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Endogenous Growth and the Labor Market

Frédérique CERISIER, Fabien POSTEL-VINAY*

ABSTRACT. – We present a multisectoral model of qualitative endogenous growth with imperfect matching on the labor market. Under these conditions, growth induces unemployment through the Schumpeterian process of creative destruction. We study the long-run link between growth and unemployment. We show that this link structurally depends on the relative importance of the degree of competition in the economy, and that of intertemporal substitutability. Moreover, although the relationship between growth and labor market variables is generally ambiguous, we show that the creative destruction effects are very likely to dominate in this kind of models, thus suggesting that a increase in the growth rate should be accompanied by a rise in long-term unemployment.

Croissance endogène et marché du travail

RÉSUMÉ. – Nous présentons un modèle multisectoriel de croissance endogène qualitative avec appariement imparfait sur le marché du travail. Dans ces conditions, la croissance est génératrice de sous-emploi par le processus schumpetérien de destruction créatrice. Nous étudions le lien de long terme entre la croissance et le chômage. Nous montrons que ce lien dépend structurellement de l'importance relative du degré de concurrence dans l'économie et du degré de substituabilité intertemporelle. De plus, bien que la relation entre la croissance et les variables du marché du travail soit en général ambiguë, nous montrons que les effets de destruction créatrice sont très probablement dominants dans ce type de modèles, ce qui suggère qu'une hausse du taux de croissance devrait s'accompagner d'une hausse du taux de chômage de long terme.

* F. CERISIER : MAD, Université de Paris 1; F. POSTEL-VINAY : MAD, Université de Paris 1.
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1 Introduction

The analysis of the link between growth and long-run unemployment has recently raised the interest of a rising number of contributors in economic research. Indeed, it appears that models of endogenous growth allow for a common determination of growth and unemployment rates (BEAN and PISSARIDES [1993], AGHION and HOWITT [1994]). In particular, models of growth through technical progress have been exploited for that purpose. In fact, the concern of economists with the effect of technical change on employment appeared, according to the historical survey of PETIT [1993], with the beginning of industrialization in the early nineteenth century. By that time, Classical economists optimistically argued that jobs destroyed in one activity would somehow be replaced in another, by means of an increase in profitability inducing a rise in the level of global demand. This view became known as the "compensation theory". Recent experience as well as recent developments in economic theory highlighted the simplism of this argument and raised a number of related questions. These questions may be roughly classified as follows.

The first issue is much of a microeconomic concern about the influence of technical change on the labor demand at the firm level. Fundamentally, the basic question to be answered here is whether technical change destroys jobs (*e.g.* through substitution of capital for labor), or merely displaces them by changing the structure of labor demand (*e.g.* by making old qualifications obsolete and introducing the need for new ones). The second issue is to study the effects of sustained innovation on the global income, and thus on aggregate demand.

The third issue is the one handled in this article. Innovation, while creating new goods and thus new production opportunities, implies the obsolescence of some production units. This process of creative destruction (primarily introduced by Joseph A. Schumpeter) implies a "recurrent rejuvenation of the productive apparatus" in the words of Schumpeter [1942]. Although Schumpeter remains rather vague on unemployment issues, one can think that the new jobs created by the innovation process are not likely to benefit those workers this process pushed into unemployment. Formally, if matching in the labor market is not frictionless, innovation will induce unemployment.

We develop a model based on the same grounds as the one by AGHION and HOWITT [1994]: to uncover the employment effects of the creative destruction process, we add imperfect matching on the labor market to an innovation-based growth model. However, Aghion and Howitt's original model rested on a somewhat peculiar structure. In the first place, they make the non standard assumption that the renewal of the production sector is due to competition for a scarce input, loosely defined as "human capital". In the second place, their economy is taken unisectoral, which hides the effects arising from imperfect competition. The main purpose of the present contribution is thus to describe the relationship between long run unemployment and growth through the creative destruction phenomenon in a general and tractable model. Our model is multisectoral, and the growth process is close to the standard framework by GROSSMAN and HELPMAN [1991]. It clearly exhibits the conflict between the three competing effects of sustained innovation on labor market variables, namely the direct and indirect creative destruction effects and the capitalization effect (*cf.* AGHION and HOWITT [1994], or d'AUTUME [1995]). An increased rate of innovation first implies higher turnover on the labor market which tends, roughly speaking, to increase altogether unemployment and the number of vacant jobs. This is the direct effect of creative destruction. Furthermore, if newer goods are preferred by consumers, an increased rate of innovation will induce a faster decline in older firms' profitability, which in turn will intensify the flow into unemployment. This is the indirect effect of creative destruction. Finally, those two effects can partially or totally be overshadowed by the positive effect of growth on firms' profitability, namely the capitalization effect. The question of knowing whether or not it will be the case is one of particular interest, which cannot be answered without ambiguity. However, our model uncovers a couple of interesting features that should be kept in mind when trying to move towards such an answer. In particular, we argue that the Schumpeterian view of the growth phenomenon is somewhat biased towards the creative destruction effects.

As a return to the multiple sectors hypothesis, we show that the way the economy looks like in steady states heavily and structurally depends on the relative importance of the degree of competition, and the willingness of private consumers to substitute intertemporally.

The model is made in the spirit of CABALLERO and JAFFE [1993]. Its particular feature is to generate growth through product variety as well as product quality increases. It implies in particular that firms are infinitely lived with heterogeneous employment levels. The creative destruction process is characterized by a continuous decline in the market shares of a given producer, consecutive to the appearance of new goods. Hence, installed producers continuously lay off workers, while a continuous flow of newborn firms hires those workers. This continuity seems relevant and convenient as a "macroeconomic approximation", although it hides the fact that technical progress is likely to have local and discontinuous effects, for an innovation probably leads to the destruction of one particular firm in one particular sector, from time to time, rather than to a smooth and homogenous shrinking of all firms.

Unlike earlier contributions treating problems of resources allocation between production and innovation (see *e.g.* GROSSMAN and HELPMAN [1991], CABALLERO and JAFFE [1993]), we cannot model explicitly the R&D sector, since labor cannot be freely allocated among the production and R&D sectors. In our model, any agent can become an entrepreneur by innovating (creating a new good), and creating a new firm to produce it. An exogenous entry cost must be paid to create a new firm. All firms find themselves in situation of monopolistic competition. The innovation rate, *i.e.* the frequency of firm creations is thus endogenously determined by a condition of free entry on the goods market.

The paper is organized as follows: section 2 presents the model and its intertemporal equilibrium. Steady states are studied and commented in section 3. Section 4 studies some issues of comparative statics. Section 5 concludes.

2 The Model

At any point of time, a continuum of imperfectly substitutable goods indexed by $j \in]-\infty, f_t]$ is produced by monopolistic competitors. At any date, any agent can become an entrepreneur by creating a firm to produce a new good. Creating a new firm implies paying a sunk cost D_t, and getting into a hiring process. To describe this hiring process, we assume as in PISSARIDES [1990] that matching on the labor market is not frictionless. A newborn firm thus has to open a certain number of vacant jobs, and find workers that fit these vacant jobs before it can start production. Hence at some time t there are N_t firms in the economy, while $f_t < N_t$ is the number of firms that did actually start production before date t. A firm that has an index between f_t and N_t is still filling its vacancies, and is not currently producing. More precisely, the sequence of a firm's creation follows four main steps:

1. an agent decides to create a new firm, pays the entry cost and invents a new good;
2. the new firm determines its demand for labor, L^d and opens exactly L^d vacant jobs;
3. vacancies are filled with an imperfect matching technology;
4. once the new firm has completed the filling of its vacancies, and only then, it starts production.

As matching is not costless, there is rent to be shared from successful matches. Wages are thus determined, once production has started, by a continuous bargaining process between entrepreneurs and workers.

We now describe the model in more detail.

2.1. The Production Process

The basic structure of our model borrows from CABALLERO and JAFFE [1993]. Time is continuous. Agents consume the following aggregate of goods in every period:

$$(1) \qquad Y_t = \left[\int_{-\infty}^{f_t} \left(x_t(j)e^j\right)^{1-1/\beta} dj \right]^{\beta/(\beta-1)}, \text{ with } \beta > 1,$$

with $]-\infty, f_t]$ the continuum of available goods in the economy, and $x_t(j)$ the demand for each of these goods. The term e^j in the definition of Y_t

reflects the hypothesis that newer goods are preferred by consumers, because of their improved quality relative to older ones [1]. We thus have a model that combines both expanding quality and expanding variety (see CABALLERO and JAFFE [1993]). β is the (static) elasticity of substitution between goods. The assumption $\beta > 1$ means that there is global substitutability. It may be useful to notice that β can be viewed as an indicator of the degree of competition in the economy: the higher β, the more substitutable the goods and hence the tougher competition.

Maximizing aggregate consumption under budget constraint, and using Y_t as numeraire yields the ensuing demand for the good of quality j at time t:

$$x_t(j) = p_t(j)^{-\beta} \cdot e^{(\beta-1)j} Y_t, \tag{2}$$

and:

$$\left[\int_{-\infty}^{f_t} \left(p_t(j)e^{-j}\right)^{1-\beta} dj\right]^{1/(1-\beta)} = 1 \tag{3}$$

with $p_t(j)$ denoting the current price of good j in terms of the consumption aggregate.

We choose a constant returns production technology, namely $x_t(j) = L_t(j)$, with $L_t(j)$ being the employment level in sector j, and assume that labor is supplied without disutility.

2.2. **The Matching Process**

A firm created at time t will have to open a certain number of job slots and fill them before it can start production. As previously announced, we assume that the labor market undergoes some frictions in the matching process between firms and job seeking workers. We represent these frictions by assuming that there is a number $m(U_t, V_t)$ of matches per unit time, with m being a standard matching function defined after PISSARIDES [1990]. More precisely, m depends on the number of unemployed, U_t, and on the total number of vacancies, V_t. It is assumed to exhibit constant returns to scale, so that m is homogeneous of degree one.

Due to these frictions, completing a firm's hiring process will take some time. Let d_t be the length of the hiring period. This firm will then start production at date $t + d_t$. Hence we should have $N_t = f_{t+d_t}$. The number of jobs this firm opens at time t depends on the anticipated market situation at time $t + d_t$, and is worth $L_{t+d_t}(N_t)$. We define this number as $V_t(N_t) = L_{t+d_t}(N_t)$.

We then need to ensure that firms get into production by rank of age. For the sake of simplicity, we thus assume that the matching process is

1. This term could equivalently appear in the production technology defined below, and thus be viewed as a Harrod-neutral labor augmenting technical progress.

deterministic and that all the matches made at some date t are made in the oldest non-producing firms, *i.e.* the firms created at time τ such that $\tau + d_\tau = t$. The state of the labor market determines the number of such firms that will enter into the productive apparatus, since it sets the total number of their hires. Formally, we have at every date:

$$m(U_t, V_t) = L_t(f_t)\dot{f}_t. \tag{4}$$

The length of the hiring delay will hence be given by [2]:

$$\int_t^{t+d_t} m(U_s, V_s)\, ds = V_t. \tag{5}$$

The total stock of vacancies V_t will decrease at every date t by the number of matches, and increase by the number of vacancies opened by newborn firms. Hence, since there are $\dot{N}_t$ new firms arriving per unit time, that stock solves:

$$\dot{V}_t = -m(U_t, V_t) + \dot{N}_t V_t(N_t). \tag{6}$$

Equilibrium on the labor market then yields:

$$\bar{L} = L_t + U_t, \tag{7}$$

with $L_t = \int_{-\infty}^{f_t} L_t(j)\, dj$ denoting total employment in the continuum of firms that are actually producing at time t, and $\bar{L}$ being total labor supply.

2.3. **Wage and Price Setting**

The achievement of its hiring process allows any firm to start producing and make profits. These profits have to be shared between entrepreneurs and workers. We assume that this is done through a bargain between the firm and its marginal worker, which takes place at every date once production has started. Let ξ be the relative bargaining power of the worker. The outcome of that bargain is classically supposed to ensure that the global surplus is shared in $(\xi, 1-\xi)$ proportions, that is:

$$\mathcal{W}_t^e(j) - \overline{\mathcal{W}}_t^u = \frac{\xi}{1-\xi} \cdot (\mathcal{V}_t^o(j) - \mathcal{V}_t^v(j)) \tag{8}$$

with $\mathcal{W}_t^e(j)$ denoting the wealth of an employed worker in firm j, $\overline{\mathcal{W}}_t^u$ that of an unemployed one [3], $\mathcal{V}_t^o(j)$ the value of firm j's marginal (occupied)

2. It could be argued here that, since the opening of a vacant job is costless, it would be in a firm's interest to open a higher (and maybe infinite) number of vacancies in order to accelerate the hiring process. To avoid that, we shall assume that there is some kind of "labor market authority" that forces the firms to commit to opening the "right" number of vacancies, *i.e.* the one derived hereabove.
3. A bar over a variable is used to denote an average value, taken as given by individuals.

job, and $\mathcal{V}_t^v(j)$ the value of a job with its worker fired in that firm. The value functions defined above may be derived in the following way: we first assume that a dismissed worker is never replaced [4], so that $\forall(j,t),\ \mathcal{V}_t^v(j) = 0$. Now let $\lambda_t(j)$ be the instantaneous probability for a job to be destroyed in firm j. Note that this probability *a priori* depends on j. $\mathcal{W}_t^e(j)$ then evolves according to the ensuing asset equation, with r_t denoting the real interest rate:

$$(9) \qquad r_t\mathcal{W}_t^e(j) = w_t(j) + \dot{\mathcal{W}}_t^e(j) - \lambda_t(j)(\mathcal{W}_t^e(j) - \overline{\mathcal{W}}_t^u).$$

Similarly, let $\mu_t = m(U_t, V_t)/U_t$ be the instantaneous probability for an unemployed worker to match a vacant job. As it was assumed zero earnings for the unemployed, one has:

$$(10) \qquad r_t\overline{\mathcal{W}}_t^u = \dot{\overline{\mathcal{W}}}_t^u - \mu_t(\overline{\mathcal{W}}_t^u - \overline{\mathcal{W}}_t^e),$$

with $\overline{\mathcal{W}}_t^e$ denoting the average wealth of an employed worker, thus taken as given by individuals. Finally, the firm's value evolves according to:

$$(11) \qquad r_t\mathcal{V}_t^o(j) = p_t(j) - w_t(j) + \dot{\mathcal{V}}_t^o(j) - \lambda_t(j)\mathcal{V}_t^o(j),$$

Replacing equations (9), (10) and (11) into the bargain outcome (8), one finally gets the wage setting equation for firm j:

$$(12) \qquad w_t(j) = \xi[p_t(j) + \mu_t\overline{\mathcal{V}}_t^o],$$

with $\overline{\mathcal{V}}_t^o$ being the average value of an occupied job.

We now turn to the price setting rule. Firms aim at maximizing their discounted expected value, so that their problem can be written as:

$$(13) \qquad \max_{\{p_s(j), s\geq t\}} \int_t^{+\infty} [p_s(j) - w_s(j)]L_s(j)\cdot e^{-\int_t^s r_u\,du}\,ds,$$

subject to the wage setting equation (12), the demand constraint (2) and their constant returns technology. The latter program is in fact statical, and if one takes the various constraints into account, it can be rewritten statically as:

$$(14) \qquad \max_{p_t(j)} [p_t(j)(1-\xi) - \xi\mu_t\overline{\mathcal{V}}_t^o]\cdot p_t(j)^{-\beta}.$$

Its first order condition is the price setting equation:

$$(15) \qquad w_t(j) = \left[1 - \frac{1-\xi}{\beta}\right]\cdot p_t(j).$$

4. Which is true at least in the steady state. We return to this assumption below.

This identity together with the wage setting equation has strong implications. The first one is that it makes us able to give the value of the current price $p_t(j)$ (or equivalently, that of the current wage $w_t(j)$) in firm j as a function of exogenous parameters and of the average value of an occupied job. Hence all firms pay the same wage and set the same price. This is important, since all workers are identical so that if the wages were to differ across firms, it could be in the interest of a poorly paid worker to quit his job and search for another match. The fact that all wages are the same, regardless of the firms' ages ensures that workers never quit unless their job is destroyed.

This homogeneity, together with our price normalization (3) yields the current price:

$$p_t = \left(\frac{1}{\beta - 1}\right)^{1/(\beta-1)} e^{f_t}. \tag{16}$$

As the goods market works under monopolistic competition, one can now derive the labor demand of producer j at date t. Indeed, we get this labor demand from equations (2), (16) and the constant returns technology:

$$L_t(j) = (\beta - 1)^{\beta/(\beta-1)} Y_t \cdot \exp\{(\beta - 1)j - \beta f_t\}. \tag{17}$$

One sees that this labor demand is decreasing in the number of firms. This reflects the fact that when a firm gets older, it looses market shares because of the consumers' taste for variety and quality. This tends to induce a decline in a given firm's demand for labor over time. However, the relative variation in firm j's labor demand has an overall ambiguous sign, since the growth of global product tends to make it positive, as shown by:

$$\frac{\dot{L}_t(j)}{L_t(j)} = \frac{\dot{Y}_t}{Y_t} - \beta \dot{f}_t. \tag{18}$$

As it is clear that this relative variation will be of negative sign in a steady state, we have assumed throughout the analysis that we stay close enough to a steady state to be sure it always remains negative. Under this assumption, the probability $\lambda_t(j)$ is given by $\lambda_t(j) = -\dot{Y}_t/Y_t + \beta \dot{f}_t$, and is positive. One sees that this probability is the same for all firms.

Replacing the price setting rule (15) into the wage setting rule (12) one gets:

$$\frac{1 - 1/\beta}{\mu_t} = \frac{\xi}{1 - \xi} \cdot \frac{\overline{\mathcal{V}}_t^o}{p_t}, \tag{19}$$

an important equation that will be referred to as (WP) in the remainder. It represents the intersection of the wage setting and the price setting schedules, and links the labor market variables to the growth and interest rates. One can see that it implies a downward sloping relationship between the value of a job (*i.e.* the surplus of an employed worker's welfare over that of an unemployed one) and the probability for an unemployed worker to find a job.

2.4. **Free Entry and Intertemporal Equilibrium**

An agent who wishes to innovate and create a firm must pay an entry cost D_t. Let G_t be the gross expected discounted value of a newborn firm. Remember that a firm gets its first positive profits at $t + d_t$. G_t is defined by the discounted sum of the profit flows accruing to that firm. In our situation, G_t can obviously be written as follows:

$$G_t = \overline{\mathcal{V}}^o_{t+d_t} L_{t+d_t}(N_t) \cdot e^{-\int_t^{t+d_t} r_u \, du}. \tag{20}$$

Free entry on the goods market ensures that this value is equal to the entry cost, namely $G_t = D_t$. To ensure the existence of a balanced growth path, D_t has to grow at the same rate as all nominal variables. Hence we assume $D_t = k \cdot e^{N_t}$, with k being an exogenous constant parameter. This assumption can be viewed as resulting from constant returns in the R&D sector. After some algebra, it is easy to check that this condition can be written:

$$k \cdot \left(\frac{1}{\beta - 1}\right)^{1-1/(\beta-1)} = \frac{\overline{\mathcal{V}}^o_{t+d_t}}{p_{t+d_t}} \cdot L_{t+d_t} \cdot e^{-\int_t^{t+d_t} r_u \, du}. \tag{21}$$

The free entry equation will be referred to as (FE) in the remainder.

The model is completed by writing equilibrium on the goods market. There are in fact two different sources of demand for goods in our model, namely agents' consumption and firms' entry costs. To precise the evolution of consumption, we assume the representative agent to be endowed with a utility function of the CIES type in an aggregate consumption index C_t with intertemporal substitutability σ, so that $U(C_t) = C_t^{1-1/\sigma}/(1 - 1/\sigma)$. He maximizes his discounted sum of utility as of time 0 under intertemporal budget constraint, facing an endogenous interest rate r_t and an exogenous discount factor, ρ. Standard optimal control arguments lead to the following value for the consumption growth rate:

$$\frac{\dot{C}_t}{C_t} = \sigma(r_t - \rho). \tag{22}$$

As there are $\dot{N}_t$ new firms per unit time, the total income of the economy will be given by $Y_t = C_t + \dot{N}_t \cdot D_t$.

The intertemporal equilibrium in our model is determined by the set of dynamic equations we have written until now. The complete dynamic system is presented in appendix A.

3 Steady State

In a steady state, the rate of innovation is constant. Let g be this rate. Furthermore, the constancy of the labor market variables, and that of the hiring delay imply that $\dot{N}_t = \dot{f}_t = g$. In order to obtain finite present values, we restrain our attention to growth rates such that $g < r$, thus taking account of the usual "transversality condition".

3.1. The Steady State System

The constancy of the number of matches [5] $m(U, V)$ together with the definition of d_t then says that:

$$d_t \equiv d(V/U) = \frac{V/U}{m(1, V/U)} \tag{23}$$

for all t, by virtue of function m's homogeneity. In the remainder, we shall write $V/U = \theta$. θ is traditionally referred to as the "tightness" parameter of the labor market. It is easy to check that d is an increasing function of θ. We shall assume, in addition, that $\theta \mapsto m(1, \theta)$ satisfies the Inada conditions, so that d bijectively maps $\mathbb{R}_+$ onto itself. One also has $f_t = N_{t-d(\theta)}$.

Stationarity implies that the total flow into unemployment must match the total flow out of unemployment. The total flow out of unemployment is defined as the total number of hires of new firms, as given by equation (4), *i.e.* the total number of matches per unit time $m(U, V)$. This leads to the well known Beveridge relation [6]:

$$(\beta - 1)g \cdot L = m(U, V), \tag{24}$$

This equation also reflects the stationarity of V. With the labor market equilibrium ($L = \bar{L} - U$), it defines a curve in the (U, V) plane that is downward sloping and concave to the origin [7] which will be denoted (BC). The presence of the growth rate in the (BC) equation uncovers a first, direct effect of "creative destruction": faster growth implies faster labor turnover.

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5. In steady states, time indexes will be omitted when unnecessary.
6. To see this point, just notice that we get from the labor demand (17) the ensuing equality: $L_t(f_t) = (\beta - 1)L_t$. Also note that this equality imposes $\beta < 2$ to make sense.
7. Differentiating equation (24) with respect to U and V yields:

$$m_1'(U, V)dU + m_2'(U, V)dV = -(\beta - 1)gdU.$$

Since the number of matches is assumed to be increasing with respect to both its arguments, our curve slopes downward. Its convexity can easily be demonstrated from the assumption of nonincreasing returns in the matching technology.

It thus shifts the (BC) curve outwards and tends to increase the stock of unemployed as well as the stock of vacant jobs.

To determine the equilibrium values of U and V, we need two more relationships between these two variables. These relationships will be given by considering equations (WP) and (FE). The first step to study (WP) and (FE) in steady states is to write the stationary value of an occupied job. This is done by writing equation (11) in a steady state:

$$\frac{\overline{\mathcal{V}}_t^o}{p_t} = \frac{(1-\xi)/\beta}{(r-g)+g(\beta-1)}. \tag{25}$$

This expression of the steady state value of $\overline{\mathcal{V}}_t^o/p_t$ calls for some comments. Note that the actual discount rate that a firm applies to its flows of profit is $r-g+g(\beta-1)$. The term $r-g$ corresponds to a "capitalization effect" (PISSARIDES [1990]): $g-r$ can be seen as the net growth rate of a firm's income. The capitalization effect tends to augment the longevity of a job. Indeed, faster growth of aggregate demand is beneficial to all firms and thus implies slower declines in old firms' demand for labor. This is the origin of the $-g$ term in the actual discount rate firms apply to their incomes. The term $g(\beta-1)$ represents an "indirect creative destruction effect" (AGHION and HOWITT [1994]): it is a measure of the speed at which an installed firm will lose market shares. The indirect creative destruction effect tends to shorten jobs' lifetimes [8]. In conclusion, an opening firm anticipating a rising rate of growth will be willing to open more vacant jobs if the capitalization effect dominates, and less of them if the indirect creative destruction effect dominates.

With these remarks in mind, one can rewrite the (WP) equation in a steady state:

$$\rho - g[(1-1/\sigma)+(1-\beta)] = \frac{\xi}{\beta-1}\cdot\frac{\theta}{d(\theta)}. \tag{26}$$

Hence the wage negotiations and the price setting behavior of firms lead to a linear relationship between the rate of growth and the probability $\theta/d(\theta)$ for an unemployed to find a job. The slope of this relationship has the sign of $(\beta-1)-$ $(1-1/\sigma)$.

We now turn to the study of the rentability condition. From equations (21) and (25), we get in a steady state:

$$G_t = \left(\frac{1}{\beta-1}\right)^{-1+1/(\beta-1)} \cdot \frac{\overline{\mathcal{V}}_t^o}{p_t} L\cdot\exp\{(g-r)d(\theta)+f_t\} = ke^{N_t}$$

8. Indeed, the average duration of a match is given by the inverse of the rate of dismissal, $1/\lambda$. It appears from the definition of λ (see equation (18) above) that this duration decreases w.r.t. the growth rate, since $\lambda = g(\beta-1)$ in steady states.

$$(27) \qquad \Leftrightarrow G_t e^{-N_t} = \left(\frac{1}{\beta-1}\right)^{1/(\beta-1)} \cdot \frac{(1-1/\beta)(1-\xi)}{\rho - g[(1-1/\sigma)+(1-\beta)]} \cdot L \cdot e^{-rd(\theta)} = k.$$

One thus sees that the (detrended) value of a firm, which free entry imposes equal to the entry cost k, is equal to the sum of constant instantaneous profit flows (this constant being equal to the initial value of the actual flow of profit accruing to the firm), discounted at rate $r-g+g(\beta-1)$. Those profit flows are proportional to the level of employment, L, as a consequence of our constant returns assumption. Furthermore, profits appear once the firm has waited $d(\theta)$, hence the extra discount term $e^{-rd(\theta)}$. This free entry condition again links the growth rate to the state of the labor market. This link works through three different channels. The first one is the length of the hiring delay which, together with the value of the interest rate, sets the "direct" cost of the labor market frictions. The second channel is the steady state level of employment, which is a measure of the value of the first profit [9] made by a new firm. The third one is the value of the discount rate the firms will apply to their profit streams, $r-g+g(\beta-1)$, which increase (decreases) with g if $(1-1/\sigma)+(1-\beta) < (>) 0$.

We now possess a system of three equations, namely (24), (26), and (27) with three unknown variables, L, g, and θ. We now turn to the study of this system.

3.2. **System Resolution**

Although not difficult, the resolution of the steady state system is a bit tedious. Hence all proofs have been confined in the appendix. Under the assumptions we made about the matching function m, it is straightforward to state the following lemma:

LEMMA 1 : Equation (26) defines θ as a C^∞ function of g for all values of g such that the left hand side of this equation is positive. Let φ be this function. Then:
— if $1-1/\sigma > \beta - 1$, then $\varphi'(g) < 0$ and $\varphi''(g) > 0$ for all $g \in [0, \rho/(2-\sigma^{-1}-\beta))$, and:
— if $1-1/\sigma < \beta-1$, then $\varphi'(g) > 0$ and $\varphi''(g) > 0$ for all $g \in \mathbb{R}_+$.

The proof stems immediately from the implicit functions theorem.

Now solving for L in the system formed by the free entry condition (27) and the Beveridge curve (24), and inserting the solution back into the wage setting/price setting equation (26) yields the following relationship between

9. That is, the first positive value taken by the instantaneous flow of profits accruing to that firm: $(p_t - w_t) \cdot x_t(f_t)$.

growth and labor market tightness:

$$(28)\qquad \left(g+\frac{m(1,\theta)}{\beta-1}\right)e^{rd(\theta)} = \left(\frac{1}{\beta-1}\right)^{1/(\beta-1)}\cdot\frac{(\beta-1)(1-\xi)}{\beta\xi k}\bar{L}=\mathcal{K}\big(\beta,\xi,\bar{L},k\big),$$

in which we introduce $\mathcal{K}\big(\beta,\xi,\bar{L},k\big)$ for the sake of notational brevity [10]. About this last equation, we can state:

LEMMA 2 : Equation (28) defines θ as a C^∞ function (say ψ) of g on the interval $[0,\mathcal{K}]$. Function ψ is strictly decreasing on this whole interval, and $\psi(\mathcal{K})=0$.

The proof is in the appendix.

With these two lemmas, we have a system of two equations with unknowns (θ, g). Our concern now is to find out whether this system has zero, one or several solutions. This must be done by studying the (virtual, at the present time) intersections of functions φ and ψ within an interval of acceptable values for g (that is $g\in[0,\rho/(1-\sigma^{-1}))$ if $\sigma>1$, and $g\in\mathbb{R}_+$ otherwise). Since function ψ is always decreasing, one sees that the "nice" situation will be the one where φ is increasing, *i.e.* the case of low intertemporal substitutability (low σ) and high static substitutability (high β, which can also be interpreted as a situation of tough competition). In the opposite case, although sufficient conditions to ensure the existence of an equilibrium couple (θ^*, g^*) may be given, nothing too precise can be said about the uniqueness of such an equilibrium without further knowledge of the shape of the matching technology. More formally, one can state the following two propositions:

PROPOSITION 3 [Case of low intertemporal substitutability and hard competition] : Assume $\beta-1>1-1/\sigma$. Let:

$$\hat{\theta}=\frac{\beta-1}{\xi}\cdot\ln\left[\frac{\mathcal{K}\xi}{\rho}\right],$$

and (whenever $\sigma>1$), $$\check{\theta}=\frac{(\beta-1)^2}{\xi}\cdot\ln\left[\frac{\mathcal{K}\xi}{\rho}\cdot\frac{1-1/\sigma}{\xi+\beta-1}\right].$$

Then:
— if $\sigma\le 1$ then a necessary and sufficient condition for the existence and uniqueness of a steady state equilibrium $(L^*,g^*,\theta^*)\in\big[0,\bar{L}\big]\times[0,\mathcal{K}]\times\mathbb{R}_+$ is $\varphi(0)<\hat{\theta}$;
— if $\sigma>1$ then a necessary and sufficient condition for the existence and uniqueness of a steady state equilibrium $(L^*,g^*,\theta^*)\in\big[0,\bar{L}\big]\times[0,\rho/(1-\sigma^{-1})]\times\mathbb{R}_+$ is $\varphi(0)<\hat{\theta}$ and $\varphi[\rho/(1-\sigma^{-1})]>\check{\theta}$.

10. To avoid uninteresting discussions, we shall additionally assume in the remainder that the total labor force is large enough to ensure $\mathcal{K}\ge\rho/\big(1-\sigma^{-1}\big)$ (whenever $\sigma>1$). This assumption is completely innocuous.

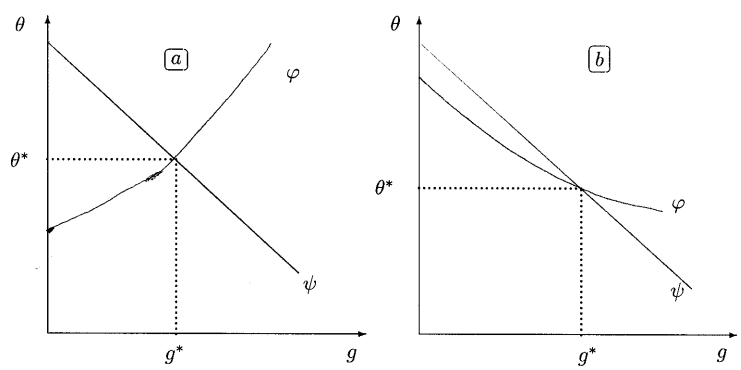
PROPOSITION 4 [Case of high intertemporal substitutability and mild competition] : Assume $\beta - 1 < 1 - 1/\sigma$. Then a sufficient condition for at least one steady state equilibrium $(L^*, g^*, \theta^*) \in [0, \bar{L}] \times [0, \rho/(1-\sigma^{-1})] \times \mathbb{R}_+$ to exist is either $\varphi(0) > \hat{\theta}$ and $\varphi[1-\sigma^{-1}] < \check{\theta}$, or $\varphi(0) < \hat{\theta}$ and $\varphi[\rho/(1-\sigma^{-1})] > \check{\theta}$.
Uniqueness is not ensured by these conditions.

A quick proof for both propositions is contained in the appendix.

Basically, a steady state equilibrium is determined by a pair of equilibrium values θ^* and g^*, given by the (eventual and eventually unique) intersection of the φ and ψ schedules in the (g, θ) plane. Those values determine a position for the Beveridge curve (24) in the (U, V) plane, and a particular point on it defined by the intersection of this curve with the $V = \theta^* U$ line. The point made in propositions 3 and 4 is to give some conditions under which the φ and ψ schedules have one (or more) intersection point(s). More precisely, what they say is that one can be in either one of the situations depicted on figure 1.

FIGURE 1

Steady State Equilibrium.



The cases described in proposition 3, which can roughly be gathered under the hypothesis $\beta - 1 \geq 1 - 1/\sigma$, are represented on panel *a* of figure 1. It is the case in which the indirect creative destruction effect offsets the capitalization effect, which makes the φ schedule slope up. The uniqueness of the steady state equilibrium is therefore always guaranteed. Other cases (one not necessarily unique steady state, proposition 4) occur when the opposite inequality holds – that is, $\beta - 1 < 1 - 1/\sigma$. In that case, the capitalization effect overcomes the indirect creative destruction effect. An example of such a case is depicted on panel *b* of figure 1 [11].

Hence we see that the relative value of parameters β and σ (static and intertemporal substitutabilities) has crucial consequences on the behavior of endogenous variables. The remainder of the paper is dedicated to the study of that behavior.

4 Comparative Statics

In this section, we turn to some issues of comparative statics. To determine the effects on growth, labor market tightness and employment of shifts in the various exogenous parameters, we note from equations (26) and (28) that the φ schedule shifts upwards in response to positive shocks on ρ and β, and downwards when ξ or σ increase, and that the ψ schedule shifts upwards in response to increases in $\bar{L}$ and σ, and downwards after a rise in ξ , ρ, k, or β [12]. These properties lead to the results of comparative statics for θ^* and g^* that are summarized in the first two lines of table 1 (the labels "case *a*" and "case *b*" refer to the same distinction as on figure 1).

TABLE 1

Comparative Statics.

positive shock on…		k	$\bar{L}$	σ	ρ	ξ	β
1. Response of g^*	case a	$-$	$+$	$+$	$-$	?	$-$
	case b	$-$	$+$	$+$	$-$	?	$-$
2. Response of θ^*	case a	$-$	$+$	?	?	$-$	?
	case b	$+$	$-$	$-$	$+$	$-$	$+$
3. Response of u^*	case a	$-$	$+$	$+$	$-$	?	$-$
	case b	$-$	$+$	$+$	$-$	?	$-$

11. Of course, the situation depicted there is not the only one possible. Other configurations are studied in detail in a discussion paper version of this article (available on request to the authors). For instance, since when $\beta - 1 < 1 - 1/\sigma$ both schedules slope downwards, one can imagine a situation with the φ schedule being steeper than the ψ schedule. The equilibrium obtained in such a case can be shown to exhibit counterintuitive comparative static properties, which make us term it "undesirable". Moreover, since φ and ψ have the same slope sign, the occurrence of multiple equilibria is quite possible – albeit requiring a rather "pathological" behavior of function ψ. This possibility can be checked in a numerical experiment, also available on request. We choose to focus on the "likely" situations, *i.e.* those depicted on figure 1, the drawing of the omitted cases being left to the fancy of the interested reader.
12. The effect of a rise in β on ψ is *a priori* ambiguous, but it can be shown to be negative for sufficiently small (*i.e.* close enough to one) values of β.

About the comparative statics of employment, notice that substituting the Beveridge relationship into the (WP) equation (26), one gets:

$$u^* = \frac{\xi}{\xi + \rho/g^* - [(1 - 1/\sigma) + (1 - \beta)]}, \tag{29}$$

with u^* denoting the equilibrium unemployment rate. Combining the results of table 1 with this equation yields the results shown in line three of table 1. Those results strongly suggest an inverse relationship between the rate of growth and the long-run level of employment, through a prevalence of the creative destruction effects.

Cutting the entry cost (diminishing k) directly encourages the entry of firms, which raises the equilibrium growth rate and, through creative destruction, augments unemployment. A larger total labor force ($\bar{L}$) raises the growth rate by a standard "scale effect", which again raises unemployment through creative destruction. Variations in σ or ρ have direct straightforward effects on the firms' total discount rate: a rise in σ or a drop of ρ both lower the rate of interest, thus diminishing the firms' discount rate. This encourages entry, and the corresponding rise in the growth rate is transferred to the unemployment rate through the creative destruction channel.

The parameters hitherto considered were basic components of the endogenous growth rate. Hence, a shift in one of those parameters only affects equilibrium employment through the growth rate — that is, through creative destruction. Things are different if one considers shifts in either the workers' bargaining power ξ, or the static elasticity of substitution β, which both affect the growth rate *and* the unemployment rate directly. Indeed, an ambiguity may appear after a rise in ξ. By raising the cost of labor, this directly raises unemployment. But it also makes jobs less profitable, which discourages the entry of firms, thus tending to lower the rate of growth. That in turn attenuates creative destruction, which raises employment and firms' profits. The overall effect on g^* (and therefore, on u^*) is ambiguous. It should be noted, however, that whenever the sign of the change in u^* that follows a rise in ξ is positive, it never results from a prevalence of the capitalization effect. Finally, a rise in the static elasticity of substitution β, which makes the economy more competitive, reinforces the creative destruction effect [13] because it accelerates the loss of market shares incurred by old firms. It also has a direct negative effect on the firms' profits, by reducing their monopoly power and thus the mark-up of prices over wages. Both these effects tend to shorten jobs' lifetimes, thus raising unemployment.

Why does the relationship between growth and unemployment seem so generally positive in this model? A quick answer would be to say that the

13. The relationship between growth and the degree of competition in an innovation-based endogenous growth model is studied in great detail in AGHION and HOWITT [1996].

“reallocative” effect of growth when the labor market undergoes frictions always overcomes its effects on profitability. Indeed, remember that the rate of job destruction, λ, is given by $\lambda = \beta\dot{f} - \dot{Y}/Y = (\beta-1)g$ (see equation (18) and footnote 8). As we already argued, this reflects the creative destruction effects. Faster growth is detrimental to older firms, since it makes newer goods appear faster, those goods being preferred and therefore substituted to theirs by consumers. $(\beta-1)g$ was therefore viewed as a measure of the speed at which those firms lose market shares: this is the indirect creative destruction effect. Gross substitutability ($\beta > 1$) implies that the rate of dismissal λ is always positive and proportional to the growth rate [14]. Hence, because of goods substitutability, faster growth always leads to a bigger rate of dismissal, which in turn means faster entry into unemployment. Because of the labor market frictions, this higher turnover rate leads to more unemployment: this is the direct effect of creative destruction. It thus appears that even though the capitalization effect can dominate the indirect effect of creative destruction (that is, even though a rise in g can make in the firms' total discount rate, $r-g+g(\beta-1)$ drop), the direct effect of creative destruction always raises unemployment. Whether or not the capitalization effect offsets the indirect creative destruction effect is only a matter of knowing whether the steady state real present value of an occupied job, $\mathcal{V}^o/p$, increases or decreases with respect to the growth rate.

The global prevalence of the creative destruction effect of growth on employment is a general feature of exogenous growth models of technological unemployment (MORTENSEN and PISSARIDES [1995], AGHION and HOWITT [1994], although the latter contribution finds a positive relationship between growth and employment at high growth rates). It seems that endogenizing the rate of growth and introducing a “keynesian” ingredient such as imperfect competition into the model does not question that result, and even reinforces it, in some way. We finally stress that the fact that creative destruction effects broadly dominate the capitalization effect in our model does not mean that this latter effect is absent from it. What we argue is that “Schumpeterian” models of growth-induced unemployment are designed to obtain that result, which does not appear as clearly in the available empirical work (see *e.g.* the study by the CEPR [1995]). Whatever the data say, it makes little doubt that creative destruction is a source of unemployment — albeit maybe small —, which is enough for making the schumpeterian models of unemployment interesting, their main purpose being to analyze that particular feature of growth rather than to fit the facts.

14. It is clear that this result would be inverted if the goods were gross complements ($\beta < 1$), for in that case, the appearance of a new good would have a positive effect on the demand for all other goods. Unfortunately, for obvious reasons of concavity of the firms' profits, the model of this article is inadequate for the study of that case.

5 Conclusion

We have developed a model of qualitative endogenous growth with imperfect matching on the labor market. In these conditions, growth induces unemployment through the schumpeterian process of creative destruction. The link between growth and labor market variables remains ambiguous after this study, although it was shown that the negative creative destruction effect of growth on employment was very generally prevalent. We also have exhibited the important structural consequences of a change in the relative value of intertemporal substitutability and static substitutability by showing that the slopes of two among the three fundamental equations of our model (those are the free entry and the wage setting - price setting equations) depended upon the relative value of those elasticities. We have shown that if competition (as measured by static substitutability) is relatively harsh, then the situation is quite clear as there may be zero or one unique steady state equilibrium, depending on the values of some parameters. On the other hand, milder competition may lead to a rather confused picture (with the possible coexistence of multiple equilibria).

Our conclusion that innovation-based growth generally favors frictional unemployment certainly calls for further discussion. Before affirming that growth is definitely bad for employment, it seems fair to acknowledge that schumpeterian models of growth fail to capture a number of positive effects of "technical change" on employment, which might well lead to more optimistic conclusions. Further research should thus focus on the possibility of reinforcing the capitalization effect in that kind of models. An attempt in that direction was already made by MORTENSEN and PISSARIDES [1995], who build a model in which investment is not completely irreversible for firms have access to technological update. Another possibility to foster the positive effects of growth on employment in a multisectoral model would be to posit some complementarity between some of the consumption goods (along these lines, see YOUNG [1993]). In the end, there are two components of the model that would certainly be worth endogenizing. The first one is the research sector (and the cost of innovation). The second one is education and the update of workers' skills during unemployment. Despite its convenience, the assumption of a homogeneous labor force that comes with the matching model is somewhat unsatisfactory from that point of view, for innovation-based unemployment could certainly be thought of as a skill mismatch problem.

APPENDIX A

The Dynamics

The unknown variables of our model at some date t are U_t, V_t, L_t, Y_t, r_t, N_t and f_t. The dynamic system ruling the model will thus consist in eight differential-difference equations.

The first one is the Keynes-Ramsey condition (22):

$$\frac{\dot{C}_t}{C_t} = \sigma(r_t - \rho), \tag{30}$$

together with the value of the GNP $Y_t - k\dot{N}_t e^{N_t} = C_t$. Another one is the law of motion of V_t, namely (6). If one notices that at any date t, $L_t(f_t) = (\beta - 1)L_t$, one can write this law of motion as:

$$\dot{V}_t = -m(U_t, V_t) + \dot{N}_t \cdot (\beta - 1)L_{t+d_t}. \tag{31}$$

The law of motion of L_t stems from equation (17):

$$\frac{\dot{L}_t}{L_t} = \frac{\dot{Y}_t}{Y_t} - \beta\dot{f}_t. \tag{32}$$

Moreover, the equilibrium condition on the labor market (7) implies:

$$\dot{U}_t + \dot{L}_t = 0. \tag{33}$$

The number of firms evolves according to the hiring rule (4):

$$\dot{f}_t(\beta - 1)L_t = m(U_t, V_t). \tag{34}$$

The definition of d_t (equation (5)) yields [15]:

$$V_t = \int_t^{t+d_t} m(U_s, V_s)\, ds. \tag{35}$$

The wage setting/price setting schedule yields:

$$\frac{\beta - 1}{\xi} = \mu_t \cdot \int_t^{+\infty} \frac{Y_s}{Y_t} e^{(f_s - f_t)(1-\beta) - \int_t^{t+d_t} r_u\, du}\, ds. \tag{36}$$

Our last equation is given by the free entry condition (21):

$$k \cdot \left(\frac{1}{\beta - 1}\right)^{1-1/(\beta-1)} = \frac{\overline{\mathcal{V}}^o_{t+d_t}}{p_{t+d_t}} \cdot L_{t+d_t} \cdot e^{-\int_t^{t+d_t} r_u\, du}. \tag{37}$$

15. Note that the derivative of this equation together with the previous one leads to the "intuitive" identity: $N_t = f_{t+d_t}$.

Proof of lemma 2

For notational simplicity, we shall work with the auxiliary parameters defined below:

$$S = 1 - \frac{1}{\sigma}, \text{ and } B = \beta - 1.$$

We can rewrite equation (28) in the following form:

$$\Gamma(g, \theta) = \mathcal{K}\big(\beta, \xi, \bar{L}, k\big).$$

Consider a given value of $g \in [0, \mathcal{K})$. Then it stems from our assumptions on function m that the function $\theta \mapsto \Gamma(g, \theta)$ bijectively maps $\mathbb{R}_+$ onto $[g, +\infty)$. Hence, equation (28) will have an unique solution $\theta = \psi(g)$ for any value of $g \in [0, \mathcal{K})$. The remainder of the lemma stems directly from the implicit functions theorem, and the fact that $\Gamma(\mathcal{K}, 0) = \mathcal{K}$ obviously solves equation (28).

APPENDIX C

Proof of propositions 3 & 4

C.1 Proposition 3

We keep the notations S, B, and Γ from the proof of lemma 2 above. Since $B < S$ in this proposition, the φ schedule is increasing. Hence existence of an equilibrium implies its uniqueness. The restrictions on g we derived earlier imply that g must be less than $\mathcal{K}$ (see lemma 2 above), and also that $g < r$, which always holds true when $\sigma \leq 1$, and is equivalent to $g \leq \rho/S$ otherwise. Hence, from the assumption $\mathcal{K} > \rho/S$ if $\sigma > 1$, the binding condition is $g \leq \rho/S$, so that solutions may be studied within such values of g. Then, since φ is increasing, a necessary and sufficient condition for the equilibrium to exist and to be unique is $\varphi(0) < \psi(0)$ and $\varphi(\rho/S) > \psi(\rho/S)$. Since Γ is increasing in both of its arguments, these conditions are respectively equivalent to $\Gamma(0, \varphi(0)) < \mathcal{K}$ and $\Gamma(\rho/S, \varphi(\rho/S)) > \mathcal{K}$. Solving the first inequality for $\varphi(0)$ yields exactly $\varphi(0) < \hat{\theta}$. The second is equivalent to $\varphi(\rho/S) > \check{\theta}$. Hence proposition 3.

C.2 Proposition 4

In the case of a decreasing φ schedule, the above conditions still ensure the existence of an equilibrium, but uniqueness is lost. Furthermore, in the decreasing case, existence also holds when φ starts higher and ends lower than ψ. Hence proposition 4.

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