ANNALES D'ÉCONOMIE ET DE STATISTIQUE. – N° 49/50 — 1998

# Equilibrium Coalition Structures in Markets for Network Goods

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**ABSTRACT.** – Firms that produce network goods have strong incentives to adhere to common technical standards. However, adhering to common standards decreases the horizontal differentiation between goods, and that increases market competition. This paper analyzes how these countervailing forces shape firms' decisions to comply to common technical standards under oligopoly. In the model, firms' outputs are identical in non-network characteristics, but firms can adhere to different compatibility standards. Consequently, a good's relative quality level is determined by the total sales of compatible goods. The technical standards coalition structures that form at equilibrium under this framework exhibit interesting characteristics. In particular, coalitions that vary greatly in total sales, profits, and prices often emerge, even though underlying products and cost structures are identical across firms.

## Les structures de coalition d'équilibre sur les réseaux de marchandises

**RÉSUMÉ.** – Les entreprises qui produisent des marchandises en réseau ont de véritables motifs à adhérer aux normes techniques communes. De cette façon, adhérer aux normes communes diminue la différentiation horizontale entre les marchandises, et ceci augmente la concurrence sur le marché. Cet article analyse comment ces forces de compensation forment des décisions fermes pour se conformer à la norme technique commune sous l'oligopole. Dans le modèle, les sorties d'entreprise sont identiques dans des caractéristiques hors réseau, mais les entreprises peuvent adhérer à différentes normes de compatibilité. En conséquence, un bon niveau relatif de qualité est déterminé par le total des ventes de marchandises compatibles. Les structures de coalition de normes techniques qui se forment à l'équilibre sous ce cadre exposent des caractéristiques intéressantes. En particulier, les coalitions qui changent considérablement dans des ventes totales, les bénéfices, et les prix émergent souvent, quoique les produits et les structures fondamentaux de coût soient identiques à travers des entreprises.

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# 1 Introduction

A good that derives a portion of its value from the consumption level of related goods is influenced by network externalities. In some cases these externalities determine nearly the entire value of the good. For example, the value of a fax machine largely depends on the number of other compatible fax machines on the network. For other goods, these externalities have more subtle influences. For example, the value of a car can increase with the overall consumption of cars, since this can increase the availability of parts, mechanics, gasoline, roads, maps, and various other related goods and services [1]. The value of nearly every good is influenced by network externalities to some extent [2].

Firms face a unique incentive structure when network externalities are present. On the one hand, a firm can choose to make its output compatible with an established standard, as this increases the value of the product to the consumer. On the other hand, by making its output incompatible with other products, the firm can gain monopoly power, even though its output is less valuable to consumers. The contrasting benefits associated with exploitation of network externalities versus exploitation of monopoly pricing power shapes the equilibrium in these markets.

In this work, a model is developed to solve for these equilibria and is applied to markets where a small number of firms compete. Firms are assumed to produce outputs that are identical in all characteristics except that they may adhere to different compatibility standards. The model is a variation of the GABSZEWICZ and THISSE [1979] or SHAKED and SUTTON [1982] models of vertical differentiation, where firms choose quality in the first stage and prices in the second. Our model differs from the traditional vertical differentiation models in two respects. First, in these vertical differentiation models quality differences reflect inherent differences in the features of products. This differs from our framework, since firms' outputs are identical with respect to inherent characteristics. Second, vertical (quality) differentiation in this model is determined solely by the level of sales of the various coalitions (the group of firms that produce compatible goods), with firms choosing coalition affiliations and output levels simultaneously. Thus, in this model quantity and quality are determined simultaneously.

The central findings of this analysis are: (1) The equilibria are often asymmetric. Despite producing identical goods in terms of inherent characteristics and having identical cost structures, firms' prices, sales and profits can vary dramatically. (2) This asymmetry is more acute for pure network goods than it is in markets where network externalities play a small

1. See KATZ and SHAPIRO [1985], ECONOMIDES [1996].
2. While these externalities need not always be positive, as many networks suffer from congestion, this paper examines the effects of positive network externalities under oligopoly.

role. (3) Firms that are in leading coalitions (those with greatest sales) have *less* incentive to make their technical standards available to others when network externalities are large. (4) Full compatiblity is the equilibrium in markets where network externalities play smaller roles. In markets where these externalities are significant, the equilibria tend to support multiple platforms.

We want to underline our result that, in markets with strong network externalities, often the equilibrium exhibits incompatibility and acute differences in production levels and prices of firms that adhere to different technical standards. This leads us to believe that in network industries, acute differences of size are a natural feature of equilibrium rather than a historical aberration. This may explain the historical (pre-divestiture) domination of the telephone industry by AT&T and the current domination of the personal computer software market by Microsoft.

The paper is organized as follows: The model and the corresponding equilibrium concepts are developed in section 2. There are three basic types of coalition structures that can arise at equilibrium [3]. The general characteristics of these structures are described in section 3. In section 4, the equilibrium coalition structures are derived for markets where two or three firms compete. We conclude in section 5.

# 2 The Model

## 2.1. Coalition Structures

Given a set of firms $S = \{1, ..., S\}$, and $i = 1, ...$ I technical standards, we identify a subset $C_i \subseteq S$ as a coalition, when the members of $C_i$ adhere to the same technical standard. The partition of $S$ into its subsets defines a coalition structure $\boldsymbol{C} = \{C_1, ..., C_I\}$. Let $c_i$ be the number of firms in coalition $C_i$. A coalition structure is represented as a vector of the cardinalities of the coalitions, $(c_1, c_2, ..., c_I)$ [4]. In this application, the coalitions are ordered in descending order according to total sales.

Product compatibility by all firms means that a single coalition includes all firms. Total incompatibility, where every firm adheres to its own unique standard, would mean that $s = I$ and every coalition is of cardinality one. Between these two extremes, there is a variety of partial incompatibility

3. The three basic types of coalition structures are full compatibility (every firm produces output that is compatible with every other firm's output), total incompatibility (no firm's output is compatible with another firm's output), and partial incompatibility (subgroups of firms, denoted coalitions, produce compatible output).
4. Specific assumptions on the demand and cost structure of our model imply that all firms realize equal profits within the same coalition.

coalition structures. For example, the coalition structure (2, 0) represents full compatibility in two-firm competition. The coalition structure (1, 1, 1) represents total incompatibility in a three-firm industry.

## 2.2. The Structure of the Game and the Equilibria

We analyze two game structures that apply to different regimes of intellectual property rights. In both game structures, firms play a two-stage Cournot-style game. In the first game structure, we assume that all technical standards are non-proprietary, so that firms can coalesce on any standard without restrictions. Thus, the decision of a firm to join a technical standard coalition is only dependent on if it achieves higher profits when it joins. We use the term "non-cooperative equilibrium coalition structure" for the equilibrium of this game. In contrast, in the second game structure, each firm has a technical standard that is proprietary to itself. Thus, if other firms want to join its technical standard coalition, they have to get the consent of the proprietary standard owner. We use the term "consensual equilibrium coalition structure" for the equilibrium of this game, noting, however, that the consent of members of the coalition that a firm leaves is not required.

In the first game structure, each firm chooses sequentially a coalition standard and a quantity of production. We will assume that in making its production decision, each firm considers the technical standard and the output level of all other firms as fixed *a-la-Cournot*. The firms make their choices known simultaneously to each other in the first stage. Each firm $j$ brings to the market its own output in the second stage, and the other firms and the consumers observe these quantities. Firms and consumers also observe which technical standard each firm has chosen, and they calculate total output of each coalition. Given this information, firms and consumers can determine the position of the demand curve for the good of each coalition. In anticipation of consumer demand determined through this process, firms choose technical standards and production levels non-cooperatively. Then output is auctioned *a-la-Cournot*. The second game structure is identical except that entry in a coalition requires the consent of other members. It is instructive to describe the equilibrium through the concept of an "adjacent coalition structure", defined next.

> DEFINITION 1 : A coalition structure that results when coalition structure $\boldsymbol{C}$ is changed by the movement of only one firm (across coalitions) is called an *adjacent coalition structure* to $\boldsymbol{C}$. For example, the coalition structure (3, 0, 0) and (2, 1, 0) are adjacent coalition structures, since the latter coalition structure can be reached from the former by the defection of only one firm to a new compatibility standard.

> DEFINITION 2 : A coalition structure $\boldsymbol{C}$ is a *non-cooperative equilibrium* when no firm in $\boldsymbol{C}$ has an incentive to change affiliations by joining a neighboring coalition to form an adjacent coalition structure.

By the last definition, at a non-cooperative equilibrium coalition structure, no firm wants to change its coalition affiliation. Let $\boldsymbol{D}_{C_i}$ be an adjacent coalition structure to $\boldsymbol{C}$, formed by the movement of firm $i$ to another coalition. Then by definition 2, $\boldsymbol{C}$ is a non-cooperative equilibrium coalition structure if and only if the profit condition $\Pi_i(\boldsymbol{C}) \geq \Pi_i(\boldsymbol{D}_{C_i})$ holds for each firm $i$ and every adjacent coalition structure $\boldsymbol{D}_{C_i}$ formed by a unilateral defection of firm $i$. By definition, this equilibrium concept considers only moves in and out of coalitions by a single firm. Thus, we do not allow movements of groups of firms.

The concept of the non-cooperative equilibrium implicitly assumes that other firms have no power to stop a firm from joining or leaving a standards coalition. This is an important assumption that applies to many but not all environments. Most importantly, it applies to areas where there are well-known but incompatible technical standards. However, there is a class of cases where an existing coalition has the ability to prevent other firms from joining. For example, if the technical standard is the intellectual property of a coalition, this coalition can prevent others from "joining it" by not authorizing others to use this standard. For such cases, we use the concept of a *consensual equilibrium.*

DEFINITION 3 : A coalition structure $\boldsymbol{C}$ is a *consensual equilibrium* when either of the following conditions hold: (a) no firm wants to move unilaterally, or (b) no coalition is willing to accept a firm that is willing to join it. We assume that a coalition of null size is willing to accept any firm.

Since condition (a) is necessary and sufficient for a non-cooperative equilibrium, this implies that *any non-cooperative equilibrium is also a consensual equilibrium*. Also note that the consensual equilibrium disregards the interests of firms in the coalition from where a firm may defect; it is assumed that firms in the original coalition are unable to stop a firm from defecting.

## 2.3. **Demand**

Let coalition $i$ have total production (market coverage) $n_i$, normalized so that $0 \leq \sum_{i=1}^{I} n_i \leq 1$. Let the willingness to pay for one unit of a good produced by a firm in coalition $i$ to person of type $\omega$ be $u(\omega, n_i) = \omega\, h(n_i)$. Consumer types $\omega$ are uniformly distributed over the interval [0, 1] [5]. The *network externalities function,* $h(n_i)$, captures the positive influence on utility associated with network size. The network externalities function is specified to be linear:

(1) $$h(n_i) = K + An_i.$$

A good's value embodies network and non-network benefits; $K$ represents the non-network benefits that a good provides, since it measures the

5. This setup is similar to ECONOMIDES and HIMMELBERG [1995], which focused primarily on perfect competition. Note that the multiplicative specification implies that consumer types vary in the value they attach to the network externality in a network of fixed size.

willingness to pay for a unit of the good when there are no other units sold. A benchmark case of the above function is when $K = 0$. This describes a market for a *pure network good,* since the good has no value in a network of zero size.

## 2.4. **Price Equilibrium for any Coalition Structure**

In the model, an industry has $S$ firms, each producing a single good. All firms are assumed to produce goods of equal inherent value (the parameters of the network externality function, $K$ and $A$, are constant across firms' output), however goods can vary with respect to their compatibility standard. When a collection of firms comply with a common technical standard (a coalition), every firm in the coalition reaps the network externality associated with the coalition's total sales. Since goods are identical in other respects, they are differentiated in quality only by the size of sales of the coalition to which their producer belongs. Let coalition $C_i$, $i = 1, ..., I$, have $c_i$ firms, total output $n_i$, with a typical firm in $C_i$ producing output $n_{ci}$ so that $n_i = \sum^{ci} n_{ci}$ [6]. Without loss of generality, we assume that the index $i$ of a coalition is inversely related to the amounts of sales of that coalition; *i.e.*, $n_i > n_{i+1}$, $i = 1, ..., I - 1$ [7]; coalition $C_1$ has the highest sales. Let $\omega_i$ be the marginal consumer who is indifferent between buying good $i$ and good $i + 1$. This indifference implies:

$$\begin{aligned} &\omega_i\, h\,(n_{i+1}) - p_{i+1} = \omega_i\, h\,(n_i) - p_i \\ &\Leftrightarrow \omega_i = (p_i - p_{i+1})/[A\,(n_i - n_{i+1})], \\ &\qquad i = 1, ..., I, \end{aligned} \tag{2}$$

where we define $p_{I+1} \equiv 0$ and $h\,(n_{I+1}) \equiv 0$. Consumers of types $\omega > \omega_i$ derive greater consumer surplus from good $i$ than from good $i + 1$ (at the going prices). Conversely, consumers of types $\omega < \omega_i$ prefer good $i + 1$ over good $i$. Thus, consumers of types $\omega$, $\omega_i < \omega < \omega_{i-1}$, buy good $i$ [8]. It follows that higher $\omega$ types buy network goods that belong to coalitions with higher sales. Goods from coalitions of higher coverage have higher prices, $p_i > p_{i+1}$ [9]. Sales of each good are:

$$n_i = \omega_{i-1} - \omega_i, \quad i = 1, ..., I, \tag{3}$$

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6. For notational simplicity, we suppress an index for firms, although firms within the same coalition may produce different outputs. We will show that at equilibrium, all firms within the same coalition produce the same output.
7. In general we write $n_i \geq n_{i+1}$, but equality is never part of an equilibrium as explained below in footnote 11.
8. These are standard results in models of vertical differentiation.
9. If this were not true, a good from the coalition with lower market coverage would not be purchased.

where $\omega_0 \equiv 1$. Summing these we have

(4) $$\omega_i = 1 - \Sigma_{j=1}^{i} n_j, \quad i = 1, ..., I,$$

and therefore market coverage is:

(5) $$\Sigma_{i=1}^{I} n_i = 1 - \omega_I = 1 - p_I/(A\,n_I + K).$$

Inverting the demand system (2), the general form of prices is:

(6) $$p_i = (K + A\,n_i)\,(1 - \Sigma_{j=1}^{i} n_j) - \Sigma_{j=i+1}^{I+1} (K + A\,n_j)\,n_j, \quad i = 1, ..., I.$$

Given constant marginal cost $m$, profits of a firm in $C_i$ are:

(7) $$\Pi_{ci} = n_{ci}\,(p_i - m).$$

Profits are maximized when [10, 11]

(8) $$\partial \Pi_{ci}/\partial n_{ci} = p_i - m + n_{ci}\,[A\,(1 - \Sigma_{j=1}^{i} n_j) - (K + A\,n_i)] = 0,$$

The solution to the system of equations (6), (7), and (8) defines the equilibrium production levels and prices. It follows directly from equation (8) that all firms in the same coalition produce equal amounts, and therefore $n_i = c_i\,n_{ci}$. Equilibrium profits are

(9) $$\Pi_{ci}^{*} = n_{ci}^{*}\,(p_i - m) = (p_i - m)^2/[(K + A\,n_i) - A\,(1 - \Sigma_{j=1}^{i} n_j)].$$

In the following sections, the model is used to analyze alternative market structures; this provides a basis for determining equilibrium coalition structures.

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10. The second order condition is $\partial^2\,\Pi_{ci}/\partial n_{ci}^2 = 2\,[(1 - \Sigma_{j=1}^{i} n_j)\,h'\,(n_i) - h\,(n_i)] + n_{ci}\,[(1 - \Sigma_{j=1}^{i} n_j)\,h''\,(n_i) - 2\,h'\,(n_i)] < 0$. The term in the first brackets is negative because price is greater than marginal cost at the first order condition. The second term in brackets is also negative since $h''\,(n_i) = 0$.

11. We can now explain why equal production of two coalitions is not possible at equilibrium. If two coalitions $C_i$ and $C_{i'}$ had exactly the same total amount of expected sales, $n_i^e = n_{i'}^e$, then they would define the same marginal consumer $\omega_i = \omega_{i'}$ and would command the same price $p_i = p_{i'}$ givne by (6). (In equation (3) we would have $n_i + n_{i'} = \omega_{i-1} - \omega_i$, and $n_i + n_{i'}$ would similarly appear in the first sum of (6).) Now, given this tie, a firm in coalition $C_i$ has an incentive to expand its output so that the output of the coalition increases from $n_i$ to $n_i + \varepsilon$. If it does so, it reaches a higher quality, and all consumers in $[\omega_i + \delta,\, \omega_{i-1}]$ switch to it. Since this is better for the firm that expands output, it has a unilateral incentive to deviate from equilibrium. It follows that equal production levels by two or more coalitions are ruled out at equilibrium.

# 3 Potential Market Structures

## 3.1. Full Compatibility

Full compatibility refers to the case where all firms in the industry produce compatible output [12]. This coalition structure is denoted $(S, 0)$, since all firms belong to the leading platform. In this case, $I = 1$. The total size of the network is $\Sigma_{s=1}^{S} n_S$, and the willingness to pay by consumer of type $\omega$ is $\omega\,(K + A\,\Sigma_{s=1}^{S} n_S)$. At equilibrium there is a unique price for all goods, since goods from different firms are identical in every attribute. Given a common price, the marginal consumer $\omega$ who purchases the good is defined by

(10) $$\omega^* = p/(K + A\,\Sigma_{s=1}^{S} n_s).$$

Since consumers of indices higher than $\omega^*$ buy the good, the size of the network (demand) at price $p$ is $\Sigma_{s=1}^{S} n_S = 1 - \omega^*$, or equivalenlty,

(11) $$\Sigma_{s=1}^{S} n_s = 1 - p/(K + A\,\Sigma_{s=1}^{S} n_s).$$

The willingness to pay of the last consumer is:

(12) $$p\,(\Sigma_{s=1}^{S} n_s,\, \Sigma_{s=1}^{S} n_s) = (K + A\,\Sigma_{s=1}^{S} n_s)\,(1 - \Sigma_{s=1}^{S} n_s).$$

The $j$-th oligopolist maximizes the profit function,

(13) $$\Pi_j = n_j\,[p\,(\Sigma_{s=1}^{S} n_s,\, n_j + \Sigma_{s\neq j}^{S} n_s) - m],$$

by solving the following first order condition [13]:

(14) $$d\Pi_j/n_j = p\,(\Sigma_{s=1}^{S} n_s,\, n_j + \Sigma_{s\neq j}^{S} n_s) + n_j\,(p^1 + p^2) - m = 0.$$

At the full compatibility equilibrium, all firms produce equal quantities, since identical first order conditions are solved. Substituting for $p$, $p^1$ and $p^2$ in equation (14) and re-writing it for the typical firm $s$ gives [14]:

(15) $$(K + Asn_s^*)\,(1 - sn_s^*) + n_s^*\,[A\,(1 - 2\,sn_s^*) - K] - m = 0.$$

Note that when the externality becomes insignificant, *i.e.*, as $A \rightarrow 0$, the market equilibrium converges to the traditional symmetric Cournot equilibrium, since

$$\lim_{A\rightarrow 0} n_s^* = [1 - (m/K)]/(S + 1).$$

12. This discussion follows ECONOMIDES and HIMMELBERG [1995].
13. We use the notation $p^k$ to signify the partial derivative of the $p$ function with respect to its $k$-th argument.
14. If the first order condition has two admissible (*i.e.*, non-negative) solutions, we assume that consumers will coordinate to the higher one. It is easy to show that, under full compatibility, there is only one equilibrium with positive sales as long as $m < K$.

For any positive externalities ($A > 0$) the compatiblity equilibrium production is greater than at the traditional Cournot equilibrium [15].

## 3.2. Total Incompatibility

In this case, each firm produces a good that is incompatible with output from every other firm. Therefore, all standards coalitions are of size 1, the number of firms in any coalition $c_i$ equals one, and the number of firms $S$ equals the number of coalitions $i$. Sales, prices, and profits are ordered according to rank in the index of firms, with firm (coalition) 1 having the highest sales, prices and profits.

Notice that in the total incompatibility case, the market equilibrium also converges to the symmetric Cournot equilibrium as the size of the network externality tends to zero. This result is obtained by substituting equation (6) into equation (8), setting $n_{ci} = n_i$, $s = I$, and taking the limit as $A$ tends to zero:

$$\lim_{A \to 0} n_i^* = [1 - (m/K)]/(S + 1),$$

which is the quantity per firm at the $S$-firm Cournot equilibrium without externalities. Note that as externalities tend to zero, output tends to the same limit under either compatibility or incompatibility; thus, when externalities are very small, whether firms are compatible or not makes little difference.

In the following cases to be analyzed, numerical methods are used to determine equilibrium structures. Without loss of generality, the network externality function is normalized so that $h(n_i) = k + n_i$, where $k = K/A$. The marginal cost is also set equal to zero $(m = 0)$. While the simplification regarding marginal costs does not qualitatively affect any of the findings, it does provides a computationaly more convenient structural form. In this specification, the index $1/k$ measures the intensity of the marginal network externality. Thus, a good with small $k$ provides benefits primarily through its associated network externality, while goods with large $k$ have relatively low network externality effects. At the extremes, a pure network good is represented by setting $k$ equal to zero and the standard Cournot case of no externalities is approached when $k$ tends to infinity.

The effects of positive network externalities on market structure are analyzed by solving numerically for the total incompatibility equilibrium for various values of $k$ and $s$. The first result is that entry has greater effects on incumbent coalitions' (firms') outputs and profits when $k$ is large. The relative effects on profits of entry is seen by comparing figures 1 and 2. These figures depict profits of the leading 3 firms in the market given various number of firms in the industry. In figure 1, $k$ is set equal to 1. In figure 2, $k$ equals 0. Notice that as the number of firms increase, the profits for the leading firms in the industry where $k$ is large (fig. 1) are more dramatically

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15. Equation (15) is a quadratic in $n_s$ with a well-defined solution $n_s^*$ that is continuous in $A$. Defining the LHS of equation (15) as $F(A)$, it is easy to show that $F(\lim_{A \to 0} n_s^*) > 0$ and $F'(\lim_{A \to 0} n_s^*) < 0$. It follows that $n_s^*(A) > \lim_{A \to 0} n_s^*$ when $A > 0$.

affected by entrant firms. The intuition is straightforward: the greater non-network benefits associated with high-$k$ industries make goods of different compatibility standards closer substitutes. Therefore, the effect of increased competition on profits is more pronounced for high-$k$ industries.

FIGURE 1

***Number of Firms in Marke.***

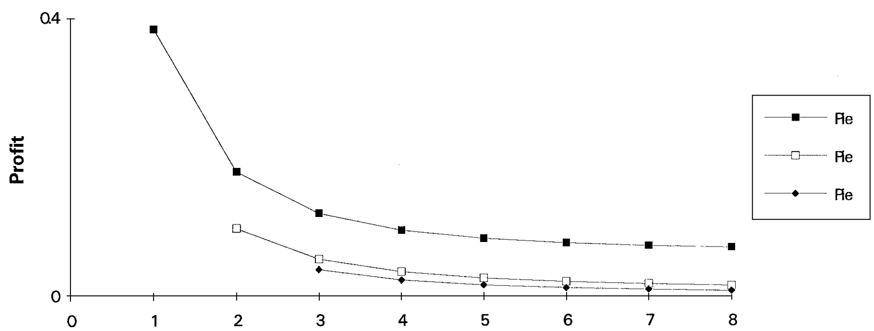
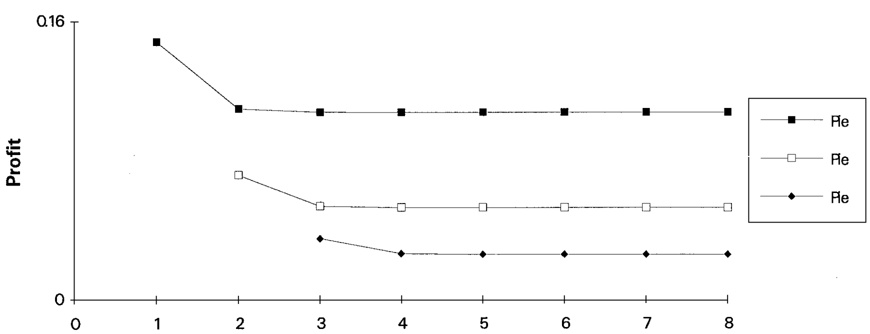



FIGURE 2

***Number of Firms in Marke.***




Under total incompatibility, the relative effects that entry has on firms' output is seen in Table 1, which shows the Herfindahl-Hirschman (H) index of market concentration, $H = \Sigma_{i=1}^{S}(n_i/\Sigma_{i=1}^{S} n_i)^2$. Table 1 shows that when $k$ is small there is greater inequality in firms' outputs. The $H$ index decreases in $k$ (increases in $1/k$) for all fixed $S$ [16]. This indicates that the inequality across firms' outputs is larger for markets where network externalities play larger roles. In other words, for the total incompatibility

16. Reflecting on the earlier result of convergence to a symmetric Cournot equilibrium as marginal network externalities become negligible, the last column of Table 1 at $k = 5$ gives a concentration index almost equal to that of a symmetric Cournot oligopoly.

case, market concentration, output, and price inequality increase with the extent of the network externality.

The $H$ index is also naturally decreasing in $s$ for fixed $k$, reflecting more intense competition as more firms compete in the industry. A finding of greater interest is that the $H$ index decreases more significantly in $s$ for markets that exhibit lower network externalities (when $k$ is large). This is because neither the output of firms in leading coalitions or the their prices change very much as more firms enter when $k$ is small. Goods with large network externalities provide large incentives to organize consumers into few platforms. This, however, provides high monopoly power to leading platforms, which are not significantly affected by entry of firms offering incompatible output. However, when network externalities contribute a relatively small portion to a good's value (large $k$), incompatible output provides a closer substitute to leading platform goods, and consequently have a greater effect on leading firms' output and profits.

## 3.3. **Partial Incompatibility: Two Coalitions**

Under partial incompatibility, the number of coalition structures (technical standards) is larger than one and less than the total number of firms ($1 < I < s$). The simplest case is when there are only two coalition structures. For coalition structures of two-layers, we establish that sales of a firm in a coalition of higher market coverage are higher than for any firm in a coalition of lower market coverage [17],

(16) $$n_{ci} > n_{cj} \Leftrightarrow i < j.$$

TABLE 1

***Herfindahl Index for Different Intensities of Marginal Network Externality 1/k and Numbers of Firms (Coalitions) S.***

| | | Intensity of Marginal Network Externality $1/k$ | | | | |
|---|---|---|---|---|---|---|
| | | $\infty$ | 2 | 1 | 0.5 | 0.2 |
| Number of Firms (Coalitions) $S$ | 3 | .510 | .415 | .363 | .339 | .334 |
| | 5 | .470 | .331 | .248 | .207 | .201 |
| | 10 | .464 | .287 | .172 | .106 | .100 |

17. The numerical proofs of this and subsequent sections were done in *Mathematica* and are available from the authors upon request.

Since prices are always ordered in the same direction as coalition sales, profits of a firm in a coalition of higher market coverage are also higher than for any firm in a coalition of lower market coverage,

$$\Pi_{ci} > \Pi_{cj} \Leftrightarrow i < j. \tag{17}$$

Equilibrium prices of all goods increase with $k$. this is expected because an increase in $k$ increases the value of the good to all consumers. Sales of each firm in the upper layer fall in $k$, while sales of each firm in the lower layer increase in $k$:

$$\partial n_{c1}/\partial k < 0, \qquad \partial n_{c2}/\partial k > 0. \tag{18}$$

Fixing the total number of firms in the market to add to a constant, $c_1 + c_2 = s$, we compare prices, sales per firm, sales per coalition, and profits as both $c_1$ and $k$ vary. The price quoted by firms in the coalition with the largest market share (leading platform), $p_1$, initially increases in $c_1$ for low $c_1$, and later decreases. See figure 3. Sales per firm in the leading platform, $n_{c1}$ decrease in $c_1$, as seen in figure 4. Market coverage for the leading platform, $n_1$, increase in $c_1$, as seen in figure 5.

FIGURE 3

***Number of Firms in Leading Platform.***

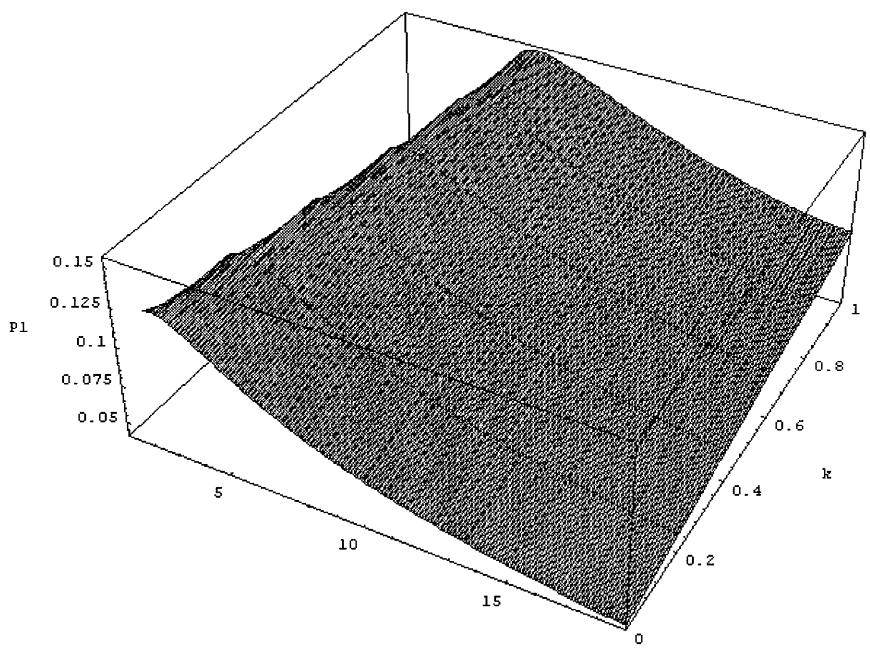


FIGURE 4

***Number of Firms in Leading Platform.***

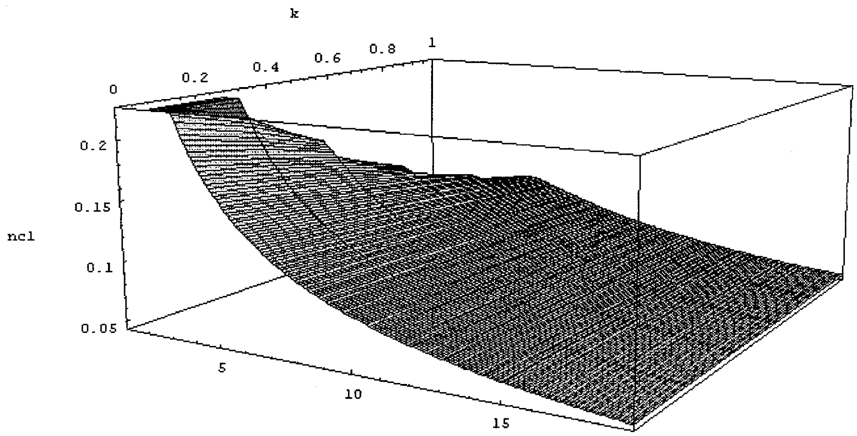



FIGURE 5

***Number of Firms in Leading Platform.***

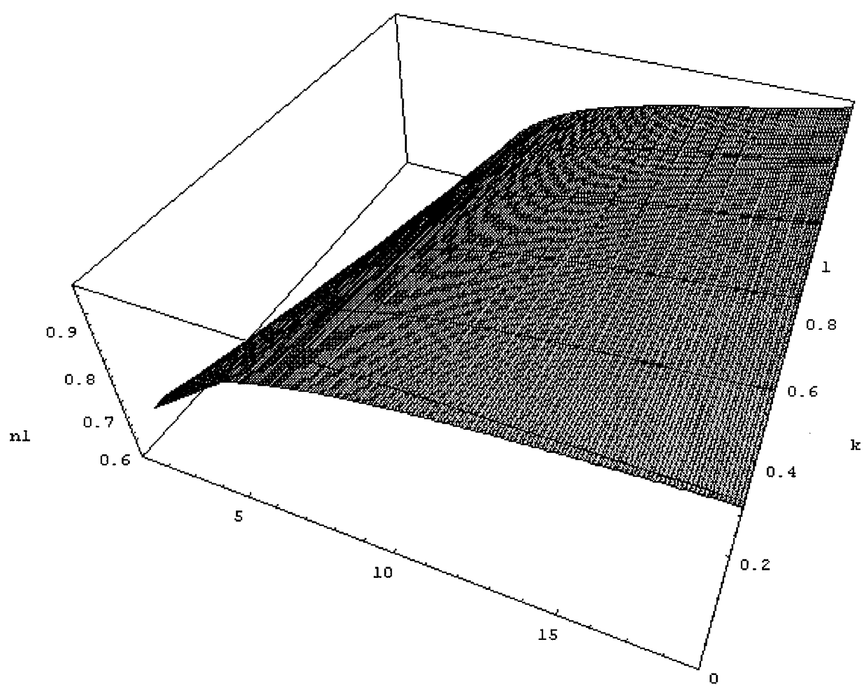
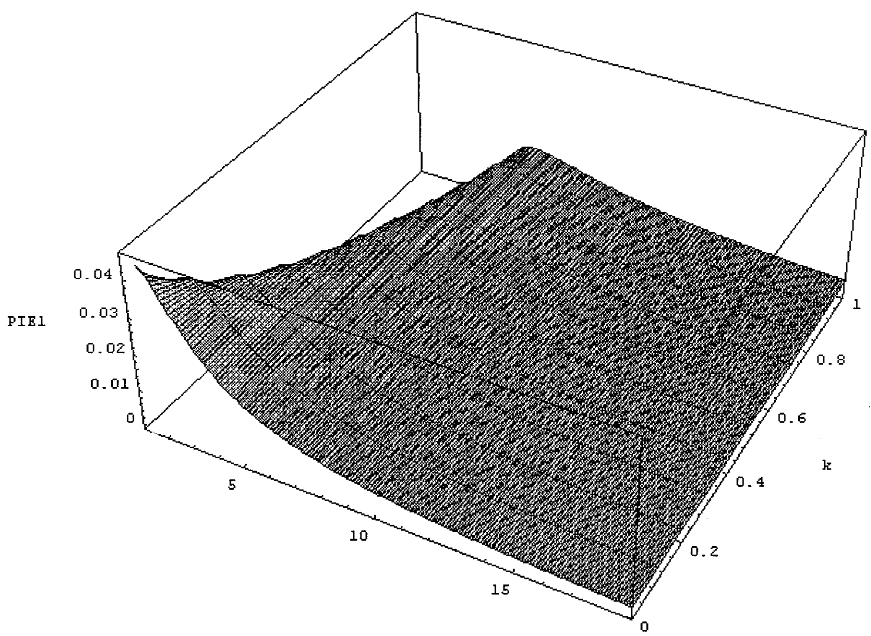



Figure 6 shows how profits in the leading coalition change with $c_1$ and $k$. Profits of a firm in the top coalition, $\Pi_1$, initially increase in $c_1$ for low $c_1$, and later decrease. Therefore, firms in the top coalition benefit from other firms joining them when the top coalition is small. The intuition behind the profit structure depicted in figure 6 is as follows: For low $c_1$, entry into the leading platform increases market coverage substantially. The network externalities for the leading platform subsequently increase sharply, while the externality benefits for the lower platform decline. This change in the network externalities more than offsets the effects of greater competition in the top coalition, and the price of the top coalition's output rises. Initially, higher prices dominate the drop in per firm output, and consequently firms' profits rise. However, as additional firms enter the leading platform, the marginal increases in total output diminish, and the effects of greater competition in the top platform dominates increases in the network externality benefits, and both prices and profits fall.

FIGURE 6

***Number of Firms in Leading Platform.***




The equilibrium profit structure, and, in particular the profit incentives of firms in the leading coalition to encourage entry into their coalition when $c_1$ is small, counters the usual oligopoly result that prices and profits decrease with additional entry. This is an important feature of network good markets, and is probably understated in figure 6 because marginal costs are assumed to be zero. If firms in the top coalition have rising marginal costs, then the economic forces behind additional entry into the top coalition will be even stronger, since this would imply efficiency gains in production.

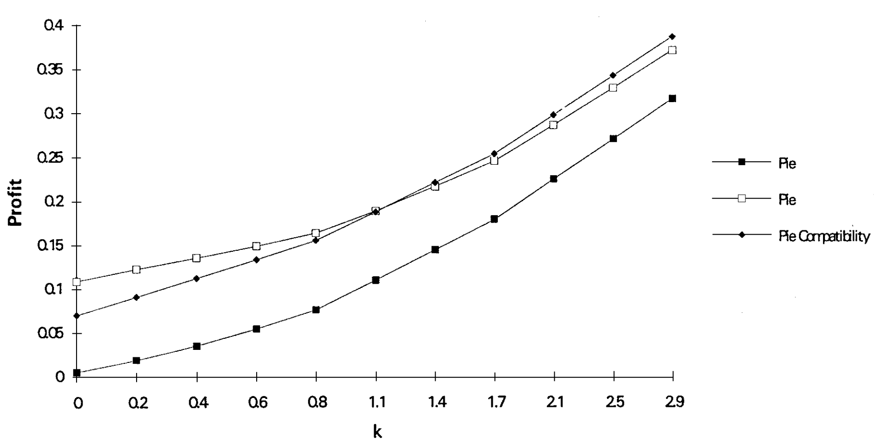
# 4 Application of Equilibrium Concepts

## 4.1. Special Case: Two Firms

In an industry with two firms, there are two possible coalition structures: (i) compatibility – denoted (2, 0); and (ii) incompatibility – denoted (1, 1). To analyze the non-cooperative equilibrium structures that arise, we calculate profits conditional on coalition affiliation. Figure 7 shows equilibrium profits under both coalition structures for each firm as $k$ varies [18]. This plot illustrates that, for large $k$, $k > 1.1$, both firms earn more under compatibility than the leading firm earns under incompatibility. Therefore for large $k$, compatibility is the coalition structure equilibrium. Since it is a non-cooperative equilibrium, compatibility is also a consensual equilibrium.

For small $k$, when there are strong network externalities, compatibility profits lie between the incompatibility profits of the first and the second firm. In this case, firm 2 wants compatibility and firm 1 wants incompatibility. Therefore, for small $k$, there is no non-cooperative equilibrium. Incompatibility is a consensual equilibrium, which arises when the leading firm has the power to restrict the second firm from entering its coalition.

FIGURE 7




These results may seem paradoxical, since the incentive to break from compatibility is higher in goods with strong network externalities, when one expects the highest benefits from compatibility. The intuition behind this

18. We can show uniqueness of all production equilibria (given coalition structure) in the two- and three-firm cases.

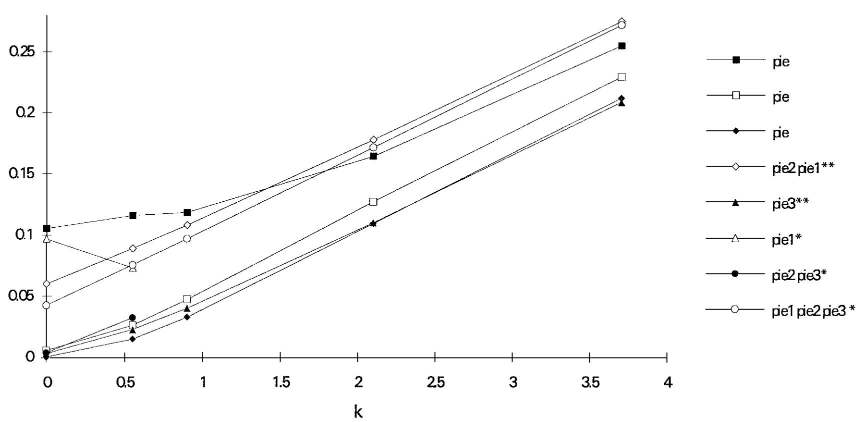
finding is as follows: The differences across firms in the equilibrium outputs and prices under incompatibility increase as network externalities play a larger role in the goods value (when $k$ is small). With large externalities, goods where $k$ is close to zero, the top firm sells to the vast majority of consumers and receives a very high price. Thus, the incentive to deviate from the equal quantities, prices, and profits compatibility equilibrium and become the top firm under incompatibility is greater when network externalities are large. Table 2 summarizes the equilibrium coalition structures for the two-firm industry as $k$ varies.

## 4.2. **Special Case: Three Firms**

### 4.2.1. **Non-Cooperative Coalition Structure Equilibria**

In an industry with three firms, the potential coalition structures are as follows: full compatibility, (3, 0, 0); total incompatibility, (1, 1, 1); and partial incompatibility, (2, 1, 0) or (1, 2, 0). Equilibrium profits associated with the different coalition structures are presented in figure 8. This graph can be used to determine which coalition structures qualify as non-cooperative equilibria. We first eliminate those coalition structures that do not qualify. Three coalition structures can be immediately eliminated as candidates for non-cooperative equilibrium. In each of these cases, a firm has an incentive to deviate and join another coalition, thus creating an adjacent coalition structure.

FIGURE 8




• (1, 1, 1) is *not* a non-cooperative equilibrium because profits at (2, 1, 0) are higher for a firm in the top layer than for the middle firm in (1, 1, 1); thus, there is an incentive for the middle firm to join the top layer.

• (2, 1, 0) is *not* a non-cooperative equilibrium because profits at (2, 1, 0) are lower for a firm in the middle layer than at full compatibility at (3, 0, 0), and therefore the middle firm has an incentive to join the top coalition.

TABLE 2

***Coalition Structure Equilibria in a Two-Firm Industry***

| Range of $k$ | Intensity of Marginal Network Externality $1/k$ | Non-Cooperative Equilibria | Consensual Equilibria |
|---|---|---|---|
| [0, 1.1] | [0.909, $\infty$] | None | (1, 1) |
| [1.1, $\infty$] | [0, 0.909] | (2, 0) | (2, 0) |

• (1, 2, 0) is *not* a non-cooperative equilibrium because profits for a firm at the top layer at (2, 1, 0) are higher than for a firm in the lower layer of (1, 2, 0).

We now establish under what conditions the remaining coalition structure, (3, 0, 0) (*i.e.,* full compatibility), *is* a non-cooperative equilibrium. For $k > 0.5$, the firm in the top layer in coalition structure (1, 2, 0) is worse off than at (3, 0, 0). Thus, for $k > 0.5$, a defection from (3, 0, 0) to (1, 2, 0) or to (2, 1, 0) is not desirable for the defecting firm. Therefore, ***for k > 0.5, full compatibility is a non-cooperative equilibrium.*** For $k < 0.5$, the firm in the top layer at (1, 2, 0) is better off than at (3, 0, 0), and therefore (3, 0, 0) is *not* a noncooperative equilibrium. Since all other coalition structures have been shown not to be non-cooperative equilibria, ***for k < 0.5, there is no non-cooperative equilibrium in pure strategies.***

In summary, we find that the only non-cooperative equilibrium is at full compatibility, and that it is achieved only for goods that have some value on their own irrespective of sales to other customers. For pure two-way network goods where $k = 0$, there is no non-cooperative equilibrium coalition structure. This is because in any coalition structure, there is some firm that has an incentive to deviate from the coalition that it belongs to and join another coalition.

### 4.2.2. **Consensual Coalition Structure Equilibria**

Figure 8 is now used to determine which coalition structures are *consensual* equilibria. Since the consensual equilibrium is defined by less restrictive conditions, there are more consensual equilibria than non-cooperative equilibria. First note that ***for k > 0.5, (3, 0, 0) is a consensual equilibrium*** because it is a non-cooperative equilibrium. For $k < 0.5$, a firm in (3, 0, 0) wants to defect and be by itself in the top layer in (1, 2, 0). Therefore for $k < 0.5$, (3, 0, 0) is *not* a consensual equilibrium.

We now establish the conditions under which (2, 1, 0) is a consensual equilibrium. Profits at (2, 1, 0) are higher for a firm in the top layer than at full compatibility (3, 0, 0). Afirm in the top layer of (2, 1, 0) has an incentive *not* to accept the firm from the lower layer. As noted earlier, a firm in the top layer of (2, 1, 0) does not want to move to the middle layer of (1, 2, 0). Furthermore for large $k$, $k > 1.5$, the firm in the top layer of (1, 1, 1) prefers to be together with the firm in the second layer rather than apart. Therefore, ***for k > 1.5, partial compatibility (2, 1, 0) is a consensual***

***equilibrium.*** For small $k$, $k < 1.5$, the firm in the top layer of (1, 1, 1) prefers to be apart from the firm in the second layer. Therefore, for $k < 1.5$, $(2, 1, 0)$ is *not* a consensual equilibrium.

We now establish the conditions under which (1, 2, 0) is a consensual equilibrium. For very small $k$, $k < 0.5$, a firm in the top layer of (1, 2, 0) does not want anyone to join it (and form (2, 1, 0)), even though a firm in the second layer wants to join. For $0.1 < k < 0.5$, a firm in the second layer does not want to defect and be by itself (thus forming (1, 1, 1)). A firm in the top layer of (1, 2, 0) does not want to join the second layer, thus forming (3, 0, 0). Therefore, ***for 0.1 < k < 0.5, partial compatibility (1, 2, 0) is a consensual equilibrium.*** For $k > 0.5$, the firm in the top layer of (1, 2, 0) wants to join the middle layer, and the middle layer wants to accept it, thus forming (3, 0, 0). Therefore for $k > 0.5$, (1, 2, 0) is *not* a consensual equilibrium.

We now establish the conditions under which (1, 1, 1) is a consensual equilibrium. For extremely small $k$ (very high externalities), $k < 0.1$, the middle firm does not want the bottom firm to join it, and the top firm does not want the middle firm to join it, even though any lower firm wants to join a higher layer. This establishes that for ***k < 0.1, (1, 1, 1) is a consensual equilibrium.*** For $k > 0.1$, the middle firm will accept the lowest layer firm, and therefore (1, 1, 1) is *not* a consensual equilibrium.

In summary, we found that each coalition structure can be a consensual equilibrium for some range of $k$. In particular, coalition structures of partial compatibility are consensual equilibrium coalition structures for different values of $k$: for $k < 0.5$ the consensual equilibrium is (1, 2, 0), and for $k > 1.5$, the consensual equilibrium is (2, 1, 0). Full compatibility (3, 0, 0) is a consensual equilibrium whenever it is a non-cooperative equilibrium, *i.e.*, for $k > 0.5$. Finally, total incompatibility (1, 1, 1) is an equilibrium for $k < 0.1$. These results are summarized in Table 3.

TABLE 3

***Coalition Structure Equilibria in a Three-Firm Industry.***

| Range of $k$ | Intensity of Marginal Net work Externality $1/k$ | Non-Cooperative Equilibria | Consensual Equilibria |
|---|---|---|---|
| [0, 0.1] | [10, $\infty$] | None | (1, 1, 1) |
| [0.1, 0.5] | [2, 10] | None | (1, 2, 0) |
| [0.5, 1.5] | [0.666, 2] | (3, 0, 0) | (3, 0, 0) |
| [1.5, $\infty$] | [0, 0.666] | (3, 0, 0) | (3, 0, 0), (2, 1, 0) |

Note that, for some regions of the parameters, there is multiplicity of consensual equilibria. Given the nature of consensual equilibrium, this should not be surprising and does not create a contradiction even though the equilibria are adjacent coalition structures. For example, for high externalities, both total incompatibility (1, 1, 1) and partial compatibility

(1, 2, 0) are consensual equilibria. Although under total incompatibility (i) no one from a higher platform wants to join a lower platform, and (ii) every firm in a lower platform wants to join a higher one, (1, 1, 1) is a consensual equilibrium because (iii) no firm in a higher platform wants to admit one form a lower platform. At the same time, (1, 2, 0) is a consensual equilibrium as well, because (among other reasons) no middle platform firm wants to go lower by itself thus forming (1, 1, 1).

# 5 Conclusion

Firms that compete in markets where network externalities are present face unique tradeoffs regarding the choice of a technical standard. Adhering to a leading compatibility standard allows a firm's product to capture the value added by a large network, however, simultaneously the firm loses direct control over the market supply of the good. Alternatively, adhering to a unique standard allows the firm to control the market supply of the product, but it sacrifices the added value associated with a large network. The tension between these economic forces shapes the coalition formation equilibrium in these markets.

In this work, we developed a model that can be used to solve for the potential coalition formation equilibria in markets for network goods. The model is then implemented on several different market structures and provides some insight regarding the characteristics of the equilibria that emerge. The principal findings of this analysis are: (1) Industry output is larger under the full compatibility equilibrium than it is under the standard Cournot equilibrium when network externalities are present. (2) The coalition formation equilibria that emerge are often very asymmetric in firms' profits and output, despite firms producing identical goods in terms of inherent qualities and using the same production technology. The acuteness of these asymmetries increases as the portion of a goods value that derives from the network increases. (3) The conflicting benefits associated with joining a leading coalition versus adhering to a unique standard also influences a firm's decision on whether to make their technical standards available to competitors. In many instances, firms in leading coalitions earn higher profits by allowing additional firms enter that platform, despite the increase in direct competition and the fact that no side payments are allowed. (4) Full compatibility can be a consensual equilibrium for pure network goods in markets with two or three firms. However, full compatibility is not a non-cooperative equilibrium in these markets. This follows because the potential monopoly rents associated with the leading platform in a pure network good market are very large, and this creates an enormous incentive for firms to be the sole producer in the leading platform. (5) When a coalition is able to exclude entrants (because it holds proprietary standards), in markets with very strong externalities, the equilibrium is characterized by either total or partial incompatibility. This result indicates that market dominance by one or few firms may be an inherent characteristic of market equilibrium in network industries.

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