

A Comparison of a Production Smoothing Model and a Dynamic Factor Demand Model with Inventories: Applications to French Industrial Sectors

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ABSTRACT. – In literature two types of models exist that aim at describing the production decision(s) of entrepreneurs, taking account of production costs and costs incurred by the existence of inventory stocks of final goods. One type is called the production smoothing models and the other type the factor demand models that include inventories. In this paper both types are discussed, compared with each other and estimated. The main results are that the factor demand model is preferred to the production smoothing model since (i) costs are more “structurally” specified by which more efficient parameter estimates are obtained and (ii) arbitrary normalisation rules are not needed. GMM estimation results obtained with data from French industrial sectors also corroborate the preference of the factor demand model.

Une comparaison d'un modèle de lissage de production avec un modèle dynamique de facteurs de demande avec des stocks : Des applications aux secteurs industriels français

RÉSUMÉ. – Dans la littérature il y a deux types de modèles qui décrivent la décision d'une entreprise : les modèles de lissage et les modèles des facteurs de demande avec des stocks. Dans cet article les deux modèles sont comparés et estimés avec des données françaises. Les résultats montrent que le modèle des facteurs de demande est préféré au modèle de lissage parce que (i) les coûts sont spécifiés structurellement de telle sorte que les paramètres estimés sont plus efficaces et (ii) on n'a pas besoin des règles arbitraires pour une normalisation. Les résultats empiriques confirment la préférence du modèle des facteurs de demande avec des stocks.

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1 Introduction

Inventories have gained much attention in literature. Most attention has been paid to their high volatility and their “smoothing role”. Inventories are said to smooth production when entrepreneurs do not change their level of production at the same speed as the changes in sales that they perceive.

The smoothing role of inventories is specified by econometric models, that are called production smoothing models (PSM). Examples are WEST [1987], RAMEY [1991] and EICHENBAUM [1989]. In these models entrepreneurs are assumed to decide upon the production level and the level of inventory stocks. Sales are assumed to be exogenous. The entrepreneur minimizes costs that consist of production costs incurred by inventories.

In the literature there are also models that focus on factor demand. These models specify the level of production only indirectly. In these factor demand models (FDM) it is assumed that an entrepreneur chooses the production inputs optimally, by which the production level is determined (indirectly).

As the PSM and the FDM both aim to describe production decisions, both models should be similar. This similarity evidently should hold if the FDM, like the PSM, includes inventories (FDMI, for short).

The aim of this study is to compare both type of inventory models, *i.e.* the PSM and the FDMI. The specifications of both models and the model solutions are compared from an econometric point of view. Thereafter, estimation results obtained with French sectorial data are used to compare both models empirically.

The outline of this study is as follows. In section 2 a PSM and a FDMI are specified. Their differences from an economic and econometrical point of view are discussed. In section 3 GMM estimation results are presented for both models. These results are obtained with data from five French industrial sectors. For each sector both models are compared by nested tests. Section 4 summarizes and concludes.

2 Two Structural Models with Inventories

Inventories are defined as investment in (unsold) final goods at the end of period t by Y_t , sales by S_t and the changes in the inventory stock V_t as $\Delta V_t \equiv V_t - V_{t-1}$. The identity

$$(1) \quad Y_t = S_t + \Delta V_t$$

then holds. If there are no inventories, *i.e.* $V_t = 0$, production equals sales. The inventory stock increases (decreases), *i.e.* $\Delta V_t > 0$ ($\Delta V_t < 0$), if production exceeds (falls short of) sales.

Two types of production models are presented and discussed in this section. In the models presented a representative entrepreneur is assumed to minimize² production costs in addition to inventory costs. Production costs are incurred by using and acquiring capital stock, labour and materials. Inventory costs can result from *holding inventories* and/or *keeping inventories in line with sales*. The way to model these inventory costs adopted in the models here is according to HOLT, MODIGLIANI, MUTH and SIMON [1960].

The outline of this section is as follows. In the first part a PSM, and in the second part a FDMI is specified. In the third part differences between these two models are discussed. In the fourth part the model solutions are presented.

2.1. A Production Smoothing Model

Under neoclassical assumptions an entrepreneur is assumed to be rational, *i.e.* uses all information available at period t when making decisions, and aims at minimizing costs over an infinite horizon. The criterium function to be minimized is specified as

$$(2) \quad E \left\{ \sum_{h=0}^{\infty} \beta_{t+h} L_1(P_{M,t+h}, P_{I,t+h}, P_{N,t+h}, S_{t+h}, V_{t+h}) \mid \Omega_t \right\}$$

where

$$\beta_{t+h} \equiv \prod_{i=0}^h \frac{1}{1+r_{t+i}}.$$

L_{1t} denotes the restricted cost function, E indicates the expectations conditional on Ω_t , Ω_t the information set and r_t the going nominal interest rate³.

The entrepreneur aims at minimizing costs where the cost function in (2) is specified as

$$(3) L_{1t} \equiv [\psi_1 M_{Mt} + \psi_2 P_{It} + \psi_3 P_{Nt}] Y_t + P_t^q \left[\frac{1}{2} \omega_1 Y_t^2 + \frac{1}{3} \omega_2 Y_t^3 + \frac{1}{2} [\Delta Y_t \Delta^* V_t] \begin{bmatrix} \eta_{11} & \eta_{12} \\ \eta_{12} & \eta_{22} \end{bmatrix} \begin{bmatrix} \Delta Y_t \\ \Delta^* Y_t \end{bmatrix} \right]$$

where the definition

² Instead of a cost function to be minimized, a profit function to be maximized can be specified. In this study the cost minimizing approach is investigated. Main conclusions also uphold for the profit maximizing approach.

³ A nominal interest rate is taken here since L_{1t} in (2) is also in nominal terms.

$$(4) \quad \Delta^*V_t \equiv V_t - \theta_0 - \theta_1 S_t$$

and identity (1) hold. P_t^q , P_{Mt} , P_{It} and P_{Nt} are the nominal product price, the nominal price of materials (including energy), the nominal investment price and the nominal wage respectively. $\psi_1, \psi_2, \psi_3, \omega_1, \omega_2, \eta_{11}, \eta_{22}, \eta_{12}, \theta_0, \theta_1$ are parameters to be estimated.

The first term in (3) represents cost shocks in the factor prices, being the materials price, wages and the investment price. The shocks are proportional to production, Y_t . Together with the quadratic and cubic term of production they represent the costs associated with the level of production. The term $\eta_{11}\Delta Y_t^2$ represents the production adjustment costs.

$\eta_{22}(\Delta^*V_t)^2$ represents the objective of entrepreneurs to keep inventories in line with sales. This specification is according to HOLT et al. [1960] and in the following referred to as Holt's objective. It assumes that the level of inventories is kept in line with sales. Deviations of the "optimum" or "desired level of inventory stocks, being $V_t = \theta_0 + \theta_1 S_t$, give rise to costs. If $\theta_0 = \theta_1 = 0$, costs occur when the inventory level V_t changes.

Furthermore, costs associated with interrelations of ΔY_t and Δ^*V_t are adopted, *i.e.* $\eta_{12} \neq 0$, to catch asymmetries on both sides of the attractors $\Delta Y_t = 0$ and $\Delta^*V_t = 0$; asymmetries of the first are confirmed in the factor demand literature, for instance asymmetric adjustment costs, whereas the second asymmetry was confirmed by GRANGER and LEE [1989]. The multiplication of the latter terms in (3) by the nominal product price, indicates that (3) is in nominal terms.

Cost function (3) is more general than most other PSM's in the literature. For example WEST [1986] assumes $\psi_1 = \psi_2 = \psi_3 = \theta_0 = \omega_2 = \eta_{12} = 0$, EICHENBAUM [1989] assumes a stochastic process for the cost shocks and $\theta_0 = \omega_2 = \eta_{12} = 0$, and RAMEY [1991] assumes $\psi_2 = \theta_0 = \eta_{12} = 0$. Notice that Ramey is the only to specify a cubic term of production since $\omega_2 \neq 0$. This allows for (possible) non-convex production costs. DURLAUF and MACCINI [1993] assume $\psi_1 = \omega_2 = 0$ but include energy prices and additional inventory costs.

2.2. A Factor Demand Model with Inventories

The objective function of a FDMI can be specified as

$$(5) \quad E \left\{ \sum_{h=0}^{\infty} \beta_{t+h} L_2(P_{M,t+h}, P_{I,t+h}, P_{N,t+h}, S_{t+h}, K_{t+h}^*, K_{t+h}, N_{t+h}, V_{t+h}) \mid \Omega_t \right\}$$

where the discount factor β_t is defined as in (2). The physical capital stock used for production is represented by K_t^* , the potential physical capital stock by K_t , labour by N_t , and nominal investment by I_t . Other variables are represented by the same symbols as in the previous subsection. Materials will be represented by M_t .

In contrast with the PSM's, FDMI's specify (adjustment) costs in association with production factors. If the production factors materials, labour and capital are distinguished the cost function L_{2t} can be specified as

$$(6) \quad L_2(P_{Mt}, P_{It}, P_{Nt}, S_t, K_t^*, K_t, N_t, V_t) \\ = RC_t(P_{Mt}, S_t, K_t^*, N_t, V_t) + P_{It} I_t + P_{Nt} N_t + P_t^q AC_t.$$

Due to the multiplication of the product price by the adjustment costs, represented by AC_t (6) is nominal. RC_t represents the restricted cost function, specified as

$$(7) \quad RC_t(P_{Mt}, S_t, K_t^*, N_t, V_t) = \alpha_0 + \alpha_M P_{Mt} \\ + \frac{1}{2} \alpha_{MM} P_{Mt}^2 \\ + P_{Mt} [\alpha_{MS} S_t + \alpha_{MV} \Delta V_t + \alpha_{MK} K_t^* + \alpha_{MN} N_t] \\ + P_{Mt} \left[\frac{1}{2} \alpha_{SS} S_t^2 + \alpha_{SV} S_t \Delta V_t + \alpha_{SK} S_t K_t^* + \alpha_{SN} S_t N_t \right. \\ + \frac{1}{2} \alpha_{VV} (\Delta V_t)^2 + \alpha_{VK} \Delta V_t K_t^* + \alpha_{VN} \Delta V_t N_t \\ \left. + \frac{1}{2} \alpha_{KK} K_t^{*2} + \alpha_{KN} K_t^* N_t + \frac{1}{2} \alpha_{NN} N_t^2 \right],$$

for which regularity conditions should hold (see DIEWERT and WALES [1987]).

As in many other FDMI's it will be assumed that both capital and labour are quasi-fixed production factors. This implies that adjustment costs are occurred when capital is acquired or scrapped and employees are hired or fired. Materials are assumed to be a variable production factor.

If adjustment costs of capital concern gross investment, AC_t can be specified as ⁴

$$(8) \quad AC_t \equiv \frac{1}{2} [I_t \Delta N_t \Delta^* V_t] \begin{bmatrix} \gamma_{11} & \gamma_{12} & \gamma_{13} \\ \gamma_{12} & \gamma_{22} & \gamma_{23} \\ \gamma_{13} & \gamma_{23} & \gamma_{33} \end{bmatrix} \begin{bmatrix} I_t \\ \Delta N_t \\ \Delta^* V_t \end{bmatrix}.$$

This function AC_t is strictly convex if and only if the matrix $\Gamma := \{\gamma_{ij}\} (i, j = 1, 2, 3)$ is positive definite. The cross-coefficients in the specification allow for asymmetries in the factors. For example, if $\gamma_{12} > 0$, $\gamma_{22} > 0$ and $\gamma_{23} = 0$, the *hiring costs for labour* are higher than the *firing costs of labour* at a level where $I_t > 0$ since $\gamma_{12} I_t \Delta N_t + \gamma_{22} (\Delta N_t)^2$ is an asymmetric function around $\Delta N_t = 0$.

4. Notice that the "adjustment costs" include also the costs of keeping inventory stock in line with sales.

A further remark concerns the inclusion of capital as an endogenous production factor. Gross investment, I_t , is not included as a decision variable in the restricted cost function (7) since it is a function of K_t . Capital stock is assumed to be constructed according to the time-to-build specification of KYDLAND and PRESCOTT [1982]. See appendix C for more details.

The cost function L_{2t} can be explained as follows.

If the production factors materials, physical capital stock and labour are distinguished, the cost function is to be specified as

$$(9) \quad P_{M_t} M_t + P_{I_t} I_t + P_{N_t} N_t + P_t^q AC_t.$$

like in the PSM, the identity (1) and the definition (4) hold. The entrepreneur in this model then minimizes costs (9), over an infinite horizon, but faces the production restriction

$$(10) \quad Y_t = f(M_t, K_t^*, N_t).$$

f represents the production function that is assumed to satisfy the Inada conditions. $K_t^* \equiv K_t U_t$ is defined as the physical capital stock used for production, K_t is the potential capital stock and U_t is the utilisation rate of capital stock. It is important to account for the utilisation rate since in particular in recession periods K_t^* and K_t can differ much.

In order to substitute restriction (10) in the criterium function, materials –the production factor that does not incur adjustment costs– can be rewritten as a function of production, capital (K_t^*) and labour under regular conditions. This expression for materials is to be substituted in (9). Usually, though, the materials costs $P_{M_t} M_t$ are substituted for a “restricted cost” function (see for example DIEWERT and WALES [1987]). Such a restricted cost function is given in (6). In this way cost function (6) is similar to the cost function (9).

Cost function (5) is more general than most FDM’s in the literature. If inventories do not exist, *i.e.* $\Delta V_t = 0$ and consequently $Y_t = S_t$, the restricted cost function (7) is similar to that used by, for example, BERNDT, FUSS and WAVERMAN [1979] and MOHNEN, NADIRI and PRUCHAS [1986]. In comparison with RAMEY [1989] who investigates stage-of-processing inventory investment in a FDM *with* inventories, like the model here, two important differences exist. First, model (5) is not static but dynamics are specified by adjustment costs. Second, the capital stock is not predetermined but endogenous. The need to specify capital as decision variable and dynamics in both capital and labour is, for example, emphasized by BERNDT FUSS and WAVERMAN [1979] and PINDYCK and ROTHEMBERG [1983].

2.3. Differences between the PSM and FDMI

In most studies either a PSM or a FDMI is considered. Some PSM studies, though, refer to elements of the FDMI. For example, EICHENBAUM [1989] uses a PSM similar to (2) and concentrates on the nominal cost shocks. He compares a production-level smoothing model (without costs shocks) with a production-cost model *with* cost shocks. On the basis of

monthly time series of American sectors his main conclusion is that the latter model is preferable. Unfortunately –in my view– the cost shocks in his model are specified by a rather ad hoc stochastic process, being an autoregressive process of first order. As another example, DURLAUF and MACCINI [1993] incorporate price shocks of wages, materials and energy. In contrast with the FDMI's, they do not consider production factors but estimate coefficients (like ψ_1, ψ_2, ψ_3).

Clearly, the PSM and the FDMI aim to describe the same objective, which is the minimization of production costs and costs associated with inventories. Thus from an economic point of view both models are similar.

Also the specifications of the objective functions, L_{1t} and L_{2t} , seem to be similar at first glance. Y_t is a function of M_t, K_t^* and N_t (see (10)). By substituting (10) in (2) the criterium function (2) and (5) contain thus the same arguments. Costs of production consists of the costs of employing capital, employees, materials as well as adjustment costs. In addition to these costs, lump sum costs –for example rents or interest costs– may exist but are irrelevant in the minimisation process because they are fixed costs. Except for the lump sum costs, costs associated with the *production* should thus be equivalent to the costs associated with all *production factors*. but the specifications of the PSM (2) and the FDMI (5) are not equivalent. Neither the *production level* costs nor the *adjustment* cost specifications are the same.

Let us first compare the production level costs. To simplify, assume that $\omega_2 = 0$, and that inventories do not exist by which $\Delta V_t = 0$ and $Y_t = S_t$. In the PSM costs associated with the level of production equal (see (3))

$$[\psi_1 P_{Mt} + \psi_2 P_{It} + \psi_3 P_{Nt}] Y_t + 1/2 \omega_1 P_t^q Y_t^2.$$

In the literature on PSM's the terms between square brackets are said to represent “cost shocks” to the production. These shocks are estimated since ψ_i for $i = 1, 2, 3$ are unknown parameters. To estimate these parameters a normalisation rule is to be chosen⁵. In the FDMI the level costs equal (see (6))

$$RC_t(P_{Mt}, S_t, K_t^*, N_t, V_t) + P_{It}I_t + P_{Nt}N_t.$$

unlike the level costs in the PSM, the variable costs $P_{It}I_t + P_{Nt}N_t$ are observed. A normalisation rule is thus not necessary!

Like these “level” costs, adjustment costs (see (8)) are specified in more detail in the FDMI than in the PSM (see (3)). Only if a linear production function with *one* production factor holds in (10), for example $Y_t = \alpha_1 N_t$, the two adjustment cost specifications are equal.

Another issue concerning the production level as well as the adjustment costs is that interrelations are differently specified in the two models. This is easy to see by considering a simple example.

Suppose that production is linearly homogeneous in the production factors capital and labour and specified by $Y_t = \alpha K_t + (1 - \alpha) N_t$. in this case the level costs of production are investment costs $P_{It}I_t$ and labour costs

5. For more details on this issue, see section 3.

$P_{Nt}N_t$. Investment is thus *not* interrelated with the variable labour costs. In the PSM, however, these costs are specified as $(\psi_1 P_{It} + \psi_2 P_{Nt}) Y_t$. Here investment (labour) costs affect production and are thus interrelated with labour (investment). A similar comparison of the adjustment costs of both models shows that the specifications can only coincide for very specific parameter choices. This follows from the fact that adjustment costs in the PSM are

$$\eta(\Delta Y_t)^2 = \eta\alpha^2(\Delta K_t)^2 + \eta(1-\alpha)^2(\Delta N_t)^2 + \eta\alpha(1-\alpha)(\Delta K_t\Delta N_t),$$

whereas in the FDMI these costs are

$$\gamma_{11}(\Delta K_t)^2 + \gamma_{22}(\Delta N_t)^2 + \gamma_{12}\Delta K_t\Delta N_t.$$

To summarize, in comparison with the PSM the specification of the objective function in the FDMI is more specific. More structure is imposed on the production level costs as well as on the adjustment cost specification. Variable costs associated with the production factors are appropriately specified and arbitrary normalisation rules are not needed.

Another difference concerns the decision variables. The number of first order conditions equals four in the FDMI whereas in the PSM only one first order condition results. This is shown in the next subsection.

2.4. The Model Solutions

In the PSM sales and prices are assumed to be exogenous to the entrepreneur. For this reason only Y_t and V_t in (3) are endogenous variables. The two equality restrictions (1) and (4) can be substituted in (3). The restricted cost function L_{1t} then does not depend on Y_t . So Y_t is not an argument of L_{1t} in (2). The only endogenous variable left in the model is then the inventory stock. The first order condition of the PSM (2) is given by differentiating with respect to V_t , *i.e.*

$$(11) \quad E \left\{ \psi_1 \left[P_{Mt} - \frac{\beta_{t+1}}{\beta_t} P_{M,t+1} \right] + \psi_2 \left[P_{It} - \frac{\beta_{t+1}}{\beta_t} P_{I,t+1} \right] + \psi_3 \left[P_{Nt} - \frac{\beta_{t+1}}{\beta_t} P_{N,t+1} \right] + P_t^q \left[\omega_1 \left(Y_t - \frac{\beta_{t+1}}{\beta_t} Y_{t+1} \right) + \omega_2 \left(Y_t^2 - \frac{\beta_{t+1}}{\beta_t} Y_{t+1}^2 \right) + \eta_{11} \left(\Delta Y_t - 2 \frac{\beta_{t+1}}{\beta_t} \Delta Y_{t+1} + \frac{\beta_{t+2}}{\beta_t} \Delta Y_{t+2} \right) + \eta_{22} (V_t - \theta_0 - \theta_1 S_t) + \eta_{12} \Delta Y_t + \eta_{12} \left(V_t - \theta_0 - \theta_1 S_t - 2 \frac{\beta_{t+1}}{\beta_t} (V_{t+1} - \theta_0 - \theta_1 S_{t+1}) + \frac{\beta_{t+2}}{\beta_t} (V_{t+2} - \theta_0 - \theta_1 S_{t+2}) \right) \right] \mid \Omega_t \right\} = 0.$$

Legendre-Clebsch conditions are obtained by differentiating (11) with respect to V_t . Second order conditions for marginally increasing costs

in Y_t can be found by differentiating (2) twice with respect to Y_t (see also RAMEY [1991]). These necessary conditions for an optimal solution to (4), can be verified after estimation.

For the FDMI (6) the first order conditions for capital, labour and inventories are given by ⁶.

$$(12a) \quad E \left\{ \sum_{j=0}^3 \frac{\beta_{t+j}}{\beta_{t+2}} \varphi_{3-j} [P_{I,t+j} + \gamma_{11} P_{t+j}^q I_{t+j} + \gamma_{12} P_{t+j}^q \Delta N_{t+j} + \gamma_{13} P_{t+j}^q \Delta V_{t+j}] + \frac{\partial RC_{t+2}}{\partial K_{t+2}} \Big| \Omega_t \right\} = 0$$

$$(12b) \quad P_{Nt} + P_t^q [\gamma_{12} I_t + \gamma_{22} \Delta N_t + \gamma_{23} \Delta V_t] + \frac{\partial RC_t}{\partial N_t} - E \left\{ \frac{\beta_{t+1}}{\beta_t} P_{t+1}^q [\gamma_{12} I_{t+1} + \gamma_{22} \Delta N_{t+1} + \gamma_{23} \Delta V_{t+1}] \Big| \Omega_t \right\} = 0$$

$$(12c) \quad P_t^q [\gamma_{13} I_t + \gamma_{23} \Delta N_t + \gamma_{33} \Delta V_t] + \frac{\partial RC_t}{\partial V_t} - E \left\{ \frac{\beta_{t+1}}{\beta_t} P_{t+1}^q [\gamma_{13} I_{t+1} + \gamma_{23} \Delta N_{t+1} + \gamma_{33} \Delta V_{t+1}] + \frac{\beta_{t+1}}{\beta_t} \frac{\partial RC_{t+1}}{\partial V_t} \Big| \Omega_t \right\} = 0.$$

Shephard's lemma for the variable production factor materials,

$$(12d) \quad M_t = \frac{\partial RC_t}{\partial P_{Mt}},$$

also holds.

3 Empirical Results

The outline of this section is as follows. In the first part the data are described. In the second part attention is paid to the estimation strategy. In

6. Notice that U_t is not a decision variable since it is determined by the optimal choices of K_{t+2} , N_t , and M_t . For the derivation of (12a), see also appendix C.

the third part the GMM results for the models (11) and (12) are presented. Test statistics are given in the fourth part.

3.1. Data

The data are seasonally adjusted quarterly time series of the period 1970.I-1992.IV from five industrial sectors in France. The sectors are the intermediate goods, the professional equipment, the consumers equipment, the transport equipment and the consumption goods sector. Except for the data on the utilisation rate and the official discount rate, all series come from the National Accounts. A detailed data description is given in appendix A-B.

Before using the time series to estimate the models presented in section 2, descriptive statistics are calculated and multi-cointegration tests according to GRANGER and LEE [1989] are carried out. These results can be obtained upon request.

One finding from these analyses is that inventories (in levels) are only a small part of production but, in comparison with sales, turn out to be very erratic. Except for the intermediate goods sector, inventories seem to “smooth” production. This implies that the correlation between sales and changes in the inventory stock is *positive* in the intermediate goods sector. Furthermore, for all sectors multi-cointegration tests show that production, sales and inventory stock cointegrate. From this follows that inventory stock move in line with sales which is a necessary condition for the specification of HOLT *et al.* [1960] to hold.

3.2. Estimation Strategy

The first order conditions (11) and (12a)-(12d) are estimated by the Generalized Method of Moments⁷ of HANSEN [1982] using the GMM-routine of TSP 4.2b. A convergence tolerance of 0.001 chosen.

The PSM has only the first order condition (11). This equation contains variables two periods in the future (Y_{t+2} , S_{t+2} and V_{t+2}). So a substitution of these expected future variables by their realisations can entail a residual that is a second order moving average. The FDMI has the four first order conditions (12a)-(12d). Because of the time-to-build (see appendix C), equation (12a) contains variables of at most three periods in the future. The second and the third equation have only variables one period in the future. The fourth equation has none.

One important remark needs to be made about serial correlation. The residuals can be autocorrelated because of specification errors or for example, persistent technology or persistent sales shocks that are incorporated in Y_t

7. The PSM and FDM can also be estimated by a Full Information Maximum Likelihood method. In this case the demand process must be specified, which is difficult because much less consensus exists on demand than on cost specifications. See also WEST [1993], who compares full and limited estimation methods for the PSM, and tries to explain the differences in estimation results of various authors, that are due to the normalisation rules.

and S_t but not explicitly modelled. For example BLINDER [1986] emphasizes the existence of persistent sales shocks, in particular in macroeconomic time series. GREGOIR and LAROQUE [1992] also emphasize the importance of non-stationarities in production, sales and inventory series. Corrections for autocorrelation seem therefore necessary unless residuals do not have a unit root and consequently cointegration between all variables in the models exists.

The first model is estimated by making corrections for autocorrelation (by quasi-differencing (11)) of fourth, third, second and first order. As these experiments show that the residuals turn out to be non-stationary, (11) is further estimated in first differences for all sectors. The moving average in the PSM is thus of third order.

Corrections for autocorrelation in the FDMI are made by quasi-differencing once each equation. Their autocorrelation coefficients are referred to as ρ_1 , ρ_2 , ρ_3 and ρ_4 . The moving average order for the four equations is then four, two, and one (provided that all ρ 's are significant). It turns out that a trade-off with the dynamic part (8) clearly exists. Consequently, as the "right" order of autocorrelation in the disturbance is difficult to disentangle from the dynamics specified by adjustment costs, the assumption $\rho_1 = \rho_2 = \rho_3 = \rho_4 = 1$ is made. In line with the PSM therefore, the residuals are assumed to have a unit root. This detrending method used here differs from the deterministic detrending methods used elsewhere (see for instance EICHENBAUM [1989] and RAMEY [1991]).

Instrumental variables used in both models are an intercept, Y_t , S_t , ΔV_t , production in value added, I_t , N_t , M_t , P_{It} , P_{Nt} , P_{Mt} and P_t^q , all of each sector under investigation, GNP and national gross investment and national value added, all in constant and current prices. Except for the intercept, they are differenced once to satisfy conditions of ergodicity. For both models (11) and (12), instruments are lagged two quarters⁸ to account for the moving average error structures imposed by the theoretical model. When calculating the weighting matrix, a moving average of fourth order is accounted for in the FDMI.

Because of the moving averages, the Parzen-kernel is used in order to guarantee the positive definiteness of the weighting matrix. Corrections are made to account for the heterogeneity in the FDMI.

When estimating the PSM, a normalisation is necessary since the optimal solution for model (11) without restrictions is the solution where all parameters equal zero. Instead of fixing one parameter in the adjustment part (see for example BLANCHARD [1983] who fixes $\eta_{11} = 1$ or RAMEY [1991] who fixes $\eta_{22} = 1$), the normalisation $\psi_1 = \psi_2 = \psi_3 = 1$ is chosen. This rule is more in line with (6) because –like in (12)– the factor costs are then fully taken into account (see also (11)).

Unfortunately, the number of observations in the FDMI is less than the number of observations in the first model. The utilisation rate of capital

8. taking instruments two periods lagged, in (12a) a MA(4), in (12b)-(12c) a MA(2) and in (12d) a MA(1) is assumed.

stock exists only from 1977 onwards (see appendix B).

3.3. GMM Estimation Results

TABLE 1

GMM-Estimates PSM (see (11)) in first differences

Parameters	Interm. goods	Prof. equip.	Cons. equip.	Trans. equip.	Cons. goods
ω_1	-2.96* (-1.88)	-2.46* (-1.76)	-2.02 (-1.09)	-1.92* (-1.77)	-3.03* (-1.87)
ω_2	1.22 (1.49)	0.49 (0.66)	0.69 (0.95)	0.69* (1.55)	1.19 (1.31)
θ_0	-5.91 (-0.78)	-34.04 (-0.01)	5.19 (0.51)	-28.47 (-0.23)	-1.19 (-0.21)
θ_1	4.36 (0.86)	7.39 (0.01)	-0.84 (-0.29)	-1.11 (-0.41)	4.68 (0.28)
η_{11}	0.07 (0.51)	0.37 (1.34)	0.47* (1.50)	0.03 (0.14)	0.28 (1.29)
η_{12}	0.01 (0.34)	0.0001 (0.0001)	-0.02 (-0.21)	0.04 (0.26)	-0.03 (-0.27)
η_{22}	-0.08 (-0.92)	-0.0002 (-0.001)	0.09 (0.44)	-0.001 (-0.23)	-0.03 (-0.19)
J	10.12	11.97	3.27	10.13	6.43

* Significant at the 10%-level.

Figures in brackets are *t*-values. J is the test-statistic for overidentifying restrictions with 11 (= 18-7) degrees of freedom. Number of observations is 84.

The GMM-estimates of the PSM (11) and the FDMI (12) are presented in table 1 and table 2 respectively. In the first panel of table 1 the parameters associated with the costs shocks, in the second panel the parameters associated with Holt's objective and in the third panel the production cost parameters are presented. In the first panel of table 2 the production function parameters, in the second panel the parameters associated with Holt's objective and in the third panel adjustment costs parameters are presented.

When estimating the PSM models, for each sector separately, no convergence problems were encountered. In the FDMI cross equation restrictions exist, that were imposed during estimation. When estimating this model, for four sector no convergence could be reached without imposing additional restrictions. Experiments showed that Holt's objective, *i.e.* minimizing $\Delta^*V_t = V_t - \theta_0 - \theta_1 S_t$, is the main cause for these convergence problems. For this reason, the restriction $\theta_0 = \theta_1 = 0$ was imposed for these four sectors. Only for the transport equipment sector θ_0 and θ_1 are estimated (see table 2).

In the last row of table 1 and 2 the J-statistic is given, for each sector. This is a test statistic for the overidentifying restrictions (see HANSEN [1992]).

TABLE 2

GMM-Estimates TDM Model (see (12)) in first differences.

Parameters	Interm. goods	Prof. equip.	Cons. equip.	Trans. equip.	Cons. goods
α_{mm}	-0.19*	0.05	-0.68*	0.04	-0.23**
α_{md}	1.79	-1.06	1.96*	0.67	-3.37*
α_{mv}	-0.02	0.05**	0.01**	-0.08**	0.01
α_{mk}	-0.61	-0.82	-0.40	-1.92**	1.68*
α_{mn}	0.76	0.50	-0.91	0.40	2.00
α_{dd}	-2.03	1.52	-0.85	0.17	4.61**
α_{dv}	0.01	-0.004	-0.001	-0.01	-0.01
α_{dk}	0.11	-0.54	-0.18	-0.12	-0.78
α_{dn}	1.09**	1.09	-0.13	0.19	0.50
α_{vv}	0.0003	-0.001**	-0.00001	-0.0004	-0.00001*
α_{vk}	0.0002	-0.03**	-0.00004	0.03**	0.001
α_{vn}	-0.005	-0.04**	-0.00004	0.003**	-0.004*
α_{kk}	1.28*	1.31*	0.44	1.51**	-0.17
α_{kn}	-0.37	0.11	0.01	0.77**	-0.83
α_{nn}	-1.69**	-1.76	0.22	-1.38**	-2.29*
θ_0	[0]	[0]	[0]	3.62**	[0]
θ_1	[0]	[0]	[0]	0.30	[0]
γ_{11}	0.09*	0.16	2.19*	0.17**	0.01
γ_{12}	-0.004	0.33	0.08	-0.03	0.93**
γ_{13}	-0.0003	0.04**	0.003	-0.01**	0.02**
γ_{22}	3.31*	3.12	-0.18	0.51	-4.92
γ_{23}	0.03	-0.16	0.003	-0.05	-0.10
γ_{33}	0.005	0.04	0.002	-0.04**	-0.01
J	22.39	37.53	29.81	22.98	33.96

*Significant at 10%-level

** Significant at 5%-level

Figures in square brackets are the numbers at which the parameter is fixed. J is the test-statistic for overidentifying restrictions with 49 ($=4 \cdot 18 - 23$) degrees of freedom for the transport equipment sector and 47 for the other sectors. Number of observations in 63.

These statistics are χ^2 distributed and have 11 and 49 freedom for the PSM and the FDMI respectively. The results show that the estimated PSM and FDMI is not rejected, even at the 10%-level. This holds for all sectors.

As follows from table 1, the parameter estimates of the PSM are very poor according to the very low t-values, presented in brackets. The production cost parameters, estimated by ω_1 , ω_2 and η_1 , η_2 , η_3 are significant for some sectors, but only at a 10%-level.

The poor performance of this model is remarkable. Other studies where PSM are estimated, see for example WEST [1987], RAMEY [1991] or EICHENBAUM [1989] show highly significant parameter estimates. The model here is even a more general model of the models presented in these studies. Our major difference between the estimations here and these other studies is the use of quarterly instead of monthly data. The reason for using quarterly data is that investment and employment data are not available a higher frequency. Another difference is that the other studies use at deterministic detrend methods, whereas first differences are taken here. Experiments with other detrend methods with the French data used here, do however not give

better estimation results. Also analyses with pooling the data of the five sectors, and changing the set of instruments, do not improve the estimation results obtained with the PSM. Neither do other normalisation rules give results that are more in line with the findings in the other (American) studies.

In comparison with the PSM results in table 1, the FDMI in table 2 shows more significant parameter estimates. Many production and adjustment cost parameters are significant at the 5%-level.

Remarkable from the results obtained with the FDMI is the finding of negative adjustment costs for the intermediate goods and –slightly negative costs– for the consumption goods sector. This follows from calculating the sample average adjustment costs for each sector. These result (not presented here) are in contradiction with the objective function of the theoretical models since negative costs makes increasing production profitable. Producers of the French intermediate goods sector seem thus to bunch production, possibly because of scale advantages. These results are in line with RAMEY [1991], who also found evidence for production bunching in the American industry.

3.4. Test statistics

This comparison of the parameter estimates of the PSM and the FDMI, obtained with the same estimation method and the same set of instruments, shows thus the preference of the FDMI and corroborates the expectations mentioned in section 2.3. The results hold despite the fact that in the FDMI less observations are used, *i.e.* 63 instead of 84 (because of the utilisation data, see section 3.2).

A more direct comparison of the PSM and the FDMI is not possible. After all, imposing zero-restrictions in the FDMI does not reduce the FDMI to a PSM and, vice versa, substituting a production function in the PSM does not lead to the type of FDMI that we have under investigation here.

The two models are thus not nested and therefore non-nested tests should be carried out. But in order to do so both models must have the same number of moment conditions. One condition for carrying out non-nested GMM tests is that the same number of moment conditions, *i.e.* the number of equations times the number of instruments, holds. As the PSM has one (see (11)), and the FDMI has four first order conditions (see (12)), this non-nested test is not applicable.

In order to make a comparison, therefore a general model is specified. The two models (11) and (12), hence five equations, are jointly estimated. The hypothesis that the PSM is the correct model is then tested against the alternative hypothesis that this general model is the correct model. This is carried out as a *nested* Likelihood Ratio test. The general model is estimated under the alternative hypothesis and the general model with zero restrictions for the FDMI is estimated under the null-hypothesis. So the same number of moment restriction exists under both hypotheses. The two J-statistics are then subtracted, and a LR test statistic has been obtained that is χ^2 distributed. In a similar way, the hypothesis that the FDMI is the correct

model is tested against the general model. These tests are easy to apply and give us an indication about the performance of both models.

The results of these LR-tests show that except for the consumer equipment sector, the PSM is rejected against the general model. So for the consumers equipment sector the test is not decisive but for each other sector the general model is preferred to the PSM. The results of the LR-tests show further that the FDMI is accepted against the general model (at the 10%-level). This holds for each sector, except for the consumers equipment sector where some convergence problems were encountered when estimating the general model as well as the (nested) FDMI. So for the other sectors the FDMI is preferred to the general model.

As these results corroborate the main findings in the preceding section that the FDMI is preferred to the PSM, the test statistics are not presented but can be obtained upon request.

TABLE 3

Wald Tests with FDM.

Parameters	Interm. goods	Prof. equip.	Cons. equip.	Trans. equip.	Cons. goods
H_{01}	191.21*	50.67*	37.14*	112.57*	191.52*
H_{02}	0.02	2.71	2.56	63.40*	2.26

*Significant at 5%-level

$$H_{01} : \alpha_{SS} = \alpha_{VV} = \alpha_{SV} \wedge \alpha_{SK} = \alpha_{VK} \wedge \alpha_{SN} = \alpha_{VN}$$

$$H_{02} : \theta_0 = \theta_1 = \gamma_{13} = \gamma_{23} = \gamma_{33}$$

For the analyses here it is further important to know whether the explanatory power of inventories in the model is high. If the inventory costs modelled in (6) are not significant, there is no necessity to take them into account.

In order to test for the significance of inventories, Wald tests are applied. The results of two tests are given in table 3. The first hypothesis imposes the restriction that there are no inventory costs in the restricted cost function, *i.e.* $H_{01} : \alpha_{SS} = \alpha_{VV} = \alpha_{SV}, \alpha_{SK} = \alpha_{VK}, \alpha_{SN} = \alpha_{VN}$. The second hypothesis imposes the restriction that there are no inventory costs in the adjustment cost function, *i.e.* $H_{02} : \theta_0 = \theta_1 = \gamma_{13} = \gamma_{23} = \gamma_{33} = 0$. This latter test is a test for the importance of the objective of HOLT et al. [1960] of keeping inventories in line with sales.

The results in table 3 show that the first hypothesis is rejected at the 5%-level for all sectors. Specifying inventory costs as a part of the restricted cost function is thus important. The results in table 3 show that the second hypothesis is only rejected for the transport equipment sector. This is the only sector where θ_0, θ_1 did not cause convergence problems and were estimated. The objective of Holt et al., *i.e.* $\Delta V_t^* = V_t - \theta_0 - \theta_1 S_t$, can thus be omitted for the intermediate goods, the professional equipment, the consumers equipment and the consumer goods sector.

4 Conclusions

In literature PSM's are often used to analyze the decisions of entrepreneurs concerning production and inventory decisions, see for example EICHENBAUM [1989], DURLAUF and MACCINI [1989], WEST [1986, 1993) and WEST and WILCOX [1993]. In this study PSM's are compared with FDMI's.

A comparison of the model specifications shows that the production level and the production adjustment costs are clearly different. The production level costs are more appropriately specified in the FDMI. Data information on factor prices is taken into account by which the cost structure is observed instead of estimated. Normalisation rules are therefore not needed in FDMI's. This is important since in particular WEST [1993] emphasizes the strong sensitivity of the estimation results in PSM's due to these rules. As another advantage, production adjustment costs are more "structurally" specified in the FDMI.

From an econometric point of view, the FDMI seems thus preferred to the PSM. The estimation results with French sectorial data confirm this. The parameter estimates obtained with the FDMI are more efficient than those obtained with the PSM. Casting the two models in a general model and carrying out nested LR-tests also shows that the FDMI turns out to perform better than the PSM.

The empirical results in this study could of course be improved. For example, the models estimated here could be analyzed by using seasonally unadjusted, higher frequency and less aggregated data. Questions like "do inventories smooth production", or "do inventories smooth production factors" can then be investigated. As shown in this study, an investigation of such issues are preferably carried out with a FDMI rather than with a PSM.

APPENDIX

Quarterly Data for French Sectors 1970.I-1992.IV

A Variables

- Y* Production in constant prices;
S Sales in constant prices;
V Inventory investment in constant prices;
I Gross fixed capital formation (GFCF), in constant prices;
N Average weekly working hours, that in $NP \star NH$ where *NP* and *NH* equal respectively the number of all employees and the average weekly hours of work;
M Materials, including energy;
P_I Nominal price of gross investment, that is investment (GFCF) in current prices divided by investment (GFCF) in constant prices;
W Nominal hourly wages;
P_M Nominal price of materials (including energy);
P^q Producer price, calculated as the sales in values divided by the sales in volumes;
U Physical capital stock utilisation rate;
r Nominal official discount rate, yield of government bonds.

B Data description

The interest rate is from (Tendance de la Conjoncture INSEE). *TC*, *U* is from an INSEE database and all other variables are from (les Comptes Nationaux Trimestriels, INSEE) *CNT*. The data are seasonally adjusted with the X11-ARIMA method. Constant prices have as a base year 1980. In the econometric analyse (see section 3) variables are indexed at 1985.II.

The following sectors are included in the analyses:

- S2* Industry of intermediate goods;
S3 Industry of professional equipment;
S4 Industry of consumers equipment;
S5 Industry of transport equipment;
S6 Industry of consumption goods.

These sectors can be found in *CNT* with the French branch codes *U04*, *U05A-U05C* *U06*. Sectors *S3-S5* together equal the industry of investment goods. The manufacturing industry comprises all sectors *S2-S6*.

Each sector has a national account, in constant and current prices, that looks as:

Assets	Liabilities
Consumption	Production (=Value added+Intermediate goods)
Gross investment	Imports
Government spending	Commercial margin
Exports	Taxes
Variations in stocks	

In the analyses, $Y \equiv$ production, $\Delta V \equiv$ variation in inventories and $S \equiv Y - \Delta V$, all in constant prices. The utilisation rate, U , is calculated from quarterly (unadjusted) margins of the available physical capital stock capacity for 1976.I-1992.IV. To obtain a value for the inventory stock a benchmark V_0 is taken and $V_t = V_0 + \sum_{i=1}^t \Delta V_i$ is generated. The benchmark is the value of inventories in 1980, obtained from surveys on the individual firm level. Each firm in this survey is classified according to its main activity, the so called “sector” classification. On the contrary, the data of the national accounts are divided into parts according to the different final products made. Each part is classified according to the type of product. Because of this so called “branch” classification a translation had to be made from sectors to branches to use the benchmarks V_0 of the sector classification. As a consequence of the use of these branch inventory data, backlogs of ordered but not yet delivered goods are not included.

C Calculation of physical capital stock series

It is assumed that capital is built stagewise according to KYDLAND and PRESCOTT [1982]. Strong further motivations for assuming time-to-build and the time-to-build scheme discussed below are found in ALTUG [1989] and PEETERS [1995].

To construct series that account for gestation lags it is assumed that the construction period is three quarters. This is close to KYDLAND and PRESCOTT [1982] who use four quarters for macroeconomic capital series, and to ALTUG [1989] who uses four quarters and one quarter for macroeconomic structures and equipment series respectively. For each sector the capital stock of 1970 is used as a benchmark (CNT) and $\kappa = 0.025$ (so $D_{t-1} = 0.025 K_{t-1}$). Gross investment are from CNT. The distribution of investment during construction is fixed at $\delta_1 = 1/6$, $\delta_2 = 2/6$, $\delta_3 = 3/6$, so proportionally decreasing during the construction period. This declining investment scheme during the construction period is chosen according to findings in Peeters (1995). For the initial quarter it is assumed that $\mathbf{P}_{1,1970.1} = \mathbf{P}_{2,1970.1} = \mathbf{P}_{3,1970.1} = \mathbf{P}_{4,1970.1} = \mathbf{P}_{1970.}$, As the multi-period time-to-build specification is used it holds that

$$I_t = \sum_{j=0}^3 \varphi_j K_{t+j-1} \quad \text{where} \quad \varphi_0 \equiv \delta_1 (\kappa - 1)$$

$$\varphi_j \equiv \delta_{j+1} (\kappa - 1) + \delta_j \quad \text{for} \quad j = 1, 2$$

$$\varphi_3 \equiv \delta_3.$$

κ is the constant depreciation rate. $\delta_3, \delta_2, \delta_1$ describe the distribution of investment during construction and are fixed at 3/6, 2/6 and 1/6. This implies that half of the investment occurs during the first period of the construction. As a consequence of this time-to-build, the variable K_{t+2} instead of K_t is the decision variable at t . It follows further that

$$\frac{\partial I_{t+i}}{\partial K_{t+2}} = \sum_{i=0}^3 \varphi_{3-i} \quad \text{for} \quad i = 0, 1, \dots, 3,$$

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