reduced-rank form with only five free coefficients:

$$\begin{pmatrix} y_{1t} \\ y_{2t} \end{pmatrix} = \begin{pmatrix} \alpha_1 \\ 1 \end{pmatrix} \left[\begin{pmatrix} \theta_{1.21} & \theta_{1.22} \end{pmatrix} \begin{pmatrix} \varepsilon_{1t-1} \\ \varepsilon_{2t-1} \end{pmatrix} + \begin{pmatrix} \theta_{2.21} & \theta_{2.22} \end{pmatrix} \begin{pmatrix} \varepsilon_{1t-2} \\ \varepsilon_{2t-2} \end{pmatrix} \right] + \begin{pmatrix} \varepsilon_{1t} \\ \varepsilon_{2t} \end{pmatrix}$$

One easily sees that the linear combination $y_{1t} - \alpha_1 y_{2t} = \varepsilon_{1t} - \alpha_1 \varepsilon_{2t}$ is a white noise. The normalized codependence vector $\alpha' = [1, -\alpha_1]$ is, up to a scalar factor, the only one which can yield this result. ■

This strong form of codependence, illustrated in example 2.1, in which shocks adjust instantaneously and where the impulse responses are perfectly correlated has been labelled serial correlation common feature (SCCF) by ENGLE and KOZICKI [1993] and previously emphasized by VELU *et al.* [1986].

3. In this paper we are only interested in linear combinations that yield the same lowest degree of persistence even if a complete set of nested reduced rank structures can be found (AHN and REINSEL [1988]).

The main advantage provided by a moving average framework, as the one proposed by GOURIÉROUX and PEAUCELLE [1989, 1993], is that a linear combination which reduces the dynamics delivers a meaningful interpretation in terms of persistence and adjustment speed. This is not necessarily the case in a VAR(p) analysis (except for the strong SCCF case).

Remark 1. VAHID and ENGLE (1997) use the term "codependence" to characterize a situation in which linear combinations of VARMA(p,q) are ARMA$(0,q)$ with q non-negative and small. Like the GOURIÉROUX and PEAUCELLE framework, this model turns out to be a particular case of Scalar Components Models (TIAO and TSAY [1989]) and is an interesting generalization of ENGLE and KOZICKI [1993]. VAHID and ENGLE (1997), ANDERSON and VAHID (1988) also present the connection between GMM and canonical correlation estimators for a common feature analysis. However codependence still exhibits several properties. In addition to the one exposed above concerning the reduction of the degree of persistence, the use codependence with restrictions on moving average processes instead of general VARMA(p,q), allows to avoid the difficulty of exchangeable structure, i.e. when two different models give rise to the same covariance structure (see HANNAN [1969]; TIAO and TSAY [1989]). Morever, codependence tests are in some way nested because if $[\Theta_1 \cdots \Theta_q]$ is of reduced rank then $[\Theta_{1+i} \cdots \Theta_q]$ is also of reduced rank for the same codependence vector because of the overlapping property of their left null spaces. This is not the case in general VARMA processes. ■

2.2. Testing for Codependence

The first step when testing for codependence is to determine a suitable multivariate MA process by inspecting the autocovariance function. Indeed, as in the univariate framework, autocovariance matrices become non significantly different from zero for the order $q+1$ in a pure MA(q). If one notes $\Gamma(i)$ the covariance matrices, $i = 0 \dots q$, where $\Gamma(0)$ is the variance-covariance matrix at time t, testing for codependence is equivalent to testing for the null hypothesis: H_0: $\alpha'\Gamma(q-b+1) = \alpha'\Gamma(q-b+2) = \cdots = \alpha'\Gamma(q) = 0$. Thus, the codependence analysis is an investigation of the rank and the left null spaces of matrices $\Gamma^* = [\Gamma(q-b-1) : \cdots : \Gamma(q)]$ or $\Theta^* = \left[\Theta_{q-b+1} : \cdots : \Theta_q\right]$ for $b = q$ to 1. When the codependence vector as well as its degree are unknown, GOURIÉROUX and PEAUCELLE [1993] propose to study the canonical correlation between the $(n \times T)$ matrix $W_1 \equiv Y_t = \{y_1^{'} \dots y_T^{'}\}^{'}$ and the $(n \times K) \times T$ matrix $W_2 = \{Y_{t-k}^{'} \dots Y_{t-q}^{'}\}^{'}$ for $k = 1,\dots,q$ and $K = q-k+1$ with $t = 1 \dots T$. Note that W_1 and W_2 have to be adjusted for their mean. See also TIAO and TSAY [1985, 1989] and REINSEL [1993].

Because the multivariate process has been supposed Gaussian covariance stationary, the sequence of codependence likelihood ratio test statistics, for fixed q and fixed k (and therefore also K, is, for H_0:

rank$[\Theta_k : \cdots : \Theta_q] \leqslant n - s$ against H_a: rank$[\Theta_k : \cdots : \Theta_q] > n - s$ (ANDERSON [1984]; LÜTKEPOHL [1991]) [4]:

$$\zeta_{K,s} = -T \sum_{i=1}^{s} \log(1 - \lambda_i), \qquad s = 1,\ldots,n \tag{3}$$

where $0 \leqslant \lambda_1 \leqslant \lambda_2 \leqslant \ldots \leqslant \lambda_n \leqslant 1$ are the ordered eigenvalues of the symmetric matrix:

$$\Lambda_K = (W_1 W_1')^{-1/2} W_1 W_2' (W_2 W_2')^{-1} W_2 W_1' (W_1 W_1')^{-1/2} \tag{4}$$

Test statistic (3) implies that we test for the non-significance of the sum of the s smallest eigenvalues of the empirical counterpart of Λ_K. These eigenvalues, estimated in the usual way[5], are the squared canonical correlations and the smallest of them measures the relationship between the linear combination of the components of W_1 and the linear combination of the components of W_2 that is the least correlated. If the null hypothesis cannot be rejected, the columns of the matrix $\alpha \equiv (W_1 W_1')^{-1/2} \upsilon_K$, where υ_K are the eigenvectors of Λ_K (see footnote 5) associated with this smallest correlation, span the codependence space. This matrix α whose columns span the left null space of $\Theta_K^* = [\Theta_k : \cdots : \Theta_q]$ is such that $\alpha' \Theta_K^* = 0_{(s \times q-k+1)}$.

In practice, the test statistics for zero eigenvalues are based on a backward sequential reduced rank test of the empirical counterparts of the Λ_K matrices for K going from $K = 1$ (i.e. a test on the last moving average matrix) up to $K = q$ (i.e. a test for a reduced rank in all matrices). If the null is not rejected, a reduced rank structure in all matrices correspond to the SCCF analysis in which a linear combination yields a white noise. Because of the covariance-stationary hypothesis, the test statistics asymptotically follows, under the null, a χ^2 with $snK - s(n-s)$ degrees of freedom (see VAHID and ENGLE, 1993), where $K = (q - k + 1)$, $k = 1,\ldots,q$.

3 Some Monte Carlo Simulations

In this section, we first study the size and power of codependence tests when the Data Generating Process (DGP) is a second order VMA. Then, we

4. Notice that TIAO and TSAY [1985, 1989] propose to correct the test statistic (3) in order to take into account moving average error processes. Indeed, test statistic (3) is a kind of multivariate portmanteau statistic whose asymptotic χ^2 distribution is likely to be valid under the white noise hypothesis and not under general codependence null hypotheses, i.e when $b \neq q$. Nevertheless, we will use (3) because we generated a white noise process under the null. Moreover, via Monte Carlo experiments, it is shown in HECQ [1997] that the corrected test does not behave differently as the non corrected one in the presence of MA residuals for the reduced rank hypothesis.

5. One sees that the canonical correlation problem has been transformed into a standard determination of the eigenvalues. Indeed for two matrices W_1 and W_2, ANDERSON [1984] shows that the squared canonical correlations are given by finding the zeros of the determinant $|\lambda W_1 W_1' - W_1 W_2' (W_2 W_2')^{-1} W_2 W_1'|$. To avoid a non standard determination of the eigenvalues one gets a usual eigenvalue determination by solving $|\lambda I - (W_1 W_1')^{-1/2} W_1 W_2' (W_2 W_2')^{-1} W_2 W_1' (W_1 W_1')^{-1/2}|$ in which $W_1 W_1'$ has been factorized. The initial eigenvectors are recovered by multiplying the eigenvectors we obtain by $(W_1 W_1')^{-1/2}$.

consider the impact of misspecification, taking into account the fact that the DGP is actually a VAR process as well as when the analysis is carried out on first differences of cointegrated $I(1)$ variables. We finally investigate the effectiveness of codependence when using temporally aggregated data and models involving non-normal disturbances such as Student's *t* distributions and GARCH processes. Note that throughout the rest of the paper, we consider only bivariate processes, leaving for further investigation the issues related to multiple codependence relationships such as identification.

3.1. **The basic DGP**

DGP(1) is a multivariate moving average process of order two, i.e. a VMA(2) while in the DGP(2), the moving average matrices are of reduced rank and the first row is equal to two times the second row. Consequently, DGP(1) is used to analyse the power of the test while DGP(2) is devoted to the study of size properties. Therefore the linear combination $y_{1t} - 2y_{2t} = \varepsilon_{1t} - 2\varepsilon_{2t}$ is, under the null of codependence, a white noise with a normalized codependence vector $\alpha' = [1, -2]$. This is formally written as:

$$\text{(5)}\quad \text{DGP(1)} : \begin{pmatrix} y_{1t} \\ y_{2t} \end{pmatrix} = \begin{pmatrix} .3 & -.6 \\ -.1 & -.5 \end{pmatrix} \begin{pmatrix} \varepsilon_{1t-1} \\ \varepsilon_{2t-1} \end{pmatrix} + \begin{pmatrix} .6 & .2 \\ .4 & -.3 \end{pmatrix} \begin{pmatrix} \varepsilon_{1t-2} \\ \varepsilon_{2t-2} \end{pmatrix} + \begin{pmatrix} \varepsilon_{1t} \\ \varepsilon_{2t} \end{pmatrix}$$

$$\text{(6)}\quad \text{DGP(2)} : \begin{pmatrix} y_{1t} \\ y_{2t} \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \end{pmatrix} \left[\begin{pmatrix} -.1 & -.5 \end{pmatrix} \begin{pmatrix} \varepsilon_{1t-1} \\ \varepsilon_{2t-1} \end{pmatrix} + \begin{pmatrix} .4 & -.3 \end{pmatrix} \begin{pmatrix} \varepsilon_{1t-2} \\ \varepsilon_{2t-2} \end{pmatrix} \right] + \begin{pmatrix} \varepsilon_{1t} \\ \varepsilon_{2t} \end{pmatrix}$$

$$\text{(7)}\qquad \begin{pmatrix} \varepsilon_{1t} \\ \varepsilon_{2t} \end{pmatrix} \sim N\left[\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 & .6 \\ .6 & 1 \end{pmatrix} \right]$$

Except for the means of the processes which have been fixed to zero, none of the coefficients are null in the DGP. Without lost of generality, we set both variances equals to 1 and assume that the errors are correlated with a coefficient of .6, i.e. that shocks are not independent at all point of time. The parameters of the MA part are chosen in order to obtain strong MA(2) effects and to yield an invertible multivariate process.

3.2. **The Pure MA(*q*) Case**

In the estimated model, we retain successively an order equal to 1, 2, 3 and 6 for the VMA(q), knowing the true one is a VMA(2)[6]. Table 1 gives the empirical sizes under the null of codependence while Table 2 presents the power results under the alternative of no codependence. Two sample sizes are taken into account: $T = 100$ and $T = 500$. We consider 10000 replications in each case, using the routine RNDN in GAUSS. $T + 50$ observations are generated and the first 50 are dropped to initialize the processes.

TABLE 1
Size of Codependence Tests in the* MA*(q) Case

$T = 100$	$q = 1$		$q = 2$		$q = 3$		$q = 6$	
	$s = 1$	$s = 2$	$s = 1$	$s = 2$	$s = 1$	$s = 2$	$s = 1$	$s = 2$
rank($\Theta_1 \ldots \Theta_q$)	4.45	100	4.48	99.97	4.60	99.95	4.87	99.89
rank($\Theta_2 \ldots \Theta_q$)			2.92	61.78	2.42	53.11	2.01	40.81
rank($\Theta_3 \ldots \Theta_q$)					.54	6.47	.34	7.14
rank($\Theta_4 \ldots \Theta_q$)							.31	6.47
rank($\Theta_5 \ldots \Theta_q$)							.44	5.74
rank(Θ_q)							.40	5.05
$\bar{\alpha}_2$	1.987		1.986		1.986		1.986	
s(α_2)	.284		.196		.191		0.189	
$T = 500$	$q = 1$		$q = 2$		$q = 3$		$q = 6$	
	$s = 1$	$s = 2$	$s = 1$	$s = 2$	$s = 1$	$s = 2$	$s = 1$	$s = 2$
rank($\Theta_1 \ldots \Theta_q$)	4.61	100	4.77	100	4.81	100	5.21	100
rank($\Theta_2 \ldots \Theta_q$)		4.68	99.99		4.57	99.98	4.28	99.86
rank($\Theta_3 \ldots \Theta_q$)					.62	6.17	.39	7.65
rank($\Theta_4 \ldots \Theta_q$)							.47	7.56
rank($\Theta_5 \ldots \Theta_q$)							.48	7.18
rank(Θ_q)							.53	6.11
$\bar{\alpha}_2$	1.998		1.999		1.999		1.999	
s(α_2)	.092		.079		.076		.073	

The tables have to be read in the following way. The columns labelled $q = 1,2,3,6$ refer to the autocovariance structure in the estimated model.

We look for the presence of s codependence vectors. Reduced-rank tests analyze the nullity of these autocovariances as a multivariate correlogram when one tests the null hypothesis that the sum of the two eigenvalues of $\hat{\Lambda}_K$ is equal to zero. We denote this situation $s = 2$ and a rejection means $s = 2 = n$. A rejection frequency of, say 100%, for $s = 2$ means that we

6. The corresponding χ^2 critical values at a 5% level for one zero eigenvalue in Λ_K $(s = 1)$ or for both null eigenvalues ($s = 2$) are given in the following table:

	$q = 1$	$q = 2$	$q = 3$	$q = 4$	$q = 5$	$q = 6$
$s = 1$	19.67	16.92	14.06	11.07	7.81	3.84
$s = 2$	36.41	31.41	26.30	21.02	15.51	9.49

These critical values allow to test for the non significance of autocovariance matrices up to a lag order equal to 6. If, for instance, one wishes to test the significant matrices in an MA(2), only the last two columns are used.

TABLE 2
Size of Codependence Tests in the* MA*(q) Case.

$T = 100$	$q = 1$		$q = 2$		$q = 3$		$q = 6$	
	$s = 1$	$s = 2$	$s = 1$	$s = 2$	$s = 1$	$s = 2$	$s = 1$	$s = 2$
$\text{rank}(\Theta_1 \ldots \Theta_q)$	22.98	100	99.81	100	99.51	100	99.25	100
$\text{rank}(\Theta_2 \ldots \Theta_q)$			46.41	99.96	35.58	99.99	34.51	99.91
$\text{rank}(\Theta_3 \ldots \Theta_q)$					.64	5.72	.44	7.29
$\text{rank}(\Theta_4 \ldots \Theta_q)$							.45	6.29
$\text{rank}(\Theta_5 \ldots \Theta_q)$							.42	5.63
$\text{rank}(\Theta_q)$							.52	4.41
$T = 500$	$q = 1$		$q = 2$		$q = 3$		$q = 6$	
	$s = 1$	$s = 2$	$s = 1$	$s = 2$	$s = 1$	$s = 2$	$s = 1$	$s = 2$
$\text{rank}(\Theta_1 \ldots \Theta_q)$	90.90	100	100	100	100	100	100	100
$\text{rank}(\Theta_2 \ldots \Theta_q)$			99.81	100	99.18	100	99.56	100
$\text{rank}(\Theta_3 \ldots \Theta_q)$					.72	5.81	.55	7.92
$\text{rank}(\Theta_4 \ldots \Theta_q)$							.52	7.43
$\text{rank}(\Theta_5 \ldots \Theta_q)$							.52	6.84
$\text{rank}(\Theta_q)$							.54	5.70

retain in each case a multivariate model of that specific order. The values under the column $s = 1$ are used to analyze the reduced-rank structure, i.e. to see whether there exists a codependence vector which reduces the order of the VMA process (for instance to a white noise in our case). Therefore, in this pure MA framework, under the null of codependence, the empirical size should not be too far from 5% (the nominal size) in the column $s = 1$, while the probability of rejecting the null autocovariance should be the highest as possible for $s = 2$ up to the lag order equal to 2. Under the null, the tables give the empirical mean as well as the standard deviation of the codependence vector which produces a white noise process[7]. Thus, we can study the bias of the estimated normalized coefficient. Regarding the power analysis, up to the 2nd order, the test statistics have to reject the null for both $s = 2$ and $s = 1$ as often as possible.

Under the null of codependence, we do not face size distortions in this pure VMA(q). The main source of worries definitely comes from the underestimation case under the alternative. As pointed out in Table 2, the lack of power may be important in a sample size ($T = 100$) which is usual in empirical studies. In small sample we also may notice a decrease of power if we maintain an overidentified dynamic structure[8].

3.3. The MA(∞) case

In this subsection, we consider the impact of misspecification, as the DGP is actually a VAR(1) and not a finite order MA. This case is quite interesting,

7. We will see that, with GARCH errors, there are strong size distortions for the null of a white noise.
8. As for the usual identification problem it may be preferable to overestimate the order instead of underestimate it. In a similar way it may be preferable to underestimate the degree of codependence instead of overestimate it. This result, not specific to codependence test statistics, are also found in Augmented DICKEY FULLER unit root tests for cointegration (see BOSWIJK and FRANSES [1992] for instance).

as it allows the use of codependence in a broader statistical context. This extension has already been analyzed by KUGLER and SCHWENDENER (1992) and by KUGLER and NEUSSER (1993). These papers do not however consider the issue of estimating the codependence vector but rather address the question of testing the validity of a well-known codependence restriction.

In (8), (9) and (10), we consider a VAR(1) in which the multivariate error process is the same as before. Note that the autoregressive matrix is chosen to get a stable process. Under the null of codependence considered through DGP(4), we generate the variables as before, i.e. the first row is two times the second row:

$$\text{(8)} \qquad \text{DGP(3)}: \begin{pmatrix} y_{1t} \\ y_{2t} \end{pmatrix} = \begin{pmatrix} .5 & .1 \\ .4 & .5 \end{pmatrix} \begin{pmatrix} y_{1t-1} \\ y_{2t-1} \end{pmatrix} + \begin{pmatrix} \varepsilon_{1t} \\ \varepsilon_{2t} \end{pmatrix}$$

$$\text{(9)} \qquad \text{DGP(4)}: \begin{pmatrix} y_{1t} \\ y_{2t} \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \end{pmatrix} \begin{pmatrix} .4 & .5 \end{pmatrix} \begin{pmatrix} y_{1t-1} \\ y_{2t-1} \end{pmatrix} + \begin{pmatrix} \varepsilon_{1t} \\ \varepsilon_{2t} \end{pmatrix}$$

$$\text{(10)} \qquad \begin{pmatrix} \varepsilon_{1t} \\ \varepsilon_{2t} \end{pmatrix} \sim N\left[\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 & .6 \\ .6 & 1 \end{pmatrix} \right]$$

Tables 3 and 4 give respectively the size and power in this situation. As one could expect, the autocovariances are significant for lag orders higher than two. From table 3, one observes no size distortions nor bias under the null[9].

TABLE 3
Size of Codependence Tests in the* MA*(q) Case.

$T = 100$	$q = 1$		$q = 2$		$q = 3$		$q = 6$	
	$s = 1$	$s = 2$	$s = 1$	$s = 2$	$s = 1$	$s = 2$	$s = 1$	$s = 2$
rank($\Theta_1 \dots \Theta_q$)	4.85	100	4.61	100	4.45	100	4.91	100
rank($\Theta_2 \dots \Theta_q$)			4.55	99.70	4.52	99.27	4.09	97.07
rank($\Theta_3 \dots \Theta_q$)				4.44	84.	30	3.05	65.89
rank($\Theta_4 \dots \Theta_q$)							2.05	35.64
rank($\Theta_5 \dots \Theta_q$)							1.50	21.01
rank(Θ_q)							1.62	16.33
$\bar{\alpha}_2$	1.959		1.958		1.957		1.956	
s(α_2)	.108		.102		.103		.105	
$T = 500$	$q = 1$		$q = 2$		$q = 3$		$q = 6$	
	$s = 1$	$s = 2$	$s = 1$	$s = 2$	$s = 1$	$s = 2$	$s = 1$	$s = 2$
rank($\Theta_1 \dots \Theta_q$)	5.01	100	5.06	100	5.41	100	5.31	100
rank($\Theta_2 \dots \Theta_q$)			5.01	100	5.27	100	5.12	100
rank($\Theta_3 \dots \Theta_q$)					5.10	100	4.64	100
rank($\Theta_4 \dots \Theta_q$)							4.57	99.19
rank($\Theta_5 \dots \Theta_q$)							4.11	85.68
rank(Θ_q)							3.80	64.20
$\bar{\alpha}_2$	1.992		1.992		1.992		1.992	
s(α_2)	.039		.039		.039		.039	

9. Note also that in this case, the empirical sizes are more stable even in small samples than for the MA(2) DGP process. This is related to the fact that, by definition in the case of a stable VAR, autocovariances steadily decrease (see remark 2).

TABLE 4
Power of Codependence Tests in the VAR(1).

$T = 100$	$q = 1$		$q = 2$		$q = 3$		$q = 6$	
	$s = 1$	$s = 2$	$s = 1$	$s = 2$	$s = 1$	$s = 2$	$s = 1$	$s = 2$
rank($\Theta_1 \ldots \Theta_q$)	73.01	100	56.74	100	47.75	100	34.36	100
rank($\Theta_2 \ldots \Theta_q$)			10.30	98.93	7.72	97.68	6.25	93.35
rank($\Theta_3 \ldots \Theta_q$)					5.78	80.82	3.44	61.57
rank($\Theta_4 \ldots \Theta_q$)							2.32	37.57
rank($\Theta_5 \ldots \Theta_q$)							1.65	25.80
rank(Θ_q)							2.20	23.08
$T = 500$	$q = 1$		$q = 2$		$q = 3$		$q = 6$	
	$s = 1$	$s = 2$	$s = 1$	$s = 2$	$s = 1$	$s = 2$	$s = 1$	$s = 2$
rank($\Theta_1 \ldots \Theta_q$)	100	100	100	100	100	100	100	100
rank($\Theta_2 \ldots \Theta_q$)			38.70	100	26.60	100	17.69	100
rank($\Theta_3 \ldots \Theta_q$)					9.78	100	7.43	100
rank($\Theta_4 \ldots \Theta_q$)							6.22	98.22
rank($\Theta_5 \ldots \Theta_q$)							5.07	83.55
rank(Θ_q)							5.01	65.45

However, with respect to DGP(3), the analysis concludes in favor of the existence of a serious decrease of power and a strong tendency to spuriously find out a significant degree of codependence. This last problem tends to become more important the higher the fitted lag length, even though in no single case we accept the white noise with 500 observations.

Remark 2. *On the stability of codependence vectors through q*: some important power losses may suggest to pay attention to other features like the stability of vectors corresponding to the significant eigenvalues (close to the first non different one from zero which is associated to the estimated codependence vector). This procedure works quite well for pure MA. However, such a procedure may be misleading for stable VAR(p) for which, even under the alternative, some regularity conditions lead to seemingly stable "pseudo-true codependence vectors". To see that, let us consider a VAR(1) $y_t = \Phi_1 y_{t-1} + \varepsilon_t$. The process is called stable if all eigenvalues of Φ_1 have modulus less than 1. Consequently, this can be rewritten as a VMA(∞) $y_t = y_0 + \Phi_1^i \varepsilon_{t-i}$ where the matrices Φ_1^i converge to zero as $T \to \infty$. The autocovariances of the VAR(1) are $\Gamma(h) = \sum_{i=1}^{\infty} \Phi_1^{h+i} \Omega_\varepsilon (\Phi_1^i)'$ or $\Gamma(0) = \Phi_1 \Gamma(1)' + \Omega_\varepsilon$ for $h = 0$ and $\Gamma(h) = \Phi_1 \Gamma(h-1)$ for $h > 0$ (LÜTKEPOHL [1991]). Consequently, when the autocovariances converge towards zero, the sum of the eigenvalues of Λ_K converge towards zero. As its first eigenvalues converge quicker towards zero, this gives the impression of the presence of a codependence vector. The sequence of pseudo-true codependence vectors converges towards the vector associated to a zero eigenvalue in the last autocovariance matrix. Depending on the variance-covariance matrix of errors and the eigenvalues (real or complex roots) of Φ_1, one gets different kinds of convergence (exponential or oscillatory) and speeds (which may regarded as a measure of the memory of the process). Table 5 illustrates this point by giving the eigenvectors associated with the smallest eigenvalues for different Φ_1 and Ω_ε matrices, the estimated model being an VMA(6).

TABLE 5
Autocovariances in VAR(1).

$T = 500$	case 1		case 2		case 3		case 4	
Φ_1	$\begin{bmatrix} .5 & .1 \\ .4 & .5 \end{bmatrix}$		$\begin{bmatrix} .5 & .1 \\ .4 & .5 \end{bmatrix}$		$\begin{bmatrix} .5 & .1 \\ -.4 & .5 \end{bmatrix}$		$\begin{bmatrix} .5 & .1 \\ -.4 & .5 \end{bmatrix}$	
Ω_ε	$\begin{bmatrix} 1 & .6 \\ .6 & 1 \end{bmatrix}$		$\begin{bmatrix} 1 & -.9 \\ -.9 & 1 \end{bmatrix}$		$\begin{bmatrix} 1 & .6 \\ .6 & 1 \end{bmatrix}$		$\begin{bmatrix} 1 & -.9 \\ -.9 & 1 \end{bmatrix}$	
	$s = 1$	$s = 2$	$s = 1$	$s = 2$	$s = 1$	$s = 2$	$s = 1$	$s = 2$
$\mathrm{rk}(\Theta_1 \cdots \Theta_q)$	99.61 [1, –.59]	100	100 [1, –1.41]	100	100 [1, 1.09]	100	100 [1, 25]	100
$\mathrm{rk}(\Theta_2 \cdots \Theta_q)$	17.69 [1, –.54]	100	14.52 [1, –.77]	100	68.36 [1, .58]	100	44.58 [1, .07]	100
$\mathrm{rk}(\Theta_3 \cdots \Theta_q)$	7.43 [1, –.51]	100	7.02 [1, –.53]	100	21.50 [1, .30]	99.94	16.72 [1, –.10]	100
$\mathrm{rk}(\Theta_4 \cdots \Theta_q)$	6.22 [1, –.50]	98.22	6.26 [1, –.50]	99.96	10.98 [1, .11]	80.84	12.56 [1, .28]	98.00
$\mathrm{rk}(\Theta_5 \cdots \Theta_q)$	5.07 [1, –.50]	83.55	5.74 [1, –.45]	94.76	5.82 [1, –.08]	40.80	9.52 [1, –.02]	59.54
$\mathrm{rk}(\Theta_q)$	5.01 [1, –.50]	65.45	5.42 [1, –.35]	79.86	5.16 [1, –.17]	27.90	8.46 [1, .41]	32.38

It comes out that when the roots of $\det(I_n - \Phi_1 z)$ are real, the pseudo-true codependence vector seems very stable. This is reinforced if the memory of the process is shorter (compare case 1 and case 2). Figures 1 and 2 give the distribution of case 1 both under the alternative and under the null with $\alpha' = [1, -2]$. One can expect that the non-codependent case will appear stable in empirical studies.

FIGURE 1
Sequence of Codependence Vectors (under the null).

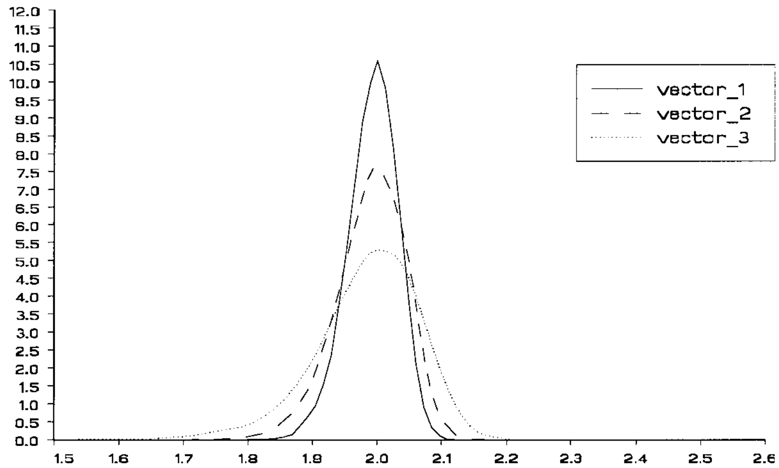
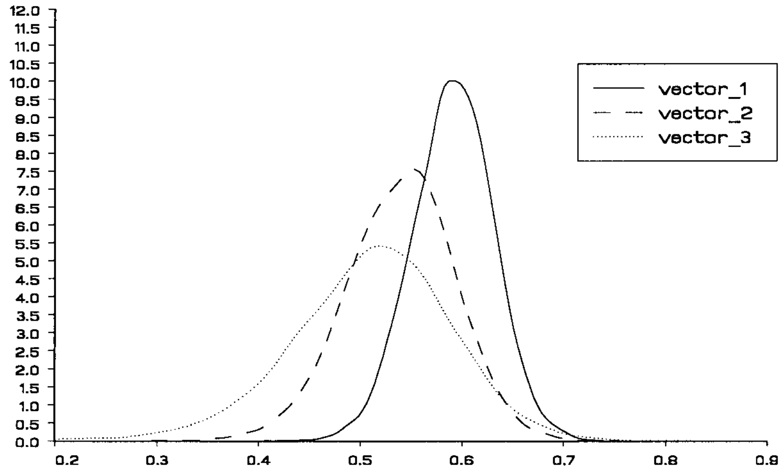


FIGURE 2

Sequence of Pseudo-Codependence Vectors (under the alternative hypothesis).



On the other hand, when the roots of det($I_n - \Phi_1 z$) are complex, the autocovariances converge in an oscillatory way towards zero, and the pseudo-codependence vector does not appear stable. ■

3.4. **Omission of a cointegrating vector**

We now investigate the power of codependence tests when omitting to account for a cointegrating relationship. We therefore generate two $I(1)$ variables y_{1t} and y_{2t} which are cointegrated with a cointegrating vector equal to [1 ,–1]. The dynamic structure is the same as for the VMA(2) in DGP(5) with errors structure (7). We assume that we do not take into account this long run relationship between the variables in level and carry out the codependence analysis on the first differences of these $I(1)$ variables. Table 6 gives the power of the test statistic in this situation. We obtain somewhat intuitive results. Indeed, the testing procedure spuriously concludes in favor of a codependence relationship much more often than in the case of non cointegrated series. Moreover, the value of the normalized pseudo-true codependence vectors (given in Table 6 under power figures) suggests that under the correct dynamic structure, i.e. a MA(2), the cointegrating vector drives both variables so that they exhibit a common structure in their first difference.

3.5. **Presence Non-Normal Errors**

In order to analyze the impact of non-normal disturbances on codependence test statistics, we consider two statistical cases often encountered in the analysis of financial data like interest rates, stock prices, exchange rates.

We first analyze the case of fatter tails than the gaussian ones by considering a bivariate Student's t distribution with 3 degrees of freedom and a

TABLE 6
Power of Codependence Tests in the presence of a Cointegrating Vector

$T = 100$	$q = 1$		$q = 2$		$q = 3$		$q = 6$	
	$s = 1$	$s = 2$	$s = 1$	$s = 2$	$s = 1$	$s = 2$	$s = 1$	$s = 2$
$\text{rk}(\Theta_1 \dots \Theta_q)$	44.68 [1, –2.46]	100	86.02 [1, –2.60]	100	86.83 [1, –2,71]	100	87.70 [1, –2.95]	100
$\text{rk}(\Theta_2 \dots \Theta_q)$			5.33 [1, –1.08]	53.88	4.28 [1, –1.11]	48.47	3.30 [1, 1.13]	41.54
$\text{rk}(\Theta_3 \dots \Theta_q)$					.86 [1, –1.80]	10.49	.61 [1, –1.71]	10.99
$rk(\Theta_4 \dots \Theta_q)$							.53 [1, –1.73]	10.30
$\text{rk}(\Theta_5 \dots \Theta_q)$							.42 [1, –1.74]	10.03
$\text{rank}(\Theta_q)$							.42 [1, –1.79]	9.03

$T = 500$	$q = 1$		$q = 2$		$q = 3$		$q = 6$	
	$s = 1$	$s = 2$	$s = 1$	$s = 2$	$s = 1$	$s = 2$	$s = 1$	$s = 2$
$\text{rk}(\Theta_1 \dots \Theta_q)$	99.63 [1, –2.46]	100	100 [1, –2.61]	100	99.91 [1, –2.73]	100	100 [1, –3.03]	100
$\text{rk}(\Theta_2 \dots \Theta_q)$			9.53 [1, –1.00]	99.73	9.63 [1, –1.00]	99.72	9.59 [1, –1.00]	99.59
$\text{rk}(\Theta_3 \dots \Theta_q)$					.84 [1, –1.78]	11.17	.66 [1, –1.59]	11.93
$\text{rk}(\Theta_4 \dots \Theta_q)$							.77 [1, –1.71]	11.74
$\text{rk}(\Theta_5 \dots \Theta_q)$							.70 [1, –1.75]	11.90
$\text{rk}(\Theta_q)$							.89 [1, –1.82]	10.81

correlation equal to .6. The data have been generated by dividing a multivariate Normal distribution by $\sqrt{S/\upsilon}$, where S is a independent χ^2 with υ degrees of freedom.

We also consider the very popular autoregressive conditionally heteroscedastic case. More precisely, we assume IGARCH(1,1) processes with normal conditional distributions and parameter values currently encountered with volatile data, i.e. a small parameter of volatility and a high MA one (see for instance BOLLERSLEV [1986]). The retained specification may be summarized as follows. We first generate two GARCH processes:

$$(11) \quad \begin{aligned} u_{1t} &= \eta_{1t}(h_{1t})^{1/2},\ h_{1t} = 1 + .25u_{1t-1}^2 + .75h_{1t-1},\ \eta_{1t} \sim N(0,1) \\ u_{2t} &= \eta_{2t}(h_{2t})^{1/2},\ h_{2t} = 1 + .1u_{2t-1}^2 + .9h_{2t-1},\quad \eta_{2t} \sim N(0,1) \end{aligned}$$

the two dimensional error process $\varepsilon_t = [\varepsilon_{1t}, \varepsilon_{2t}]'$ is then obtained by multiplying $u_t = [u_{1t}, u_{2t}]'$ by the Cholesky factorization of Ω_ε defined in (7) or (10). We retain the same specification (5) and (6) for the VMA(2) concerning the remaining of the process.

Because this type of misspecification specifically concerns financial data, we only present the results for $T = 500$. Table 7 and table 8 show size and power results for respectively the Student's t and the GARCH cases. Some comments are in order.

With respect to a higher kurtosis degree than the one associated with the Normal distribution, it comes out that there is neither size distortion, nor additional power losses with the Student's t in comparison.

TABLE 7
Size and Power of Codependence Tests with Student's t.

$T = 500$	$q = 1$		$q = 2$		$q = 3$		$q = 6$	
size	$s = 1$	$s = 2$	$s = 1$	$s = 2$	$s = 1$	$s = 2$	$s = 1$	$s = 2$
rank($\Theta_1 \dots \Theta_q$)	4.46	100	4.97	100	5.30	100	5.33	100
rank($\Theta_2 \dots \Theta_q$)			4.43	99.92	4.84	99.96	4.53	99.58
rank($\Theta_3 \dots \Theta_q$)					.60	6.73	.52	7.93
rank($\Theta_4 \dots \Theta_q$)							.48	7.33
rank($\Theta_5 \dots \Theta_q$)							.39	6.86
rank(Θ_q)							.48	5.92
$\bar{\alpha}_2$	1.999		1.999		1.999		1.999	
s(α_2)	.092		.077		.074		.071	
$T = 500$	$q = 1$		$q = 2$		$q = 3$		$q = 6$	
Power	$s = 1$	$s = 2$	$s = 1$	$s = 2$	$s = 1$	$s = 2$	$s = 1$	$s = 2$
rank($\Theta_1 \dots \Theta_q$)	81.59	100	100	100	100	100	100	100
rank($\Theta_2 \dots \Theta_q$)			98.50	100	97.14	100	98.27	100
rank($\Theta_3 \dots \Theta_q$)					.65	5.77	.65	7.88
rank($\Theta_4 \dots \Theta_q$)							.54	7.09
rank($\Theta_5 \dots \Theta_q$)							.54	6.81
rank(Θ_q)							.52	5.66

TABLE 8
Size and Power of Codependence Tests in the prense of GARCH(1,1).

$T = 500$	$q = 1$		$q = 2$		$q = 3$		$q = 6$	
size	$s = 1$	$s = 2$	$s = 1$	$s = 2$	$s = 1$	$s = 2$	$s = 1$	$s = 2$
rank($\Theta_1 \dots \Theta_q$)	18.46	100	28.88	100	35.09	100	45.69	100
rank($\Theta_2 \dots \Theta_q$)			18.04	98.90	26.18	99.12	38.43	93.22
rank($\Theta_3 \dots \Theta_q$)					4.98	28.82	10.75	45.91
rank($\Theta_4 \dots \Theta_q$)							8.86	40.21
rank($\Theta_5 \dots \Theta_q$)							6.56	34.53
rank(Θ_q)							3.81	25.57
$\bar{\alpha}_2$	1.987		1.992		1.992		1.993	
s(α_2)	.186		.118		.111		.107	
$T = 500$	$q = 1$		$q = 2$		$q = 3$		$q = 6$	
Power	$s = 1$	$s = 2$	$s = 1$	$s = 2$	$s = 1$	$s = 2$	$s = 1$	$s = 2$
rank($\Theta_1 \dots \Theta_q$)	65.65	100	100	100	99.91	100	100	100
rank($\Theta_2 \dots \Theta_q$)			87.31	100	94.70	100	96.99	100
rank($\Theta_3 \dots \Theta_q$)					6.21	28.02	12.60	44.55
rank($\Theta_4 \dots \Theta_q$)							10.27	39.41
rank($\Theta_5 \dots \Theta_q$)							8.56	33.70
rank(Θ_q)							5.57	25.50

The things are however quite different with GARCH errors. Firstly, the analysis points out that the use of the test statistics leads to an overestimation of the lag order. This is a multivariate extension of the results obtained by DIEBOLD [1986] who showed that in heteroscedastic models, the standard tests for autocorrelation like the portmanteau statistics are not efficient and that the asymptotic critical values may be strongly biased. See also HECQ [1996] for an extension of this point to the use of information criteria. Secondly, it is also found out in Table 8 that GARCH errors are responsible for huge size distortions in the sense that traditional critical values are too low for rejecting the null [10]. It is a little bit more tricky to compare the power because table 8 presents non size adjusted figures. So the power looks as high as in the non GARCH case on table 2. If we adjust the power for the empirical size (results not reported) it appears that we would retain a VMA of order two but the power of the test for the reduced rank structure decreases from 96.99 to 60.97 for $q = 6$ in Table 8.

3.6. Temporal aggregation

Finally, we briefly analyze the impact of temporal aggregation for stock and flow variables. For stock variables (unemployment rates, interest rates), it is usually advised to take one data on three for instance if one wants to obtain quarterly data from monthly ones. For flow variables (GDP, consumption), it is preferable to take the average or the sum. In order to compare size and power of temporal aggregation with our preceding results, we generate 500 observations and take one data out of five or a five order average (depending on the type of data) to work with a final sample size equal to 100. Actually, we only need to consider here the case of flow variables : the process trivially reduces to a multivariate white noise if we take one observation out of five, as in the case of stock variables. For flow variables, Tables 9 and 10 show that the unconstrained system becomes a VMA(1), a well known result. There exists in this case a codependence vector which reduces the system to a white noise without size distortions. However, the analysis reveals a serious lack of power, as the empirical figures do not reach the 15% level. This means that, unlike for non stationary variables (see PIERSE and SNELL [1995], SHILLER and PERRON [1985]), the reliability of the test statistics depends not only on the data span but also on the absolute sample size.

10. Non reported results with smaller sample size shows that test statistics are not consistent, as the distortions increase with the sample size. This is due to the fact that GARCH errors need time to spread out, especially because of the MA part of the GARCH. This result is also found for the impact of GARCH errors on unit root tests (see HECQ [1995], HECQ and URBAIN [1993]). Note that the results are amplified if we allow for a more leptokurtic conditional distribution, as a Student's *t* for instance, instead of a normal one.

TABLE 9

Size and Power of Codependence Tests with Temporal Aggregation.

$T = 100$ size	$q = 1$ $s = 1$	$s = 2$	$q = 2$ $s = 1$	$s = 2$	$q = 3$ $s = 1$	$s = 2$	$q = 6$ $s = 1$	$s = 2$
rank($\Theta_1 \ldots \Theta_q$)	4.44	99.12	4.74	99.76	4.36	99.73	4.24	98.55
rank($\Theta_2 \ldots \Theta_q$)			.55	7.17	.40	7.20	.27	5.57
rank($\Theta_3 \ldots \Theta_q$)					.58	6.84	.23	6.55
rank($\Theta_4 \ldots \Theta_q$)							.26	6.29
rank($\Theta_5 \ldots \Theta_q$)							.24	6.14
rank(Θ_q)							.41	6.14
$\bar{\alpha}_2$	1.894		1.929		1.929		1.923	
s(α_2)	1.143		.491		.473		.543	
$T = 500$ Power	$q = 1$ $s = 1$	$s = 2$	$q = 2$ $s = 1$	$s = 2$	$q = 3$ $s = 1$	$s = 2$	$q = 6$ $s = 1$	$s = 2$
rank($\Theta_1 \ldots \Theta_q$)	13.28	97.41	9.55	98.10	8.13	100	6.30	91.39
rank($\Theta_2 \ldots \Theta_q$)			.48	5.96	.37	6.38	.23	6.90
rank($\Theta_3 \ldots \Theta_q$)					.45	5.72	.22	6.30
rank($\Theta_4 \ldots \Theta_q$)							.18	5.54
rank($\Theta_5 \ldots \Theta_q$)							.19	5.28
rank(Θ_q)							.36	5.16

TABLE 10

Engle-Granger Cointegration Tests.

Country	ADF Statistic	Lag order
France	– 1.98	1
Netherlands	– 1.73	1
Germany	– 2.54	3
UK	– 1.27	2
Italy	– 1.60	1
Sweden	– 1.33	3
Spain	– 2.33	2
United States	– 1.39	1
Japan	– 0.682	1

Note: Residuals of regression of GNP (expressed in log) on unemployment rate. Residuals from the inverse regression did not give any modification of diagnostic about the presence of a cointegration relationship.The lag order necessary to make the residuals white noise is reported in the last column.

4 Empirical Illustration: OKUN's Law revisited

In his 1962 seminal paper, Arthur OKUN considered the link between unemployment changes and output changes. It is however important to mention that the so-called OKUN's Law may be read through two versions that are quite different from each others, both in an empirical and a theoretical perspectives. The first version relates the change in the unemployment rate to the growth rate of output while the second one is based on a relationship between the departure of the unemployment rate from its natural level and the GNP gap (the deviate of output from its potential level)[11].

Considering the regression of changes in the unemployment rate on the GNP growth rate, OLS estimates reported by HENIN and JOBERT [1993] range from – 0.17 to about – 0.41, reflecting a stronger relationship in the North-American labor market than in Europe. As emphasized by the authors, these estimates unfortunately abstract from a dynamic specification of the joint process unemployment-economic activity. This is precisely this point which can be tackled by the codependence approach, as all the significant dynamic correlations are taken into account in order to obtain the short run comovements.

Therefore, we conduct in this section a codependence analysis applied to the changes in the unemployment rate and the growth rate of GNP for major OECD economies. As we think that since the first oil shock, the interaction between activity and unemployment has been significantly altered, we conduct the analysis with seasonally adjusted quarterly data ranging from 74Q1 to 93Q4[12]. As we use data which are in fact first differences of possibly integrated variables, we test for the presence of a unit root on the unemployment rate and on the GNP level. Not surprisingly, using the traditional ADF tests (DICKEY and FULLER [1979 and 1981]), we find both variables $I(1)$ for all countries and the variations of unemployment as well as the GNP growth rates $I(0)$[13]. As stressed by the Monte-Carlo results of section 3, it is of high importance in this case to test for the presence of a cointegrating relationship before conducting the codependence analysis.

In order to detect such a relationship, we rely on the ENGLE-GRANGER [1987] two-step procedure even if we reach the same conclusions using the JOHANSEN (1988) maximum likelihood procedure. Results of the tests are reported in Table 10. These tests point out the absence of a long run relationship between the variables in level. LAGRANGE multiplier tests do not reject the absence of GARCH errors. As a consequence, we face a context appropriate to a codependence analysis in first difference.

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11. We do not consider the second version for several reasons. First, as pointed out by PRACHOWNY [1993], in order to avoid some misleading misspecification as well as to use a model consistent with a general production function, it is necessary to estimate not only potential output but also long term equilibrium values of additional variables determining the unemployment gap (such as the capital utilisation rate, the supply of workers or the number of working hours). These estimates in turn rely on quite different estimation methods and may influence greatly the estimated values of the parameter of interest. In fact, we consider the OKUN's law more as a reduced form relationship deemed appropriate for a first glance at the situation of the labour market and for international comparisons rather than a completely specified equilibrium relationship. Secondly, as stressed by HENIN and JOBERT [1993], this version is consistent with the view of an unemployment rate stationary around a reference level (thought to be 4%, the estimated natural level) and assumes a GNP level stationary around a potential GNP often in turn modelled as following a deterministic trend, either linear or quadratic (as in PRACHOWNY [1993]). This view is of course rather inappropriate, especially in the European context where the high persistence and the stochastic trend of the unemployment rate have become stylised facts.

12. All the data come from the OECD BSD database. Output is the gross domestic product (at factor cost), less government wages, expressed in volume.

13. As we suspect the presence of MA terms, we prefer to carry out DICKEY-FULLER tests rather than tests based on a non parametric correction (see SCHWERTZ [1989]).

For the codependence estimation procedure, we consider a lag order up to 12, which still takes into account a delay of 3 years in the joint process activity-unemployment. In order to carry out the correct procedure, we need to fit suitable VMA processes. The simulation results show that, in small samples, one has to exclude underestimated VMA process in order to prevent strong power losses. Therefore, we apply the following lag length selection procedure. We start with the maximal order and restrict the order if the sum of the two eigenvalues are null. We also look if each element of the autocorrelation matrix corresponding to this lag is not significantly different from zero (using the bounds of the confidence interval approximated by $2\sqrt{T}$). The results of the codependence analysis are summarised in Table 11.

TABLE 11

Codependence Tests Statistics - OKUN's Law.

Country	France		Netherlands		Germany		UK		Spain	
	$s=1$	$s=2$	$s=1$	$s=2$	$s=1$	$s=2$	$s=1$	$s=2$	$s=1$	$s=2$
$\text{rk}(\Theta_1 \dots \Theta_q)$	8.25	77.71*	29.21*	300*	24.61*	118*	36.70*	159*	49.61*	182*
$\text{rk}(\Theta_2 \dots \Theta_q)$	6.95	38.76*	11.99	168*	21.70*	77.72*	30.25	121*	41.32*	134*
$\text{rk}(\Theta_3 \dots \Theta_q)$	1.65	21.52*	11.91	110*	20.39*	52.55*	30.17	86.10*	30.56*	106*
$\text{rk}(\Theta_4 \dots \Theta_q)$	.35	8.40*	7.78	68.97*	5.65*	35.52*	20.16	63.61*	21.56	87.33*
$\text{rk}(\Theta_5 \dots \Theta_q)$			7.29	47.53*			18.01	58.64*	20.64	71.64*
$\text{rk}(\Theta_6 \dots \Theta_q)$			6.29	33.37*			17.52	56.37*	17.85	61.75*
$\text{rk}(\Theta_7 \dots \Theta_q)$			4.50	29.59*			17.38	51.18*	16.10	54.45*
$\text{rk}(\Theta_8 \dots \Theta_q)$			1.21	6.16			16.88	48.06*	15.36	48.77*
$\text{rk}(\Theta_9 \dots \Theta_q)$							8.34	36.98*	10.10	33.46*
$\text{rk}(\Theta_{10} \dots \Theta_q)$							7.59	28.39*	5.01	26.02*
$\text{rk}(\Theta_{11} \dots \Theta_q)$							6.49	19.78*	2.17	12.92
$\text{rk}(\Theta_{12})$							.35	10.57*	.19	9.62*
Codep. vector	[1, .545]		[1, 1.131]		none		[1, 970]		[1, 1.167]	

TABLE 11 (continued)

Codependence Tests Statistics - OKUN's Law.

Country	Sweden		USA		Japan		Italy	
	$s=1$	$s=2$	$s=1$	$s=2$	$s=1$	$s=2$	$s=1$	$s=2$
$\text{rk}(\Theta_1 \dots \Theta_q)$	17.54**	71.66*	13.83**	78.09*	4.80	26.27*	23.61*	59.75*
$\text{rk}(\Theta_2 \dots \Theta_q)$	8.35	37.09*	3.03	41.59*	1.19	22.31*	.24	10.97*
$\text{rk}(\Theta_3 \dots \Theta_q)$	7.16	21.61	1.03	23.06*	.34	17.85*		
$\text{rk}(\Theta_4 \dots \Theta_q)$	2.21	13.71	.29	5.67*	.12	10.60*		
$\text{rk}(\Theta_5 \dots \Theta_q)$	.64	10.22						
$\text{rk}(\Theta_6)$	.309	2.96						
Codep. vector	[1, –.599]		[1, .739]		[1, .232]		[1, –.555]	

A * indicates a rejection at a 5% level while ** means a rejection at 10% level

First, notice that we certainly face the VAR case for Spain, the U.K. and maybe the Netherlands. Our results illustrate the idea that the strength of the relationship is not reflected only by the value of the estimated elasticity (as in an OLS regression) but also by the estimated degree of codependence and the remaining persistence of the shocks. For instance, while the smallest estimated elasticity is found in the case of Japan, it is worth noting that this vector reduces the process to a white noise. On the contrary, in the case of Spain, the high elasticity is associated with a persistence of shocks over three quarters. Relying on these criteria, we find out strong relationships for France, the Netherlands, the UK, the United States and for Japan. The sign of the estimated elasticities is consistent with OKUN's Law. It is however worth re-emphasising that our estimates provide information on long run elasticities which are not to compare with OLS estimates of static regressions. These values are actually found higher than the values of the OLS estimates provided by the recent empirical studies mentioned above. For Germany, Italy and Sweden, our results do not conclude in favour of the existence of a strong relationship between the changes in unemployment and the GNP growth rates. For Germany, no codependence relationship is found[14]. For Italy and Sweden, while we find a degree of codependence equal to 1 and 2 respectively (1 for Sweden if we retain a 10% nominal level), it is quite dangerous to conclude in favour of codependence. In each case, the degree of codependence is rather small. Furthermore, the instability of successive estimated codependence vectors is indicative of the weakness of the link[15]. Given the weak power of the tests in small samples stressed by the Monte-Carlo simulations, one may question the actual existence of the detected relationship.

5 Conclusion

In this paper, we have studied the use of the codependence approach which allows to capture more accurately than standard techniques the dynamic interaction between stationary variables. We have illustrated the advantages of codependence through the estimation of OKUN's type relationships for the main OECD countries.

Through Monte Carlo simulations, we have investigated the robustness of the testing procedure in various statistical contexts. The results may be summarised as follow.

In most cases, our simulations do not provide evidence of size distortions or of a bias in the estimates as a result of misspecification. Regarding empirical sizes, one has however to pay attention to the occurrence of GARCH errors, as they induce size distortions which, furthermore, increase with the sample

14. Note that for Germany, we would fit a starting VMA(6) if looking strictly at all coefficients of the autocorrelation matrices. In this case, we obtain a codependence degree of 1, restricting the persistence of shocks of 4 to 3 with very unstable codependence vectors corresponding to the various values of k (the estimated codependence vector, i.e. the one associated to the smallest non significant eigenvalue) gives an elasticity of – 0.85. Once more, this instability is indicative of the absence of codependence.
15. Remember that, while stability does not imply codependence (see section 3.3), pronounced instability in the successive eigenvectors rules out the existence of a codependence relationship. For Italy, for $k = 1$, the implied elasticity is equal to 0.555 but found to be – 0.739 for $k = 2$. We find out the same feature for Sweden, going from an elasticity equal to 0.599 for $k = 1$ to a value of -1.115 for $k = 2$ (which is found moreover to be the appropriate vector if one adopts a 10% nominal level test rather than a 5% level).

size. In practice, more appropriate critical values could be derived by bootstrap like for instance in the unit root tests procedure in the presence of GARCH effects (HECQ [1995]).

Except for the pure MA(q) case, one has to be cautious in accepting codependence, as our simulations provide evidence of quite important power losses which may be due to various statistical reasons. Moreover, some results suggest that the stability of the estimated vectors corresponding to the non significant eigenvalues is not a discriminating feature for assessing the existence of a codependence relationship.

When working with data expressed in first differences like in our illustration, it is advised to preliminary check for the existence of a cointegrating relationship between the variables taken in level. Finally, unlike for integrated variables, data aggregation is shown to induce huge power distortions.

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