

# Segregation and Labor Discrimination in Cities

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**ABSTRACT.** – The aim of this paper is to analyze segregation and labor discrimination in a monocentric city. We first show that, if some categories are systematically discriminated in the labor market (such as blacks, young people, long-run unemployed...), they will also be spatially segregated at the vicinity of the CBD. Second, we perform comparative statics to examine how urban configuration is modified in equilibrium by changes in labor market characteristics. Finally, we contemplate two policies that the (local) government can settle in order to alleviate residential segregation and labor discrimination.

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## Ségrégation et discrimination sur le marché du travail dans les villes

**RÉSUMÉ.** – L'objet de cet article est d'analyser la ségrégation et la discrimination sur le marché du travail dans une ville monocentrique. Nous montrons tout d'abord que si certaines personnes sont systématiquement discriminées sur le marché du travail (comme par exemple les jeunes, les minorités...), elles se retrouveront confinées autour du centre des villes. Ensuite, nous faisons des exercices de statistique comparative afin d'analyser la modification de la configuration urbaine d'équilibre suite à des variations de certaines caractéristiques sur le marché du travail. Enfin, nous étudions deux types de politique que le gouvernement (local) peut mettre en œuvre pour éviter les problèmes de ségrégation résidentielle et de discrimination sur le marché du travail.

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# 1 Introduction

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The working of the labor market has not yet been extensively taken into account by theoretical urban economists. Indeed, the urban literature has focused mostly on optimal locations of agents and their interactions in the city. Consequently, the labor market is supposed to clear and no unemployment appears in equilibrium. Recently, some authors have tried to introduce unemployment in an urban setting by either considering it as a risk (DESALVO and EECKHOUDT [1982] or ZENOU and EECKHOUDT [1997]) or by giving it microfoundations (ZENOU and SMITH [1995] and SMITH and ZENOU [1997])<sup>1</sup>. The present paper sets itself in this line of research by studying a specific problem which links urban economics and labor market, namely residential segregation and labor discrimination in a monocentric city.

It is commonly observed in North American cities that segregation is correlated with socioeconomic differences between central city and suburban residents. On average, suburbanites are better-educated, have higher incomes, hold higher-status jobs, are more likely to be employed, and are less likely to belong to racial minorities than central-city dwellers (see for e.g. Summers and Luce [1987]). One of the explanations commonly put forward is that residential segregation is sustained primarily by differentials in the price of housing between the two types of communities (Kain [1968]). Recently, Bénabou [1993] shows that different social classes tend to segregate because of a tendency toward social homogenization, reinforced by local externalities such as human capital investment.

Our paper aims to analyze both segregation and labor discrimination in a monocentric city. Since in the former approaches the modelization of the urban configuration is quite crude, we propose here to explicit the location of heterogeneous workers in the city and to determine the city equilibrium. Workers living in the city are assumed to belong to one of the following three categories: the employed, the discriminated unemployed and the non-discriminated unemployed. Each worker faces uncertainty in the next period since he does not know which occupational status he will have. Each employed worker has a (low) probability to lose his job. The main difference between the discriminated and the non-discriminated unemployed workers is their access to employment due to the external appearance of the discriminated (women, black people...): the probability of finding a job is thus lower for the discriminated unemployed workers.

We first show that the discrimination of a category of workers leads to residential segregation at the vicinity of the central-city. Second, we perform comparative statics exercises and examine how urban configuration is modified at equilibrium by changes in labor market characteristics. In particular, it is shown that *lowering the probability of being unemployed for discriminated unemployed workers rises their utilities but reduces the*

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1. For a survey of the literature integrating labor and urban economics, see ZENOU [1996].

*utilities of the other categories of workers.* This reduction in the probability has another effect which is to spread the city out. Finally, we analyze what type of policies the (local) government can perform in order to fight against segregation and labor discrimination. Helping the prejudiced unemployed through housing or hiring wage subsidy schemes prevents residential segregation only, whereas setting up skill-training programs allows to decrease both segregation and labor discrimination in the city.

The remainder of the paper is the following. Section 2 presents the urban model of uncertainty. In section 3, we show that discriminated unemployed workers are segregated around the CBD and we analyze the comparative statics. Section 4 is devoted to the sketching of two distinct policies by the local government, aiming to fight against segregation and labor discrimination in the city. Section 5 concludes.

## 2 The Model

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The city is monocentric with a center called the Central Business District (CBD hereafter) where all – identical – firms are located. Individuals locate outside of the CBD on a featureless homogeneous plain and travel to the CBD for work and shopping. If there were neither uncertainty nor unemployment, agent  $i$  in the city would solve the following program

$$(1) \quad \begin{aligned} & \text{Max}_{z_i, q_i, x} U(z_i, q_i) \\ & \text{s.t. } w_i = z_i + R(x)q_i + T(x), \end{aligned}$$

where  $z_i$  is the composite good for agent  $i$  taken as the numeraire,  $q_i$ , the consumption of land or the lot-size of the house for agent  $i$ ,  $x$ , distance from the CBD,  $w_i$ , the urban wage for agent  $i$ ,  $R(x)$ , the land rent price at distance  $x$  from the CBD and  $T(x)$ , the pecuniary commuting cost at distance  $x$  from the CBD, with  $T'(x) > 0$  and  $T''(x) \leq 0$ . We assume that  $U(\cdot)$  is continuous, twice differentiable, increasing in its arguments and strictly concave. Observe that the three types of agents have identical pecuniary travel costs and identical preferences for land and thus the same utility functions. Replacing  $z_i$  by its expression given by the budget constraint, (1) rewrites

$$(2) \quad \text{Max}_{q_i, x} U(w_i - T(x) - R(x)q_i, q_i).$$

We assume now that there is a durable level of unemployment in the city because CBD firms set a wage  $w$  at a higher level than the market clearing wage (due for example to the constraint of a legal minimum wage). We assume also that there are two types of unemployed workers differentiated according to their probability of finding a job. According to

what we observe in the labor market, we can interpret this differentiation as short run unemployed versus long run unemployed or as young versus older unemployed or female versus male or lastly as black versus white unemployed. The only thing that matters in the discrimination process in that firms observe perfectly workers' characteristics in term of external appearance. It is indeed commonly observed that first, one of the major problem in the labor market is the difficulty of access to employment for certain categories of unemployed workers, and second, these unemployed tend to be spatially segregated (for e.g., in North America, these "ghettos" are found generally at the vicinity of the CBD). Consequently, we will explicitly assume hereafter that urban workers belong to one of the three following groups: employed, non-discriminated unemployed, discriminated unemployed, respectively labelled group 0, group 1, group 2.

Our problem is the following. Each individual in the city faces uncertainty for the next period in term of occupational status. If someone is employed today, will he still be employed tomorrow? What is the probability for a discriminated unemployed to be employed tomorrow? In order to answer these questions, we use the model of Drèze and Modigliani [1972] of consumption decisions under uncertainty. They develop a delayed uncertainty model where the individual chooses today (ex-ante) an optimal value of consumption before knowing his future income. In an urban setting, this means that the individual decides ex-ante the level of his optimal housing demand, his location in the city and his bid rent, and cannot change his decision ex-post when the risk is known. Consequently, in an adverse state (being unemployed), he will reduce his consumption of non-housing non-transportation goods because his location in the city is already set (see DESALVO and EECKHOUDT [1982] and ZENOU and EECKHOUDT [1997]). Here, *households base their location decisions on expected utility and make their decision before the realization of uncertainty.*

In our model with three type of workers, uncertainty arises since nobody knows for sure what his situation will be in the next period, and can only make expectations. As observed by DESALVO and EECKHOUDT [1982] and ZENOU and EECKHOUDT [1997], in order to avoid unnecessary complications, we have collapsed the two periods into a single one so that we discuss only income uncertainty and not future income uncertainty. Formally, each agent  $i$  ( $i = 0, 1, 2$ ) possesses a strictly concave von Neumann Morgenstern (VNM) utility function  $E[U]_i$  which is supposed to be continuous and twice differentiable. It writes

$$(3) \quad E[U]_i = \pi_i U(I_u - R(x)q_i, q_i) + (1 - \pi_i) U(I_e - R(x)q_i, q_i), \\ i = 0, 1, 2$$

where  $\pi_i$  is the probability of being unemployed for agent  $i$ ,  $I_u = b - T(x)$  ( $b$  stands for the unemployment benefits), the net income for the unemployed worker,  $I_e = w - T(x)$ , the net income for the employed worker.

Observe that in (3), we do not take explicitly into account the present employment status of workers. Actually, this is done implicitly through the probability  $\pi_i$ . Indeed, we assume that

$$(4) \quad 0 < \pi_0 < \pi_1 < \pi_2 < 1,$$

which captures two facts: first, employed workers who earn more today ( $w > b$ ) have a lower probability to become unemployed tomorrow than the today unemployed workers ( $\pi_0$  can actually be interpreted as a turnover rate), and second, that belonging to the discriminated group means to be more likely to stay unemployed tomorrow than any non-discriminated unemployed worker.

Furthermore, since we focus on a bid rent approach (see e.g. FUJITA [1989]), agent  $i$  will choose only  $q_i$  (and not  $x$ ) to maximize (3). Recall that the bid rent is defined as the maximum rent per unit of land that the individual can pay for residing at distance  $x$  while enjoying a fixed level of utility. Bid rent is then defined at a given distance  $x$  from the CBD. Agent  $i$  solves the following program

$$(5) \quad \begin{aligned} \text{Max}_{q_i} E[U]_i &= \pi_i U(I_u - R(x)q_i, q_i) \\ &+ (1 - \pi_i)U(I_e - R(x)q_i, q_i), \\ &i = 0, 1, 2 \end{aligned}$$

Denoting first derivative of  $E[U]_i$  by  $E_{q_i}$ , first order condition (FOC) is given by

$$(6) \quad E_{q_i} = \pi_i (U_{q_i} - RU_{z_i})_u + (1 - \pi_i)(U_{q_i} - RU_{z_i})_e = 0$$

where  $U_{q_i}$  and  $U_{z_i}$  are respectively the partial derivative of  $U$  with respect to  $q_i$  and  $z_i$  and the indexes  $u$  and  $e$  mean that the derivatives are evaluated for the unemployed and the employed worker respectively.

Since we assume the normality of the housing  $q_i$ , we must have that <sup>2</sup>

$$(7a) \quad \frac{\partial}{\partial z_i} \left( \frac{U_{q_i}}{U_{z_i}} \right) = U_{q_i z_i} - \left( \frac{U_{q_i}}{U_{z_i}} \right) U_{z_i z_i} > 0$$

$$(7b) \quad (U_{q_i} - RU_{z_i})_u < 0$$

$$(7c) \quad (U_{q_i} - RU_{z_i})_e > 0$$

Let us denote the second derivative of  $E[U]_i$  by  $E_{q_i q_i}$ . Then, the second order condition (SOC) is satisfied for

$$(8) \quad \begin{aligned} E_{q_i q_i} &= \pi_i (U_{q_i q_i} - 2RU_{q_i z_i} + R^2 U_{z_i z_i})_u \\ &+ (1 - \pi_i)(U_{q_i q_i} - 2RU_{q_i z_i} + R^2 U_{z_i z_i})_e < 0. \end{aligned}$$

If the FOC and SOC are satisfied for any  $R$ , then there exists a unique  $q_i^*$ , equals to

$$(9) \quad q_i^* = q_i^*(\pi_i, I_u(x), I_e(x), R)$$

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2. For a rigorous proof, see DESALVO and EECKHOUDT [1982], p. 102.

The indirect utility function,  $V_i$ , can be fixed at a utility level,  $v_i$  and thus writes

$$(10) \quad V_i(\pi_i, I_u(x), I_e(x), R) \equiv v_i$$

and, the bid rent function is

$$(11) \quad R_i = \Xi_i(\pi_i, I_u(x), I_e(x), v_i)$$

By performing comparative statics exercises, we are able to establish the following proposition:

PROPOSITION 1:

$$(12) \quad \begin{aligned} \frac{\partial q_i^*}{\partial \pi_i} < 0, \quad \frac{\partial q_i^*}{\partial I_u} > 0, \\ \frac{\partial q_i^*}{\partial I_e} > 0, \quad \frac{\partial q_i^*}{\partial v_i} > 0, \quad i = 0, 1, 2 \end{aligned}$$

$$(13) \quad \begin{aligned} \frac{\partial \Xi_i}{\partial \pi_i} < 0, \quad \frac{\partial \Xi_i}{\partial I_u} > 0, \quad \frac{\partial \Xi_i}{\partial I_e} > 0, \\ \frac{\partial \Xi_i}{\partial v_i} < 0, \quad \frac{\partial \Xi_i}{\partial x} < 0, \quad i = 0, 1, 2 \end{aligned}$$

*Proof:* see the appendix.

The following comments are in order. First, not surprisingly, increasing the unemployment benefit or the current wage rises both the housing consumption and the bid rent, and thus workers locate further away from the CBD. Second, since workers are risk averse, when the probability of being unemployed rises, housing consumption as well as bid rent decreases and thus causes the household to move closer to the CBD. Moreover, since workers differ only with respect to the probability of unemployment, the ones with the greater probability (*i.e.*, the discriminated unemployed) consumes less housing and live closer to the CBD.

### 3 Agents' Location Before the Government Policy

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We have characterized the problem of each type of agent in the city and calculated his optimal level of housing consumption and his bid rent, which has been proved both to be lowered in case of an increase in the probability

of being unemployed, in equilibrium. We now inquire where each category of workers locates in the city. It is well known that households with greater income locate at the outskirts of the city if land is a normal good and if the city is monocentric (HARTWICK, SCHWEIZER and VARAIYA [1976], MIYAO [1975], WHEATON [1976]). Here, households differ only in term of their probabilities, *i.e.*, their access to employment in the next period. Intuitively, and since households can be ranked in exactly the same way as in the income classes model, our result must yield to the same urban configuration. Indeed, we have:

**PROPOSITION 2:** Employed workers locate at the periphery of the city whereas the discriminated unemployed reside in the vicinity of the CBD. The non-discriminated unemployed locate in between these two categories of agents. This location pattern stems from the fact that bid rents get steeper with an increase in probability  $\pi$ .

*Proof:* By using (A.10) in the appendix, we have

$$-\frac{\partial}{\partial \pi} \left( \frac{\partial \Xi}{\partial x} \right) = -\frac{\partial}{\partial \pi} \left( \frac{-T'(x)}{q^*} \right) = \frac{\partial}{\partial \pi} \left( \frac{T'(x)}{q^*} \right) = -\frac{T'(x)}{(q^*)^2} \frac{\partial q^*}{\partial \pi}$$

which is strictly positive by using (12) in Proposition 1. We have shown that the larger the probability  $\pi$ , the steeper the bid rent. This means that agents with greater  $\pi$  locate closer to the CBD than agents with lower  $\pi$  (see e.g. Rule 2.4., p. 30 in FUJITA [1989]).  $\square$

The urban configuration is described in Figure 1. Therefore, despite the fact that all unemployed workers get the same level of unemployment

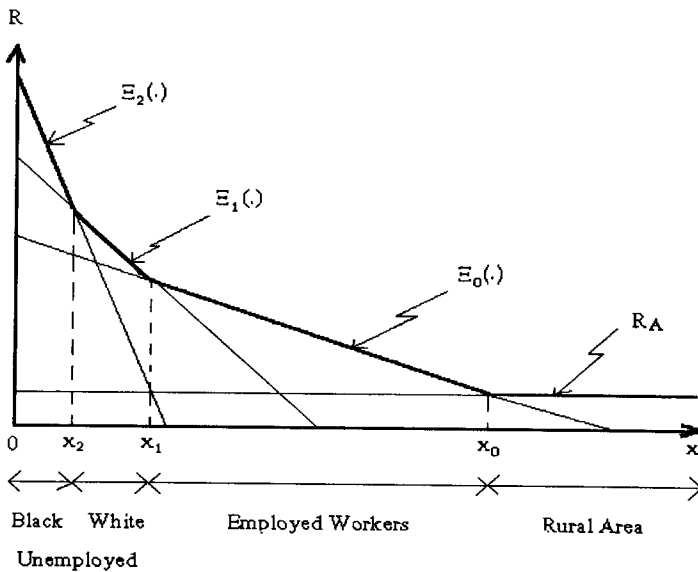


FIGURE 1

*Workers' Locations before the Government Policy*

benefit, they are not consuming the same quantity of housing and they are not locating in the same area of the city because discriminated unemployed have a lower expected utility or expected income than the non-discriminated ones.

We can now complete this model by determining the city equilibrium. We assume that our city is a CCA-type, *i.e.* a closed city with absentee landlords. Land market being competitive, the market land rent curve is given by

$$(14a) \quad R(x) = \Xi_2(\pi_2, I_u(x), I_e(x), v_2^*) \quad \text{for } 0 < x \leq x_2^*$$

$$(14b) \quad R(x) = \Xi_1(\pi_1, I_u(x), I_e(x), v_1^*) \quad \text{for } x_2^* < x \leq x_1^*$$

$$(14c) \quad R(x) = \Xi_0(\pi_0, I_u(x), I_e(x), v_0^*) \quad \text{for } x_1^* < x \leq x_0^*$$

$$(14d) \quad R(x) = R_A \quad \text{for } x > x_0^*$$

where  $x_2^*$  is the endogenous boundary between discriminated unemployed and non-discriminated unemployed workers,  $x_1^*$ , the endogeneous boundary between non-discriminated unemployed and employed workers,  $x_0^*$ , the endogeneous city fringe and  $R_A$ , the agricultural rent. Equations (14a)-(14d) means that, within the city, the land rent will be determined by the absentee landlords whose rational behavior will consist in taking the highest bid rents (thus, in the city, each location is occupied by the highest bidder). Outside of the city, the land rent will be equal to the rural one. Therefore, the market rent curve,  $R(x)$ , is the upper envelope of the equilibrium bid rent curves  $\Xi_i(\pi_i, I_u(x), I_e(x), v_i^*)$  of all household types ( $i = 0, 1, 2$ ) and the agricultural rent line  $R_A$  (see Figure 1 for an illustration with black and white unemployed workers). The next conditions to determine the city equilibrium are

$$(15a) \quad \Xi_2(\pi_2, I_u(x_2^*), I_e(x_2^*), v_2^*) = \Xi_1(\pi_1, I_u(x_2^*), I_e(x_2^*), v_1^*)$$

$$(15b) \quad \Xi_1(\pi_1, I_u(x_1^*), I_e(x_1^*), v_1^*) = \Xi_0(\pi_0, I_u(x_1^*), I_e(x_1^*), v_0^*)$$

$$(15c) \quad \Xi_0(\pi_0, I_u(x_0^*), I_e(x_0^*), v_0^*) = R_A$$

$$(16a) \quad \int_0^{x_2^*} \frac{L(x)}{q_2^*(\pi_2, I_u(x), I_e(x), v_2^*)} dx = N_2$$

$$(16b) \quad \int_{x_2^*}^{x_1^*} \frac{L(x)}{q_1^*(\pi_1, I_u(x), I_e(x), v_1^*)} dx = N_1$$

$$(16c) \quad \int_{x_1^*}^{x_0^*} \frac{L(x)}{q_0^*(\pi_0, I_u(x), I_e(x), v_0^*)} dx = N_0$$



where  $L(x)$  is the land distribution in the city assumed to be continuous for all  $x \geq 0$  and positive at each  $x > 0$  (for example,  $L(x) = 1$  for a linear city). We assume that  $L(x)$  is increasing in  $x$  which is a natural hypothesis for cities (see FUJITA [1989], ch. 3 and ch. 4 for a discussion).  $N_0$ ,  $N_1$  and  $N_2$  are respectively the number of employed, non-discriminated unemployed and discriminated unemployed workers living in the city.

Border conditions (15a)-(15c) state that  $x_2^*$  is determined by the intersection between non-discriminated unemployed and discriminated unemployed's bid rents,  $x_1^*$  by the intersection between employed and non-discriminated unemployed's bid rents, and  $x_0^*$  by the intersection of the agricultural bid rent and the bid rent of employed workers located at the city fringe. Observe that the border conditions (15a)-(15c) assure that the market land rent  $R(x)$  is continuous in  $x$  or in other words that  $R(x)$  is continuous across the boundary between land occupied by adjacent classes (or categories). Moreover, (16a)-(16c) are simply the land market clearing conditions, or population constraints, stating that everybody must be accommodated in one's "bidding zone".

The six equilibrium conditions (15a)-(15c) and (16a)-(16c) are then solved for the six endogenous variables,  $x_0^*$ ,  $x_1^*$ ,  $x_2^*$ ,  $v_0^*$ ,  $v_1^*$ ,  $v_2^*$ . It is easily shown that the city equilibrium exists and is unique (see e.g. FUJITA [1989], ch. 4) or Hartwick *et al.* [1976].

The equilibrium  $(x_0^*, x_1^*, x_2^*, v_0^*, v_1^*, v_2^*)$ , defined by (15a)-(15c) and (16a)-(16c), can be characterized as implicit functions of the exogeneous parameters  $\pi_0$ ,  $\pi_1$ ,  $\pi_2$ ,  $b$ ,  $w$ ,  $N_0$ ,  $N_1$ ,  $N_2$ . We can therefore perform comparative statics exercises and obtain easily <sup>3</sup> the following result:

PROPOSITION 3: The following table summarizes our results:

	$\pi_0$	$\pi_1$	$\pi_2$	$b$	$w$	$N_0$	$N_1$	$N_2$
$v_0^*$	-	+	+	+	+	-	-	-
$v_1^*$	-	-	+	+	+	-	-	-
$v_2^*$	-	-	-	+	+	-	-	-
$x_0^*$	-	-	-	+	+	+	+	+
$x_1^0$	-	-	-	+	+	-	+	+
$x_2^*$	-	-	-	+	+	-	-	+

The following comments are in order. First, a variation of  $b$  (the unemployment benefit) or  $w$  (the current wage) has an unambiguous effect on both utility levels and borders: an increase in any of these two variables rises  $v_i^*$  and  $x_i^*$  ( $i = 0, 1, 2$ ). If any group of agents in the city becomes richer (in expectation), then all classes are more suburbanized and better off. Second, if a class increases in size, then all classes are worse off. Moreover, if the

3. One can refer to HARTWICK *et al.* [1976] for a similar comparative statics analysis. The proof of Proposition 3 is therefore similar to the one found in HARTWICK *et al.* [1976].

discriminated-unemployed group increases in size, all borders rise and thus the city spreads out whereas if the number of employed workers increases, the city spreads even outer but other workers are pushed in: borders between any two groups shift inward. Lastly, increasing (decreasing) the probability of being unemployed of group  $i$  reduces (increases) the borders and thus the city-size whereas it rises (reduces) the utility levels of the outer classes and reduces (increases) the utility levels of the inner classes. First effect stems from the fact that, in a period of recession (prosperity), when  $\pi$  is higher (lower) for any group of agents, workers reduce (increase) their housing consumption and their bid rent get lower (greater), so every household moves inward (outward).

## 4 The Government Policy

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So far, we have shown that in the absence of a government policy aiming to help discriminated unemployed workers, the latter have, first, difficulties in finding a job and, second, locate close to the CBD. This can be a possible explanation for the observation that some categories of population such as blacks, young people... tend to locate at the vicinity of the CBD. Therefore, a government policy which is set up to eradicate labor discrimination and residential segregation must help these individuals to "leave the ghetto". How can this be done?

According to our model, the local government can design two types of such policies. First one would provide the prejudiced unemployed workers (the target urban population) with either a lump-sum housing subsidy (this type of policy has been study in a different context by CARLTON and FERREIRA [1977], CARLTON [1981] and TURNBULL [1993]) or a hiring-wage subsidy, the latter consisting in filling the gap of perceived efficiency between the two groups of unemployed workers by the firms. The goal of the second policy is to modify  $\pi_2$  so that discrimination disappears, *i.e.*,  $\pi_2 = \pi_1$ . This can be done through some special training programs for discriminate unemployed. The purpose of the latter policy is to make the CBD firms indifferent between hiring type 1 and type 2 unemployed workers.

Let us first study the second policy. What are the consequences on the urban configuration of equalizing  $\pi_1$  and  $\pi_2$ ? In order to achieve this policy, the government must adequately decrease  $\pi_2$ . According to Proposition 3 and the corresponding table, decreasing  $\pi_2$  spreads the city (and all intra-city borders) out, rises  $v_2^*$  but reduces the equilibrium utilities  $v_0^*$  and  $v_1^*$  of respectively the employed and the non-discriminated unemployed. If the government can achieve this policy, *i.e.*,  $\pi_1 = \pi_2$ , there will be only two categories of workers in the city, namely employed workers (group 0) and unemployed workers (labelled now group 1). Optimal equations (9)-(10) will still hold with  $i = 0, 1$  and  $\pi_1 > \pi_0$ . The equilibrium conditions (14a)-(14d), (15a)-(15c), (16a)-(16c) will be modified by the fact that there

will be only two borders,  $x_1^*$ , the border between unemployed and employed workers, and  $x_0^*$ , the city fringe. Of course,  $x_1^*$  and  $x_0^*$  have no reason to be the same as in the previous section. Therefore, we will have the following 4 equilibrium conditions:

$$(17a) \quad \Xi_1(\pi_1, I_u(x_1^*), I_e(x_1^*), v_1^*) = \Xi_0(\pi_0, I_u(x_1^*), I_e(x_1^*), v_0^*)$$

$$(17b) \quad \Xi_0(\pi_0, I_u(x_0^*), I_e(x_0^*), v_0^*) = R_A$$

$$(18a) \quad \int_0^{x_1^*} \frac{L(x)}{q_1^*(\pi_1, I_u(x), I_e(x), v_1^*)} dx = N_1$$

$$(18b) \quad \int_{x_1^*}^{x_0^*} \frac{L(x)}{q_0^*(\pi_0, I_u(x), I_e(x), v_0^*)} dx = N_0$$

The city equilibrium exists and is unique (see Figure 2). Moreover, the equilibrium  $(x_0^*, x_1^*, v_0^*, v_1^*)$  can be also characterized as implicit functions of the exogenous parameters  $\pi_0, \pi_1, I_u, I_e, N_0, N_1$ .

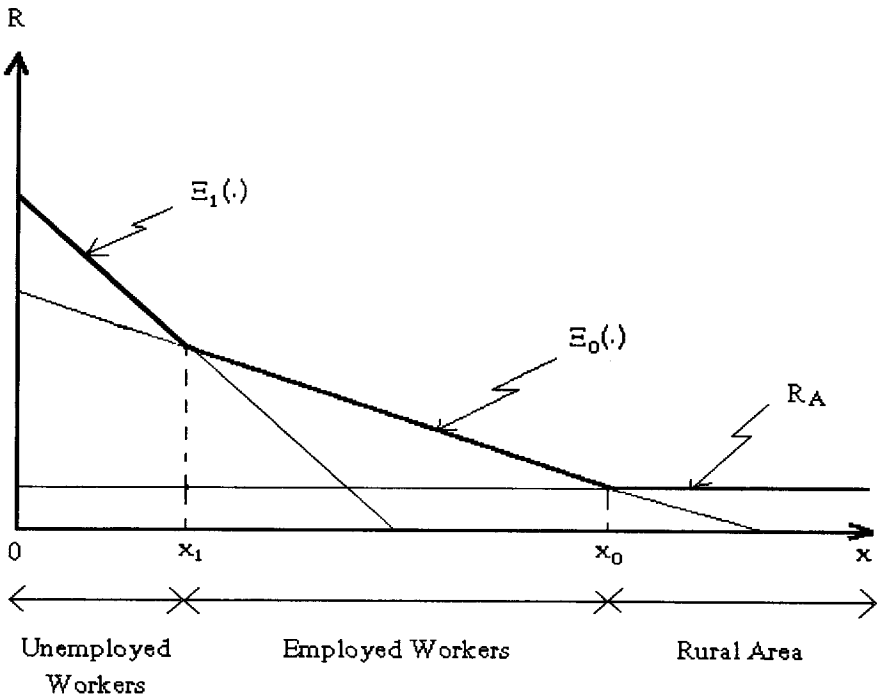


FIGURE 2

*Workers' Locations after the Government Policy*

The alternative consists in providing discriminated unemployed people either with a lump-sum subsidy for housing or a hiring-wage subsidy. Let us analyse the lump-sum subsidy for housing (the other policy is exactly the same). In this case, for type 2 workers, (5) writes

$$(19) \quad \begin{aligned} \text{Max}_{q_2} E[U]_2 &= \pi_2 U(I_u + \gamma - R(x)q_2, q_2) \\ &+ (1 - \pi_2) U(I_e - R(x)q_2, q_2) \end{aligned}$$

where  $\gamma$  is the lump-sum subsidy. The optimal  $q_2^*$  is

$$(20) \quad q_2^* = q_2^*(\pi_2, I_u(x) + \gamma, I_e(x), R)$$

The indirect utility function,  $V_2$ , can be fixed to a utility level,  $v_{2,\gamma}$  and thus writes

$$(21) \quad V_2(\pi_2, I_u(x) + \gamma, I_e(x), R) \equiv v_{2,\gamma}$$

and, the bid rent function is

$$(22) \quad R_2 = \Xi_2(\pi_2, I_u + \gamma, I_e, v_{2,\gamma}, x)$$

It is readily verified from Proposition 1 that

$$(23) \quad \frac{\partial q_2^*}{\partial \gamma} > 0, \quad \frac{\partial \Xi_2(\gamma)}{\partial \gamma} > 0$$

Moreover, in order to see the impact of  $\gamma$  on the location of discriminated unemployed workers, we have to compare the steepness of bid rents between types 1 and 2 unemployed workers. This yields

$$(24) \quad \begin{aligned} & - \frac{\partial \Xi_2(\pi_2, I_u(x_2^*) + \gamma, I_e(x_2^*), v_{2,\gamma}^*)}{\partial x} \\ &= \frac{T'(x)}{q_2^*(\pi_2, I_u(x_2^*) + \gamma, I_e(x_2^*), v_{2,\gamma}^*)} \\ &> \frac{T'(x)}{q_1^*(\pi_1, I_u(x_2^*), I_e(x_2^*), v_1^*)} \\ &= - \frac{\partial \Xi_1(\pi_1, I_u(x_2^*), I_e(x_2^*), v_1^*)}{\partial x} \end{aligned}$$

which is equivalent to

$$(25) \quad q_2^*(\pi_2, I_u(x_2^*) + \gamma, I_e(x_2^*), v_{2,\gamma}^*) \leq q_1^*(\pi_1, I_u(x_2^*), I_e(x_2^*), v_1^*)$$

From (23) and (25), the local government can easily find an optimal  $\gamma^*$  such that

$$(26) \quad \begin{aligned} & - \frac{\partial \Xi_2(\pi_2, I_u(x_2^*) + \gamma^*, I_e(x_2^*), v_{2,\gamma^*}^*)}{\partial x} \\ &= - \frac{\partial \Xi_1(\pi_1, I_u(x_2^*), I_e(x_2^*), v_1^*)}{\partial x} \end{aligned}$$

which leads to an urban configuration such as the one depicted in Figure 2. Of course, here again the city borders will change and in particular, city-size will be greater.

Observe that the lump-sum housing subsidy (or the hiring wage subsidy which has exactly the same impact) policy only deals with residential segregation since it does not affect  $\pi_2$  which remains greater than  $\pi_1$ . Thus, this policy does not alleviate labor discrimination. On the contrary, the other policy allows to modify simultaneously housing demand emanating from the previously discriminated unemployed workers and their probability to stay unemployed, since it yields the annulation of the probability gap for the two groups of job-seekers. This can be summarized by the following proposition.

**PROPOSITION 4:** By subsidizing discriminated unemployed workers via a lump-sum housing or a hiring wage transfer, the local government only manages to alleviate residential segregation, whereas by setting special training programs for the same prejudiced people, it allows to eradicate both their residential segregation and discrimination in the city.

This proposition has some interesting implications if we interpret the discriminated versus the non-discriminated unemployed as long run versus short run unemployed<sup>4</sup>. It is indeed well-known that long run unemployment is deeply demoralizing for the great majority of unemployed, who lose their skills and their work habits. Empirical observations suggest that the longer the unemployment spell, the less likelihood of getting a job, reflecting demoralisation, deskilling, reduced information about jobs, reduced job search, employer discrimination (see e.g., NICKELL [1979], HECKMAN and BORJAS [1980] and JACKMAN and LAYARD [1991]). In this context, the necessity of a high quality training for long-term unemployed and of the financial inducement to employers hiring long-term unemployed is advocated (ACEMOGLU [1995], WAUTHY and ZENOU [1996]). In our proposition the same results arise but the first policy is more efficient because it fights both discrimination and spatial segregation.

## 5 Conclusion

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This paper validates the idea that incorporating labor market in a standard urban model sheds light on spatial labor analysis by improving our understanding of the working of the city. Whenever there is no valuation of central locations (as prevails in North American cities, in opposition to European ones) and housing is a normal good, any sort of discrimination on the job market yields the prejudiced to be confined downtown, in a

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4. Observe that, in our model, discrimination is due to external appearance.

monocentric city. So residential segregation and labor discrimination are closely linked urban socioeconomic problems, which local governments aim to reduce as much as possible, because of the drastic consequences of the formation of such “ghettos” in contemporaneous cities: very long term unemployment, criminality, urban decay, etc. We have shown that the best policy to deal with these two connected problems consists in the enlisting of discriminated unemployed people in training schemes, so as to make firms indifferent between all job-seekers.

Different extensions can be contemplated. First, the government policy can be detailed, by stating its objective function and its budget constraint, to address the crucial problem of the financing of such a policy as well as its effects on urban configuration. Second, allowing the city to be open instead of closed implies to take in and out migration of workers into account, hence to reformulate residential equilibrium. Finally, one can relax the assumption of a monocentric city allowing for sub-centers. In this case, specific recruitment policies by firms from different centers would lead to interactions on both land and labor markets. This would complete existing models on job decentralization (see WHITE [1988] or FUJITA, THISSE and ZENOU [1997]).

*Proof of Proposition 1*

*Proof of (12)*

From the differentiation of (9), we obtain

$$(A.1) \quad \frac{\partial q_i^*}{\partial \pi_i} = -\frac{1}{E_{q_i, q_i}} [(U_{q_i} - RU_{z_i})_u - (U_{q_i} - RU_{z_i})_e]$$

which by (7c), (7d) and (8) is strictly negative.

$$(A.2) \quad \frac{\partial q_i^*}{\partial I_u} = -\frac{1}{E_{q_i, q_i}} \pi_i [U_{q_i, z_i} - RU_{z_i, z_i}]_u$$

which is positive by (7a).

$$(A.3) \quad \frac{\partial q_i^*}{\partial I_e} = -\frac{1}{E_{q_i, q_i}} (1 - \pi_i) [U_{q_i, z_i} - RU_{z_i, z_i}]_e$$

From the differentiation of (9) and by using the Envelope Theorem, we obtain

$$(A.4) \quad \frac{\partial q_i^*}{\partial v_i} = \frac{-1}{\partial \Xi_i / \partial v_i q_i^* [\pi U_{z_i} | u + (1 - \pi) u_{z_i} | e]}$$

which by (A.13) is positive.

*Proof of (13):*

From the differentiation of (11), we have

$$\frac{\partial \Xi_i}{\partial I_u} = -\frac{\partial V_i / \partial I_u}{\partial V_i / \partial \Xi_i}$$

$$\frac{\partial \Xi_i}{\partial I_e} = -\frac{\partial V_i / \partial I_e}{\partial V_i / \partial \Xi_i}$$

First, by applying the Envelope Theorem, we have

$$(A.5) \quad \frac{\partial V_i}{\partial I_u} = \pi_i U_{z_i} | u > 0$$

$$(A.6) \quad \frac{\partial V_i}{\partial I_e} = (1 - \pi_i) U_{z_i} | e > 0$$

Second, by also using the Envelope Theorem, we obtain

$$(A.7) \quad \frac{\partial V_i}{\partial \Xi_i} = -q_i^* [\pi_i U_{z_i} | u + (1 - \pi_i) U_{z_i} | e] < 0$$

Thus, we have

$$(A.8) \quad \frac{\partial \Xi_i}{\partial I_u} = \frac{\pi_i U_{z_i} |u}{q_i^* (\pi_i U_{z_i} |u + (1 - \pi_i) U_{z_i} |e)} > 0$$

$$(A.9) \quad \frac{\partial \Xi_i}{\partial I_e} = \frac{(1 - \pi_i) U_{z_i} |e}{q_i^* (\pi_i U_{z_i} |u + (1 - \pi_i) U_{z_i} |e)} > 0$$

From the differentiation of (11), we obtain

$$(A.10) \quad \frac{\partial \Xi_i}{\partial x} = -T'(x) \left[ \frac{\partial \Xi_i}{\partial I_u} + \frac{\partial \Xi_i}{\partial I_e} \right] = \frac{-T'(x)}{q_i^*} < 0$$

From differentiation of (11), we have

$$\frac{\partial \Xi_i}{\partial \pi_i} = - \frac{\partial V_i / \partial \pi_i}{\partial V_i / \partial \Xi_i}$$

and, by the use of the Envelope Theorem, we obtain

$$(A.11) \quad \frac{\partial V_i}{\partial \pi_i} = U(\cdot) |u - U(\cdot) |e < 0$$

Then

$$(A.12) \quad \frac{\partial \Xi_i}{\partial \pi_i} = \frac{U(\cdot) |u - U(\cdot) |e}{q_i^* [\pi U_{z_i} |u + (1 - \pi_i) U_{z_i} |e]} < 0$$

From the differentiation of (11), we have

$$(A.13) \quad \frac{\partial \Xi_i}{\partial v_i} = \frac{-1}{q_i^* (\pi_i U_{z_i} |u + (1 - \pi_i) U_{z_i} |e)} < 0$$

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