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Marketless Set-Up vs Trading Posts: A Comparative Analysis

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ABSTRACT. – This paper studies the efficiency properties with respect to transactions cost, of a trading post set-up vis-à-vis a marketless trading arrangement under the well-known frame-work of OSTROY and STARR [1974]. It is revealed from this exercise that the social institute of money works better when coupled with the social institution of markets and further shows the extent of inefficiency that can be potentially associated with marketless trade in large economies. This gives some insight into why, historically speaking, a market set-up has evolved over time replacing a marketless set-up.

Absence de marchés contre structure de lieux de commerce. Une analyse comparative.

RÉSUMÉ. – Cet article étudie les propriétés d'efficacité en ce qui concerne les coûts de transaction d'une structure de lieux de commerce contre des arrangements d'échanges bilatéraux en l'absence de structure de marchés dans le cadre bien connu d'Ostroy et Starr (1974). Cet exercice montre que l'institution sociale d'une monnaie fonctionne mieux en présence d'une institution sociale de marchés et souligne l'importance des inefficacités qui peuvent être potentiellement associées à des échanges sans structure de marchés dans de grandes économies. Ceci donne quelques indications sur les raisons pour lesquelles, d'un point de vue historique, une structure de marchés s'est substituée au cours du temps à une structure sans marchés.

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1 Introduction

The last two decades have witnessed the gradual emergence of a variety of models to explain the superiority of monetary over non-monetary trade from the point of view of transaction costs. In order to capture the essence of the problem, however, these models have generally dealt with institutionally vacuous economies, at least insofar as the traders are seen to confront each other in the absence of organized markets (OSTROY and STARR [1974], JONES [1976], KIYOTAKI and WRIGHT [1989] etc.). On the other hand, there is another stream of literature (see FOLEY [1970], WALLACE [1972] etc.) which concentrates exclusively on models which recognize the existence of markets or trading posts (of different forms, e.g. wholesale or retail) to deal with exchanges of different pairs of goods.

In the present paper we would incorporate social institution of markets in a framework involving monetary trade and try to examine the effect (on transaction costs) of combining the social institution of markets with monetary exchanges. In a recent work involving a different approach, STARR and STINCHCOMBE [1999] have shown how a set-up with trading posts using a medium of exchange can dominate a trading post set-up involving barter trades alone (see also Rajeev [1997]).

To study this issue we consider a pioneering contribution to the literature, viz. that of OSTROY and STARR [1974, 1990]. As is well-known, OSTROY and STARR deal with the question of *feasibility*. More specifically, they look for a condition to be satisfied by a good which, if used as a medium of exchange by the individual traders, can attain their desired allocations within a reasonable period of time. By introducing trading posts in the OSTROY-STARR set-up one can conclude that the time taken to attain the agents' desired allocations *to each given degree of approximation* is bounded above irrespective of the size of the economy. However no such bound may exist in a marketless economy. This result is established here for an economy involving only two goods and thus having direct barter exchanges. This can be easily extended to an economy with more goods and involving monetary trade.

The paper is arranged as follows. The next section compares the trading post set-up with the marketless trading arrangement. A concluding section sums up the findings.

2 Transaction Costs: Trading Posts vs Marketless Set-up

We consider an economy consisting of K traders (indexed by k) and n commodities (indexed by c), the n-th commodity, denoted by M, being earmarked to serve as the medium of exchange. The set of all traders is denoted by K also and the set of commodities other than the n-th good by C. The initial endowments of the K traders are depicted by the $K \times n$ matrix $\underset{\sim}{W} = ||w_{kc}||$, where w_{kc} represents the level of good c in the initial endowment of agent k. The utility function of agent k is U_k: $\mathbb{R}^n_+ \to \mathbb{R}$, $k = 1, 2, \ldots, K$. Prices

$$\underset{\sim}{p}' = (p_1, \ldots, p_n)$$

are exogenously given at the equilibrium levels. Given the prices and the initial endowments, the Walrasian excess demands for the economy are given by the $K \times n$ matrix $\underset{\sim}{Z} = ||z_{kc}||$, where z_{kc} is the level of agent k's excess demand for (or supply of) good c.

$z_{kc} > 0$, if agent k is an excess demander of good c

$z_{kc} < 0$, if agent k is an excess supplier of good c

Similarly, the final demands are given by the $K \times n$ matrix $\underset{\sim}{X} = \underset{\sim}{W} + \underset{\sim}{Z} = ||x_{kc}||$, where x_{kc} is the level of good c finally consumed by agent k.

Thus, the economy under study is characterized by $(\underset{\sim}{W}, \underset{\sim}{Z}, \underset{\sim}{p})$, where p is the transpose of $\underset{\sim}{p}'$. The following properties are assumed:

(U.1) The value of total excess demand equals zero, which can be represented as $Z \cdot p = 0$, where the price of the n-th good p_M is assumed to be positive.

(U.2) The total quantity of a particular good purchased is exactly equal to the aggregate amount of that good sold. In other words, $\sum_k \underset{\sim}{Z}_k = \underset{\sim}{0}$, where Z_k represents the k-th row vector of Z.

(U.3) $\underset{\sim}{Z} \geqslant -\underset{\sim}{W}$, the inequality is assumed to hold entrywise. It implies that the final consumption of each good by every agent is non-negative.

The trading arrangement taken up by OSTROY and STARR in a marketless environment is based on the assumption that in order to explore the trading possibilities, agents meet each other in pairs following an arbitrary sequence of meetings. This ad hoc sequence in no way depends on the goods an agent desires to purchase or sell. Further, when one pair meets, other pairs are allowed to meet simultaneously. Most importantly when trade takes place through such *decentralized, sequential, bilateral* meetings each agent would insist the value of his incomings through trade to be atleast as large as the value of his outgoings. More precisely,

(U.4) each bilateral trade must satisfy the *quid-pro-quo* between incomings and outgoings.

In the absence of perfect mutual coincidence of wants between the traders, in order to maintain this *quid-pro-quo*, an agent may have to accept a good from his trading partner for which he does not have final demand. In that case the good takes the form of a medium of exchange as the agent accepts it not because he finally wants to consume the good but he thinks that he can pass it on to others in exchanges for goods he finally wants to have.

For the system described so far suppose the following condition introduced by OSTROY and STARR [1974] is satisfied

(1) $$\sum_{c \neq M} p_c [z_{kc}]^+ \leqslant p_M w_{kM}, \ \forall k$$

where, c is the index for commodities, k the index for agents, M the commodity earmarked to denote the medium of exchange, p_c is the equilibrium price of good c, z_{kc} is the excess demand (supply) for(of) good c for agent k, $[z_{kc}]^+ = \max[0, z_{kc}]$, w_{kM} is the initial endowment of good M for agent k.

Therefore, the left hand side of inequality (1) shows the total value of agent k's desired purchase of goods other than M. On the other hand, the right hand side of (1) expresses the value of the initial endowment of good M for agent k. Hence, condition (1) guarantees the fact that an agent possesses enough money in his initial endowment to cover his desired purchases of all the remaining goods.

It is shown in the literature that if there exists a commodity M satisfying condition (1) above in any give competitive economy with a finite population, then using that good as a medium of exchange an agent can fulfill his/her excess demands within *one round* i.e., the time required for each agent to meet every other agent once and only once. The result holds irrespective of the sequence in which the agents meet in a marketless arrangement [1].

Suppose now we ask whether a medium of exchange satisfying such a condition can function *better* in a trading post set-up. Thus, we consider an economy with trading posts or separate markets for dealing with each good against M. Each agent visits only those markets which are relevant for him.

Depending on the levels of excess demand vectors and a specific sequence of meeting between the agents or visiting markets a marketless arrangement may sometime take less time than a trading post set-up. However, in general for economies with *large population* the advantages of the social institution of markets is clearly revealed. To compare the case of a large economy vis a vis a small one, an appropriate framework to consider may be that of DEBREU and SCARF [1963]. More precisely, the economy consists of specific classes of agents where the agents belonging to a particular class have identical characteristics with respect to utilities and initial endowments. In other words, the set-up under consideration is a replication of a smaller economy. We represent an economy of this nature satisfying condition (1) by E and its v times replication by E_v.

Now before going to deal with the question of efficiency of a market set up let us first describe the frame work in some detail. As mentioned above in an n good economy there exists $(n-1)$ markets or equivalently trading posts to exchange each good against M and all these markets are functioning simultaneously. At a particular point of time t an agent chooses to visit any *one* of these trading posts in which he wishes to trade at that time and this depends on the number of goods for which he has unsatisfied demand or supply. Let η_k^t denote the number of goods (excluding M) agent k wishes to either buy or sell at the beginning of period t. In other words he has unsatisfied demand for (supply of) η_k^t goods at the beginning of period t. We *assume* that he would choose any one of these η_k^t trading posts dealing with the η_k^t goods at *random*

1. A more plausible feasibility criterion compared to (1) is that a good, say M, is demanded in positive quantity by every agent i.e.,

$$z_{kM} + w_{kM} > 0, \text{ for all } k$$

Such a good can be used as a medium of exchange to attain equilibrium in finite time. (See, RAJEEV [1990] and DASGUPTA and RAJEEV [1997]).

with *equal* probability. Looking at reality we assume that when an agent visits a trading post he can *distinguish* between buyers and sellers present in that trading post. It takes *one* unit of time to visit a trading post and exchange with *one* trader. On the other hand if an agent (say, a buyer of good *c*) when visits the trading post for good *c* is unable to meet a seller (this can happen if number of buyers present at that time is more than the sellers) he looses *one* unit of time [2]. Suppose at a particular point of time say, *t*, δ_c^t buyers and ψ_c^t sellers have chosen to visit the trading post for good *c*. If $\min(\delta_c^t, \psi_c^t) = \delta_c^t$, probability that a buyer meets a seller is unity and a seller meeting a buyer is $\frac{\delta_c^t}{\psi_c^t}$. When a trader *i* meets a complementary trading partner *j* at the trading post for good *c*, exchange that take place between them can be in general represented by the following formulation. Let e_{kc}^t denote the amount of good *c* bought or sold (i.e., exchanged) by agent $k = i, j$ at time period *t*. We would use the convention that

$e_{kc}^t > 0$ if good *c* is purchased by agent *k* at period *t*.

$e_{kc}^t < 0$ if good *c* is sold by *k* at *t*

$$(2) \qquad \left.\begin{aligned} e_{ic}^t = -e_{jc}^t &= \min(|V_{ic}^t|, |V_{jc}^t|), \text{ if } V_{ic}^t > 0 \text{ and } V_{jc}^t < 0 \\ &= -\min(|V_{ic}^t|, |V_{jc}^t|), \text{ if } V_{ic}^t < 0,\ V_{jc}^t > 0 \end{aligned}\right\}$$

where, V_{kc}^t is the level of unsatisfied demand for good *c* for agent *k* at the beginning of period *t*. In particular,

$V_{kc}^t > 0$, if *k* has excess demand for *c*

$V_{kc}^t < 0$, if *k* has excess supply of *c*.

We would like to note that when an agent sells good *c* he gets paid by good *M* such that quid-pro-quo has been maintained. From this trading rule (2) it is clear that V_{kc}^t and hence η_{kc}^t change over time.

On the other hand, in the *marketless* trading arrangement (see OSTROY and STARR [1990] and KIYOTAKI and WRIGHT [1989]), which we would consider for the purpose of comparison, an agent meets another agent at *random* without any reference to the goods they want to exchange. More precisely if a pair of agents meet in a marketless arrangement they scan their excess demands and supplies of all the $(n-1)$ goods (excluding *M*). If an agent has excess supply of a particular good and his trading partner has excess demand, exchange for that good takes place and so on. Finally quid-pro-quo ((U.4)) has been maintained by transferring good *M* appropriately. If π is the number of goods traded by a pair of trading partners, $T(\pi)$ denotes the time involved in this process of exchange. We *assume* that $T(1) = 1$ and $T(\pi)$ is a non decreasing function of π. For notational simplicity, we also assume that when a pair of agents meet and scan their demands and they find that no trade can take place, it takes one unit of time [3].

In order to compare the time requirement under the above two trading arrangements we would first choose rational numbers from $[z_{ic} - \varepsilon, z_{ic} + \varepsilon]$ and

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2. We note that all our results hold if we assume the time requirement in this case to be any positive number.
3. Note that all our results hold if $T(0)$ takes any positive value.

$[z_{jc} - \varepsilon, z_{jc} + \varepsilon]$ for all c and denote them respectively by χ_i^c and Ω_j^c. We then consider the problem of achieving these rational values instead of the original z_{ic}'s and z_{jc}'s. The total time needed to arrive at the ε-neighbourhood of a competitive allocation under a trading post set-up would be represented by $[\widehat{T}_{TP}](\varepsilon, \nu)$. A parallel notation, viz. $[\widehat{T}_{ML}](\varepsilon, \nu)$ would be used to denote the same for the alternative marketless framework, where ν represents the number of replications of the economy and thus acts as an index of size of the set-up. For a set-up of this nature we have the following results.

PROPOSITION 1 : For an economy E_ν described above, there exists an integer $\overline{N}(\varepsilon)$, independent of ν s.t

$$E(\widehat{T}_{TP}(\varepsilon, \nu)) < \overline{N}(\varepsilon), \ \forall \nu$$

and hence

$$\text{Prob}\{\widehat{T}_{TP}(\varepsilon, \nu)) < \infty, \quad \text{for any } \nu\} = 1.$$

Proof : See Appendix.

Thus, the time taken to attain ones desired bundle of goods is independent of the size of the economy.

Note that if the original excess demands were themselves rational numbers, then the above result would go through without the need for the ε approximation.

On the other hand, a conclusion as general as the above does not hold for the marketless trading arrangement. In fact, it is possible to construct a large number of economies (with $z_{kc} \in Q$, the set of rational numbers, $\forall k, \forall c$) where expected time taken in a marketless trading scheme diverges as ν increases. More precisely,

PROPOSITION 1 : There exists at least one economy of the type E_ν for which

$$E(\widehat{T}_{ML}(\nu)) \to \infty \text{ as } \nu \to \infty$$

Proof : See Appendix.

The above two results show clearly the advantages of a trading post set-up for large economies.

Also, from Remark 2 (in Appendix) it is clear that one can construct large number of competitive economies where Proposition 2 holds.

Remark 1 : It is important however, to emphasize the fact that in the case of the above result (Proposition 1) the rational approximation played a crucial role. In fact, it would be impossible in general to establish the uniform bound if the agents' excess demand vectors were comprised of irrational components and they tried to achieve these demand points exactly. As Example 1 below shows clearly, no such uniform bound would exist even if the economy with trading posts consists of only two goods.

Example 1 : Consider an economy $\widetilde{E}$ having two goods (say, c and M) and 3 demanders and 3 suppliers with excess demands and supplies for good c

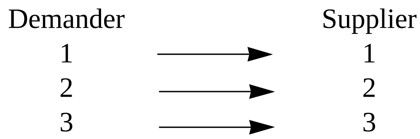
against M. Let z_i^d and z_j^s (i, j = 1,2,3) denote respectively the excess demands and excess supplies of the i-th and j-th agents for good c with

$$(z_1^d, z_2^d, z_3^d) = (1, \sqrt{2}, \sqrt{3})$$
$$(z_1^s, z_2^s, z_3^s) = (1, \sqrt{2}, \sqrt{3})$$

Initial endowments of the (excess) demanders are represented by $w_1^d = (0, 1)$, $w_2^d = (0, \sqrt{2})$, $w_3^d = (0, \sqrt{3})$, where the first components refer to good c and the second components refer to good M. Similarly initial endowments of the (excess) suppliers are given by, $w_1^s = (1, 0)$, $w_2^s = (\sqrt{2}, 0)$ and $w_3^s = (\sqrt{3}, 0)$. It can be easily checked that condition (1) of OSTROY and STARR is trivially satisfied here (where the equilibrium price vector is $p' = (1, 1)$).

From z_i^d's and z_j^s's it is clear that $z_2^d, z_3^d, z_2^s, z_3^s \in Q^c$, the set of irrational numbers. Also rank $(1, \sqrt{2}, \sqrt{3}) = 3$, in the field of integers.

Given the above competitive economy, consider the following sequence of meetings. The first demander meets the first supplier, the second demander meets the second supplier and so on as depicted through the following diagrammatic scheme:



Let the time taken in attaining equilibrium (starting from the initial endowments) when this economy replicates ν times be denoted by $\widehat{T}_{TP}(\nu)$. Clearly, $\widehat{T}_{TP}(1) = 1$ for the aforementioned sequence of meetings.

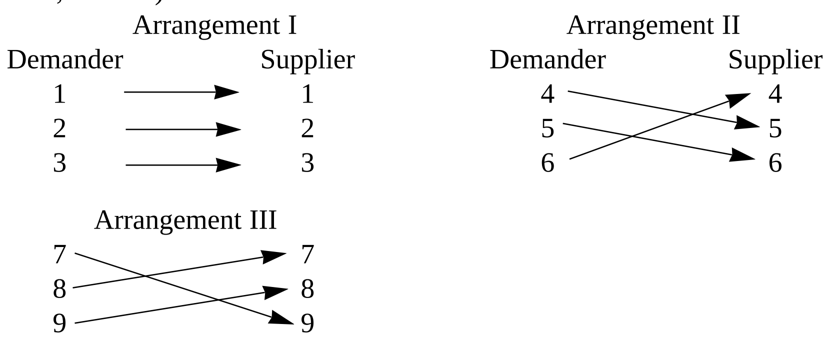
Let us now consider a $\nu = 3$ times replication of this economy. That is, there are 9 demanders as well as suppliers. Let

$$(z_1^d, z_2^d, \ldots, z_9^d) = (1, \sqrt{2}, \sqrt{3}, 1, \ldots, \sqrt{3}).$$

Similarly

$$(z_1^s, z_2^s, \ldots, z_9^s) = (1, \sqrt{2}, \sqrt{3}, 1, \ldots, \sqrt{3})$$

Consider the following sequence of meetings shown by the arrows in the figure below. Arrangement I is related to the first 3 demanders and suppliers. Similarly, Arrangement II is related to the next 3 demanders and suppliers (viz. 4, 5 and 6) and so on.



Let V_{id}^{t} (V_{is}^{t}) denote demander (supplier) i's (j's) unsatisfied demand (supply) for (of) good c at the beginning of time period t. Thus, from trading arrangement I,

$$V_{1d}^{2} = V_{2d}^{2} = V_{3d}^{2} = V_{1s}^{2} = V_{2s}^{2} = V_{3s}^{2} = 0.$$

From trading arrangement II

$$V_{4d}^{2} = V_{5d}^{2} = V_{4s}^{2} = 0$$

$$V_{6d}^{2} = \sqrt{3} - 1,\ V_{5s}^{2} = \sqrt{2} - 1,\ V_{6s}^{2} = \sqrt{3} - \sqrt{2}$$

Thus, 3 agents get satisfied and 3 agents remain to be satisfied. The reason behind this is quite simple. When two agents meet, as least one of them would get satisfied. But not more than 3 traders would get satisfied because $|1 - \sqrt{2}| \neq |\sqrt{2} - \sqrt{3}| \neq |\sqrt{3} - 1| \neq 0$.

In trading arrangement III for the same reason, only 3 agents would be satisfied and

$$V_{8d}^{2} = \sqrt{2} - 1,\ V_{9d}^{2} = \sqrt{3} - \sqrt{2},\ V_{9s}^{2} = \sqrt{3} - 1$$

In period 2, consider the following sequence of meetings.

Arrangement IV

Demander		Supplier
6	⟶	9
8	⟶	5
9	⟶	6

This would result in all the agents reaching equilibrium in two periods. Thus,

$$\widehat{T}(3) = 2$$

As a next step consider the $\nu = 3^2$ times replication of this economy and a similar sequence of meetings. One can easily check that $\widehat{T}(3^2) = 3$ so on.

Proceeding similarly it can be shown that in general there exists at least one sequence of meetings s.t. $\widehat{T}(3^l) = l + 1$ and hence the time taken for attaining equilibrium rises unboundedly as the economy replicates indefinitely.

Notice that in the above example if in the second period (arrangement IV) demanders 6, 8 and 9 meet suppliers 5, 6 and 9 respectively (or demanders 6, 8 and 9 meet suppliers 6, 9 and 5 respectively) time taken to reach equilibrium will increase even more. Similarly, time requirement for larger replication will also increase.

Note 1: When the demand and supply figures are rational numbers, the time taken to attain ones desired bundle cannot go to infinity due to the following reason. Let g be the greatest common divisor of the non-zero z_{kc} values, for all k and for all c. That is, $z_{kc} = g.\ I_{kc}$, where I_{kc} is an integer.

Hence any pairwise meeting would reduce the agents' excess demands and supplies at least by g. Since z_{kc}'s are finite, after a finite number of stages every agent would reach equilibrium.

A greatest common divisor need not exist when some z_{kc}'s are irrational numbers. Therefore, a *fixed* reduction in excess demand and supply after each pairwise meeting cannot be ensured for all such competitive economies.

4 Conclusion

This paper formally establishes the intuitively clear fact that from the point of view of time as a transactions cost, the trading post set-up dominates a marketless trading arrangement. This we believe, shades some light on why, historically speaking a market set-up has evolved over time replacing a marketless set-up.

Proof of Proposition 1

We choose rational numbers from $[z_{ic} - \varepsilon, z_{ic} + \varepsilon]$ and $[z_{jc} - \varepsilon, z_{jc} + \varepsilon]$ for all c and denote them respectively by χ_i^c and Ω_j^c. We then consider the problem of achieving these rational values instead of the original z_{ic}'s and z_{jc}'s. The associated time cost in a trading post set-up would then be denoted by $[T_c]_\varepsilon$. An exactly analogous problem would be considered for each of the $(n-1)$ trading posts that exists for dealing with each good against M (the medium of exchange). Let K_c^S and K_c^D represent respectively the sets of suppliers and demanders of good c.

Let g be the greatest common divisor of the elements of the set

$$A = \{\chi_i^c, \Omega_j^c : c \in C, i \in K_c^D, j \in K_c^S\}$$

That is, g is the largest real number such that $z_{kc} = g \cdot I_{kc}$, $\forall k$, $\forall c$, where I_{kc} is an integer.

At each period, an agent chooses a trading post at random from the set of trading posts he wishes of visit. Let η_k^t denote the number of trading posts which k wishes to visit at the beginning of time period t. Let

$$B_k^t = \{c : |V_{kc}^t| > 0, c \neq M\}$$

then

$$\eta_k^t = \# B_k^t$$

Since $|V_{kc}^t|$ is a decreasing function of t, therefore, η_k^t decreases in time.

According to our assumption, at period t, agent k chooses a trading post with equal probability $= 1/\eta_k^t$.

Let us denote the probability for a particular demander $i\,(\in K_c^{Dt}$, the set of unsatisfied demanders of good c at the beginning of time period t) being present in trading post (c, M) at period t by ρ_{ic}^t. Hence,

$$\rho_{ic}^t = \frac{1}{\eta_i^t} \geqslant \frac{1}{n-1} \tag{1}$$

Similarly, for a supplier $j\,(\in K_c^{St})$

$$\rho_{jc}^t \geqslant \frac{1}{n-1} \tag{2}$$

We denote by δ_c^t and ψ_c^t respectively the actual number of demanders and suppliers *present* in trading post (c, M), at time period t. The following relations can hold between δ_c^t and ψ_c^t

(i) $\min(\delta_c^t, \psi_c^t) = \delta_c^t$

This implies each buyer would meet a seller with probability 1 and hence it is ensured that unsatisfied demand of *each* buyer present would reduce at least by g.

(ii) $\min(\delta_c^t, \psi_c^t) = \psi_c^t$

This implies unsatisfied supply of *each* seller present would be reduced at least by g.

(iii) $\delta_c^t = \psi_c^t$

This implies excess unsatisfied supply as well as demand of the respective agents would be reduced at least by g.

Note that at any period t the *probability* that both a supplier as well as a demander being present at this market $\geqslant \frac{1}{(n-1)^2}$.

From the above discussion it is now clear that at each period t *expected* reduction of unsatisfied demand or supply of any arbitrary demander (indexed by i) or supplier (indexed by j) or both will be $\geqslant \frac{1}{(n-1)^2} \cdot g$. Thus the maximum possible time to be taken to clear the market will be less than or equal to the total time needed to reduce the unsatisfied demand as well as supply of the largest buyer and seller to zero.

Let at time period t, $\widetilde{R}^t$ be the reduction that occurs to the expression

$$\widetilde{V}_c^t = V_{lc}^t + |V_{hc}^t|$$

where

$$z_{lc} \geqslant z_{ic}, \forall i \in K_c^D, \; i \neq l \in K_c^D$$

and

$$|z_{hc}| \geqslant |z_{jc}|, \forall j \in K_c^S, \; j \neq h \in K_c^S$$

The (c, M) trading post would be completely cleared (i.e. $\widetilde{V}_c^t = 0$) after atmost τ periods if

$$\sum_{t=1}^{\tau} \widetilde{R}^t = \max_t \widetilde{V}_c^t = |z_{lc}| + |z_{hc}|$$

We have,

$$E\left(\sum_{t=1}^{\tau} \widetilde{R}^t\right) = EE\left(\sum_{t=1}^{\tau} \widetilde{R}^t | \tau\right)$$

$$= EE(\widetilde{R}^1 + \ldots + \widetilde{R}^\tau)$$

$$= E\{E\widetilde{R}^1 + E\widetilde{R}^2 + \ldots + E\widetilde{R}^\tau\}$$

$$\geqslant E\left(\frac{g}{(n-1)^2}\tau\right)$$

$$\therefore E(\tau) \leqslant \frac{(n-1)^2[z_{lc} + |z_{hc}|]}{g}$$

i.e.

$$E(T_C) \leqslant \frac{(n-1)^2[z_{lc} + |z_{hc}|]}{g}.$$

which is independent of ν.

A similar exercise can be performed for other trading posts. Adding over all c we get,

$$E(\widehat{T}_{TP}(\nu)) \leqslant \overline{N}(\varepsilon), \text{ independent of } \nu.$$

Coming now to the second part of the result, consider the sequence of random variables $\widehat{T}_{TP}(\nu)$. We have shown that

$$E(\widehat{T}_{TP}(\nu)) \leqslant \overline{N}(\varepsilon) \quad \forall \nu$$

$\therefore$ Prob $\{\widehat{T}_{TP}(\nu) < \infty$, for any $\nu\} = 1.$ □

Proof of Proposition 2

Consider an economy with 2 types of agents, viz. type 1 and type 2 and two goods, to be called *c* and *M*. The total number of type 1 agents is equal to the total number of type 2 agents. Each type 1 agent has one unit of good *c* and demands in return 1 unit of good *M*. A type 2 trader on the other hand has one unit of good *M* and demands in return 1 unit of good *c*.

It is clear that when a type 1 agent meets a type 2 agent, they can trade and attain their desired commodity bundles. Suppose each type consists of *X* number of agents. Then, the probability that an arbitrary type 1 agent, say *k*, meets a complementary trading partner (i.e. a type 2 agent) in the first period is

$$p_1 = \frac{X}{X + X - 1} = \frac{1}{2 - \dfrac{1}{X}} \leqslant \frac{2}{3} \text{ if } \quad X \geqslant 2.$$

The denominator of this probability (i.e. $X + X - 1$) refers to the total number of agents excluding *k* and the numerator (i.e. *X*) represents the number of complementary trading partners.

The probability of not meeting any type 2 agent by *k* in the first period is

$$1 - p_1 = 1 - \frac{X}{2X - 1} \geqslant \frac{1}{3}.$$

Let X_t^U denote the number of type 1 agents (or, equivalently type 2 agents) remaining unsatisfied at the beginning of time period *t*.

$$\therefore X_1^U = X$$

Clearly, X_t^U, $t > 1$, is a random variable and hence we would be concerned here with EX_t^U. Since the probability of remaining unsatisfied during the first period is $(1 - \dfrac{X}{2X-1})$ and there are a total of *X* type 1 traders in the economy

$$E(X_2^U) = X \cdot \left(1 - \frac{X}{2X - 1}\right) \geqslant \frac{1}{3}X.$$

Let us define an indicator function

$$I_{X_2^U = x} = 1, \text{ if } X_2^U = x$$
$$= 0, \text{ otherwise}$$

Then we can write

$$E(X_2^U) = E(X_2^U I_{X_2^U = 1} + X_2^U I_{X_2^U \geqslant 2})$$
$$= E(X_2^U I_{X_2^U = 1}) + E(X_2^U I_{X_2^U \geqslant 2})$$

Let us now calculate

$$E(X_2^U I_{X_2^U \geqslant 2}) = \sum_{\gamma=2}^{X} \gamma Pr\{X_2^U = \gamma\}$$

$$= \sum_{\gamma=2}^{X} \gamma \binom{X}{\gamma} (1-p_1)^{\gamma} p_1^{X-\gamma}$$

$$= X(1-p_1) \sum_{\gamma-1=1}^{X-1} \binom{X-1}{\gamma-1} (1-p_1)^{\gamma-1} p_1^{X-\gamma}$$

$$= X(1-p_1)(1-p_1^{X-1}) \geqslant \frac{1}{3} X(1-p_1), \text{ if } X \geqslant 2$$

(3) $$E(X_2^U I_{X_2^U \geqslant 2}) \geqslant \left(\frac{1}{3}\right)^2 X$$

Let us consider period $t = 3$. Suppose at the beginning of the third period, X_3^U is the number of unsatisfied agents. The probability that a type 1 agent gets satisfied during period 2 is

$$p_2 = \frac{X_2^U}{X_2^U + X_2^U - 1}$$

$$\therefore 1 - p_2 = 1 - \frac{X_2^U}{2X_2^U - 1} \geqslant \frac{1}{3}, \text{ if } X_2^U \geqslant 2$$

At the beginning of period $t = 2$, X_2^U agents have remained unsatisfied and they have a propability $(1 - p_2)$ of remaining unsatisfied during the third period

$$\therefore E(X_3^U) = EE(X_3^U | X_2^U)$$

$$= E\left(X_2^U\left(1 - \frac{X_2^U}{2X_2^U - 1}\right)\right)$$

$$= E\left[X_2^U\left(1 - \frac{X_2^U}{2X_2^U - 1}\right) I_{X_2^U = 1}\right]$$

$$+ E\left[X_2^U\left(1 - \frac{X_2^U}{2X_2^U - 1}\right) I_{X_2^U \geqslant 2}\right]$$

$$= 0 + E\left[X_2^U\left(1 - \frac{X_2^U}{2X_2^U - 1}\right) I_{X_2^U \geqslant 2}\right]$$

$$\geqslant \frac{1}{3} E\left[X_2^U I_{X_2^U \geqslant 2}\right]$$

$$\geqslant \frac{1}{3}\left(\frac{1}{3}\right)^2 X = \frac{1}{3^3} X \quad \text{(using (3))}$$

As before in an exactly parallel manner let us calculate

$$
\begin{aligned}
& E\left[X_3^U I_{X_3^U \geqslant 2}\right] \\
=& E E\left[X_3^U I_{X_3^U \geqslant 2} | X_2^U\right] \\
=& E\left[\sum_{\gamma=2}^{X_2^U} \gamma Pr\{X_3^U = \gamma\}\right] \\
=& E\left[X_2^U (1-p_2)\left(1-p_2^{X_2^U-1}\right)\right] \\
=& E\{\left[X_2^U\left(1-\frac{X_2^U}{2X_2^U-1}\right)\left(1-p_2^{X_2^U-1}\right)\right]\left[I_{X_2^U=1}+I_{X_2^U \geqslant 2}\right]\} \\
=& 0+E\left[X_2^U\left(1-\frac{X_2^U}{2X_2^U-1}\right)\left(1-p_2^{X_2^U-1}\right) I_{X_2^U \geqslant 2}\right] \\
\geqslant & \frac{1}{3} E\left[X_2^U (1-p_2) I_{X_2^U \geqslant 2}\right] \\
\geqslant & \left(\frac{1}{3}\right)^2 E\left(X_2^U I_{X_2^U \geqslant 2}\right) \geqslant \left(\frac{1}{3}\right)^2 \left(\frac{1}{3}\right)^2 X
\end{aligned}
$$

(4) $$\therefore \left(X_3^U I_{X_3^U \geqslant 2}\right) \geqslant \left(\frac{1}{3}\right)^4 X$$

Using (4), we get as before

$$
\begin{aligned}
E\left(X_4^U\right) &\geqslant \frac{1}{3} E\left(X_3^U I_{X_3^U \geqslant 2}\right) \\
&\geqslant \frac{1}{3} \cdot \left(\frac{1}{3}\right)^4 X = \frac{1}{3^5} X
\end{aligned}
$$

In general,

$$E\left(X_t^U\right) = \frac{1}{3^{2(t-2)+1}} = \frac{1}{3^{2t-3}}$$

If we replicate the economy ν times

$$E(X_t^U(\nu)) \geqslant \frac{X}{3^{2t-3}} \cdot \nu$$

Let us define $\overline{T}_\nu$ as

$$\overline{T}_\nu = \min\left\{t | \frac{\nu X}{3^{2t-3}} \leqslant 1\right\}$$

Expected time to attain equilibrium will be greater than or equal to the time needed for *expected* number of unsatisfied type 1 agents to go to zero. Hence,

$$E\widehat{T}_{ML}(\nu) \geqslant \overline{T}_\nu$$

Now given any t, we can find a ν s.t.

$$\frac{\nu X}{3^{2t-1}} \geqslant 1.$$

$\therefore$ as $\nu \to \infty$, $\overline{T}_\nu \to \infty$, and hence $E\widehat{T}_{ML}(\nu) \to \infty$. □

Remark 2 (i) From this example it is clear that the result holds even when we consider a similar economy with 4 types of agents (each type having *X* agents) and 4 commodities. The type 1 and the type 2 agents are as before. A type 3 agent demands one unit of good 3 against one unit of good 4. A type 4 agent demands one unit of good 4 against one unit of good 3 and so on. Each type of agent (type 3 and 4) has one unit of good *M* so that condition (1) is satisfied. Thus, a type 3 agent can trade only when he meets a type 4 agent and vice-versa.

In a parallel fashion we can consider 6 types of agents with 6 goods in the economy and so on. Thereby, one can construct a large member of competitive economies where Proposition 2 holds.

(ii) The result is established here for a simple two good economy involving direct barter only. It can be easily extended to a more complicated economy having a large number of goods and involving monetary trade. Time requirement would in general increase when there are more than two goods coupled with the absence of mutual coincidence of wants.

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