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Free Mobility and the Optimal Number of Jurisdictions

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ABSTRACT. – In a free mobility equilibrium with voting for pure public goods within jurisdictions and equal cost sharing, consumers will partition themselves such that high-demand jurisdictions are much larger than low-demand jurisdictions. We compare the welfare implications of a change in the number of jurisdictions. We find in a fairly simple but natural model of a large economy that if one restricts to odd numbers of jurisdictions, a smaller number is better, but among even numbers of jurisdictions the reverse holds. Further, any odd number is preferable to any even number.

Sur le nombre optimal de clubs sous l'hypothèse de libre accès

RÉSUMÉ. – Le papier s'intéresse au problème de partitionnement d'une population entre différents clubs en supposant une mobilité parfaite des individus entre clubs. Chaque club fournit un bien public (pur) dont le niveau est établi par vote à la majorité simple en supposant que les coûts sont partagés de façon égale entre les membres du club. Le papier analyse l'effet du nombre de clubs sur le bien-être social dans un modèle simple d'utilité transférable.

Le caractère pur du bien public assure que le bien-être social est maximisé en présence d'un seul club, mais le papier montre que la monotonie attendue à savoir que le bien-être social est une fonction décroissante du nombre de clubs est inexacte. En particulier, il s'avère que tout nombre impair de clubs est préférable à n'importe quel nombre pair de clubs.

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1 Introduction

In modern economies allocative decisions are taken not only by individuals, but also by governments that act on their behalf. If free mobility is possible, an important aspect of individual choice is to decide which government jurisdictions best represent their interests. Of course free mobility is not possible among all types of jurisdictions. Mobility is typically possible among states, counties and cities within a country, but the defining characteristic of the country itself is typically the right to exclude.

In this paper we explore some consequences of free mobility, but do not investigate the prior question of whether free mobility is a good idea. There is already a large literature devoted to the latter question. For example international trade theorists have extensively discussed whether labor mobility is required for productive efficiency, and following Tiebout [1956], public finance economists have emphasized the idea that free mobility permits citizens with similar tastes for public services to agglomerate.

The nature of the equilibrium that results from free mobility will depend among other things on the internal governance of jurisdictions. We follow WESTHOFF [1977] and EPPLE, FILIMON and ROMER [1984, 1993], and JEHIEL and SCOTCHMER [1994] in assuming that the residents of a jurisdiction vote on the public goods provided and that the costs are shared in a pre-specified way, in this paper equally. For the equal-sharing aspect, see also GUESNERIE and ODDOU [1981], GREENBERG and WEBER [1986] and GREENBERG and SHITOVITZ [1992]. We assume that when an individual decides to migrate he realizes that his own vote on public goods may change the provision of public goods. The cited authors have investigated the existence and stability of equilibrium, the nature of the equilibrium partition, and whether the equilibrium partition depends on whether immigration requires the consent of previous residents of a jurisdiction.

We depart from the previous literature in that we ask a new question. The cited papers have either taken the number of jurisdiction as exogenous or have assumed that new jurisdictions can form freely and therefore that the number of jurisdictions is endogenous. However neither of these hypotheses reflects the policy question that is posed when a jurisdiction wishes to subdivide, as in the cases of Belgium, Czechoslovakia, or California. In those cases a higher governing authority decides the exogenous number of jurisdictions, presumably in the realization that an equilibrum partition of the population will subsequently be achieved. Our objective in this paper is to characterize when subdivision enhances social welfare and when not.

We consider an economy with transferable utility, and we therefore take the sum of utilities as a measure of social welfare. Following most of the previous literature, we assume that public goods are "pure" within jurisdictions. That is clearly an important restriction, but seems useful as a benchmark. Even though the population is heterogeneous, our model has the property that the voting rule leads to an efficient provision of public goods within jurisdictions, and it follows that the first best allocation would be achieved with one jurisdiction. However if more than one jurisdiction is available, residents will typically partition themselves. The citizens with low demand for public goods will migrate to another jurisdiction rather than share the high costs of public goods in a jurisdiction with high-demand citizens.

Despite the impetus toward disagglomeration that comes from equal cost sharing, one might guess that social welfare declines when the number of jurisdictions is increased, since social welfare is maximized when there is one jurisdiction. We find so but in a rather surprising sense. In large economies, when the number of jurisdictions is restricted to be odd, welfare decrease with the number of jurisdictions. (That is, one jurisdiction is better than three, which is better than five...) However when the number of jurisdictions is even the reverse holds; more jurisdictions are better. (Two are worse than four, which are worse than six...) Moreover, any odd number of jurisdictions yields higher welfare than any even number. Therefore social welfare is enhanced by subdiving a jurisdiction if the previous number was even, but not if odd. As a byproduct of the welfare analysis we also observe that there is a maximum finite number of jurisdictions that will be occupied in equilibrium even if the number of potential jurisdictions is unbounded [1]. If the potential number of jurisdictions is already above this maximum, then increasing it further will have no effect.

While the welfare distinction between even and odd numbers seems surprising, an economic idea underlies it. As in the literature cited, consumers will be grouped in equilibrium according to their tastes for public goods, which we index by a parameter θ. A further characterization of equilibrium is that the sizes of the jurisdictions grow geometrically with their average taste parameter. The largest jurisdictions are also those with the highest taste for public goods, and for both reasons those jurisdictions matter most for social welfare. In free mobility equilibrium jurisdictions grow at rates that alternate between being larger and smaller than some limit rate of growth. Thus there is a "parity" aspect to the partitioning into jurisdictions: If the highest-demand jurisdiction is very large when the number of jurisdictions is odd, then the largest jurisdiction will be relatively smaller when the number is even. This feature of the allocation drives the result.

Of course we are not emphatic in defending the specific result that an odd number is always better than an even number, since a specific model underlies the result. FERNANDEZ [1997] presents an even/odd result where a policy intervention in, say, the kth jurisdiction makes everyone better off if the number of "higher" jurisdictions is even, but makes everyone worse off if it is odd. However we wish to emphasize that there is nothing unreasonable about the model, and that skeptics might therefore consider the possibility that it contains some kernel of truth. The kernel of truth seems to be nonmonotonicity: As in so many second-best problems, moving one of the choice variables (the number of jurisdictions) toward its first-best optimum might not improve the objective function if another choice variable (the division of population among jurisdictions) is not under one's control.

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1. The maximum number of jurisdictions is reminiscent of the GABSZEWICZ and THISS [1979] result on markets with vertically differentiated products, which says that only a finite number of firms will enter the market even if an unlimited number is available.

In the next two sections we present a simple model and characterize the free mobility equilibrium. In the subsequent section we present the welfare comparisons for different numbers of jurisdictions.

2 The Model

Preferences and Costs: We suppose that the consumers have different preferences indexed by $\theta \in (\theta_0, \theta^0)$, distributed uniformly with measure one on each unit interval, and with $0 < \theta_0 < \theta^0$. The preferences of a consumer of type θ can be represented $\theta z - t$ where z is the public good he consumes and t is what he pays. The cost of z is $C(z)$. For simplicity we will present our results for $C(z) = z^2$. The restriction to such a cost function makes preferences single peaked within jurisdictions and ensures a voting equilibrium within jurisdictions.

Jurisdictions: In general we allow a jurisdiction A_i to be a finite union of intervals [2]. A collection $\{A_i\}_{i=1,\ldots,k}$ is a *partition* of the population (θ_0, θ^0) if $A_i \cap A_j = \emptyset$, $i \neq j$ and $\cup A_i = (\theta_0, \theta^0)$.

Internal Governance: We assume that the constitution restricts the internal governance of jurisdictions in two ways that reflect democratic ideals. These restrictions lead to inefficient partitioning. First we assume that each consumer's tax t is an equal share of the cost of the public good within his jurisdiction; *i.e.*, $t = C(z)/n$ where n represents the number of members of the jurisdiction. Second, the members of each jurisdiction vote on the total public expenditures for public goods (or on the per-capita tax levy), knowing that they will share the costs equally. For a jurisdiction A_i we let $z(A_i)$ represent the public goods that will be provided under the voting rule, and we let $t(A_i)$ represent the per-capita expenditures. We let $U^\theta(A_i) = \theta z(A_i) - t(A_i)$. If jurisdiction A_i contains only the citizens whose preferences θ lie in an interval, say (θ_{i-1}, θ_i), then the number of members of jurisdiction A_i is $\theta_i - \theta_{i-1}$, and the public good provided in the jurisdiction is $z(\theta_{i-1}, \theta_i) = (1/4)(\theta_i^2 - \theta_{i-1}^2)$. Therefore $U^\theta(\theta_{i-1}, \theta_i) = \theta z(\theta_{i-1}, \theta_i) - C(z(\theta_{i-1}, \theta_i))/(\theta_i - \theta_{i-1}) = (1/16)(\theta_i^2 - \theta_{i-1}^2)(4\theta - \theta_{i-1} - \theta_i)$.

The model has been chosen so that the rules for internal governance lead to efficient provisions of public goods within jurisdictions, and this permits us to focus on the inefficiency of partitioning. The internal efficiency follows from the form of the utility function, the uniform distribution of θ, and the fact that jurisdictions are intervals.

2. We use a continuum of consumers for convenience, but to avoid technical problems we will identify jurisdictions that have the same members except for sets of measure zero. A *jurisdiction* or a *coalition* is a finite union of intervals, each in (θ_0, θ^0), and we will write each interval as an open interval, since no powers are granted to the closed interval that are not also granted to the open interval.

We use some technical results recorded in the next lemma, which follow from simple manipulations of the utility and cost functions presented above.

LEMMA 1: Suppose preferences are as above and $C(z) = z^2$. Then
1. The function $\theta_1 \rightarrow U^\theta(\theta_1, \theta_2)$ is decreasing for all $\theta \in (\theta_1, \theta_2)$. (All members of a coalition would be better off if the coalition expanded at the lower boundary).
2. For all (θ_{i-1}, θ_i), the function $\theta \rightarrow U^\theta(\theta_{i-1}, \theta) - U^\theta(\theta_{i-1}, \theta_i)$ is strictly convex at $\theta \in (\theta_{i-1}, \theta_i)$.
3. For all θ_{i-1}, the function $\theta_i \rightarrow U^{\theta_{i-1}}(\theta_{i-1}, \theta_i)$ is concave and maximal at $w\theta_{i-1}$ for some $w > 1$. (If a jurisdiction beginning at θ_{i-1} has an upper bound smaller than $\theta_i = w\theta_{i-1}$, then the lowest-θ member and all other members would be made better off by enlarging the jurisdiction at the upper boundary.)
4. $U^{\theta_{i-1}}(\theta_{i-1}, \theta_i) \geq U^{\theta_{i-1}}(\emptyset) = 0$ if and only if $\theta_i \leq w'\theta_{i-1}$, for some $w' > w$. (If a coalition is too large, the lowest θ individual would prefer a jurisdiction containing only himself.)
5. Homogeneity: $U^{q\theta}(q\theta_{i-1}, q^{\theta_i}) = h(q)U^\theta(\theta_{i-1}, \theta_i)$ for $\theta \in (\theta_{i-1}, \theta_i)$ and some $h(\cdot)$ function.

In the case where $C(z) = z^2$, $w = 1 + 2/\sqrt{3}$, $w' = 3$, and $h(q) = q^3$.

3 Free Mobility Equilibrium

A partition of the individuals into jurisdictions is a free mobility equilibrium if no individual wants to migrate to another jurisdiction. We also require a weak stability condition, namely that if the partition is perturbed some agents will move in such a way as to restore it. The formal definition that follows is a bit cumbersome because of the continuum of agents, although the intuitive idea should be clear. (The continuum simplifies the characterization of the equilibrium partition and also makes it easier to describe an enlargement of the set of agents.)

A *free mobility equilibrium* FME is a partition $\{A_i\}_{i=1,\dots,k}$ of the population (θ_0, θ^0) such that

(1) for all A_i and A_j in the partition and for all $\theta \in A_i, U^\theta(A_i) \geq U^\theta(A_j)$;

(2) $\exists \varepsilon > 0$ such that for every pair A_i and A_j in the partition and for any $B \subset A_i$ with $\mu(B) < \varepsilon$, $\exists \theta \in B$ such that $U^\theta(A_i \cup B) < U^\theta(A_j)$.

Condition (1) states that each individual prefers his own jurisdiction to any existing jurisdiction. Condition (2) is a weak stability condition saying that if a small group B migrates from A_j to A_i then at least one of them wants to return to the original jurisdiction A_j.

Proposition 1 characterizes FME, and equilibrium utilities are graphed in Figure 1. With an exogenous number of jurisdictions the utility of a low-θ individual can be negative, since he cannot avoid making subsidies by forming a singleton jurisdiction. (See our [1994] paper for a discussion of equilibrium with an endogenous number of jurisdictions.)

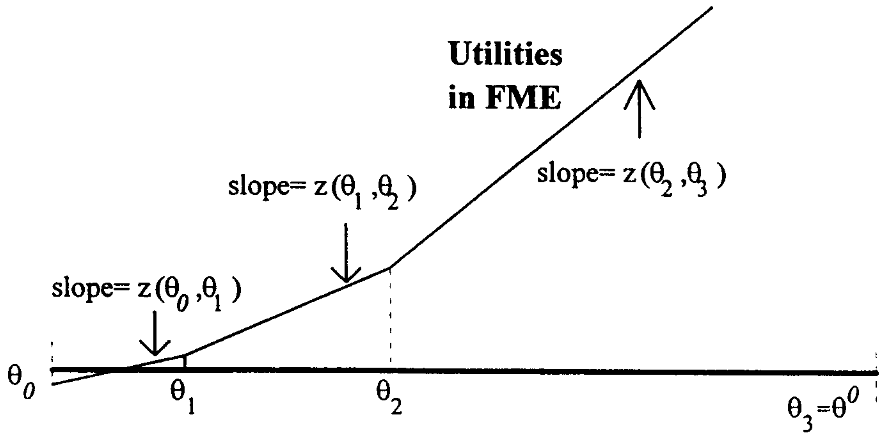

FIGURE 1

PROPOSITION 1: (i) FME exists, and jurisdictions in the FME partition are intervals [3]. (ii) There exists k^{FM} such that if $k > k^{FM}$, only k^{FM} of the jurisdictions will be occupied.

Proof: (i) We first show that if FME exists each jurisdiction i is a single interval. If not then there is another jurisdiction, say j, such that $z_i = z_j$ and i and j have the same number of members. We then show that this situation is not an equilibrium. It follows from the utility function, $\theta z_i - t_i$, that if two jurisdictions i and j are both occupied, and if any consumer strictly prefers jurisdiction i to jurisdiction j, then $z_i \neq z_j$. (If $z_i = z_j$ then $t_i < t_j$ so that every citizen prefers i to j, and j would not be occupied.) But if $z_i \neq z_j$ then almost every consumer has a strict preference between i and j. Further, if $z_i > z_j$ and a type-θ consumer strictly prefers jurisdiction i to j, then if $\theta' > \theta$, a consumer of type-θ' also prefers jurisdiction i. It follows that if $z_i \neq z_j$ for all i and j, then jurisdictions in the equilibrium partition are intervals. Thus, if jurisdiction

3. This feature is stressed by WESTHOFF [1977] and GREENBERG and WEBER [1986]. It is also discussed by EPPLE, FILIMON and ROMER [1984, 1993] and GUESNERIE and ODDOU [1981]. In all cases the consecutive property depends delicately on the exact definition of equilibrium, whether a stability requirement is imposed, and whether the population contains "atoms" that can move. EPPLE, FILIMON and ROMER particularly stress the stability problem, and point out that free mobility equilibrium might not segregate according to type, but that such a partition would be unstable. GREENBERG and SHITOVITZ [1988] present voting rules such that equilibrium exists. In contrast to these models, CUKIERMAN, HERCOWITZ and PINES [1984] have presented a model in which segregation by a taste parameter does not occur.

i is not an interval, $z_i = z_j$ and $t_i = t_j$ (hence $n_i = n_j$) for some j, in which case all members of i and j are indifferent between i and j. Of these two jurisdictions let i be the one with the highest-θ members, and let C be a small coalition of these highest-θ members. Since the citizen in C have higher θ than the median voter in their jurisdiction i, they would prefer more public goods. It follows that they can improve their utility by moving to jurisdiction j, since $U^\theta(A_j \cup C) = \theta z(A_j \cup C) - C(z(A_j \cup C))/\mu(A_j \cup C) > \theta z(A_j \cup C) - C(z(A_j \cup C))/\mu(A_j) > \theta z(A_j) - C(z(A_j))/\mu(A_j) = \theta z(A_i) - C(z(A_i))/\mu(A_i)$ for all $\theta \in C$. Thus each jurisdiction must be a single interval.

To show existence we therefore consider partitions such that each jurisdiction is an interval (θ_{i-1}, θ_i) where $\theta_{i-1} < \theta_i$ for all i. Given a partition we will refer to the *growth rates of the boundary points* $\{\lambda_i\}$ where $\lambda_i = \theta_i/\theta_{i-1}$.

We later need a characterization of the boundary points $\{\theta_i\}$ that separate jurisdictions in the equilibrium partition, and their growth rates $\{\lambda_i\}$. To characterize the growth rates of boundary points we first consider an interval of citizens $(\theta_\alpha, \theta_\beta)$, and characterize an equilibrium with two jurisdictions. We characterize the dividing point between the two jurisdictions, namely θ^{FM}, and then characterize how the ratio θ_β/θ^{FM} depends on the ratio $\theta^{FM}/\theta_\alpha$. This characterization is given by the function f defined below, where $\theta_\beta/\theta^{FM} = f(\theta^{FM}/\theta_\alpha)$, which will enable us to characterize the whole sequence of boundary points in the equilibrium partition, that is, for each i, $\theta_{i+1}/\theta_i = f(\theta_i/\theta_{i-1})$.

Using Lemma 1 (4), if $w'\theta_\alpha > \theta_\beta$, free mobility equilibrium will have one jurisdiction, namely the interval $(\theta_\alpha, \theta_\beta)$. Thus we assume that $w'\theta_\alpha < \theta_\beta$. We let $\Delta(\theta; \theta_\alpha, \theta_\beta) = U^\theta(\theta, \theta_\beta) - U^\theta(\theta_\alpha, \theta)$. This function is depicted in Figure 2. We define $\theta^{FM}(\theta_\alpha, \theta_\beta)$ such that (1) $\Delta(\theta^{FM}(\theta_\alpha, \theta_\beta); \theta_\alpha, \theta_\beta) = 0$ and (2) the function $\Delta(\cdot; \theta_\alpha, \theta_\beta)$ crosses the axis from below at $\theta^{FM}(\theta_\alpha, \theta_\beta)$. These two requirements follow from the two requirements for FME. Thus if the FME had only two jurisdictions, the partition would be $\{(\theta_\alpha, \theta^{FM}), (\theta^{FM}, \theta_\beta)\}$.

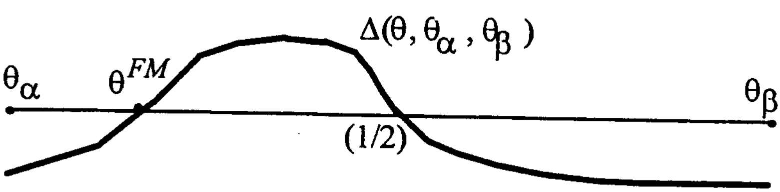

FIGURE 2

The following Lemma characterizes the function θ^{FM}. The fact that θ^{FM} decreases with its first argument is a direct consequence of the stability condition.

LEMMA 2: θ^{FM} is continuous, decreases with respect to its first argument and increases with respect to its second argument. $\theta^{FM}(\theta_\alpha, \theta_\beta) \leq 1/2(\theta_\alpha + \theta_\beta)$.

Proof: The continuity of θ^{FM} is established easily. To see that θ^{FM} decreases in θ_α, consider $\theta'_\alpha > \theta_\alpha$ such that θ'_α is close to θ_α and let $\theta^* = \theta^{FM}(\theta_\alpha, \theta_\beta)$. Letting U^{FM} refer to the utility of the boundary individual θ^{FM} we have $U^{FM}(\theta'_\alpha, \theta_\beta) < U^{FM}(\theta_\alpha, \theta_\beta)$ by Lemma 1 (1). Hence $\Delta(\theta^*; \theta'_\alpha, \theta_\beta) > 0$. By the stability requirement, condition (2) of the definition of θ^{FM}, we get that $\theta^{FM}(\theta'_\alpha, \theta_\beta) < \theta^* = \theta^{FM}(\theta_\alpha, \theta_\beta)$. That θ^{FM} increases in θ_β follows from the homogeneity condition (Lemma 1 (5)) and the monotonicity with respect to θ_α just proven. (Use $\theta^{\mathrm{FM}}(\theta_\alpha, \theta_\beta) = \theta_\beta \theta^{\mathrm{FM}}(\theta_\alpha/\theta_\beta, 1)$.) □

The function f discussed above is well defined on the domain $(1, \infty)$, and is given by equation (1).

(1) $$f(\lambda) = \frac{1 + \lambda + \sqrt{13\lambda^2 + 6\lambda - 3}}{2\lambda}$$

The function f is depicted in Figure 3, and has properties given in the following lemma, where $f^j(\lambda)$ represents j compositions, $f \circ f \circ f \ldots f(\lambda)$.

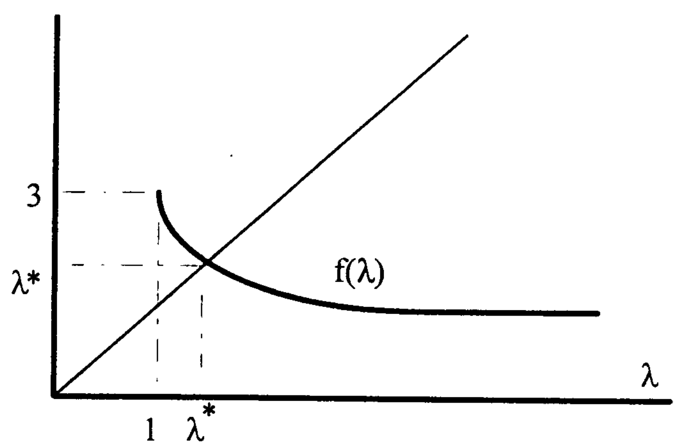

FIGURE 3

LEMMA 3: The function f is decreasing, continuous, has a fixed point λ^*, and $|f'(\lambda^*)| < 1$. Starting from any λ, the sequence $\{f^i(\lambda)\}_{i=1\ldots}$ converges to $\lambda^* > 1$, and the values in the sequence alternate between values larger than λ^* and values smaller than λ^*. The value of $\lambda f(\lambda) f^2(\lambda) \ldots f^k(\lambda) \equiv \lambda \Pi_{i=2}^k f^{i-1}(\lambda)$ is increasing with λ for all k.

Proof: The properties follow from the facts that $f'(\cdot) < 0$ and $\lambda f(\lambda)$ increases with λ, which can be proven using Lemma 2 and homogeneity. Otherwise one can use the functional form for f to show these properties.

The fixed point λ^* solves $\lambda^3 - 2\lambda^2 - 2\lambda + 1 = 0$. One can see immediately from (1) that $\lambda f(\lambda)$ increases with λ. In addition, since $f' < 0$, $f \circ f(\lambda) = f^2(\lambda)$ increases with λ, and $f^j(\lambda)$ increases with λ for any even j. Since $\lambda\Pi_{i=2}^{k+1} f^{i-1}(\lambda) = [\lambda\Pi_{i=2}^{k} f^{i-1}(\lambda)][f^k(\lambda)]$, and since $f^k(\lambda)$ increases with λ when k is even, it is enough to show that $\lambda\Pi_{i=2}^{k} f^{i-1}(\lambda)$ increases with λ when k is even. But because k is even, $\lambda\Pi_{i=2}^{k} f^{i-1}(\lambda)$ can be expressed as $[\lambda f(\lambda)][f^2(\lambda)f(f^2(\lambda))][f^4(\lambda)f(f^4(\lambda))]\ldots[f^{k-2}(\lambda)f(f^{k-2}(\lambda))]$. Each of these terms is increasing in λ. By putting $f(f^j\lambda))$ on the lefthand side of (1) and substituting $f^j(\lambda)$ for λ on the righthand side of (1), and then cross-multiplying, one can see that since $f^j(\lambda)$ increases with λ (since j is even), the product $[f^j(\lambda)f(f^j\lambda))]$ increases with λ. □

We define k^{FM} as the number k such that $\theta^k(1) \leq \theta^0 < \theta^{k+1}(1)$ where for each j and λ_1, $\theta^j(\lambda_1)$ is defined as $\theta^j(\lambda_1) \equiv \theta_0\lambda_1\Pi_{i=2}^{j} f^{i-1}(\lambda_1)$. That k^{FM} is finite and exists follows because by Lemma 3 since $\theta^k(\lambda_1)$ is monotone, and because every second growth rate is larger than $\lambda^* > 1$.

We can now complete the proof of Proposition 1.

Suppose first that $k \leq k^{FM}$. Since the boundary points are monotone in λ_1 by Lemma 3 there is a unique equilibrium λ_1 for which $\theta^k(\lambda_1) \equiv \theta^0$, and the solution satisfies $1 < \lambda_1$ since $k < k^{FM}$. The partition $\{\theta^j(\lambda_1)\}, j = 1, \ldots, k$, is an equilibrium because the growth rates satisfy the equation of motion f. Suppose next that $k > k^{FM}$. Then since by definition $\lambda_1 = \theta_1/\theta_0 \geq 1$, only k^{FM} jurisdictions are occupied and FME(k) coincides with FME (k^{FM}). □

Assuming that $k < k^{FM}$, we now investigate how social welfare depends on the sizes and on the number of jurisdictions. We might infer from the optimality of $k = 1$ (with pure public goods one group is best) that social welfare decreases monotonically with k, since increasing the number of jurisdictions decreases their average size. However the average size turns out not to be the most important consideration. In our example the sizes of groups grow geometrically and social welfare is dominated by the size of the highest-θ group, which is also the largest. The question of which k is best turns on the size of the highest-θ group. Proposition 2 says that the size of the highest-θ group is always larger for an odd number of groups than for an even number, and Proposition 3 says that social welfare reflects the size of the largest group. In particular it is best to have one group, but two groups is worst! This is because the size of the highest-θ group is smaller when there are 2 groups than for any other number, as shown in Figure 4. As one can see from the proof of Proposition 2, this ordering depends only on the properties of equilibrium summarized in Lemma 2 above.

We will let $L(k, \theta_0, \theta^0)$ represent the size of the largest group in equilibrium, namely $\theta^0 - \theta_{k-1}$ or $\theta_k - \theta_{k-1}$ where k is the number of jurisdictions.

PROPOSITION 2: Let (θ_0, θ^0) be given. (i) If k and k' are even and $k^{FM} \geq k' > k$, then $L(k, \theta_0, \theta^0) < L(k', \theta_0, \theta^0)$. (ii) If k and k' are odd, and $k^{FM} \geq k' > k$, then $L(k, \theta_0, \theta^0) > L(k', \theta_0, \theta^0)$. (iii) If k is even and k' is odd, $k^{FM} \geq k', k$, then $L(k, \theta_0, \theta^0) < L(k', \theta_0, \theta^0)$.

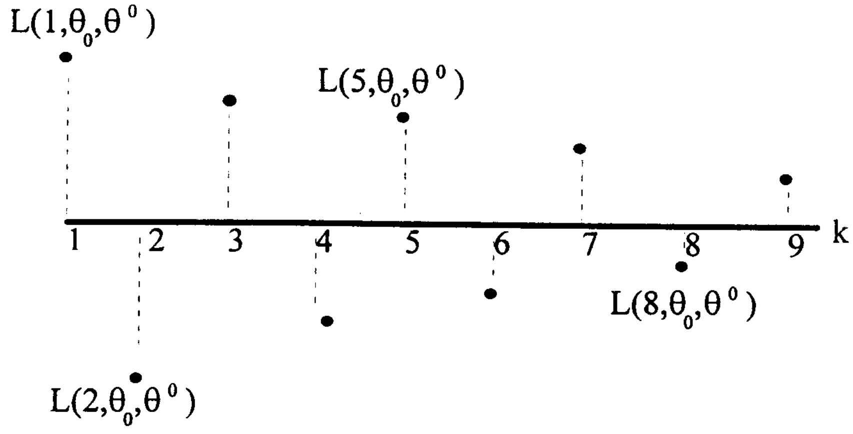

FIGURE 4

Proof: We cannot have $L(k, \theta_0, \theta^0) = L(k', \theta_0, \theta^0)$ if $k \neq k'$, as then the partitions would coincide, and that would contradict the premise that the partitions have different numbers of occupied jurisdictions. In the following argument we use the notation FME(k) to mean the equilibrium partition when there are $k \leq k^{FM}$ jurisdictions.

It suffices in (i) and (ii) to argue that the inequalities hold for $k' = k + 2$. (i) Suppose to the contrary that $L(k, \theta_0, \theta^0) > L(k + 2, \theta_0, \theta^0)$. Let $\{\theta_i\}_{i=0}^{k}$ be FME(k) and let $\{\theta'_i\}_{i=0}^{k+2}$ be FME($k + 2$). Then

$$\theta_{k-1} < \theta'_{k+1} < \theta_k = \theta'_{k+2}$$

But this implies that

$$\theta_{k-2} > \theta'_k$$

since otherwise we would have that

$$\theta_{k-1} = \theta^{FM}(\theta_{k-2}, \theta_k) \geq \theta^{FM}(\theta'_k, \theta'_{k+2}) = \theta^{FM}(\theta'_k, \theta_k) = \theta'_{k+1},$$

which would contradict either Lemma 2 or the first inequality. Further,

$$\theta_{k-3} < \theta'_{k-1}$$

since otherwise we would have (using the first inequality) that $\theta_{k-2} = \theta^{FM}(\theta_{k-3}, \theta_{k-1}) \leq \theta^{FM}(\theta'_{k-1}, \theta'_{k+1}) = \theta'_k$, which contradicts either Lemma 2 or the second inequality. Further,

$$\theta_{k-4} > \theta'_{k-2}$$

since otherwise (using the second inequality) $\theta_{k-3} = \theta^{FM}(\theta_{k-4}, \theta_{k-2}) \geq \theta^{FM}(\theta'_{k-2}\theta'_k) = \theta'_{k-1}$, which would contradict either Lemma 2 or the third inequality. Further,

$$\theta_{k-5} < \theta'_{k-3}$$

since otherwise (using the third inequality) that $\theta_{k-4} = \theta^{FM}(\theta_{k-5}, \theta_{k-3}) \leq \theta^{FM}(\theta'_{k-3}, \theta'_{k-1}) = \theta'_{k-2}$, which would contradict either Lemma 2 or the fourth inequality.

The pattern that emerges has the following property: For every odd $i \in \{3, \ldots k-1\}$, $\theta_{k-1} < \theta'_{k-i+2} < \theta'_{k-i+3} < \theta_{k-i+1}$ and for every even $i \in \{2, \ldots, k\}$, there does not exist $j \in \{1, \ldots, k+2\}$ such that $\theta'_j \in (\theta_{k-i}, \theta_{k-i+1})$. This implies that $\theta_1 < \theta'_3 < \theta'_4 < \theta_2$ and that θ'_1 and θ'_2 are not in the interval (θ_0, θ_1). Thus $\theta'_0 = \theta'_1 = \theta'_2 = \theta_0$, which means there could not be $k+2$ occupied jurisdictions in FME($k+2$). From this contradiction we conclude that the premise $L(k, \theta_0, \theta^0) > L(k'+2, \theta_0, \theta^0)$ must be incorrect. This completes (i).

(ii) Suppose to the contrary that $L(k, \theta_0, \theta^0) < L(k+2, \theta_0, \theta^0)$. Let $\{\theta_i\}_{i=0}^{k}$ be FME(k) and let $\{\theta'_i\}_{i=0}^{k+2}$ be FME($k+2$). Then

$$\theta_k = \theta'_{k+2} > \theta_{k-1} > \theta'_{k+1}.$$

Then

$$\theta_{k-2} < \theta'_k$$

since otherwise we would have $\theta_{k-1} = \theta^{FM}(\theta_{k-2}, \theta'_{k+2}) = \theta^{FM}(\theta_{k-2}, \theta_k) \leq \theta^{FM}(\theta'_k, \theta'_{k+2}) = \theta'_{k+1}$ which would contradict either Lemma 2 or the first inequality. But then

$$\theta_{k-3} > \theta'_{k-1}$$

since otherwise we would have (using the first inequality) that $\theta_{k-2} = \theta^{FM}(\theta_{k-3}, \theta_{k-1}) \geq \theta^{FM}(\theta'_{k-1}, \theta'_{k+1}) = \theta'_k$, which would contradict either Lemma 2 or the second inequality. Then

$$\theta_{k-4} < \theta'_{k-2}$$

since otherwise (using the second inequality) $\theta_{k-3} = \theta^{FM}(\theta_{k-4}, \theta_{k-3}) \leq \theta^{FM}(\theta'_{k-2}, \theta'_k) = \theta'_{k-1}$, which would contradict either Lemma 2 or the third inequality. Then

$$\theta_{k-5} > \theta'_{k-3}$$

since otherwise (using the third inequality) $\theta_{k-4} = \theta^{FM}(\theta_{k-5}, \theta_{k-3}) \geq \theta^{FM}(\theta'_{k-3}, \theta'_{k-1}) = \theta'_{k-2}$, which would contradict either Lemma 2 or the fourth inequality.

The pattern that emerges is that for even indices $i \in \{2, \ldots, k-1\}$, $\theta_{k-i} < \theta'_{k-i+2} < \theta'_{k-i+3} < \theta_{k-i+1}$ and for odd indices $i \in \{3, \ldots, k\}$ there does not exist $j \in \{1, \ldots, k+2\}$ such that $\theta'_j \in (\theta_{k-i}, \theta_{k-i+1})$. But then $\theta_1 < \theta'_3 < \theta'_4 < \theta_2$, and $\theta'_1 < \theta'_2$ are not in (θ_0, θ_1). Hence $\theta'_0 = \theta'_1 = \theta'_2 = \theta_0$, which contradicts the fact that in the FME($k+2$) there are $k+2$ occupied jurisdictions. This completes (ii).

(iii) Part (i) of this proof shows that if k is even, then $L(k+2, \theta_0, \theta^0) > L(k, \theta_0, \theta^0)$. The same proof can be used to show that if k is even then $L(k+1, \theta_0, \theta^0) > L(k, \theta_0, \theta^0)$. In the proof one must substitute θ'_{k-i-1} for θ'_{k-i}, for each i. The inequality at the end becomes $\theta_1 < \theta'_2 < \theta'_3 < \theta_2$, and θ'_1 is not in the interval (θ_0, θ_1), since that would violate Lemma 2. Thus $\theta_0 = \theta'_0 = \theta'_1$, which contradicts the premise that there are $k+1$ occupied jurisdictions in FME($k+1$). But now part (iii) follows. Suppose on the contrary that for an odd integer k and an even integer k', $L(k', \theta_0, \theta^0) > L(k, \theta_0, \theta^0)$. We can assume that $k' > k$, since otherwise, from part (i), we can substitute for k' the smallest even number larger than k, and the inequality still holds. Then from part (i),

$L(k'+1,\theta_0,\theta^0) > L(k',\theta_0,\theta^0) > L(k,\theta_0,\theta^0)$, but this contradicts part (ii) since $k'+1$ is an odd number larger than k. □

We know compare the welfare in FME(k), $k < k^{FM}$.

Letting $W^{FM}(k,\theta_0,\theta^0)$ represent the sum of utilities in FME(k), which is $\Sigma_{i=1}^{k}(1/16)(\theta_{i-1}^2 - \theta_i^2)^2$ according to the above arguments, the conjecture would be that $W^{FM}(1,\theta_0,\theta^0) > W^{FM}(3,\theta_0,\theta^0) > W^{FM}(5,\theta_0,\theta^0)\ldots W^{FM}(2k+1,\theta_0,\theta^0) > W^{FM}(2k,\theta_0,\theta^0) > W^{FM}(2k-2,\theta_0,\theta^0) > \ldots > W^{FM}(2,\theta_0,\theta^0)$ where $2k$ or $2k+1$ is k^{FM}. We show these inequalities for the limit case as θ^0 becomes large.

Indexing the equilibrium by θ^0 and holding θ_0 and k fixed, let $\{\theta_0,\theta_0\lambda_1,\{\theta^j(\lambda_1)\}, j = 2,\ldots,k\}$ be the FME, where $\theta^k(\lambda_1) = \theta^0$. As θ^0 becomes large λ_1 must become large, and since the partition boundary points are monotone in λ_1 (Lemma 3) all the boundary points become large. The limit of the growth rates $\{\lambda_i\}_{i=1}^k = \{\lambda_1,\{f^i(\lambda_1)\}, i = 1,\ldots,k\}$ as λ_1 becomes large are the boundary growth rates $\{\lambda_i\}_{i=1}^k$ of FME for the game with players in $(\theta_0,\theta^0) = (0,1)$, since if $\theta_0 = 0$ and $\theta_1 > 0$, then $\lambda_1 = \infty$. The limit of $W^{FM}(k,\theta_0,\theta^0)/W^{FM}(1,\theta_0,\theta^0)$ as θ^0 becomes large is therefore $W^{FM}(k,0,1)/W^{FM}(1,0,1) = W^{FM}(k,0,1)$, and to study the limit welfare it is enough to study the game with domain $(0,1)$. Thus the following claim implies that if θ^0 is large, social welfare increases with k if k is even and decreases with k if k is odd. Further, any odd number of jurisdictions is welfare superior to any even number of jurisdictions.

PROPOSITION 3: $W^{FM}(k,0,1)$ increases with k if k is even and decreases with k if k is odd. Further, $W^{FM}(k,0,1) > W^{FM}(k',0,1)$ for all even integers k' and odd integers k.

Proof: It follows from the expression for welfare that

$$(2)\quad 16\,W^{FM}(k+1,0,1) = (1+(1/\lambda_{k+1})^2)^2 + 16\,W^{FM}(k,0,1)/(\lambda_{k+1}^4)$$

where $\lambda_k = f^{k-1}(+\infty)$. Since $(\lambda_k)_{k\to\infty} \to \lambda^* > 1$ we can show from (2) that $(W^{FM}(k,0,1))_{k\to\infty}$ converges, say to W^*. Equation (2) yields

$$(3)\quad \begin{aligned} &16[W^{FM}(k+2,0,1) - W^{FM}(k,0,1)]\\ &\quad = (1-(1/(\lambda_{k+1}^4\lambda_{k+2}^4))[-16\,W^{FM}(k,0,1) + h(\lambda_{k+1})] \end{aligned}$$

$$(4)\quad \begin{aligned} &16[W^{FM}(k,0,1) - W^{FM}(k-2,0,1)]\\ &\quad = ((\lambda_{k+1}^4\lambda_{k+2}^4) - 1)[-16\,W^{FM}(k,0,1) + h(\lambda_{k+1})] \end{aligned}$$

where $h(\lambda) = \frac{1-f(\lambda)^2\lambda^4+\lambda^4-\lambda^2}{f(\lambda)^4\lambda^4-1}$. One can show that h is decreasing.

Since λ_k alternate around λ^* and since $\lambda_1 = \infty$, it follows that for all $k = 1,2\ldots$

$$(5)\quad \lambda^* < \lambda_{2k+1} < \lambda_{2k-1}$$

and

(6) $$\lambda^* > \lambda_{2k+2} > \lambda_{2k}$$

Since h is decreasing, it follows from (5) and (6) that

(7) $$h(\lambda_{2k+1}) > h(\lambda_{2k-1})$$

(8) $$h(\lambda_{2k}) > h(\lambda_{2k+2}).$$

It follows that $W^{FM}(2k, 0, 1) > W^{FM}(2k-2, 0, 1)$ implies $h(\lambda_{2k-1}) > 16\, W^{FM}(2k, 0, 1)$ by (4), which implies $h(\lambda_{2k+1}) > 16\, W^{FM}(2k, 0, 1)$ by (7), which implies $W^{FM}(2k+2, 0, 1) > W^{FM}(2k, 0, 1)$ by (3). One can check that $W^{FM}(2, 0, 1) < W^{FM}(4, 0, 1)$. We conclude that $(W^{FM}(2K, 0, 1))_k$ is increasing and converges to W^*. Similarly, using (3), (8) and (4), we can show that $W^{FM}(2k-1, 0, 1) > W^{FM}(2k+1, 0, 1)$ implies $W^{FM}(2k+1, 0, 1) > W^{FM}(2k+3, 0, 1)$. Since $W^{FM}(1, 0, 1) > W^{FM}(3, 0, 1)$, we conclude that $(W^{FM}(2k+1, 0, 1))_k$ is decreasing and converges to W^*. □

COROLLARY: If k is even, then welfare can be improved by adding one jurisdiction. But if k is odd, it should be decreased.

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