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Democratic Acceptability of Free Trade A Dynamic Approach

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ABSTRACT. – The usual models of international trade focus attention on the optimality of free trade versus any type of barrier. The present paper considers in addition a criterion of "acceptability": the choice of a trade pattern must be accepted by a majority of the population concerned, since they are also interested in preserving different elements of social protection such as a minimal wage or unemployment benefits.

In the framework of a very simple model (two zones, two factors of production, two goods), where a system of quotas permits the anlaysis of a continuum of situations between autarky and free trade, it is shown that many configurations occur where exists a maximal level of import, maximal meaning with respect to a progressive opening of the economy with at each step majority approval. It is shown also that when a dynamic process is introduced where unskilled manpower is trained over time, then, after a sufficient lapse, the pure free trade situation may be obtained and accepted.

These results thus give support to those who consider that progressive liberalization sometimes has more virtues than a brutal elimination of all trade barriers.

Acceptabilité démocratique du libre échange. Une approche dynamique

RÉSUMÉ. – Les modèles habituels de commerce international s'intéressent surtout à l'optimalité du libre échange. Dans le présent papier, l'accent est mis sur « l'acceptabilité » : une organisation du commerce pour être mise en œuvre doit être acceptée par une majorité de la population concernée, également intéressée à préserver des éléments de protection sociale comme un salaire minimum ou des allocations chômage.

Dans le cadre d'un modèle très simple (deux zones, deux facteurs de production, deux biens), où un système de quotas permet l'analyse d'un continuum de situations entre autarcie et libre échange, il est montré qu'un niveau maximal d'importations peut exister, maximal par rapport à des quotas croissants acceptés à chaque étape à la majorité. Il est montré également que, si est introduit un processus de formation transformant le travail non qualifié en travail qualifié, alors, après un délai suffisant, le libre échange peut être atteint et accepté.

Ces résultats confortent la position de ceux qui défendent un passage progressif et non instantané au libre échange.

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1 Introduction

Traditional models of international trade focus attention on the flows of goods and services resulting from the contact between economies with different characteristics. The distortion in the flows which may result from different barriers (custom duties, levies, quotas...) are studied. The general conclusion favours free trade either as an optimal organization or, at least, as an optimal policy (see KRUGMAN [1993]).

Such an approach however completely leaves aside the question of the "acceptability" of free trade in terms of majority support. Especially, for countries where social rigidities such as a minimal wage prevail, free trade may generate unemployment and be rejected on this basis. A more complex situation is when a high level of social protection, such as an unemployment benefit, induces a redistribution which negatively affects the potential winners of free trade. Too often, the classical litterature ignores the problem of the redistributions of income induced by trade (see McCULLOCH [1993]).

We have already shown (in FUCHS [1994]) examples where situations such as those above lead a majority to prefer autarky to free trade. The framework was a simple two countries, two goods, two factors model, the factors being two types of labour, unskilled and skilled. The present paper extends the previous analysis in three respects:

– it deals with a continuum of situations between autarky and free trade, each defined by a level q of authorized imports, equilibria (with rationing) in the markets of the goods, different levels of unemployment;

– it analyses the majority positions when the level of q is increased, possibly through a dynamic process lifting barriers (blocking majorities then may appear);

– it considers last a more complex dynamic process including training that transforms unskilled into skilled labour and shows that, after some time, free trade will always obtain majority support.

More precisely, Section 2 introduces the details of the model. Section 3 defines an equilibrium concept which can take several forms according to the assumptions made on the capacities of training. Section 4 compares, from the point of view of majority voting, the different situations which can be considered between autarky and free trade, corresponding to different levels of quotas and minimal social protection. Section 5 presents the same analysis when unemployment benefits exist. Section 6 introduces a training procedure allowing initially unskilled workers to have access to skilled positions and the main results of the paper, that depend on the characteristics of the economy and different choices of policy, are presented. Last, Section 7 presents a global discussion and comment of all results and argues that they support the idea of "organized" international trade.

2 The Model

The basic model used here considers exchanges between two zones, North and South, both consuming and producing a good number one, named "textile" just for the image, while a good number two, more sophisticated, is consumed in the two zones but only produced in the North.

Goods 1 and 2 are produced from labour only, in constant returns to scale technologies, with coefficients k_1 and k_1' for good 1 in N and S respectively and coefficient k_2 for good 2. An essential feature of the model is then the existence of two types of labour: unskilled labour, available in integer quantities ℓ_1 and ℓ_1' in N and S respectively, for the production of textile; and skilled labour, available in N only in quantity ℓ_2, for the production of good 2.

Skilled labour implies a higher level of education or training than unskilled labour so that logically, w_i $(i = 1, 2)$ being the corresponding wages in N, we shall suppose that

(1) $$w_2 > w_1 > 0$$

a characteristic which implies, as we shall see later on, that unskilled workers try to become skilled.

Now w_1' being the wage paid to the unskilled in S (expressed in the same numeraire as in N) we also supose that

(2) $$w_1 > w_1' > 0$$

just because the unskilled labour force is plentiful in S and supposed not to be mobile.

Production of goods 1 and 2 will then take place if their local prices are related to wages through:

(3) $$p_i = w_i/k_i \qquad p_1' = w_1'/k_1'$$

We shall suppose that even if $k_1 > k_1'$ (a higher productivity of unskilled labour in N than in S) the gap between w_1 and w_1' in (2) is such that still

(4) $$p_1 > p_1'$$

Last, we shall suppose that workers of N are endowed with identical utility functions u of the form:

(5) $$u\left(c_i\right) = \left(c_1\right)^{\mu}\left(c_2\right)^{1-\mu}$$

where c_i is the individual consumption of good i and $0 < \mu < 1$. Workers are in fact wage earners trying to sell a unit of labour, i.e. they look for the maximal level of u under a budget constraint where the only income is wage and, possibly, positive or negative transfer revenues.

The situation in S, where a high level of unemployment and a large informal sector are supposed to exist, will not be described in detail. It will only be supposed that the income S can earn by possibly selling textile to N is totally used to buy good 2 from N.

To introduce time considerations and a continuum of situations between autarky and free trade, we shall use as the main policy tool at the disposal of N the definition of quotas [1]: the relations between N and S will be characterized by the volume of textile $q \geq 0$ authorized to be introduced into N at price p_1'.

Because of its lower price, consumers of N will then try to buy imported textile before "local" textile. For small values of q (the word "small" will receive a precise definition later on) demand for imported textile will be higher than supply so that we have to consider a "rationing scheme" (see BENASSY [1982]). For simplicity we shall consider a scheme proportional to demands, i.e. we suppose that, given q, a worker of type j ($j = 1$ or 2 according to whether he is unskilled or skilled) can buy a quantity c_q^j of textile at price p_1' with

$$\text{(i)} \qquad c_q^j = \alpha(q)\, c_1(p_1', p_2, w_j)$$

where $c_i(p_1', p_2, w_j)$ is the solution of the program:

$$\max\ u(c_1, c_2)$$

under

$$p_1'\, c_1 + p_2\, c_2 \leq w_j$$

which gives

$$\text{(6)} \qquad c_1(p_1', w_j) = \frac{\mu w_j}{p_1'} \qquad c_2(p_2, w_j) = \frac{(1-\mu)\, w_j}{p_2}$$

For the c_q^j to define a rationing scheme we then must impose in addition:

$$\text{(ii)} \qquad \lambda_1\, c_q^1 + \lambda_2\, c_q^2 = q$$

where $\lambda_j \geq 0$ is the number of active workers of type j in a situation with a given q (unemployed wage earners can buy nothing at this stage; we shall in what follows forget about the integer character of the λ_j and consider them as percentages of the ℓ_j).

Given (i) and (ii) one then gets easily

$$\text{(7)} \qquad c_q^j = w_j\, d(q)$$

1. Quotas may also be seen however as the result of a policy of voluntary export restriction by S.

with

(8) $$d(q) = \frac{q}{\lambda_1 w_1 + \lambda_2 w_2} \equiv \frac{q}{W(q)}$$

where $W(q) = \lambda_1 w_1 + \lambda_2 w_2$ is nothing but the total wages, and thus income, in N given q: the share of q that a worker of type j can buy is just his share in total income.

We then have all necessary elements to introduce a concept of equilibrium.

3 Definition of *q*-Equilibria

Let us first consider the behaviour of a worker of type j, supposing observed wages and thus, from (3), prices are those considered in Introduction. He will choose a consumption bundle $c_i(p_i, p_1', q, w_j)$ which maximizes u given the facts that:

– he can buy textile at price p_1', up to the quantity c_q^j,

– he can go on buying textile at price p_1 beyond c_q^j.

Clearly then, according to the level of q, there will be different situations.

Let q_2 be the quantity, from (6) and (7) independant of j, defined by

(9) $$c_1(p_1', w_j) = c_{q_2}^j$$

Then, if $q \geq q_2$ all workers can buy all the textile they wish at price p_1', thus

(10) $$\begin{cases} c_1(p_i, p_1', q, w_j) = c_1(p_1', w_j) \\ c_2(p_i, p_1', q, w_j) = c_2(p_2, w_j) \end{cases}$$

and we are in a pure free trade situation.

Next, if $q < q_2$, a worker of type j will solve the program:

(11) $$\begin{cases} \max u(c_q^j + \gamma_1, c_2) \\ \text{under} \quad p_1 \gamma_1 + p_2 c_2 \leq R_j \equiv w_j - p_1' c_q^j \\ \gamma_1 \geq 0 \end{cases}$$

(note that, from (9), $R_j > 0$). Considering, to begin, only the first constrainst and defining $\pi = \frac{p_1'}{p_1}$, calculation then gives

(12) $$\begin{cases} c_1(p_i, p_1', q, w_j) = c_1(p_1, w_j) + \mu(1-\pi) c_q^j \\ c_2(p_i, p_2, q, w_j) = c_2(p_2, w_j) + (1-\mu) \dfrac{p_1 - p_1'}{p_2} c_q^j \end{cases}$$

The sum of the values of these two consumptions, at price p_1 and p_2 respectively, is easily seen to be $w_j + (p_1 - p_1') c_q^j$, the last term of the sum representing the extra purchasing power resulting for j from the access to c_q^j at price $p_1' < p_1$.

Now considering in addition the constraint $\gamma_1 \geq 0$ i.e.

$$c_1\,(p_i,\,p_1',\,q,\,w_j) - c_q^j \geq 0$$

we get from (12)

$$c_q^j \leq \frac{1}{\varrho}\,c_1\,(p_1,\,w_j)$$

where

(13) $$\varrho = 1 - \mu\,(1-\pi)$$

Let then q_1 be the value of q, independant of j, defined by:

(14) $$c_1\,(p_1,\,w_j) = \varrho\,c_{q_1}^j$$

Clearly $q_1 < q_2$ (because $p_1' < p_1 - \mu\,(p_1 - p_1')$). Then, if $q \leq q_1$, the demand of a worker of type j is indeed given by (12). If $q_1 \leq q \leq q_2$ we have, using to calculate c_2 the budget constraint of (11):

(15) $$\begin{cases} c_1\,(p_i,\,p_1',\,q,\,w_j) = c_q^j \\ c_2\,(p_i,\,p_1',\,q,\,w_j) = \frac{1}{1-\mu}\,c_2\,(p_2,\,w_j) - \frac{p_1'}{p_2}\,c_q^j \end{cases}$$

Having specified the behaviours of consumers and knowing those of producers we can then introduce

DEFINITION 1: Wages w_i and w_1' and the quota q with $0 \leq q < q_2$ define a q-*equilibrium* if, given these data
– consumption of textile in N equals production in N plus the import q,
– production of good 2 in N matches consumption of N plus the demand that S addresses to N using its exports receipts of textile to buy good 2.

Precisely that means:

(16) $$\begin{cases} k_1\,\lambda_1 + q = \lambda_1\,c_1\,(p_i,\,p_1',\,w_1,\,q) + \lambda_2\,c_1\,(p_i,\,p_1',\,w_2,\,q) \\ k_2\,\lambda_2 = \lambda_1\,c_2\,(p_i,\,p_1',\,w_1,\,q) + \lambda_2\,c_2\,(p_i,\,p_1',\,w_2,\,q) + \dfrac{p_1'}{p_2}\,q \end{cases}$$

PROPOSITION 1: There exists an infinite family of q-equilibria characterized by pairs $(\lambda_1,\,\lambda_2)$ with $\lambda_1 + \lambda_2 \leq \ell_1 + \ell_2$ such that:
– for $0 \leq q \leq q_1$

(17) $$(\mu - 1)\,w_1\,\lambda_1 + \mu\,w_2\,\lambda_2 = \varrho\,p_1\,q$$

– for $q_1 \leq q \leq q_2$

$$\lambda_1 = 0$$

Proof: According to the value of q, the c_i are given by (12) or (15) and (6) (with p_1 instead of p_1'). Using (3) in addition and (ii) in the definition of c_q^j, one can easily see either that both equations (16) reduce to (17) or that they imply $\lambda_1 = 0$ and λ_2 undetermined (the equivalence of both equations (16) just reflects the facts that total wages sum up to the value of total production while external trade between N and S is balanced). Of course, the active population cannot be greater than the total population. □

Note that, from (17), $q = 0$ i.e. the autarkic situation appears to be compatible with full employment iff:

$$(\mu - 1)\, w_1\, \ell_1 + \mu\, w_2\, \ell_2 = 0 \tag{18}$$

a condition (already used in [3]) that we shall suppose to be fulfilled in what follows and which fixes the ratio w_2/w_1. This means that, finally, our only exogenous data are w_1 and w_1'.

Now, among the family of q-equilibria we are going to specify two limit elements of particular interest.

DEFINITION 2: We call *q-equilibrium with perfect mobility* a pair $(\lambda_1,\, \lambda_2)$ satisfying the conditions of Proposition 1 and, in addition, the equality

$$\lambda_1 + \lambda_2 = \ell_1 + \ell_2 \equiv \ell \tag{19}$$

To comment this definition, let us first solve (17) and (19) in λ_1 and λ_2. There is a unique solution which is, with the use of (18):

$$\begin{cases} \lambda_1 = \dfrac{\mu w_2\, \ell - \varrho\, p_1\, q}{(1-\mu)\, w_1 + \mu\, w_2} = \ell_1 - \dfrac{\varrho\, p_1\, q}{(1-\mu)\, w_1 + \mu\, w_2} \\ \lambda_2 = \dfrac{(1-\mu)\, w_1\, \ell + \varrho\, p_1\, q}{(1-\mu)\, w_1 + \mu\, w_2} = \ell_2 + \dfrac{\varrho\, p_1\, q}{(1-\mu)\, w_1 + \mu\, w_2} \end{cases} \tag{20}$$

One can then interpret these λ_j as follows: a level q of imports of textile in N creates a level $\ell_1 - \lambda_1$ of unemployment among unskilled workers but, simultaneously, creates $\lambda_2 - \ell_2 = \ell_1 - \lambda_1$ new jobs in the production of good 2, which is increased due to the purchases of S. This is possible only if unskilled workers can be transformed instantaneously in skilled workers, hence our reference to perfect mobility. Of course (20) is only valid for $q \leq q_1$, between q_1 and q_2 one has $\lambda_1 = 0$ and $\lambda_2 = \ell$.

DEFINITION 3: We call *q-equilibrium with perfect rigidity* a pair $(\lambda_1,\, \lambda_2)$ satisfying the conditions of Proposition 1 and, in addition, $\lambda_2 = \ell_2$. Solving (17) in λ_1 then immediately gives, with the use of (18) again:

$$\lambda_1 = \ell_1 - \frac{\varrho\, p_1\, q}{(1-\mu)\, w_1} \tag{21}$$

The interpretation now can read as follows: a level q of imports of textile in N creates a level $\ell_1 - \lambda_1$ of unemployment among unskilled workers; the constancy of the number ℓ_2 of skilled workers means that, this time, there is no mobility between unskilled and skilled manpower, hence the notion of perfect rigidity. Again (21) is only valid for $q \leq q_1$: between q_1 and q_2 one has $\lambda_1 = 0$, $\lambda_2 = \ell_2$.

Clearly, definitions 2 and 3 have to be interpreted as describing limit situations where the speed of introduction of the quota q is either much slower than the speed of training of unskilled workers (case of perfect mobility) or much higher than this speed of training (case of perfect rigidity). After section III and IV, section V will deal with an other type of situation where this relative speed is neither 0 nor ∞.

Now equations (20) and (21) easily allow to calculate the associated functions $d(\,.\,)$ and the values q_1 and q_2. One gets for instance:

– For q-equilibrium with perfect mobility:

(22) $$q_1^M = \frac{\mu w_2\,\ell}{\varrho\, p_1} \qquad q_2^M = \frac{\mu w_2\,\ell}{p_1'}$$

– For q-equilibrium with perfect rigidity:

(23) $$q_1^R = \frac{\mu w_2\, l_2}{\varrho\, p_1} \qquad q_2^R = \frac{\mu w_2\, l_2}{p_1'}$$

Two comments then. First, one can check that $\lambda_1(q_1) = 0$ for both equilibria, which is not a surprise: indeed, from its definition, q_1 is the level of q for which all demands for “local” textile vanish; this means that all unskilled workers are unemployed then. Next, for both equilibria also, q_2 is equal to the total demand for imported textile of the skilled wokers (in number either ℓ or ℓ_2).

4 Acceptability with Minimal Social Protection

We shall now introduce the first social and political assumptions which will allow us to begin to compare, from the point of view of majority voting, the different situations that may occur for different values of q between autarky and free trade.

First, it is clear that the definition we gave of q-equilibria implies the implicit assumption that, however high is the volume of q, the wage level w_1 of unskilled workers (and so thus p_1) remains rigid: the downward pressure on wages which results from competition of N with S has no consequence. We shall justify this fact as an explicit choice of policy by the authorities of N namely:

Assumption 1 (minimal wage)

Wage w_1 is a minimal wage level in N.

If one considers that this minimal wage has to be a real wage, i.e. allows to buy a definite basket of goods, then rigidity of w_1 implies rigidity of

w_2: in the model, adjustment to go to q-equilibria is only the result of unemployment.

Then, we shall introduce a simple democratic rule which will be the base for the choice by the authorities of N of a definite level of q. Let $S^{\cdot}(q)$ be the situation, described in the previous section, corresponding to a q-equilibrium with perfect mobility or rigidity (. being then replaced by M or R). $S^{\cdot}(0)$ is the autarkic situation, $S^{\cdot}(q)$ with $q \geq q_2$ the free trade situation. We define:

Assumption 2 (democratic rule)

The auhorities of N will choose a situation $S^{\cdot}(q)$ rather than a situation $S^{\cdot}(q')$ (q and q' being two levels of quota between 0 and q_2) if and only if the number of workers whose utility level is strictly higher in $S^{\cdot}(q)$ than in $S^{\cdot}(q')$ is strictly greater than the number of workers whose utility level is strictly lower.

We shall note the aggregate preference defined above through:

$$S^{\cdot}(q) \succ S^{\cdot}(q')$$

Now it is worth paying some attention to this preference in terms of information and vote.

One can first develop a purely static interpretation where, given q and q', all the elements characterizing situations $S(q)$ and $S(q')$ (i.e. beyond wages w_1 and w_1', $\lambda_j(.)$ and $d(.)$) are exactly known. A first case then is when they are known by the authorities who, if they also know the utility function u, can directly aggregate individual preferences and take the decision. In a second case, when the authorities ignore u, they will send their information to the agents and ask for a vote. Of course, in case of perfect information of the agents, the authorities can directly organize the vote. It has to be noted though, that in the last two cases where a vote is actually organized, a rule has to exist to fix who explicitely among the ℓ_1 initially unskilled workers will get skilled or unemployed (for instance the rule can be: are trained first those who have the longest experience, or are fired first those who have been hired last).

One can also develop a dynamical interpretation, where the aggregate preference is used to defined a succession of acceptable situations $S^{\cdot}(q_t)$ with $t \in \mathbb{N}$. At time t it is then logical to suppose that all the elements of $S^{\cdot}(q_t)$ are known by everyone (it is mere observation) while the information about $S^{\cdot}(q_{t+1})$ falls into one of the three cases discussed above.

We are then in position to state the following first results:

PROPOSITION 2: Let q be such that $0 \leq q \leq q_2$. Then:

– in case of perfect mobility of labour or in case of perfect rigidity of labour if $\ell_2 > \ell_1$, one has $S^{\cdot}(q) \succ S^{\cdot}(0) \,\forall q$;
– in case of perfect rigidity of labour if $\ell_1 > \ell_2$ there exists $q_{\max} < q_1^R$ such that:

$$q < q_{\max} \Rightarrow S^R(q) \succ S^R(0)$$
$$q > q_{\max} \Rightarrow S^R(0) \succ S^R(q)$$

Proof: Let us look at the utility level of a worker of type j. Let us define:

$$u(q,\,w_j) = u(c_i(p_i,\,p'_1,\,q,\,w_j))$$

First for $w_j = 0$ (i.e. in case of unemployment) we have from (5) and (6) $u(q,\,0) = 0$ (this is the only situation where $u = 0$).

Next for a given q, the utility level of a skilled worker is always higher than the utility level of an unskilled one (this can be seen directly looking at (6), (12) and (15)).

Then, for $0 < q < q_1$ and $w_j > 0$, the c_i are strictly increasing functions of q and so is also u (see Annex 1).

For $q_1 < q < q_2$ the situation is more subtle. However, without surprise because a greater q means a larger choice of opportunities for workers, one can check (see Annex 1 again) that still, in the range above, $\frac{\partial u}{\partial q} > 0$ (obviously, for $q \geq q_2\,u$ remains constant).

Thus, all individual utility levels increase with q between 0 and q_2 (either from the only increase in q or from the increase in income for those who, from (20), become skilled workers) except for the unskilled workers who become unemployed. This proves the first indent of Proposition 2, unemployed workers either not existing or being for any value of q a minority.

Now in case of perfect rigidity with $\ell_1 > \ell_2$, the number of unemployed people for a given q is, using (21):

$$\begin{aligned} &\ell_1 - \lambda_1 = \frac{\varrho\,p_1\,q}{(1-\mu)\,w_1} && \text{if} \quad 0 \leq q \leq q_1^R \\ &\ell_1 && \text{if} \quad q_1^R \leq q \leq q_2^R \end{aligned}$$

Then clearly $S^{\cdot}(q) \gtrless S^{\cdot}(0)$ is equivalent to the fact that the number of active workers is strictly greater or smaller than the number of unemployed.

This implies first that, for $q \geq q_1^R$, $S^{\cdot}(0) \succ S^{\cdot}(q)$. For $0 \leq q \leq q_1^R$, the condition is equivalent to

$$\ell_2 + \lambda_1 \gtrless \ell_1 - \lambda_1$$

or

$$\ell_1 + \ell_2 \gtrless \frac{2\,\varrho\,p_1\,q}{(1-\mu)\,w_1}$$

Using (18) and (23), this is equivalent to:

(24) $$q \gtrless \frac{\ell_1 + \ell_2}{2\,\ell_1}\,q_1^R \equiv q_{\max} < q_1^R$$

which achieves the proof. □

Remark 1: Obviously from the proof, in case of perfect mobility or perfect rigidity with $\ell_2 > \ell_1$,

$$q > q' \Rightarrow S^{\cdot}(q) \succ S^{\cdot}(q') \quad \forall\, q,\, q'$$

so that Proposition 2 extends the results of FUCHS [1994] which only compared autarky and free trade.

Remark 2: The case of perfect rigidity with $\ell_1 > \ell_2$ leads to an amusing extension of the Condorcet paradox since, with $q > q_{\max} > q'$, one can have simultaneously:

$$S^R(q) \succ S^R(q'),\, S^R(q') \succ S^R(0) \qquad \text{and} \qquad S^R(q) \prec S^R(0)$$

Remark 3: In all cases the model in its present form can already be used to build interesting dynamic policies defined through a succession $q_t\,(t \in \mathbb{N})$ of quota. For instance the authorities can propose an arbitrary q_0, organize a vote to know whether $S^{\cdot}(q_0) \gtrless S^{\cdot}(0)$ and then define a sequence of q_t either increasing until the free trade situation, or moving towards $q_{\max}$ which is the maximal opening accepted by majority voting. The paradox above then just reflects the myopic behaviour of the workers, who do not consider the future since they cannot transfer value from one period to another.

Rather than considering such dynamics though, we shall go further in the introduction of social assumptions so as to deal with more realistic and interesting situations.

5 Acceptability with High Social Protection

We shall now add two social assumptions:

Assumption 3 (unemployment benefit)

Any unemployed person of N receives an unemployment benefit equal to the fraction $\alpha\,(0 < \alpha < 1)$ of the minimal wage w_1.

Assumption 4 (solidarity tax)

The financing of the unemployment benefit is obtained through a solidarity tax proportional to the income of active workers.

Assumptions 3 and 4 of course only make sense for q-equilibria with perfect rigidity. We shall thus consequently forget, in what follows, about the superscript R.

Compared with the previous section, the distribution of income, given q, then becomes:

– the resources of the $\ell_1 - \lambda_1$ unemployed unskilled workers change from 0 to

(25) $$w_1^u(\alpha) = \alpha\, w_1$$

which means that the total amont of solidarity tax is

(26) $$I_S(\alpha) = (\ell_1 - \lambda_1)\, \alpha\, w_1$$

– the resources of the λ_1 active unskilled workers change from w_1 to

$$w_1(\alpha, q) = w_1 - \frac{w_1}{W(q)} I_S(\alpha)$$

where

$$W(q) = \lambda_1 w_1 + \ell_2 w_2 \tag{27}$$

is the total wages, and thus income, in N so that:

$$w_1(\alpha, q) = w_1 \left[1 - \frac{\alpha w_1 (\ell_1 - \lambda_1)}{W(q)} \right] \tag{28}$$

– the ressources of the ℓ_2 active skilled workers change from w_2 to

$$w_2(\alpha, q) = w_2 \left[1 - \frac{\alpha w_1 (\ell_1 - \lambda_1)}{W(q)} \right] \tag{29}$$

Of course the new income distribution keeps the same sum $W(q)$ as the initial one. For this redistribution to make full sense however, it has to be proven that it has no effect on the employment distribution. This is the consequence of the following result:

LEMMA 1: Total demands for goods 1 and 2, and thus levels of employment, are not affected by income redistribution.

Proof: It has to be noted first that one can extend in an obvious manner the rationing scheme considered in (7) to situations where there are more than two levels of income (which is the case of the situation just above). Indeed, one can keep the same function $d(q) = \frac{q}{W(q)}$ and just define $c_q^{j'} = w_{j'} d(q)$ for $j' = 1, 2, 3, ...$

Then the demand functions associated with the three situations arising from the existence of an unemployment benefit keep the same analytical form as given in (10), (12) or (15). This means, using (6), that they satisfy:

$$c_i(p_i, p_1', q, w_{j'}) = w_{j'} c_i(p_i, p_1', q, 1) \qquad \forall j'$$

Then, obviously, total demands for goods 1 and 2 remain constant because using identity above:

$$(\ell_1 - \lambda_1) c_i(p_i, p_1', q, w_1^u(\alpha)) + \lambda_1 c_i(p_i, p_1', q, w_1(\alpha, q)) \\ + \ell_2 c_i(p_i, p_1', q, w_2(\alpha, q)) = W(q) c_i(p_i, p_1', q, 1)$$

so that equation (16) defining q-equilibria and the levels of unemployment is not affected. □

It is worth to be noted that lemma 1 does not depend on the specific form of the utility function that we have introduced in (5) but only relies on the fact that demand functions deriving from u are homogeneous of degree 1 in income.

We can then prove the following central proposition:

PROPOSITION 3: Let assumptions (A1) to (A4) be satisfied. Then in case of perfect rigidity of labour and with $\ell_2 > \ell_1$ one has:

– for $0 \leq q, q' \leq q_1$
  • if $\pi < 1 - \frac{\alpha}{1-\mu-\alpha\mu}$ then $q' > q \Leftrightarrow S(q') \succ S(q)$
  • if $\pi > 1 - \frac{\alpha}{1-\mu+\alpha\mu}$ then $q' > q \Leftrightarrow S(q) \succ S(q')$
  • if $1 - \frac{\alpha}{1-\mu-\alpha\mu} < \pi < 1 - \frac{\alpha}{1-\mu+\alpha\mu}$ then $\exists\, q_{\max} \in]0, q_1[$ such that
  . with $q, q' < q_{\max}$ $q' > q \Leftrightarrow S(q') \succ S(q)$
  . with $q, q' > q_{\max}$ $q' > q \Leftrightarrow S(q) \succ S(q')$

– for $q_1 \leq q, q' \leq q_2$

$$q' > q \quad \Leftrightarrow \quad S(q') \succ S(q)$$

Proof: The assumption $\ell_2 > \ell_1$ has the nice effect that the choice of a skilled worker is the choice of the whole community N. As can be read from the Proposition though, it does not mean that things are simple. Let us examine successively the assertions above.

From the discussion in the first part of Annex 1, it can first be seen that for $0 \leq q \leq q_1$ three types of situations may occur according to the way the utility of a skilled worker:

$$u_2(q) \equiv u(c_i(p_i, p_1', q, w_2(\alpha, q)))$$

behaves as a function of q or $Q = \frac{q}{W(q)}$. These three situations are drawn in figure 1, 2 and 3 and arise from the fact that u_2 as a function of Q has, between 0 and Q_1, a graph which is a piece of parabola with a maximum in $Q_{\max}$ which can be greater than Q_1 (Figure 1), smaller than 0 (Figure 2) or between 0 and Q_1 (Figure 3). The economic intuition behind these situations is whether the gain in u_2 due to the positive price effect (access to a larger amount of textile at low price) is or not greater than the loss due to the negative income effect (obviously, from (29), w_2 decreases when q increases). Then the use of (A5) proves the first three assertions of Proposition 3 (the values of $Q_{\max}$ or $q_{\max}$ can be obtained from (A3) and (A4)).

Last, the discussion of the second part of Annex 1 proves that, for $q_1 < q < q_2$, the utility of a skilled worker is always a strictly increasing function of q, which allows to complete the drawing of the figures and ends our proof. □

Remark 4: Except for the situation of figure 1, where obviously $u_2(q_2) > u_2(0)$ so that $S(q_2) \succ S(0)$ i.e. a majority prefers free trade to autarky, Proposition 3 says nothing about the relative values of $u_2(q_2)$ and $u_2(0)$.

Using however in Annex 1 (A2) with $q = 0$ and (A6) with q_2 given by (23) and w given by (29) allows to prove, after a straightforward calculation, that:

(30) $$u_2(q_2) > u_2(0) \Leftrightarrow \pi^{\mu} < 1 - \frac{\alpha\mu}{1-\mu}$$

which corresponds to a result already obtained in Proposition 2 in FUCHS [1994].

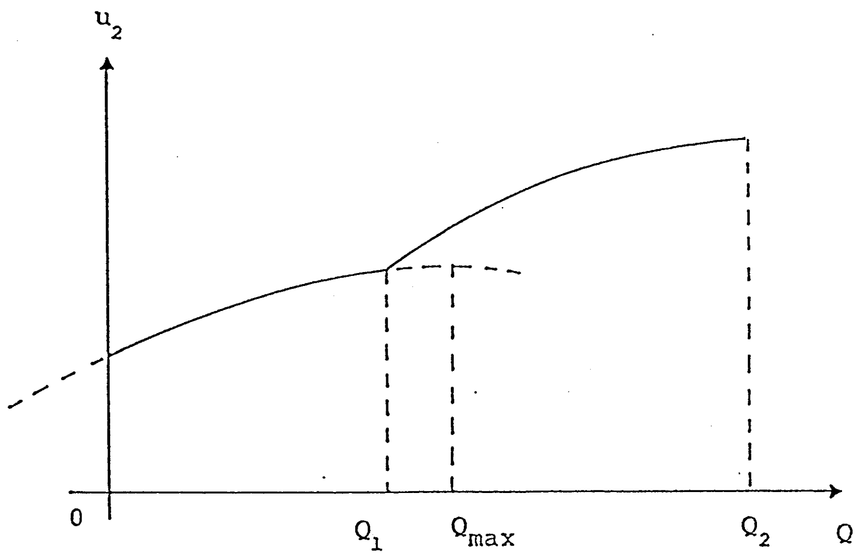

FIGURE 1

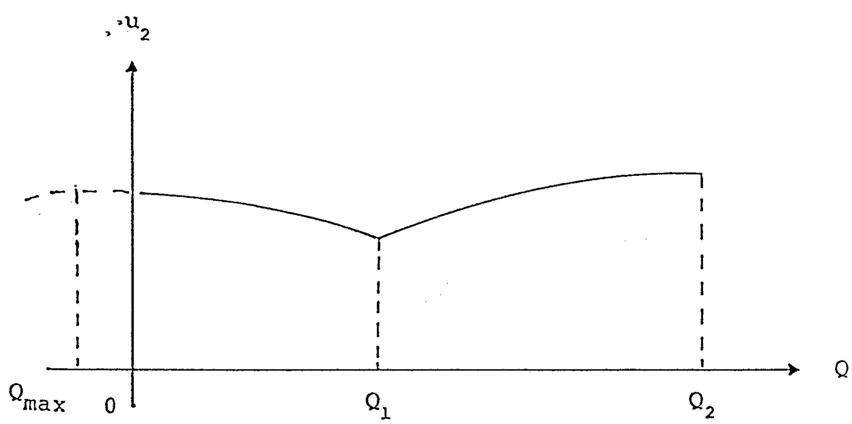

FIGURE 2

Remark 5: The economic interpretation of the inequalities above can be given in two ways:

– for a given α (level of unemployment benefit), π (the relative price of imported versus local textile) has to be small enough for the positive price effect to be larger than the negative income effect and so for u_2 to increase with q;

– since the different bounds on π can easily be seen to be decreasing in α, the larger is the unemployment benefit, the smaller has to be the relative price for u_2 to increase with q.

Remark 6: The main interest of Proposition 3 of course lies in the dynamic policies that it shows to be possible. In the situation of figure 1, the authorities of N can choose a succession q_t of quotas leading from 0

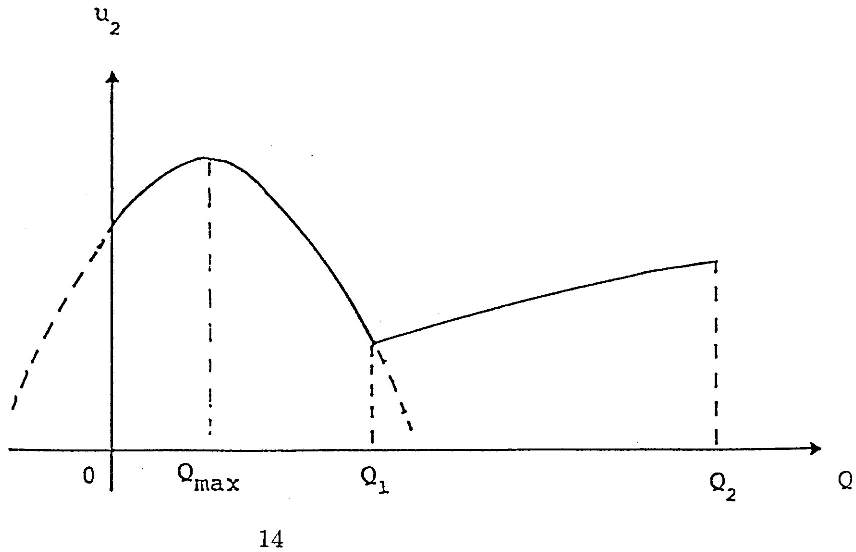

FIGURE 3

to q_2 (i.e. from autarky to free trade) with at each step a majority assent. In case of figure 2 at the opposite, no progressive opening of the economy will be possible while in figure 3 a majority veto will appear after $q_{\max}$ is reached. In the last two cases though, if (30) is satisfied, a direct jump from 0 to q_2 would still appear to be acceptable! Proposition 3 thus appears as particularly interesting to illustrate the traditional debate between gradualists and adepts of a "bing bang" liberalization.

The common conclusion from Sections 4 and 5 however is that for various reasons – a majority increase in unemployment or a negative income effect more important than the positive price effect – the question of acceptability of free trade is related to complex inequalities relating the parameters of the economy and no a priori general simple policy rule can be given which would guarantee for sure a majority support.

We are thus going to consider in Section 6 a more subtle approach allowing unskilled workers to become, after some time, skilled workers. We shall see that then, a much more simple conclusion can he obtained.

6 A Dynamic Training Process for Acceptability

Instead of considering only, as up to now, q-equilibria with perfect mobility or perfect rigidity in the instant, we shall introduce here a training process defined through the following assumption:

Assumption 5 (training conditions)

There exists in N a training system allowing unskilled workers to become skilled workers, system with the following two characteristics:

(i) the training period has a definite duration that we shall choose as our unit of time;

(ii) during a training period, at most the fraction $\frac{\ell_1}{n}$ (n integer greater than 2) of the unskilled workers can be trained.

From this starting point, we shall now build the following discrete dynamics for $t \in \mathbb{N}$:

– at time t the authorities of N allow for the introduction of a quota $q(t)$ of textile given by

$$q(t) = \frac{q_1^M}{m}\, t \tag{31}$$

where m is a positive integer and q_1^M is given by (22); clearly $t = 0$ corresponds to the autarkic situation;

– at time $t \geq 2$ the skilled active population is given by:

$$\lambda_2(t) = \lambda_2(t-1) + [\lambda_1(t-2) - \lambda_1(t-1)] \tag{32}$$

i.e. it is the sum of the skilled active population at time $t-1$ and of the number of unemployed unskilled workers created between $t-2$ and $t-1$ by the increase in the allowed quota, workers who, at time t and thanks to (i) of assumption 5 can now occupy a skilled position (this supposes also that (ii) is satisfied; a condition for that will be given later on);

– at time t again, the unskilled active population is then defined by equation (17) characterizing q-equilibria, i.e. using (31):

$$(\mu - 1)\, w_1\, \lambda_1(t) + \mu\, w_2\, \lambda_2(t) = \mu\, w_2\, \ell\, \frac{t}{m} \tag{33}$$

– the dynamics is then completely defined with the two initial conditions $\lambda_2(0) = \lambda_2(1) = \ell_2$; indeed then from (18) and (33) $\lambda_1(0) = \ell_1$,

$$\lambda_1(1) = \ell_1 - b \tag{34}$$

with $b = \frac{\ell_1\, \ell}{\ell_2\, m}$ so that the double recurrence defined by (32) and (33) can unwind in an obviously unique way;

– however a last specification will be introduced later on so as to express the natural constraints that

$$\forall\, i,\, t \quad 0 \leq \lambda_i(t) \leq \ell \tag{35}$$

In ordinary language, the dynamics can then be described as follows. At time 0 is autarky. At $t = 1$ the introduction of a volume $\frac{q_1^M}{m}$ of imported textile creates a level b of unemployment for unskilled workers. At $t = 2$, unemployed workers of $t = 1$, after training, find a skilled job but new unemployed unskilled workers appear because now a volume $\frac{2\, q_1^M}{m}$

of textile is entering into N. And so on until the bounds defined by (35) are possibly hit.

Now the double recurrence for λ_1 and λ_2 can be solved explicitely. A mixture of calculation and intuition indeed gives:

$$(36)\qquad \begin{cases} \lambda_1(t) = \ell_1 - b \sum_{\tau=0}^{t-1} (-1)^\tau (t-\tau) \left(\frac{\ell_1}{\ell_2}\right)^\tau & (t \geq 1) \\ \lambda_2(t) = \ell_2 + b \sum_{\tau=0}^{t-2} (-1)^\tau (t-\tau-1) \left(\frac{\ell_1}{\ell_2}\right)^\tau & (t \geq 2) \end{cases}$$

(insertion of the λ_i into (32) and (33) allows to check (36)).

We can then prove (see Annex 2):

> LEMMA 2: If $\ell_2 > \ell_1$ the level λ_1 of active unskilled population is a strictly decreasing function of t. The condition $m \geq \frac{\ell}{\ell_2} n$ is sufficient for (ii) in assumption 5 to be satisfied for all t. Then at time $m - 1\, \lambda_1(m-1) > 0$ and at time $m\, \lambda_1(m) < 0$.

Respect of (35) is now an easy task. Indeed from Lemma 2 it is first sufficient to define $\lambda_1(t)$ by (36) until $t = m - 1$ and then by:

$$(37)\qquad \lambda_1(t) = 0 \qquad \text{for} \quad t \geq m$$

Then, as adding equations (32) for times between 2 and t gives:

$$(38)\qquad \lambda_2(t) = \ell - \lambda_1(t-1)$$

(which by the way proves that λ_2 is an inereasing function of t if $\ell_2 > \ell_1$) we can define $\lambda_2(t)$ by (36) until $t = m$ and then by

$$(39)\qquad \lambda_2(t) = \ell \qquad \text{for} \quad t \geq m+1$$

The fact that our dynamics is finite and not infinite cannot appear as a surprise: intuitively the fact that $\lambda_1(t) = 0$ for $t \geq m$ comes from the fact that the authorized quota of textile is then equal to or greater than q_1^M where local production vanishes (the delay for λ_2 to adjust to ℓ of course comes from the duration of training). Our process in fact converges, after $m+1$ periods of time, to a q-equilibrium with full mobility where all unskilled workers have been trained.

To study then the acceptability properties of this dynamics, we shall now calculate the income distribution which, at each t, is generated by the pattern of activity above:

– first, there is a (positive from Lemma 2) number $\lambda_1(t-1) - \lambda_1(t)$ of unemployed unskilled workers with, from assumption (3) an income equal to:

$$w_1^u(\alpha) = \alpha\, w_1$$

The necessary volume of solidarity tax is thus:

$$I_S(\alpha, t) = [\lambda_1(t-1) - \lambda_1(t)]\, \alpha\, w_1 \leq \alpha\, b w_1 \tag{40}$$

(the inequality being a consequence of (A11);

– the resources of the $\lambda_1(t)$ active workers are then, from assumption 4:

$$w_1(\alpha, t) = w_1 - \frac{w_1}{W(t)}\, I^S(\alpha, t) \tag{41}$$

where the total wages $W(t)$ are now:

$$W(t) = \lambda_1(t)\, w_1 + \lambda_2(t)\, w_2 \tag{42}$$

– the resources of the $\lambda_2(t)$ skilled workers are similarly:

$$w_2(\alpha, t) = w_2 - \frac{w_2}{W(t)}\, I^S(\alpha, t) \tag{43}$$

Of course, the new income distribution keeps $W(t)$ as its sum but $W(t)$ is now a rather complex function of t. Still we can obtain:

PROPOSITION 4: Let assumptions 1 to 5 be satisfied. Suppose $\ell_2 > \ell_1$. Consider the training dynamics defined by (32), (33), (37) and (39). Then one has for $0 \leq q_t,\, q_{t-1} \leq q_2^M$ and m large enough:

- if $\pi < 1 - \frac{\alpha}{1-\mu+\alpha\mu}\; S(q_{t+1}) \succ S(q_t)\; \forall t$
- if $\pi > 1 - \frac{\alpha}{1-\mu+\alpha\mu}\; S(q_1) \prec S(0)$ but
  . $\exists T \in [2, m]$ such that:
$$t \geq T \text{ implies } S(q_{t+1}) \succ S(q_t)$$
  . $\exists T' > T$ such that:
$$t \geq T' \text{ implies } S(q_t) \succ S(0)$$

Proof: Again $\ell_2 > \ell_1$ means that we can concentrate on skilled workers. The first remark then is that for $t < m$ (i.e. $\lambda_1(t) > 0$), their consumption function remains given by (12), where w_2 is now defined by (43) and (40) and where q is given by (31).

Let us then consider

$$u_2(\alpha, t) \equiv u\left[c_i\left(p_i,\, p_1',\, \frac{q_1^M}{m}\, t,\, w_2(\alpha, t)\right)\right]$$

The variation in t of this function will define the properties of our aggregate preference $\succ$.

Now for $t = 1$, as $\lambda_2(1) = \ell_2$ and $\lambda_1(1)$ is given by (33), $u_2(\alpha, 1)$ coincides with the value $u_2\left[\frac{q_1^M}{m}\right]$ of the function $u_2(q)$ considered in the proof of Proposition 3 and in Annex 1. Thus, from Proposition 3, if m is large enough for $u_2\left[\frac{q_1^M}{m}\right]$ to be smaller then $q_{\max}$, one has well $S(q_1) \gtrless S(0)$ according to the values of π considered in Proposition 4.

Then for higher values of t still below m, one gets a formula similar to (A2) namely:

$$u_2(\alpha, t) = AW_2 \left[1 - \frac{I_S(\alpha, t)}{W(t)}\right] \left[1 + (p_1 - p_1')\,\frac{q_1^M\, t}{m\, W(t)}\right] \tag{44}$$

The first bracket has no obvious variation with t but, using (40) and (A11) and (A14) in Annex 2, one can see that it has as lowest value:

$$1 - \frac{I_S(\alpha, 1)}{W(1)}$$

(taking the lowest value of W and the highest value of I_S). On the other hand, first, from (A10) and (A14) again, for m large and $t\,(< m)$ large enough also, the first bracket goes to a limit which can be made as near from 1 as one wishes; next, a tedious but straightforward calculation shows that, for m large enough, the second bracket is a strictly increasing function of t (see Annex 2, (A15) and the following discussion). Thus $\exists\, T \in [2, m]$ such that:

$$u_2(\alpha, t+1) > u_2(\alpha, t) \quad \forall\, t > T$$

Then, for $t \geq m$ (which means $q \geq q_1^M$) W and w_2 remain constant and the results of the second part of Annex 1 apply.

It remains thus to be proven that, if $u_2(\alpha, 1) < u_2(\alpha, 0)$, $\exists\, T' > T$ such that $u_2(\alpha, T') > u_2(\alpha, 0)$. But from (44), this result can always be obtained whatever $p_1 - p_1'$ or π is, just by choosing m suitably large enough again. □

7 Comments

The model presented in the paper, despite its (relative) simplicity, already allows to single out and clarify some important elements discussed in international trade theory and in current trade negotiations.

Proposition 2 of course is without surprise when it shows that, so long skilled workers with no risk of becoming unemployed are the decisive majority, any opening of the economy, including free trade, is prefered to autarky. But it also shows that, if there is a risk of a majority of unemployed (which supposes of course the existence of some rigidities, here in wages) there appears a limit degree of opening, in the paper in the form of a maximum quota. This quota then can be found out through dynamics of progressive opening or tatonnement, including a sequence of votes.

Proposition 3 sticks further to reality by considering a rule which defines how to share the burden of possible unemployment between active workers. Then, all possible situations appear, even with a majority of skilled workers. According to the characteristics of the situation (in particular the importance of the price gap for textile between North and South) and according to the policy chosen (the level of the unemployment benefit paid by active workers) dynamics of progressive opening or tatonnement can appear to be or not useful to obtain the limit quota; sometimes they are helpful to reach it gradually with majority support, sometimes a big bang change from autarky to free trade would be a better solution. But anyhow, in a large variety of situations, free trade appears not to be acceptable from a majority point of view.

This gives of course all it interest to the final and unambiguous Proposition 4. There it is no more dynamics of tatonnement but real dynamics of training of manpower which are considered. And it thus appears that, in the worst case, if political authorities are able to go through a first period of time without being blocked by the feeling of a majority (of course preferably by using conviction: "let us try once more" than through dictatorship !) then, after a suitably chosen delay, one can obtain in the end both free trade and unanimity support for it.

Without denying of course that our model is far from the complexity of actual economic world, it gives evidence of situations where taking time to achieve structural changes can be of real interest, and in fact can be a necessity to move towards a politically stable free trade situation. This example thus gives an interesting support to those who consider that trade agreements introducing mutually agreed time delays and paths for a progressive opening of economies and liberalization can have some sound theoretical justifications.

ANNEX 1

Study of $u\,(q,\,w)$

1. For $0 \leq q \leq q_1$

Then demands are given by (12), which can be rewritten using (6) and (7)

$$c_{\cdot}\,(p_i,\,p'_1,\,q,\,w) = c_{\cdot}\,(p_i,\,w)\,[1 + (p_1 - p'_1)\,d^{\cdot}\,(q)]$$

From the definition (5) of u one then has:

$$\text{(A1)} \qquad u\,(c_1,\,c_2) \equiv u\,(q,\,w) = \frac{\mu^{\mu}\,(1-\mu)^{1-\mu}}{p_1^{\mu}\,p_2^{1-\mu}}\,w\,[1 + (p_1 - p'_1)\,d^{\cdot}\,(q)]$$

If w is constant (to $w_1,\,w_2,\,\alpha\,w_1, \ldots$), u as a function of q has the same properties as $d^{\cdot}$ i.e. is strictly increasing.

In case w is given by (28) or (29) (case of q-equilibria with perfect rigidity), using these formula, (21) and the definition (27) of $W\,(q)$ one has (with $A = \frac{\mu^{\mu}\,(1-\mu)^{1-\mu}}{p_1^{\mu}\,p_2^{1-\mu}}$).

$$u_j\,(q) \equiv u\,(q,\,w_j\,(\alpha,\,q)) = A\,w_j \left[1 - \frac{\alpha\,w_1\,(\ell_1 - \lambda_1)}{W\,(q)}\right] \times \left[1 + \frac{(p_1 - p'_1)\,q\,(1-\mu)}{\ell_2\,w_2 - \varrho\,p_1\,q}\right]$$

$$\text{(A2)} \qquad u_j\,(q) = A\,w_j \left[1 - \frac{\alpha\,\varrho\,p_1\,q}{(1-\mu)\,W\,(q)}\right] \left[1 + \frac{(p_1 - p'_1)\,q}{W\,(q)}\right]$$

(since $\ell_2\,w_2 - \varrho\,p_1\,q = (1-\mu)\,W\,(q)$).

Defining $Q = \frac{q}{W\,(q)}$ one has

$$u_j\,(q) = A\,w_j \left[1 - \frac{\alpha\,\varrho\,p_1}{1-\mu}\,Q\right] \quad [1 + (p_1 - p'_1)\,Q]$$

which is clearly a parabola in Q with a maximum in

$$\text{(A3)} \qquad Q_{\max} = \frac{1}{2}\left[\frac{1-\mu}{\alpha\,\varrho\,p_1} - \frac{1}{(p_1 - p'_1)}\right]$$

Now using the definition of W again one has easily

$$\text{(A4)} \qquad q = \frac{\ell_2\,w_2\,Q}{1 - \mu + \varrho\,p_1\,Q}$$

so that $Q_{\max}$ also defines a maximum $q_{\max}$ of u in q because q is clearly an increasing function of Q.

The last question of course is to know when $q_{\max}$ belongs to $[0,\, q_1^R]$ i.e. $Q_{\max}$ belongs to $[0,\, Q_1^R]$ where $Q_1^R = \frac{q_1^R}{\ell_2\, w_2} = \frac{\mu}{\varrho\, p_1}$. A direct calculation then gives:

(A5) $$\begin{cases} Q_{\max} > 0 & \Leftrightarrow \quad \pi < 1 - \frac{\alpha}{1-\mu+\alpha\mu} \\ Q_{\max} < Q_1^R & \Leftrightarrow \quad \pi - \frac{\alpha}{1-\mu-\alpha\mu} \end{cases}$$

2. For $q_1 \leq q \leq q_2$

Then demands are given by (15), i.e. with (8) and (ii) again:

$$c_1\,(p_i,\, p_1',\, q,\, w) = w d^{\cdot}\,(q)$$
$$c_2\,(p_i,\, p_1',\, q,\, w) = \frac{w}{p_2}\,[1 - p_1'\, d^{\cdot}\,(q)]$$

so that

(A6) $$u\,(c_1,\, c_2) \equiv u\,(q,\, w) = \frac{w}{p_2^{1-\mu}}\, d^{\cdot}\,(q)^{\mu}\,[1 - p_1'\, d^{\cdot}\,(q)]^{1-\mu}$$

In this domain, all possible incomes are independant of q (since $\lambda_1 = 0$ and W equals w_2 multiplied by the constant active population). Thus u as a function of q behaves as:

$$d^{\cdot}\,(q)^{\mu}\,[1 - p_1'\, d^{\cdot}\,(q)]^{1-\mu}$$

where $d^{\cdot}$ is proportional to q.

Taking the derivative in q of the logarithm of the quantity above one gets:

$$\mu\,\frac{d^{\cdot\prime}\,(q)}{d^{\cdot}\,(q)} - (1-\mu)\, p_1'\,\frac{d^{\cdot\prime}\,(q)}{1 - p_1'\, d^{\cdot}\,(q)}$$

which has the same sign as

$$\mu\,[1 - p_1'\, d^{\cdot}\,(q)] - (1-\mu)\, p_1'\, d^{\cdot}\,(q) = \mu\,[1 - p_1'\, d^{\cdot}\,(q)] > 0$$

since

(A7) $$d^{\cdot}\,(q) \leq d^{\cdot}\,(q_2) = \frac{\mu}{p_1'}$$

from (22) or (23).

ANNEX 2

The Dynamics of Trained Population

Our point of departure in this annex is (36):

$$(36)\qquad \begin{cases} \lambda_1(t) = \ell_1 - b\sum_{\tau=0}^{t-1}(-1)^\tau (t-\tau)\left[\frac{\ell_1}{\ell_2}\right]^\tau & (t \geq 1) \\ \lambda_2(t) = \ell_2 + b\sum_{\tau=0}^{t-2}(-1)^\tau (t-\tau-1)\left[\frac{\ell_1}{\ell_2}\right]^\tau & (t \geq 2) \end{cases}$$

Then one has:

$$(A8)\qquad \lambda_1(t-1) - \lambda_1(t) = b\sum_{0}^{t-1}\left[-\frac{\ell_1}{\ell_2}\right]^\tau$$

Using the identity

$$(A9)\qquad \sum_{0}^{n-1} x^i = \frac{1-x^n}{1-x}$$

we get:

$$(A10)\qquad \lambda_1(t-1) - \lambda_1(t) = b\,\frac{1-\left[-\frac{\ell_1}{\ell_2}\right]^t}{1+\frac{\ell_1}{\ell_2}}$$

If $\ell_1 < \ell_2$ then $\forall t \geq 1$

$$(A11)\qquad 0 < \lambda_1(t-1) - \lambda_1(t) \leq b$$

which implies that λ_1 is a strictly decreasing function of t, and that $m \geq \frac{\ell}{\ell_2}n$ is sufficient for (ii) in assumption 5 to be satisfied for all t. Adding equations (32) for times between 2 and t then gives:

$$(A12)\qquad \lambda_2(t) = \ell - \lambda_1(t-1)$$

which means that λ_2 is itself a strictly increasing function of t.

Now using the derivative of identity (A9) for $n+1$:

$$\sum_{1}^{n} ix^{i-1} = \frac{nx^{n+1} - (n+1)\,x^{n+1}}{(1-x)^2}$$

one can, after some calculation, give to the two sums of (36) the more compact expression:

$$(A13)\qquad \begin{cases} \lambda_1(t) = \ell_1 - \frac{b}{\ell^2}\left[t\ell_2^2 + (t+1)\,\ell_1\ell_2 + \ell_1^2\left[-\frac{\ell_1}{\ell_2}\right]^{t-1}\right] \\ \lambda_2(t) = \ell_2 - \frac{b}{\ell^2}\left[(t-1)\,\ell_2^2 + t\ell_1\ell_2 + \ell_1^2\left[-\frac{\ell_1}{\ell_2}\right]^{t-2}\right] \end{cases}$$

A direct calculation, using (34) again, gives $\lambda_1(m-1) > 0$ and $\lambda_1(m) < 0$ which finishes the proof of Lemma 2. We are then in position to derive a series of inequalities which will be useful to prove Proposition 4.

First, using (A12) and (A10) one has

$$A(t) = \lambda_1(t) + \lambda_2(t) = \ell - [\lambda_1(t-1) - \lambda_1(t)]$$
$$= \ell - b\,\frac{1 - [1 - \frac{\ell_1}{\ell_2}]^t}{1 + \frac{\ell_1}{\ell_2}}$$

Then one sees that for t odd $A(t)$ is increasing with t from $A(1) = \ell - b$ while for t even $A(t)$ is decreasing with t from $A(0) = \ell$, both in the limit going to $l - b\,\frac{\ell_2}{\ell} = \ell - \frac{\ell_1}{m}$. Thus, t being any odd value

$$t' > t \quad \Rightarrow \quad \ell > A(t') > A(t) \geq \ell - b$$

Next let us consider $W(t) = \lambda_1(t)\,w_1 + \lambda_2(t)\,w_2$ One has

$$W(t) = A(t)\,w_1 + \lambda_2(t)\,(w_2 - w_1)$$

As λ_2 is strictly increasing in t and smaller than ℓ, t being any odd value:

(A14) $$t' > t \quad \Rightarrow \quad \ell w_2 > W(t') > W(t) \geq W(1) = W(0) - bw_1$$

Last, let us consider the variation with t of the rationing scheme d defined by (8), which is now

$$d(t) = \frac{q_1^M\,t}{m\,W(t)}$$

One sees easily that the sign of $d(t+1) - d(t)$ is the sign of:

$$D(t) = (t+1)\,W(t) - t\,W(t+1)$$

A lengthy but straightforward calculation, using (A10), (A13) and (34) then leads to:

(A15) $$D(t) = W(0)\left[1 - \frac{\ell_1}{\ell m}\right] + \frac{\ell_1\,W(0)}{\ell m}\left[-\frac{\ell_1}{\ell_2}\right]^t \left[1 + \frac{\ell t}{\ell_2}\right]$$

The first term of the sum is positive. The second term is obviously positive for t even. For t odd, forgetting about $(-1)^t$, it is proportional to:

$$\left[1 + \frac{\ell t}{\ell_2}\right] e^{tLn\,\frac{\ell_1}{\ell_2}}$$

As $\ell_1 < \ell_2$ the logarithm is negative so that the function has a maximum somewhere in t. Choosing m large enough in (A15) with respect to this maximum then allows to have $D(t) > 0\ \forall\, t$ which means that $D(t)$ is strictly increasing in t.

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