

Endogenous Differentiation Strategies, Comparative Advantage and the Volume of Trade

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ABSTRACT. – We present a trade model in which producer differentiation strategies are endogenous. Firms can influence the brand image of their products through a trade-off between cost and product quality. Comparative advantage depends on both variable costs and the ratio of perceived product quality to total costs. Firms sell either a small range of standard varieties or a large range of more expensive high-quality varieties. Empirical trade equations including differentiation variables are derived from this model.

Stratégies de différenciation endogènes, avantages comparatifs et volume du commerce international

RÉSUMÉ. – On présente un modèle d'échanges avec stratégies de différenciation des producteurs endogènes. Les firmes influencent l'image de marque de leurs produits par un arbitrage entre coûts et qualité. Les avantages comparatifs dépendent des coûts variables et du rapport coût/qualité des produits. Les producteurs vendent soit des produits standardisés peu chers, soit un large éventail de produits moins économiques mais de qualité supérieure. Des équations d'échanges intégrant des effets de différenciation découlent du modèle.

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1 Introduction

Several empirical studies underline the role played by product differentiation in comparative advantage and export performances of industrial countries¹. However, as product differentiation is generally unobserved, empirical studies have to use *proxies* of differentiation, which are usually based on investment, R and D expenses, industrial production or miscellaneous dispersion indicators. Unfortunately, some of these *proxies* seem somewhat different from what they are supposed to represent. Above all, most of them lack real theoretical microfoundations, and this in turn casts doubt on the results of analyses based upon them.

Yet some microfoundations for this type of empirical work should be found in imperfect competition trade models, which shed light on new sources of trade and comparative advantage. These models do, however, require further development if they are to be more directly operational for empirical work. The DIXIT-STIGLITZ [1977] monopolistic competition framework extended by KRUGMAN [1980] and HELPMAN and KRUGMAN [1985] to the explanation of intra-industry trade seems an interesting analytical tool in this respect due to its appealing tractability. This model nonetheless lacks certain features that could make it an operational basis for empirical work. More particularly, deriving trade equations requires geographically differentiated products (so that separate import demand functions can be identified) and therefore market segmentation, as well as differences across countries (in order to be able to isolate certain factors of comparative advantage), i.e. three features that are not modelised in the Dixit-Stiglitz-Krugman (hereafter DSK) framework. Allowing for country differences and their relation to market segmentation is therefore a possible improvement explored in this paper, as well as the possibility of multi-product firms, incorporating other dimensions of product differentiation strategies (including geographical product differentiation).

More precisely, we present a theoretical model of trade with differentiated products that extends the DSK framework in two directions. First, it introduces endogenous horizontal differentiation strategies for individual multi-product firms. The second original feature of our model is that it partly endogenises the source of comparative advantage by allowing firms to influence the brand image of their products (through a trade-off between cost and quality). We show that endogenising differentiation strategies in these two ways allows for better microfoundations in imperfect competition trade models and establishes more direct links with some of the usual *proxies* of product differentiation.

1. Cf. BISMUT and OLIVEIRA-MARTINS [1989], the FKSEC macroeconomic model of the Central Planning Bureau of the Netherlands [1990], OLIVEIRA-MARTINS [1992], ERKEL-ROUSSE [1992, 1993], BROOKS [1993], MAGNIER and TOUJAS-BERNATE [1993], OWEN and WREN-LEWIS [1993], FONTAGNÉ, FREUDENBERG and PÉRIDY [1997], and ERKEL-ROUSSE, GAULIER and PAJOT [1997], among others.

It is worth noting that justifying trade equations with differentiation effects would in principle require a general equilibrium framework. However, our model deals with partial equilibrium only. Moreover, it is based on monopolistic competition, which can also be criticised. These two simplifications are also made in the DSK framework. Like DIXIT and STIGLITZ [1977], and KRUGMAN [1980], we think that interesting results can be derived from very simple frameworks as a first approximation. This is why we decided to make only minimal modifications to the DSK framework, so as to address the issue of the microfoundations of empirical work with differentiation effects. At a second stage, we might be tempted both to make more sophisticated assumptions about market structure and to endogenise factor markets. Such alternative hypotheses would lead to a notably more complex model, which would have to be calibrated and computed. This might be an interesting path of future research. However, the present paper is a more modest and preliminary attempt to improve the foundations of recent empirical work on trade.

Section 2 presents the basic version of the model with endogenous differentiation strategies. Each national firm decides on the number of products and export markets, given consumer preferences which are characterized, in particular, by the perceived brand image of each product. Section 3 analyses the two sources of comparative advantage in this model: the relative variable costs and the differences in ratios of costs to the perceived quality of products. Then, in section 4, comparative advantage is partly endogenised through the strategic choice between low price and high quality. Finally, in the last section, we linearise the model in order to derive trade equations. We show that these equations give an *a posteriori* justification to some of the previous empirical attempts to include differentiation *proxies* in trade equations.

2 The Basic Model

2.1. The Main Assumptions about Supply and Demand

Assume there are two countries, one ‘domestic’ and the other ‘foreign’, and K (small) sectors of differentiated goods. There is no migration of production factors from one country to the other. More particularly, the labour force is immobile and predetermined, as are consumers. Factors endowments and technologies may be different across countries. In order to simplify the specification of the model, factor markets are exogenous, therefore all the results below are only valid in a partial equilibrium context.

Positive fixed costs lead to increasing returns to scale. Therefore, a given variety of good k is assumed (as usual) to be produced by only one firm. However, contrary to usual models of monopolistic competition, ours does not assume that any firm systematically produces only one variety of good. The number of varieties per firm is the result of an optimisation program.

Nonetheless, differentiation cost is assumed to be high enough to limit the level of individual differentiation.

Each sector k in country i is characterised by F_{ki} firms. The latter produce at the conditions of production prevailing in country i . They can sell their varieties of good on both domestic and foreign markets at different prices notably because of transportation costs (segmented markets). The total cost function of a representative firm in sector k and country 1 is:

$$C_{k1} = \mathcal{F}_{k1} + d_{k1} V_{k1}^2 + c_{k1} \sum_{v=1}^{V_{k1}} [y_{kv11} + (1 + t_k) y_{kv12}]$$

and symmetrically for country 2

where $y_{kvi j}$ is the quantity of variety v produced by the representative firm of country i in sector k and exported to country j ; \mathcal{F}_{ki} are fixed costs, c_{ki} variable costs by unit of product (both supposedly depending on technology of country i in sector k only); V_{ki} is the number of varieties produced by the representative firm of country i in sector k ; $d_{ki} V_{ki}^2$ is the cost of horizontal differentiation ², where d_{ki} is supposed to be high enough so that no firm ever produces a significant part of total production; t_k is the transportation cost, which is supposedly the same for every exported variety of good k . Transportation costs are thus equivalent to the destruction of some output along the way (“iceberg” representation).

The number of domestic and foreign firms by sector is assumed to be very large and the substitutability between varieties high enough for competition to be strong. Consequently, no firm ever gets a large market share and, assuming no entry barriers, no firm can durably make a strictly positive profit. Therefore, market structure does remain close to monopolistic competition with a large number of firms. However, technological gaps and production cost differentials between countries may lead to differences between domestic and foreign varieties beyond pure horizontal differentiation. Those are reflected in the demand side of the model.

In a given country j ($j = 1, 2$), there are N_j identical consumers (with same tastes and same revenues ³). Across countries, consumers are assumed

2. This quadratic form is the only unusual feature of this cost function. One could very easily replace this function with a more general formulation of the differentiation cost, such as: $d_{ki} V_{ki}^{\delta_{ki}}$, with $\delta_{ki} \geq 2$, without qualitatively modifying our results. On the contrary, notice that the $\delta_{ki} = 1$ case would not be interesting here in so far as it would become equivalent to considering F_{ki} multi-product firms producing V_{ki} varieties or $F_{ki} V_{ki}$ firms producing only one variety. The quadratic form of the horizontal differentiation cost (or equivalently the more general $d_{ki} V_{ki}^{\delta_{ki}}$ form) is therefore important because it makes the assumption of multi-product firms effective. It also differentiates our model from the DSK framework (which corresponds to the $\delta_{ki} = 1$ case).

3. In other words, we consider an average consumer whose demand represents that of domestic consumers having identical preferences, but possibly different levels of revenue.

to have identical tastes, but they may differ by their revenue levels ⁴. The representative consumer's utility function in country j is:

$$U_j = \sum_{k=1}^K \xi_k \text{Log}(u_{kj}), \quad \text{where (normalisation): } \sum_{k=1}^K \xi_k = 1$$

u_{kj} refers to the sub-utility derived from the consumption of good k . Parameters $(\xi_k)_{k=1,\dots,K}$ reflect relative preferences for each good. Each sub-utility u_{kj} is assumed to be a CES function of the quantities $x_{k\nu j}$ of consumed varieties ν of good k :

$$u_{kj} = \left[\sum_{\nu=1}^{\mathcal{V}_{kj}} \alpha_{k\nu} x_{k\nu j}^{\frac{\sigma_k-1}{\sigma_k}} \right]^{\frac{\sigma_k}{\sigma_k-1}}$$

where \mathcal{V}_{kj} represents the total number of varieties available on market (k, j) , and $\sigma_k > 1$ the elasticity of substitution between the different varieties of good k . Index ν represents a given variety ν produced in country i ($\nu \equiv (v, i)$). As in the DSK model, variety is valued *per se*. Domestic and foreign varieties have different prices because of transportation costs, but also because of technological gaps and production cost differentials between the two countries. Apart from prices, domestic and foreign varieties also differ by some perceived characteristics resulting from national differences. These differences determine the relative “desirability” of domestic and foreign varieties (i.e. the $(\alpha_{k\nu})_{\nu=1,\dots,\mathcal{V}_{kj}}$). This can be viewed as a sort of “brand image” of each national firm. At this stage, each $(\alpha_{k\nu})_{\nu=1,\dots,\mathcal{V}_{kj}}$ is exogenous ⁵. These coefficients are normalised so that: $\sum_{\nu=1}^{\mathcal{V}_{kj}} \alpha_{k\nu}^{\sigma_k} = \mathcal{V}_{kj}$. This normalisation is a direct generalisation of the DIXIT-STIGLITZ utility function, where all utility weights are equal to one. This point is crucial

4. This assumption may seem quite extreme at first sight. Obviously, it could not hold if we intended to modelise trade across very different countries. However, in such a case, the traditional Heckscher-Ohlin model would be the appropriate framework to consider. Here, we are implicitly interested in trade between industrialised countries having relatively similar (even though not strictly identical) levels of development and economic structure. We know for instance that on the one hand French manufactured trade is conducted for the most part with other European countries, while on the other hand the import demand structures of European countries are very much alike. The assumption of identical preferences of consumers across countries seems to be an acceptable approximation in such a context. Moreover, the fact that consumers do not prefer *a priori* national products to foreign ones, everything else being equal, seems a reasonable assumption, especially (but not only) as far as European consumer behaviour is concerned.

5. This assumption looks acceptable in the short run. In the middle run, one feels that national firms can influence consumers' opinions about their brand images, by improving the quality of their varieties, as well as through advertising and promotion. That is why the assumption of exogeneity will be dropped in section 4.

and deserves to be noted. Indeed, the normalisation to a constant (say, to one), would not be compatible with the exogeneity of the α terms ⁶.

As was mentioned above, within a given country and a given sector, firms face the same cost function. Therefore, as will be shown more precisely in section 4, the varieties produced by these firms can only be differentiated horizontally, which means: $\alpha_{k\nu} \equiv \alpha_{ki}$. In other terms, the brand image of domestic (resp. foreign) varieties coincides with the brand image of the representative firm of the domestic (resp. the foreign) country. These coefficients represent some sort of “made in” labels perceived in consumer preferences as indicators of relative “quality” of varieties.

It is worth noting that this type of differentiation does not correspond to the pure vertical differentiation case, as in GABSZEWICZ, SHAKED, SUTTON and THISSE [1981] ⁷. However, adding weights such as the α terms in Dixit-Stiglitz utility functions enables the incorporation of another dimension for product differentiation. FEENSTRA, TZU-HAN and HAMILTON [1993] developed a similar demand framework, but without relating the α weights to the endogenisation of the product differentiation of firms ⁸. Finally, our utility function differs from the Armington-DSK formulation of BISMUT and OLIVEIRA MARTINS [1989]. In fact, in our model, the place of production is not distinguished in the intermediate level of the utility function. As we are particularly interested in analysing trade across similar industrialised countries, whose products may not differ radically, we consider that place of production cannot be treated as a relevant criterion of preference level *per se*.

2.2. Consumers’ Optimum

Consumers’ optimum is solved in two stages. Maximising the first level of utility determines the share R_{kj} of total revenue R_j^T attributed to the consumption of product k in country j : $R_{kj} = p_{kj} y_{kj} = \xi_k R_j^T$, where y_{kj} and p_{kj} denote respectively the total demand for composite product k expressed in physical quantity and its price. Following HICKMAN and LAU [1973], and taking our normalisation of the α terms into account, we have:

$$y_{kj} \stackrel{\text{def.}}{=} N_j \left[\sum_{\nu=1}^{\nu_{kj}} \frac{\alpha_{k\nu}}{\nu_{kj}^{1/\sigma_k}} x_{k\nu j}^{\frac{\sigma_k-1}{\sigma_k}} \right]^{\frac{\sigma_k}{\sigma_k-1}} \quad \text{and} \quad p_{kj}^{1-\sigma_k} \stackrel{\text{def.}}{=} \sum_{\nu=1}^{\nu_{kj}} \frac{\alpha_{k\nu}^{\sigma_k}}{\nu_{kj}} p_{k\nu j}^{1-\sigma_k}.$$

6. In other words, it is impossible to have the number of products endogenously determined and both the utility weights $(\alpha_{k\nu}^{\sigma_k})_{\nu=1, \dots, \nu_{kj}}$ and their sum exogenous: one degree of freedom is missing. This point was not explicit in the original Dixit and Stiglitz’s paper.

7. In the pure vertical differentiation case, identical consumers would have a strictly positive demand for only one quality of a differentiated product, depending on their common level of revenue. As in the DSK framework, in our model, identical consumers simultaneously demand all types of varieties – domestic and foreign.

8. Our α terms will be endogenised in section 4 below, while they remain exogenous in Feenstra *et alii*. Besides, from an empirical point of view, our α terms would not be incorporated in estimation residuals as in Feenstra *et al.*

At a second stage, maximising each sub-utility subject to the budget constraint gives the total demand for variety $\nu = (v, i)$ addressed to the producer (in country i) on market j :

$$y_{kvi j} = N_j x_{kvi j} = \left(\frac{\alpha_{ki}^{\sigma_k}}{\mathcal{V}_{kj}} \right) \left(\frac{p_{kvi j}}{p_{kj}} \right)^{-\sigma_k} \frac{R_{kj}}{p_{kj}}$$

Therefore, in country j , the demand for each variety is a decreasing function of its relative price and an increasing function of both its relative brand image and the real national revenue. The higher the elasticities of substitution between varieties, the more sensitive the demands to relative prices and brand images.

Hereafter, when referring exclusively to the k -th sector, the index k will be omitted in order to simplify the notation.

2.3. Producers' Optimum

Prices and the number of produced varieties derive from profit maximisation in country 1:

$$\Pi_1 = \sum_{v=1}^{V_1} [p_{v11} y_{v11} + p_{v12} y_{v12}] - C_1 \quad \text{and symmetrically for country 2.}$$

The solution of the optimum is calculated in two stages. At the first one, profits are maximised with respect to prices, the number of varieties being considered to be fixed. At the second one, profits corresponding to optimal prices are maximised with respect to the number of varieties.

First stage:

For a firm in country i , the first stage maximisation yields the first-order conditions:

$$\frac{\partial \Pi_i}{\partial p_{vij}} = y_{vij} + [p_{vij} - c_i (1 + t_{ij})] \frac{\partial y_{vij}}{\partial p_{vij}}$$

where :

$$y_{vij} = \left(\frac{\alpha_i^\sigma}{\mathcal{V}_j} \right) \left(\frac{p_{vij}}{p_j} \right)^\sigma \frac{R_j}{p_j}, \quad j = 1, 2$$

$$t_{ij} = 0 \Leftrightarrow i = j \quad \text{and} \quad t_{ij} = t \Leftrightarrow i \neq j.$$

Taking account of the usual property of price elasticities in monopolistic competition (large number of firms, and consequently large number of varieties⁹), the j -th first-order condition for variety ν implies either $y_{vij} = 0$ or that variety ν is sold in country j at a quantity and a price which depend

9. The corresponding simplification $\varepsilon_{vij} = -(\frac{\partial y_{vij}}{y_{vij}})/(\frac{\partial p_{vij}}{p_{vij}}) = \sigma - \frac{\sigma-1}{\mathcal{V}_j} (\frac{p_{vij}}{p_j})^{1-\sigma} \approx_{\mathcal{V}_j \rightarrow +\infty} \sigma$ means that, as firms are small, decisions of a single firm about prices cannot influence prices p_j .

strictly on i and j (and not on v itself). More precisely, prices at the optimum are:

$$(1) \quad p_{vij} = c_i (1 + t_{ij}) \frac{\sigma}{\sigma - 1} = p_{ij}$$

Corresponding supplied quantities y_{ii} and y_{ij} result from demand functions.

PROPOSITION 1: It is optimal for every firm to diversify its sales geographically. Consequently, firms choose to sell all their varieties on both markets ($\forall v, i, j, y_{vij} \neq 0$).

Proof: The profit corresponding to optimal prices $(p_{ij})_{j=1,2}$ is given by:

$$\Pi_i(p_{ij}) = \sum_{v=1}^{V_i} \sum_{j=1}^2 \lambda_{vij} - d_i V_i^2 - \mathcal{F}_i$$

where $\forall v = 1, \dots, V_i$, $\lambda_{vij} = 0$ (if variety v is not sold on market j) or $\lambda_{vij} = \mu_{ij} y_{ij}$ (if it is sold on market j), $\mu_{ij} y_{ij}$ being the margin obtained by supplying variety v on market j :

$$\mu_{ij} y_{ij} = (p_{ij} - c_i (1 + t_{ij})) y_{ij} = \Phi_{ij} \psi_j^{-1}$$

with:

$$\Phi_{ij} \stackrel{\text{def.}}{=} \frac{R_j}{\sigma} \alpha_i^\sigma p_{ij}^{1-\sigma} = (c_i (1 + t_{ij}))^{1-\sigma} \left(\frac{\sigma}{\sigma - 1} \right)^{1-\sigma} \alpha_i^\sigma \frac{R_j}{\sigma} > 0,$$

$$\psi_j \stackrel{\text{def.}}{=} \mathcal{V}_j p_j^{1-\sigma} = \sum_{i=1}^2 F_i V_i \alpha_i^\sigma p_{ij}^{1-\sigma} > 0$$

Therefore, $\mu_{ij} y_{ij} > 0$.

As a strictly positive additional margin $\mu_{ij} y_{ij}$ can be obtained by supplying every variety on a new market, the firm chooses to sell all its varieties on both markets. \square

Consequently:

$$\Pi_i(p_{ij}) = V_i \sum_{j=1}^2 \mu_{ij} y_{ij} - d_i V_i^2 - \mathcal{F}_i$$

Proposition 1 expresses how firms take account of consumers' taste for variety in their optimisation programs. In fact, the ability to get a positive margin by supplying any variety on any market originates from the systematic existence of a corresponding potential demand, due to consumer taste for variety (in a simplified world without entry barriers). This property seems consistent with observed diversification strategies of firms in the world economy today, as part of the so-called "globalisation process". It is easy to apply proposition 1 generally to any number of markets (*Cf.* appendix).

Second stage:

The optimal horizontal differentiation strategy of firms is derived from the maximisation of $\Pi_i(p_{ij})$ with respect to V_i . A single firm having a negligible impact on each national market, it is assumed that its decisions cannot modify prices p_j . Therefore:

$$\frac{\partial \Pi_i(p_{ij})}{\partial V_i} = \sum_{j=1}^2 \mu_{ij} y_{ij} - 2 d_i V_i$$

and the first-order condition with respect to V_i is:

$$(2) \quad 2 d_i V_i = \sum_{j=1}^2 \mu_{ij} y_{ij} = \mu_{ii} (y_{ii} + (1+t) y_{ij}) = \mu_{ii} y_i. \Leftrightarrow V_i^2 = \frac{\mu_{ii} Y_i}{2 d_i}$$

with $Y_i = V_i y_i$.

As in KRUGMAN [1980], the number of varieties is an increasing function of the total production Y_i , but here this property is satisfied at the firm level.

2.4. Existence and Unicity of the Zero Profit Equilibrium

The zero profit equilibrium resulting from the first rank optima of consumers and firms is defined by conditions (1), (2) and the zero profit condition^{10, 11}:

$$(3) \quad d_i V_i^2 = \mathcal{F}_i \quad \forall (i, j) \in \{1, \dots, I\}^2, \quad \forall \text{implicit } k \in \{1, \dots, K\}.$$

A generalisation of this system of equations is solved in the *appendix*. Results concerning the basic model are given in tables 1 and 2 below, introducing the following notations (in tables 1 and 2, index k is omitted):

$$c_k = \frac{c_{k1}}{c_{k2}}, \quad f_{ki} = d_{ki} V_{ki} = \sqrt{d_{ki} \mathcal{F}_{ki}}, \quad f_k = \frac{f_{k1}}{f_{k2}},$$

$$\alpha_k = \frac{\alpha_{k1}}{\alpha_{k2}}, \quad T_k = (1 + t_k)^{1-\sigma_k},$$

10. We neglect the difference of profits to zero, due to the fact that both the total number of varieties ($V_i F_i$) and the number of firms (F_i) are integers. These approximations can be used without entailing a strong drawback to our analysis because both numbers are very large. Notice that V_i represents in reality the average number of varieties per firm, which does not have to be an integer.

11. As our discussion focuses on a limited number of small sectors of the world economy, we may omit the macroeconomic condition in terms of which each country has a balanced trade position.

$$r = \frac{R_1}{R_2} = \frac{R_1^T}{R_2^T}, \quad \rho_{ki} = \frac{f_{ki} C_{ki}^{\sigma_k - 1}}{\alpha_{ki}^{\sigma_k}} \quad \text{and} \quad \rho_k = \frac{\rho_{k1}}{\rho_{k2}} = \frac{f_k C_k^{\sigma_k - 1}}{\alpha_k^{\sigma_k}}.$$

If it does exist, the zero profit equilibrium is unique and characterized by intra-industry trade in sector k as long as ρ_k belongs to the following interval ¹², the relative revenue r being given:

$$(4) \quad \rho_k \in]\rho_{km}, \rho_{kM}[$$

$$(4') \quad \Leftrightarrow T_k < r \cdot \varphi_T(\rho_k) < \frac{1}{T_k}$$

$$\text{where:} \quad \rho_{km} = \frac{T_k(r+1)}{rT_k^2+1} > T_k \quad \text{and} \quad \rho_{kM} = \frac{r+T_k^2}{T_k(r+1)} < \frac{1}{T_k}$$

and $\varphi_T(\rho_k) = \frac{1-\rho_k T_k}{\rho_k - T_k}$ is a decreasing function of ρ_k so that $\varphi_T(1) = 1$.

These inequalities show that the equilibrium with intra-industry trade in sector k exists if and only if the two countries are similar enough in terms of revenues and ρ_{ki} ratios (r and ρ_k close to 1), as is the case when the two countries are identical or, alternatively, when r and ρ_k are both high, or both low. Now, according to tables 1 and 2, when ρ_k increases, *ceteris paribus*, market shares, export and penetration ratios in sector k evolve to the benefit of country 2. In other words, the comparative advantage of country 2 in sector k appears to increase while that of the other country decreases (*see section 3 below for a comprehensive analysis of the factors of comparative advantage*). Therefore, intra-industry trade takes place in sector k when neither of the two countries has a low relative revenue together with a relative disadvantage “in the ρ sense” (on the contrary, a low relative revenue has to be counterbalanced by a relative advantage “in the ρ sense”, and *vice versa*).

It seems that, when ρ_k exceeds ρ_{kM} , firms of country 1 are not competitive enough in terms of ρ_k to be able to export good k to country 2. If ρ_k is so high that it exceeds $1/T_k$, firms in country 1 even stop producing good k ; the good is then entirely provided by firms of country 2. Symmetrical cases appear in sector k when ρ_k is lower than ρ_{km} , or even lower than T_k : in these cases, country 1 is the only country to export, if not to produce good k . To sum up, when ρ_k does not belong to the interval $] \rho_{km}, \rho_{kM} [$, pure inter-industry trade takes place in sector k instead of intra-industry trade: the most competitive country exports good k to the other, which may even not produce it.

12. This interval derives from the conditions of positivity of y_{ki} and F_{ki} , expressed with respect to ρ_{ki} , T_{ki} and r in table 1 (Cf. *appendix* and see table 1).

TABLE 1

Basic Variables and their Relations to the ρ Ratios

Indicator	Formula	Brand image $\alpha \downarrow$ $\rho \uparrow$ (c, f constant)	Cost $c \uparrow$ $\rho \uparrow$ (α, f constant)	Cost $f \uparrow$ $\rho \uparrow$ (c, α constant)
Domestic consumption in country 1 (per variety)	$y_{11} = \frac{2(\sigma-1)f(d_2 \mathcal{F}_2)^{1/2}(\rho-T)}{c_2 c \rho(1-T^2)}$	\uparrow	$\uparrow \Leftrightarrow \rho \leq \sigma T$	\uparrow
Imported consumption of country 1 (per variety)	$y_{21} = \frac{2(\sigma-1)(d_2 \mathcal{F}_2)^{1/2}(\rho-T)T^{\frac{\sigma-1}{\sigma}}}{c_2(1-T^2)}$	\uparrow	\uparrow	\uparrow
Domestic consumption in country 2 (per variety)	$y_{22} = \frac{2(\sigma-1)(d_2 \mathcal{F}_2)^{1/2}(1-\rho T)}{c_2(1-T^2)}$	\downarrow	\downarrow	\downarrow
Imported consumption of country 2 (per variety)	$y_{12} = \frac{2(\sigma-1)f(d_2 \mathcal{F}_2)^{1/2}(1-\rho T)T^{\frac{\sigma-1}{\sigma}}}{c_2 c \rho(1-T^2)}$	\downarrow	$\downarrow \Leftrightarrow \rho \leq \frac{\sigma}{T} *$	\downarrow
Total number of varieties produced in country 1	$F_1 V_1 = \frac{R_2}{2f\sigma(d_2 \mathcal{F}_2)^{1/2}} \left[\frac{r\rho}{\rho-T} + \frac{\rho T}{\rho T-1} \right]$	\downarrow	\downarrow	\downarrow
Total number of varieties produced in country 2	$F_2 V_2 = \frac{R_2}{2\sigma(d_2 \mathcal{F}_2)^{1/2}} \left[\frac{rT}{T-\rho} + \frac{1}{1-\rho T} \right]$	\uparrow	\uparrow	\uparrow
Total production per firm in country 1	$V_1(y_{11} + (1+t)y_{12}) = \frac{2(\sigma-1)}{c_2 c} \mathcal{F}_1$	\rightarrow	\downarrow	\uparrow
Total production per firm in country 2	$V_2(y_{21} + (1+t)y_{22}) = \frac{2(\sigma-1)}{c_2} \mathcal{F}_2$	\rightarrow	\rightarrow	\rightarrow
Domestic price in country 1	$p_{11} = \frac{c c_2 \sigma}{\sigma-1}$	\rightarrow	\uparrow	\rightarrow
Import price - country 1	$p_{21} = \frac{c_2 \sigma}{\sigma-1} T^{\frac{-1}{\sigma-1}}$	\rightarrow	\rightarrow	\rightarrow
Domestic price in country 2	$p_{22} = \frac{c_2 \sigma}{\sigma-1}$	\rightarrow	\rightarrow	\rightarrow
Import price - country 2	$p_{12} = \frac{c c_2 \sigma}{\sigma-1} T^{\frac{-1}{\sigma-1}}$	\rightarrow	\uparrow	\rightarrow

Nota: 1) The first index represents the producer country, the second index the consumer country. The sector index (k) has been omitted for simplicity. 2) c varies through c_1, c_2 being unchanged. Similarly, f and α vary through f_1 and α_1 alone. 3) Identical countries in sector k : $c = f = r = \rho = 1$.

* *Nota:* This condition is automatically verified if the two countries are identical.

TABLE 2

**Comparative Advantage Indicators and Market Shares
(for a given sector)**

Indicator	Formula	Brand image α ↓ ρ ↑ (c, f constant)	Cost c ↑ ρ ↑ (α, f constant)	Cost f ↑ ρ ↑ (c, α constant)
Relative price of domestic and imported goods in country 1	$\frac{p_{22}}{p_{12}} = \frac{1}{c} T^{\frac{1}{\sigma-1}}$	↓	↓	↓
Relative price of domestic and imported goods in country 2	$\frac{p_{11}}{p_{21}} = c T^{\frac{1}{\sigma-1}}$	↑	↑	↑
Ratio of relative perceived quality to cost in country 1	$\frac{1}{\rho}$	↓	↓	↓
Ratio of relative perceived quality to cost in country 2	ρ	↑	↑	↑
Market share of country 1*	$MS_1 = \left(1 + \frac{F_2 V_2 (y_{21} + y_{22})}{F_1 V_1 (y_{12} + y_{11})}\right)^{-1}$	↓	↓	↓
Market share of country 2*	$MS_2 = \left(1 + \frac{F_1 V_1 (y_{12} + y_{11})}{F_2 V_2 (y_{21} + y_{22})}\right)^{-1}$	↑	↑	↑
Penetration ratio country 1	$MR_1 = \left(1 + \frac{F_1 V_1 y_{11}}{F_2 V_2 y_{21}}\right)^{-1}$ $= \left(1 + \frac{\alpha^\sigma}{c^\sigma} \frac{F_1 V_1}{F_2 V_2} T^{\frac{\sigma}{\sigma-1}}\right)^{-1}$	↑	↑	↑
Penetration ratio country 2	$MR_2 = \left(1 + \frac{F_2 V_2 y_{22}}{F_1 V_1 y_{12}}\right)^{-1}$ $= \left(1 + \frac{c^\sigma}{\alpha^\sigma} \frac{F_2 V_2}{F_1 V_1} T^{\frac{\sigma}{\sigma-1}}\right)^{-1}$	↓	↓	↓
Export ratio country 1	$XR_1 = \frac{V_1 y_{12} (1+t)}{V_1 (y_{11} + y_{12} (1+t))}$ $= \frac{T}{1-T^2} \left(\frac{1}{\rho} - T\right)$	↓	↓	↓
Export ratio country 2	$XR_2 = \frac{V_2 y_{21} (1+t)}{V_2 (y_{22} + y_{21} (1+t))}$ $= \frac{T}{1-T^2} (\rho - T)$	↑	↑	↑

*Nota: The first index represents the producer country, the second index the consumer country. The sector index (k) has been omitted for simplicity. *: Market shares only take account of the production which is actually sold on both markets (i.e. that part of output ($ty_{ij}, j \neq i$) destroyed during transportation is excluded).*

3 The Factors of Comparative Advantage and the Magnitude of Intra-Industry Trade

3.1. The two Factors of Comparative Advantage

Our discussion will focus on one or two small sectors of the world economy (k and k'), both satisfying (4), all things being equal in the other sectors. Consequently, we may consider as a first approximation the case where total revenue in each country is not affected by small changes in sectors k and k' . To simplify, we shall assume that transportation costs do not depend on the goods that are exported ($T_k = T_{k'} = T$).

In a given sector k , $\rho_k > 1$ is equivalent to:

$$(5) \quad \left(\frac{f_1 c_1^{\sigma-1}}{\alpha_1^\sigma} \right)_k > \left(\frac{f_2 c_2^{\sigma-1}}{\alpha_2^\sigma} \right)_k$$

With σ taken as given, both numerators in (5) roughly represent a composite cost of production, including variable as well as fixed costs, while both denominators similarly reflect perceived quality or brand image of varieties. Thus, inequality (5) expresses the fact that country 2 has an absolute advantage over country 1 in producing good k : in fact, firms of country 2 can sell good k cheaper than firms of country 1, for any perceived quality; or equivalently firms of country 2 can produce varieties of higher perceived quality or brand image than those from country 1, but at the same cost. Note that the higher the elasticity of substitution σ , the more sensitive to differentials in relative costs or in perceived quality the ρ_k ratio.

Let x_{k1} denote the ratio of exports to imports for country 1 and sector k :

$$(6) \quad x_{k1} = \left(\frac{F_1 V_1 y_{12}}{F_2 V_2 y_{21}} \right)_k = \frac{1}{c_k} \Psi_{r,T}(\rho_k)$$

where Ψ is a decreasing function of ρ_k parametrised by r and T , but which does not depend on k :

$$\Psi_{r,T}(\rho_k) = \frac{r \cdot \varphi_T(\rho_k) - T}{\frac{1}{\varphi_T(\rho_k)} - rT}$$

and $\varphi_T(\rho_k) = \frac{1-\rho_k T}{\rho_k - T}$ is the decreasing function of ρ_k defined in (4').

The corresponding ratio x_{k2} for country 2 is equal to: $x_{k2} = \frac{1}{x_{k1}}$.

PROPOSITION 2: Each country exports relatively more than it imports in sectors of comparative advantage in terms of variable costs, given equality of both ρ ratios (of relative overall costs to perceived quality) and transportation costs.

Proof: Let us consider two sectors k and k' in which $\rho_k = \rho_{k'}$ but where:

$$c_k < c_{k'} \quad \Leftrightarrow \quad \frac{c_{k1}}{c_{k'1}} < \frac{c_{k2}}{c_{k'2}}$$

Country 1 has a comparative advantage in producing good k , while country 2 has a comparative advantage in producing good k' in terms of variable costs.

In this case:

$$\Psi_{r,T}(\rho_k) = \Psi_{r,T}(\rho_{k'}) \equiv \psi, \quad x_{k1} = \frac{1}{c_k} \psi \quad \text{and} \quad x_{k'1} = \frac{1}{c_{k'}} \psi.$$

Therefore:

$$\frac{x_{k1}}{x_{k'1}} = \frac{c_{k'}}{c_k} > 1 \quad \text{and} \quad \frac{x_{k2}}{x_{k'2}} = \frac{c_k}{c_{k'}} < 1$$

Country 1 exports relatively more than it imports in sector k where it has a comparative advantage in terms of variable costs, while country 2 exports relatively more than it imports in sector k' . \square

PROPOSITION 3: Each country exports relatively more than it imports in sectors of comparative advantage in terms of the ρ ratios of overall costs to perceived quality, given the equality of relative unit variable costs and transportation costs.

Proof: Now consider two sectors k and k' in which $c_k = c_{k'}$, but where:

$$\rho_k > \rho_{k'} \quad \Leftrightarrow \quad \frac{(f_1 c_1^{\sigma-1} / \alpha_1^\sigma)_k}{(f_1 c_1^{\sigma-1} / \alpha_1^\sigma)_{k'}} > \frac{(f_2 c_2^{\sigma-1} / \alpha_2^\sigma)_k}{(f_2 c_2^{\sigma-1} / \alpha_2^\sigma)_{k'}}$$

Even if unit variable costs are equal in both sectors, country 2 has a comparative advantage in producing good k and country 1 in producing good k' , as regards their relative ability to produce high quality varieties at a low cost.

In this case, $x_{k1} = \Psi_{r,T}(\rho_k)$ and $x_{k'1} = \Psi_{r,T}(\rho_{k'})$. Moreover:

1) As $\rho_k > \rho_{k'}$, $\varphi_T(\rho_k) < \varphi_T(\rho_{k'})$.

2) As every considered ρ is supposed to satisfy (4), the corresponding $\varphi_T(\rho)$ satisfies (4'), i.e. takes its values within the interval $]\frac{T}{r}, \frac{1}{rT}[$. It can also be shown that $\Psi_{r,T}$ is an increasing function of $\varphi_T(\rho)$ within this interval of values for $\varphi_T(\rho)$. Moreover, as $\varphi_T(\rho)$ is a decreasing function of ρ , $\Psi_{r,T}$ is also a decreasing function of ρ as long as the latter satisfies (4). Consequently:

$$\rho_k > \rho_{k'} \quad \Rightarrow \quad \Psi_{r,T}(\rho_k) < \Psi_{r,T}(\rho_{k'}).$$

And finally:

$$\frac{x_{k1}}{x_{k'1}} < 1 \quad \text{and} \quad \frac{x_{k2}}{x_{k'2}} > 1$$

Country 1 exports relatively more than it imports in sector k' where it has a comparative advantage in the ρ sense, while country 2 exports relatively more than it imports in sector k . \square

In sum, in this model, comparative advantage has two dimensions: one relating to unit variable costs regardless of national brand images; the other

concerning ratios of overall costs to perceived quality or brand image. Yet, the two dimensions of comparative advantage are not independent, as both are affected by unit variable costs. Therefore, increasing comparative advantage requires a subtle trade-off between costs and quality, in order to decrease costs without deteriorating the ρ ratios.

3.2. The Degree of Similarity between Countries and the Magnitude of Intra- and Inter-Industry Trade

HELPMAN and KRUGMAN [1985] show that the share of intra-industry trade increases with the degree of similarity across countries. We find a similar though more complex result. To exhibit it, let us study the Grubel and Lloyd index of intra-industry trade in a given sector (index k omitted) for each country $i = 1, 2$:

$$I_i = \frac{X_i + M_i - |X_i - M_i|}{X_i + M_i} \in [0, 1],$$

where X_i = export, M_i = import of country i in sector k

From section 2-4, we already know when there is pure inter-industry trade between the two countries in sector k (either $\rho \leq \rho_m$, in which case country 1 is the only exporter of good k , or $\rho \geq \rho_M$ which is the symmetrical case). Then, both I indices equal zero. When $\rho_m < \rho < \rho_M$, there is some intra-industry trade in sector k . The I indices then equal:

$$I_i = 1 - \frac{|x_i - 1|}{x_i + 1}$$

where $x_i = \frac{X_i}{M_i}$ is the indicator of “net export” of country i in sector k which has already been studied in section 3.1:

$$(6') \quad x_1 = \frac{F_1 V_1 y_{12}}{F_2 V_2 y_{21}} = \frac{1}{x_2} = \frac{1}{c} \Psi_{r,T}(\rho) = \frac{1}{c} \frac{r \cdot \varphi_T(\rho) - T}{\frac{1}{\varphi_T(\rho)} - rT}$$

The I indices reach their maximal values when “net exports” are equal to one (pure intra-industry trade in sector k). They increase with “net exports” if the latter are less than one, and decrease otherwise. From (6') we can immediately derive the following result:

$$\left| \begin{array}{l} \text{PROPOSITION 4: } \forall (r, \rho)/T < r \cdot \varphi_T(\rho) < \frac{1}{T} \text{ with } \varphi_T(\rho) = \frac{1-\rho T}{\rho-T}, \\ \exists! c > 0 \text{ so that } x_1 = x_2 = 1: \\ (7) \quad c = \varphi_T(\rho) \frac{r \varphi_T(\rho) - T}{1 - rT \varphi_T(\rho)} \end{array} \right.$$

This property means that pure intra-industry trade appears in several different cases, including the following ones:

1) The two countries are identical in terms of comparative advantage ($(c, \rho) = (1, 1)$) and revenue ($r = 1$);

2) Differences in total comparative advantage are counterbalanced by revenue differentials. For instance, equation (7) may be satisfied with high c and ρ , but also a high r . In this case, country 1, though suffering an overall comparative disadvantage in sector k , can export as much as it imports in this sector thanks to higher revenue, i.e. because it is richer, or simply bigger than country 2.

3) The two countries are similar with respect to r , but country 1 has a comparative disadvantage in sector k due to high unit variable costs. Nonetheless, country 1 can export as much as it imports in sector k if costs differentials are counterbalanced by a good brand image. In this case, country 2 can counterbalance its comparative disadvantage in terms of ρ (more precisely in terms of its brand image) through lower costs.

Pure intra-industry trade may appear even if the two countries are not identical. Nonetheless, proposition 4 reminds us that intra-industry trade cannot take place if the two countries are too different in terms of revenue and ρ ratios (Cf. inequality (4')), which is included in proposition 4).

In sum, we find that intra-industry trade is high and inter-industry trade low when the differences between the two countries in terms of unit variable costs, ρ ratios, and revenue counterbalance each other, as may be true even when the two countries are not identical. Conversely, inter-industry trade increases and intra-industry trade diminishes, or even disappears, when the average performances of the two countries in terms of (c, ρ, r) differ too much. This result generalises that of HELPMAN and KRUGMAN [1985].

4 A Model Endogenising Part of Comparative Advantage

In the medium run, it seems realistic to assume that firms can modify their brand images through regular quality improvement, innovation and brand promotion¹³. Such an effort implying a cost, firms have to choose a judicious combination of quality and costs in order to improve their individual ρ ratios. Taking account of the microeconomic–firm specific–dimension of the ρ ratios means endogenising at least part of their constitutive elements, and consequently part of comparative advantage. In our concern to keep the model as transparent as possible, we shall modify the basic model in a very simple way.

The perceived quality of a variety takes account of both its intrinsic quality and its producer's brand image. Improving perceived product quality generates increases in fixed costs (R&D, advertising), but also in variable

13. The word "regular" is very important here. In fact, the regularity of the effort (in improving quality, innovating and promoting) is essential in this static model. Remember that the word "quality" does not represent the traditional notion of vertical differentiation.

costs, through, for instance (depending on the sector) more careful finishing, specific know-how (with better paid qualified workers) or more expensive raw materials. Although improving intrinsic product quality or producer's brand image may affect variable and fixed costs differently (improving the latter possibly requiring more fixed costs), we still consider perceived product quality to be a one dimension variable.

Consumers' perception of product quality is assumed to reflect the "objective" quality aimed for by producers. Let q_i denote this "objective" quality for any representative firm of sector k in country i (index k being omitted in the whole section). We have: $\alpha_i = q_i \forall i$. As the α terms will now derive from producers' optima, we can no longer normalise them as we did in section 2 above: we need additional degrees of freedom. Therefore, we define the mean quality of varieties supplied on market j :

$$\bar{q}_j^\sigma = \sum_{i=1}^2 \sum_{f=1}^{F_i} \sum_{v=1}^{V_i} \frac{\alpha_i^\sigma}{V_j} = \sum_{i=1}^2 F_i V_i \frac{q_i^\sigma}{V_j}$$

and we modify demand functions accordingly:

$$y_{vij} = \left(\frac{q_i^\sigma}{\bar{q}_j^\sigma V_j} \right) \left(\frac{p_{vij}}{p_j} \right)^{-\sigma} \frac{R_j}{p_j}$$

Besides, we suppose that there is a standard of minimal quality for every variety of good k ($\forall i, q_i \geq q_- > 0$). Finally, producing varieties of quality q_i is supposed to require the following flows of costs:

$$(8) \quad - \text{ fixed costs : } \quad \mathcal{F}_i = F_i^0 + F_i^1 q_i^\gamma \quad \text{with } \gamma > 0 \text{ rational}$$

$$(9) \quad - \text{ variable costs : } \quad c_i = \varpi_i q_i^\theta \quad \text{with } \theta > 0 \text{ rational}$$

Every firm has to fix: its prices (p_{vij}), its degree of horizontal differentiation (V_i), and its position on the quality scale (q_i). All these decisions derive from profit maximisation:

$$\text{Max}_{(p_{vij}), V_i, q_i} \quad \Pi_i = \sum_{v=1}^{V_i} \sum_{j=1}^2 (p_{vij} - \varpi_i q_i^\theta (1 + t_{ij})) y_{vij} - d_i V_i^2 - F_i^0 - F_i^1 q_i^\gamma$$

Each individual firm is considered to be too small with respect to national markets of product k to be able to influence both prices (p_j) and the average quality of good k in country j (\bar{q}_j). Taking account of these assumptions, the producer's optimum derives from a two stage calculation. At the first stage, we repeat the same calculation as above (Cf. section 2), quality being treated as given. We obtain exactly the same results:

$$(10) \text{ Prices : } p_{vij} = c_i (1 + t_{ij}) \frac{\sigma}{\sigma - 1} = \varpi_i q_i^\theta (1 + t_{ij}) \frac{\sigma}{\sigma - 1} = p_{ij}$$

$$(10') \text{ Quantities (per variety) : } y_{vij} = y_{ij} \neq 0 \quad \forall (v, i, j)$$

$$(11) \text{ Number of varieties: } 2 d_i V_i = q_i^{\sigma(1-\theta)+\theta} \sum_{j=1}^2 \Phi_{ij} \psi_j^{-1}$$

where:

$$\Phi_{ij} \stackrel{\text{def.}}{=} \frac{R_j}{\sigma} \varpi_i^{1-\sigma} (1 + t_{ij})^{1-\sigma} \left(\frac{\sigma}{\sigma - 1} \right)^{1-\sigma} > 0$$

$$\text{and } \psi_j \stackrel{\text{def.}}{=} \bar{q}_j^\sigma \nu_j p_j^{1-\sigma} = \sum_{i=1}^2 F_i V_i q_i^\sigma p_{ij}^{1-\sigma} > 0$$

At a second stage, we set the partial derivative of profit with respect to q_i to zero. As the partial derivative of $\Phi_{ij} \psi_j^{-1}$ can be neglected (small firms), we get:

$$\frac{\partial \Pi_i(p_{ij})}{\partial q_i} = V_i (\sigma(1 - \theta) + \theta) q_i^{\sigma(1-\theta)+\theta-1} \sum_{j=1}^2 \Phi_{ij} \psi_j^{-1} - \gamma F_i^1 q_i^{\gamma-1}$$

Consequently:

$$(12) \quad \sum_{j=1}^2 \Phi_{ij} \psi_j^{-1} = \frac{\gamma F_i^1 q_i^{\gamma-\sigma(1-\theta)-\theta}}{V_i (\sigma(1 - \theta) + \theta)}$$

Notice that (11) and (12) imply a positive relation between quality and the degree of horizontal differentiation:

$$(13) \quad 2 d_i V_i^2 = \frac{\gamma F_i^1 q_i^\gamma}{(\sigma(1 - \theta) + \theta)}$$

These conditions have to be consistent with a non negative profit:

$$\Pi_i(p_{ij}, V_i, q_i) = V_i \sum_{j=1}^2 \mu_{ij} y_{ij} - d_i V_i^2 - \mathcal{F}_i = d_i V_i^2 - F_i^0 - F_i^1 q_i^\gamma \geq 0$$

If this condition is not satisfied for the quality value q_i derived from first-order conditions, we get a corner solution: $q_i = q_-$ (13'), which will be supposed to lead to a non negative profit.

Zero profit equilibria derive from conditions (10), (10'), (11), (13) or (13'), together with the zero profit condition:

$$(14) \quad d_i V_i^2 = F_i^0 + F_i^1 q_i^\gamma \quad 14$$

Prices being replaced by their values (given by (10)), we get a system with 3×2 equations and 3×2 unknown variables: F_i , q_i , and V_i , $I = 1, 2$. We first solve this system with respect to F_i and V_i ($i = 1, 2$), q_1 and q_2 being considered as parameters, in the same way as we did previously (see Appendix). Every unknown variable (p_{ij} , F_i , V_i) is then expressed with respect to q_1 and q_2 . Replacing V_i with its expression (function of q_i) given by (14) in (13), we get the condition determining each q_i , if the solution is not a corner one. To sum up, we have:

$$(15) \quad 2(F_i^0 + F_i^1 q_i^\gamma) = \frac{\gamma F_i^1 q_i^\gamma}{(\sigma(1-\theta) + \theta)} \quad \text{or} \quad q_i = q_-$$

The non trivial solution:

$$q_i^\gamma = \frac{2 F_i^0}{F_i^1} \frac{\sigma(1-\theta) + \theta}{\gamma - 2\sigma(1-\theta) - 2\theta}$$

is strictly positive when $\theta < \sigma/(\sigma - 1)$ and $\gamma > 2(\sigma(1-\theta) + \theta)$. If in addition $q_i > q_-$, there are two possible solutions: the non trivial one, and the corner one. In this case, two interesting results appear, which can be very easily generalised for any number of countries.

PROPOSITION 5: A firm choosing high quality also opts for a high degree of horizontal differentiation and vice versa ¹⁵.

Proof: If the solution q_i is the non trivial one, (13) gives the proof (and we do not need the zero profit solution to prove proposition 5). If the solution is the trivial one q_- , we need the zero profit condition 14 to give the proof. \square

We can give a more general formulation of this property, taking account of the links between costs and quality:

PROPOSITION 6: The producers' first possible strategy consists in reducing costs in order to be able to sell varieties at low prices; cost cuts lead to relatively standardised products and low quality (case $q_i = q_-$). The second and opposite strategy consists in supplying a relatively large number of high quality varieties at a higher price (non trivial solution).

Proof: Cf. proposition 5 + equations (8) and (9). \square

The result of the trade-off between costs and quality may differ from one country to the other, depending on national conditions of production.

14. Again, we neglect the fact that both the total number of varieties and the number of firms are integers.

15. This property is not a standard result in the literature.

If very specific restrictions on parameters are satisfied, the two strategies may even lead to the same profit, in which case multiple equilibria may appear. Apart from these very particular cases, only one strategy is chosen per sector within a given country. Different conditions of production across the two countries may lead to different strategies of firms from one country to the other, as well as to different levels of high quality q_1 and q_2 ¹⁶.

A country in which firms choose the first strategy is likely to have a comparative advantage in terms of variable costs, and maybe also in terms of ρ (if the quality of its national products is not too low compared to their prices). A country in which firms choose the second strategy is likely to have a comparative advantage in the ρ sense, provided that high quality and differentiation are not counterbalanced by costs which are too high. However, the type of comparative advantage that derives from national strategies may differ from the objectives of national firms, particularly if firms in both countries choose the same strategy.

Note that our assumption of monopolistic competition seems particularly questionable here. In fact, the prospect of making a higher profit is a significant incentive to produce high quality. In our model, this motivation does not hold as firms cannot make profits. We must admit that we would like to give up the zero profit condition, especially for firms which opt for the high differentiation strategy. Unfortunately, the model could no longer be solved analytically, which would change the approach of this paper. This is the main reason why we did not drop the assumption of monopolistic competition here. Nonetheless, we are conscious that the zero profit condition limits the validity of our results here.

5 What can we Derive from the Theoretical Model in Terms of Trade Equations?

Our model turns out to be more operational than most theoretical models of imperfect competition.

First, it can easily be modified so that it becomes more general and more realistic, with more than two countries and possible differences in consumer tastes across countries (Cf. Appendix). Demand functions become:

$$(16) \quad y_{ij} = \left(\frac{\alpha_{ij}^{\sigma_j}}{\mathcal{V}_j} \right) \left(\frac{p_{ij}}{p_j} \right)^{-\sigma_j} \frac{R_j}{p_j}$$

16. In a world with more than two countries, the range of different available qualities on each market may be rather large. The model can also be directly generalised so that θ and γ become θ_i and γ_i .

for any variety v of good k (index omitted) produced in country i and supplied on market j (taking account of the supply side, prices and therefore quantities do not depend on v itself).

Let then n_i be the number of varieties produced in country i and sold on every market ($n_i = F_i V_i$). Bilateral exports of good k from country i to country j are equal to:

$$(17) \quad X_{ij} = n_i y_{ij}$$

From which we derive total imports of good k to country j :

$$M_j = \sum_{i \neq j} X_{ij}$$

If $(a_{ij})_{j=1 \dots I}$ represents the geographical import structure of country j ¹⁷:

$$\forall i \neq j \quad a_{ij} = \left[\frac{X_{ij}}{M_j} \right] \Rightarrow \sum_{i \neq j} a_{ij} = 1, \quad \text{and} \quad a_{ii} = 0$$

we have:

$$(18) \quad \dot{M}_j = \sum_{i \neq j} a_{ij} \dot{X}_{ij} = \sum_{i \neq j} a_{ij} (\dot{X}_{ij} - \dot{X}_{jj}) + \dot{X}_{jj}$$

Let us replace bilateral exports with their values derived from (16) and (17). Therefore, we obtain the following equation for total imports of country j in sector k (index k being omitted):

$$(19) \quad \dot{M}_j = \dot{X}_{jj} - \sigma_j (\dot{p}_{M_j} - \dot{p}_{jj}) + (\dot{n}_{M_j} - \dot{n}_j) + \sigma_j (\dot{\alpha}_{M_j} - \dot{\alpha}_{jj}),$$

where:

X_{jj} = the domestic demand in country j and sector k ,

σ_j = the elasticity of substitution between the different varieties of good k in country j ,

$p_{M_j} = \prod_{i \neq j} (p_{ij})^{a_{ij}}$, the import price of country j in sector k ,

$n_j = F_j V_j$, the total number of varieties produced in country j ,

$n_{M_j} = \prod_{i \neq j} (n_i)^{a_{ij}}$ and $\alpha_{M_j} = \prod_{i \neq j} (\alpha_{ij})^{a_{ij}}$ represent respectively foreign

exporters' average capacity to supply a lot of varieties of good k on market j , and the average brand image of imported product in sector k .

This equation suggests that imports depend on both demand and relative prices, but also on two non traditional effects: relative horizontal differentiation and national brand images. We could very easily get similar export equations, including the same type of factors. Consequently, including differentiation effects in trade equations seems to be perfectly justified.

17. This structure is rather classically assumed to be stable in the short run.

However, an important empirical issue is to be able to define convincing differentiation *proxies*. What can our theoretical model tell us about this problem? Production as an acceptable *proxy* of horizontal differentiation can be derived from (2). As far as national brand images are concerned, one could try to isolate the proportion of fixed costs attributable to the quality effort (see equation (8)). Such an approach would mean restricting oneself to the only observable costs of that kind, namely the R&D expenses. However, a static model is not the best framework to justify such a *proxy*, endogenous growth models being more convincing in this respect. From this specific model, it is also difficult to give any theoretical foundation to *proxies* based on investment: a dynamic general equilibrium model would be required. However, our very simple model shows the way to theoretically founded trade equations with differentiation effects. This is an important issue, in so far as such equations prove to be empirically convincing: in fact differentiation effects appear to be very significant¹⁸ and seem to have played a non negligible part in the evolution of trade in the last decades.

6 Concluding Remarks

We have presented a two country theoretical model of international trade in a monopolistic competition framework. The main innovations of this model concern the supply side: differentiation strategies of firms are endogenous; firms can modify their brand images through a judicious trade-off between costs and perceived product quality. From this distinguishing feature of the model derive partly endogenous comparative advantage as well as a certain number of interesting properties, from a theoretical point of view as well as from a more operational point of view.

In the model, the ρ ratios of total costs to perceived product quality play an essential part in comparative advantage, together with variable costs. At the zero profit equilibrium, the share of intra-industry trade in total trade depends on both relative revenues and the two factors of comparative advantage. Consequently, pure intra-industry trade may appear even if countries are not identical, provided that relative revenues and the two factors of comparative advantage counterbalance each other. Nonetheless, pure inter-industry trade appears if the two countries are too different in terms of ρ ratios or in terms of relative revenues.

From a microeconomic point of view, comparative advantage derives partly from producer strategies in terms of costs and brand images. Having

18. See empirical papers quoted in note 1 for instance, especially BISMUT and OLIVEIRA-MARTINS in LAUSSEL and MONTET [1989], OLIVEIRA-MARTINS [1989], and ERKEL-ROUSSE, GAULIER and PAJOT [1987]: all of them use a differentiation *proxy* based on production. The latter show that, in France, the differentiation *proxy* would have contributed as much as relative prices to the evolution of manufactured exports during the last decade.

endogenised these strategies, we have shown that firms can either sell relatively standardised low quality products at low costs and prices or supply a relatively large range of differentiated high quality varieties at higher costs and prices. Therefore, one interesting property of the model is to establish a strong link between differentiation and product perceived quality or brand image. However, we have underlined the limits of the zero profit condition in this context. The monopolistic competition framework must therefore be considered as a tractable but questionable approximation, particularly in section 4.

Finally, we have suggested that endogenising producer strategies of differentiation and product quality could help to lay theoretical microfoundations for observable *proxies* of these (usually unobservable) variables. However, (not surprisingly), our very simple model can justify only some of the *proxies* used in empirical work: one would need more sophisticated theoretical models (especially dynamic general equilibrium ones) to confirm the validity of the other usual *proxies*. Nonetheless, our main theoretical result for empirical work is that exports and imports do seem to depend not only on traditional demand and price effects, but also on differentiation factors. We have therefore made a first significant step towards a better integration of new theoretical models of trade and testable trade equations: nonetheless, further steps still need to be taken in this direction.

In conclusion, the theoretical model which has been studied in this paper must be viewed as a preliminary attempt to improve the foundations of recent empirical work on trade. As was mentioned previously, two possible directions for future research based on the same logic could consist in transforming the model into a general equilibrium framework, which would obviously be preferable to partial equilibrium (though much more complex), and trying to release the zero profit condition, which was sometimes seen to be highly questionable, especially in section 4.

APPENDIX

Consider a generalisation of the basic model with I countries ($I \geq 2$) and possibly different consumers from one country to another (utility functions are therefore modified so that the elasticities of substitution, as well as the ξ and α weights, depend on countries). We could also very easily generalise the first levels of the utility functions to CES forms, which is left to the reader¹⁹. The demand functions that derive from these generalisations become, for a variety ν produced in country i and sold on market j , $j = 1$ to I (index k omitted):

$$y_{vij} = \left(\frac{\alpha_{ij}^{\sigma_j}}{\mathcal{V}_j} \right) \left(\frac{p_{vij}}{p_j} \right)^{-\sigma_j} \frac{R_j}{p_j}$$

where $R_j = \xi_j R_j^T$ is the budget allocated to consumption of good k by the N_j consumers of country j .

On the supply side, nothing is modified. Cost functions are formulated for the I countries as above (Cf. section 2). The only slight modifications concern demand functions (*see above*) and transportation costs, whose values now depend on distances between countries.

Let t_{ij} denote the transportation cost between country i and country j . t_{ij} is supposed to be common to every variety of product k exported from country i to market j . To simplify, we may assume that $t_{ij} = t_{ji}$ (however, this assumption can be released, if we include more general transaction costs in t_{ij} instead of pure transportation costs). In any case, we suppose that the cheapest way to ship a variety from any country to any other is the most direct way, which implies:

$$(H1) \quad (1 + t_{ij}) \leq (1 + t_{il})(1 + t_{lj}) \quad \forall (i, j, l) \in \{1, \dots, I\}^3$$

Profits become:

$$\Pi_i = \sum_{v=1}^{V_i} \sum_{j=1}^I p_{vij} y_{vij} - \sum_{v=1}^{V_i} \sum_{j=1}^I c_i (1 + t_{ij}) y_{vij} - d_i V_i^2 - \mathcal{F}_i$$

We proceed exactly as in the basic model (see above, section 2). Consequently, the first stage optimisation leads to the following prices on market j for any available variety produced in country i :

$$p_{vij} = c_i (1 + t_{ij}) \frac{\sigma_j}{\sigma_j - 1} = p_{ij} \quad \forall v, i, j / y_{vij} \neq 0$$

The p_{ij} are the equilibrium prices if and only if no wholesaler in country j can gain positive profit by importing varieties produced in country i from

19. The main interest of such a generalisation is that sectorial market shares would depend on the degree of differentiation in each sector, which is a good property.

another country. This absence of arbitrage opportunity can be formulated in the following way:

$$(H2) \quad p_{ij} \leq p_{il}(1 + t_{ij}) \quad \forall (i, j, l) \in \{1, \dots, I\}^3$$

$$\Leftrightarrow \frac{\sigma_j}{\sigma_j - 1} (1 + t_{ij}) \leq \frac{\sigma_l}{\sigma_l - 1} (1 + t_{il})(1 + t_{lj}) \quad \forall (i, j, l) \in \{1, \dots, I\}^3$$

Taking account of hypothesis (H1), this condition is satisfied if the elasticities of substitution $(\sigma_j)_{j=1, \dots, I}$ are identical, or if they do not significantly differ from each other. We shall assume that this is the case, and that (H2) is satisfied. In this case, every variety produced in country i and sold on market j is supplied at the same price p_{ij} and at the following quantity:

$$y_{ij} = p_{ij}^{-\sigma_j} \frac{\alpha_{ij}^{\sigma_j}}{\mathcal{V}_j} \frac{R_j}{p_j^{1-\sigma_j}}$$

Again, firms can get strictly positive margins from selling any variety on any market:

$$\mu_{ij} y_{ij} = (p_{ij} - c_i(1 + t_{ij})) y_{ij} = \Phi_{ij} \psi_j^{-1} > 0 \quad \text{where :}$$

$$\Phi_{ij} \stackrel{\text{def.}}{=} \frac{R_j}{\sigma_j} \alpha_{ij}^{\sigma_j} p_{ij}^{1-\sigma_j} = (c_i(1 + t_{ij}))^{1-\sigma_j} \left(\frac{\sigma_j}{\sigma_j - 1} \right)^{1-\sigma_j} \alpha_{ij}^{\sigma_j} \frac{R_j}{\sigma_j}$$

$$\text{and } \psi_j \stackrel{\text{def.}}{=} \mathcal{V}_j p_j^{1-\sigma_j} = \sum_{i=1}^I F_i V_i \alpha_{ij}^{\sigma_j} p_{ij}^{1-\sigma_j} > 0$$

This property implies that each firm chooses to diversify across markets ($\forall v, i, j, y_{vij} \neq 0$). Consequently, the subsequent stages of the optimisation program are the same as in the basic model (Cf. section 2 above). Finally, the zero profit equilibrium is defined through the following system (generalising that of section 2):

$$(1) \quad \begin{cases} p_{vij} = c_i(1 + t_{ij}) \frac{\sigma_j}{\sigma_j - 1} = p_{ij} & \forall v, i, j, \\ y_{vij} = y_{ij} = \left(\frac{\alpha_{ij}^{\sigma_j}}{\mathcal{V}_j} \right) \left(\frac{p_{ij}}{p_j} \right)^{-\sigma_j} \frac{R_j}{p_j} \neq 0 \end{cases}$$

$$(2) \quad 2 d_i V_i = \sum_{j=1}^I \mu_{ij} y_{ij} \quad \text{where } \mu_{ij} = p_{ij} - c_i(1 + t_{ij})$$

$$(3) \quad d_i V_i^2 = \mathcal{F}_i \quad \forall (i, j) \in \{1, \dots, I\}^2, \forall \text{index } k \text{ (omitted)}$$

• Resolution:

Consider equations (1) to (3) for a given sector (index k omitted). The p_{ij} are substituted with their values (1) into (2) and (3). Consider the system (2)+(3) of $2I$ equations and $2I$ unknown parameters F_i (the number of

firms in sector k and in each country i) and V_i (the number of varieties per firms), $i = 1, \dots, I$. $\forall (i, j) \in \{1, \dots, I\}^2$, we have:

$$(2) \quad 2 d_i V_i = \sum_{j=1}^I \Phi_{ij} \psi_j^{-1} \quad \text{where} \quad \Phi_{ij} = \frac{R_j}{\sigma_j} \alpha_{ij}^{\sigma_j} p_{ij}^{1-\sigma_j}$$

$$(2') \quad \text{and :} \quad \psi_j = \sum_{i=1}^I F_i V_i \alpha_{ij}^{\sigma_j} p_{ij}^{1-\sigma_j}$$

$$(3) \quad d_i V_i^2 = \mathcal{F}_i$$

The I equations of type (2) determine a system of I linear equations with I unknown parameters, the $(\psi_j^{-1})_{j=1, \dots, I}$. The determinant of the system is:

$$D_M = \left(\prod_{j=1}^I \cdot \frac{R_j}{\sigma_j} \right) \begin{vmatrix} \alpha_{11}^{\sigma_1} p_{11}^{1-\sigma_1} & \dots & \alpha_{1I}^{\sigma_I} p_{1I}^{1-\sigma_I} \\ \dots & \alpha_{ij}^{\sigma_j} p_{ij}^{1-\sigma_j} & \dots \\ \alpha_{I1}^{\sigma_1} p_{I1}^{1-\sigma_1} & \dots & \alpha_{II}^{\sigma_I} p_{II}^{1-\sigma_I} \end{vmatrix} \stackrel{\text{def.}}{=} \left(\prod_{j=1}^I \cdot \frac{R_j}{\sigma_j} \right) D_D$$

D_M would be identical to zero if transportation costs were equal to zero, which is supposedly not the case. Besides, for calculation purpose, we suppose that there are no identical countries, otherwise the system would contain redundant equations²⁰. Consequently, we consider D_M to differ from zero, which is true in general. Therefore, we have:

$$\forall j = 1, \dots, I, \quad \psi_j^{-1} = D_D^{-1} \left(\prod_{j=1}^I \cdot \frac{R_j}{\sigma_j} \right)^{-1} \sum_{i=1}^I (2 d_i V_i) \cdot (-1)^{i+j} \cdot \mathcal{M}_{ij}^{D_M}$$

where $\mathcal{M}_{ij}^{D_M}$ is the minor of D_M (derived from D_M minus its i -th line and j -th column). Let $\mathcal{M}_{ij}^{D_D}$ denote the corresponding minor of D_D . We have:

$$\mathcal{M}_{ij}^{D_M} = \mathcal{M}_{ij}^{D_D} \cdot \prod_{j' \neq j} \frac{R_{j'}}{\sigma_{j'}}$$

therefore:

$$\forall j = 1, \dots, I, \quad \psi_j^{-1} = D_D^{-1} \frac{\sigma_j}{R_j} \sum_{i=1}^I (2 d_i V_i) \cdot (-1)^{i+j} \cdot \mathcal{M}_{ij}^{D_D}$$

Definition (2') of $(\psi_j)_{j=1, \dots, I}$ is also a system of I linear equations with I unknown parameters: the numbers of varieties per country $(F_i V_i)_{i=1, \dots, I}$ in

20. In such a case, there would be no theoretical problem, nor would there be any differences in the results with respect to the non identical country case, but we would have to write things differently.

sector k . The determinant of this second system is the transpose of D_D , which like D_D usually differs from zero. Consequently, we have:

$$\forall i = 1, \dots, I, \quad F_i V_i = D_D^{-1} \sum_{j=1}^I (\psi_j) \cdot (-1)^{i+j} \cdot \mathcal{M}_{ij}^{DD}$$

Let us substitute $(\psi_j)_{j=1, \dots, I}$ with their values, which have been derived from the preceding system. We therefore get:

$$\forall i = 1, \dots, I, \quad F_i V_i = (-1)^i \sum_{j=1}^I \frac{R_j}{\sigma_j} \frac{\mathcal{M}_{ij}^{DD}}{\sum_{i'=1}^I 2 d_{i'} V_{i'} (-1)^{i'} \mathcal{M}_{i'j}^{DD}}$$

which is equivalent to:

$$\forall i = 1, \dots, I, \quad F_i = (-1)^i \sum_{j=1}^I \frac{R_j}{\sigma_j} \frac{\mathcal{M}_{ij}^{DD}}{\sum_{i'=1}^I 2 d_{i'} V_{i'}^2 \frac{V_i}{V_{i'}} (-1)^{i'} \mathcal{M}_{i'j}^{DD}}$$

As we know each V_i from equation (3), we get the number of firms in sector k in each country ²¹:

$$\forall i = 1, \dots, I, \quad F_i = \frac{1}{2 \mathcal{F}_i} \sum_{j=1}^I \frac{R_j}{\sigma_j} \left[\frac{(-1)^i (d_i \mathcal{F}_i)^{1/2} \mathcal{M}_{ij}^{DD}}{\sum_{i'=1}^I (-1)^{i'} (d_{i'} \mathcal{F}_{i'})^{1/2} \mathcal{M}_{i'j}^{DD}} \right]$$

To sum up, if it exists, the zero profit equilibrium is unique and characterised by the following features (in each sector k , index k being omitted):

(1) Prices:

$$p_{vij} = c_i (1 + t_{ij}) \frac{\sigma_j}{\sigma_j - 1} = p_{ij}$$

(2) Number of firms in country i :

$$F_i = \frac{1}{2 \mathcal{F}_i} \sum_{j=1}^I \frac{\xi_j R_j^T}{\sigma_j} \left[\frac{(-1)^i (d_i \mathcal{F}_i)^{1/2} \mathcal{M}_{ij}^{DD}}{\sum_{i'=1}^I (-1)^{i'} (d_{i'} \mathcal{F}_{i'})^{1/2} \mathcal{M}_{i'j}^{DD}} \right]$$

(3) Total number of varieties per country:

$$V_i F_i = \frac{1}{2 \mathcal{F}_i} \sum_{j=1}^I \frac{\xi_j R_j^T}{\sigma_j} \left[\frac{(-1)^i (d_i \mathcal{F}_i)^{1/2} \mathcal{M}_{ij}^{DD}}{\sum_{i'=1}^I (-1)^{i'} (d_{i'} \mathcal{F}_{i'})^{1/2} \mathcal{M}_{i'j}^{DD}} \right] \left(\frac{\mathcal{F}_i}{d_i} \right)^{1/2}$$

21. We neglect the fact that the number of firms and the total number of varieties are integers. These approximations are not significant because these numbers are very large (monopolistic competition).

(4) Quantities of any elementary variety produced in country i and supplied on market j :

$$y_{ij} = 2 \left[c_i (1 + t_{ij}) \frac{\sigma_j}{\sigma_j - 1} \right]^{-\sigma_j} \alpha_{ij}^{\sigma_j} \frac{\sigma_j}{D_D} \sum_{i'=1}^I (d_{i'} \mathcal{F}_{i'})^{1/2} (-1)^{i'+j} \mathcal{M}_{ij}^{D_D}$$

where D_D is the determinant of matrix $(\alpha_{ij}^{\sigma_j} p_{ij}^{1-\sigma})_{i=1, \dots, I, j=1, \dots, I}$ and $\mathcal{M}_{ij}^{D_D}$ its minors (dimension = $I - 1$). Total quantities of product are equal to: $y_{ij} \times F_i V_i$.

In the case $I = 2$ (two countries), we can easily calculate all these determinants with respect to prices and α terms. With some further restrictions (identical consumers, $t_{12} = t_{21} = t$) and additional notations, we obtain the results that appear in tables 1 and 2, above.

Nota: The preceding calculation and results remain valid if firms cannot supply every variety on every market (in which case, second rank equilibria appear), with the convention: $y_{vij} = 0 \Leftrightarrow p_{vij} = +\infty$.

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