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Existence of First-Order Locally Consistent Equilibria

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ABSTRACT. – This note proves the existence of a first-order locally consistent equilibrium (see, for example, BONANNO and ZEEMAN [1985], GARY-BOBO [1989a], [1989b]) with more general strategy spaces and without the "boundary conditions" employed by BONANNO and ZEEMAN [1985]. Two examples of application are also provided.

Existence d'un équilibre localement cohérent du premier ordre

RÉSUMÉ. – Cette note prouve l'existence d'un équilibre localement cohérent du premier ordre (voir, par exemple, BONANNO et ZEEMAN [1985], GARY-BOBO [1989a], [1989b]) avec des espaces de stratégies plus générales et sans la "condition de frontière" employés par BONANNO et ZEEMAN. Deux exemples d'application en économie sont aussi donnés.

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1 Introduction

A first-order locally consistent equilibrium (1-LCE) of a game is a configuration of strategies at which the first-order condition for pay-off maximization is simultaneously satisfied for all players (see SILVESTRE [1975], BONANNO and ZEEMAN [1985], GARY-BOBO [1987], [1989a], [1989b]). The economic motivation for introducing this equilibrium concept is that oligopolistic firms do not know their effective demand function, but "at any given *status quo* each firm knows only the linear approximation of its demand curve and believes it to be the demand curve it faces" (BONANNO and ZEEMAN (1985, p. 277)). In what follows, in orrder to distinguish between the abstract concept of 1-LCE (*i.e.* a configuration of a game in which tthe first-order condition for pay-off maximization is satisfied for all players) and its economic interpretation (*i.e.* a profit-maximizing configuration in a market or in an economy in which firms know only the linear approximation of their demand functions), the latter equilibrium concept will be called first-order locally consistent economic equilibrium (1-LCEE).

Within an abstract game-theoretic framework, BONANNO and ZEEMAN (henceforth BZ) have provided a general existence result of a 1-LCE under the conditions that the strategy sets of players are closed intervals in $\mathfrak{R}$, that the pay-off functions are differentiable, and two "boundary conditions" are satisfied (BONANNO and ZEEMAN [1985, p. 278]). The "boundary conditions" imply that BZ's definition of 1-LCE rules out the possibility that the first order condition for pay-off maximization is satisfied at the boundary of strategy spaces. BZ employ their existence result to prove the existence of a 1-LCEE in a monopolistic competitive industry with price-making firms. In this context, the "boundary conditions" amount to the quite restrictive hypothesis that whatever the strategy of rivals each firm is always able to earn positive profits.

The aim of this note is to prove the existence of a 1-LCE without the above-mentioned "boundary conditions" and with much more general strategy spaces (more specifically, from compact and convex subsets of $\mathfrak{R}$ to compact and convex subsets of Banach spaces). This will be done in two steps. First, we show that a 1-LCE can be rationalized in a simpler (but not necessarily more reasonable) way than the way done by the literature (see the quotation above). More specifically, we show that given a n-person, non-cooperative game in strategic form, Γ, the set of 1-LCEs of this game is equal to the set of equilibria (to be appropriately defined) of a fictitious "game", Γ_c, which differs from Γ by the fact that in game Γ_c, given a configuration of strategies (the *status quo*), players maximize the linear approximation of their *pay-off functions* (in economic terms, their *profit functions*). Then, as second step, by exploiting the above equivalence, we shall prove the existence of a 1-LCE of game Γ under our more general assumptions.

Two examples of economic applications in finite-dimensional spaces are provided. The first example deals with a model of monopolistic competition similar to the BZ's one, in which the "boundary conditions" are violated in a natural way. This extension allows to prove the existence of an oligopolistic equilibrium with price-making oligopolists under considerably weaker assumptions with respect to those employed by BZ. The second example shows that our existence result can be applied to prove in an extremely simple way the existence of a 1-LCE in a general equilibrium model with imperfectly-competitive firms (GARY-BOBO [1989a], [1989b]).

2 Existence of First-Order Locally Consistent Equilibria

Consider the following non-cooperative game: $\Gamma\left(N,\,(X_i)_{i\in N},\,(\Pi_i)_{i\in N}\right)$, where $N=\{1,\,2,...,\,n\}$ is the index set of players, X_i is the strategy set of player i, and Π_i is the pay-off function of player i. Set: $X=\Pi_{i\in N}\,X_i$, and $X_{-i}=\Pi_{j\in N\setminus\{i\}}\,X_j$. The generic element of set X (resp. X_{-i}, resp. X_i) is denoted by x (resp. x_{-i}, resp. x_i). Denote by $D_{x_i}\,\Pi_i$ the derivative of Π_i with respect to x_i. The derivative of Π_i with respect to x_i calculated at point x is denoted by $D_{x_i}\,\Pi_{i\,(x)}$.

A.1. $\forall\, i\in N : X_i$ is a convex and compact subset of a Banach space.

A.2. $\forall\, i\in N$: the function $\Pi_i : X\to\Re$ is continuous; moreover, for every $x\in X$, the derivative $D_{x_i}\,\Pi_i$ exists and is continuous [1].

Assumptions A.1 and A.2 are the same of those employed by BZ, except for the fact that A.1 and A.2 consider more general strategy spaces than BZ's ones and that they do not contain any "boundary condition". The absence of "boundary conditions" requires a more general concept of 1-LCE, which allows for boundary optimal strategies:

DEFINITION 1 : A 1-LCE for game Γ is a configuration $x^*\in X$ such that:
(i) if $x_i^*\in X_i\backslash\partial X_i$, then $D_{x_i}\,\Pi_{i\,(x^*)}=0$;
(ii) if $x_i^*\in\partial X_i$, then there exists a neighbourhood of x_i^* in X_i, $N\,(x_i^*)$, such that: $D_{x_i}\,\Pi_{i\,(x^*)}\,(x_i-x_i^*)\leq 0$, for every $x_i\in N\,(x_i^*)$.

Condition (i) in Definition 1 is the same condition employed by BZ in defining the concept of 1-LCE. Condition (ii) means that if x_i^* belongs to the boundary of the strategy set, then either it satisfies the first-order condition for pay-off maximization, or it is a local maximum. Notice that Definition 1 is in line with the usual idea that at 1-LCEs players carry out local experiments by employing the linear approximations of

1. The hypothesis that for every $x\in X$, the derivative $D_{x_i}\,\Pi_i$ exists and is continuous means that there exists an open set $\mathcal{W}_i^0\supset X_i$ and an extension of function Π_i to $\mathcal{W}_i^0$ which is continuously differentiable with respect to x_i.

some appropriate function (in this case the pay-off function) (see SILVESTRE [1977, p. 436]).

Given a configuration $x^0 \in X$, interpreted as the *status quo*, define the function $\Phi_i : X \times X_i \rightarrow \Re$ as follows: $\Phi_i(x^0, x_i) \equiv \Pi_i(x^0) + D_{x_i}\Pi_{i(x^0)}(x_i - x_i^0)$. With some abuse of language, the following fictitious n-person non-cooperative game $\Gamma_c(N, (X_i)_{i \in N}, (\Phi_i)_{i \in N})$ will be associated to game Γ. In game Γ_c, given the *status quo* x^0, the best strategy for player i is the solution to the following problem:

$$(\mathrm{P_i}) \qquad \max \Phi_i(x^0, x_i), \quad \text{s.t.} \quad x_i \in X_i.$$

Denote by $\xi_i(x^0)$ the set of solutions to problem (P_i).

If we interpret game Γ_c as an oligopolistic game among firms which choose, for example, the level of production, then the behavioural hypothesis underlying problem (P_i) is that given the *status quo*, firms maximize the linear approximation of their profit functions.

DEFINITION 2 : An equilibrium for game Γ_c is a configuration $x^* \in X$ such that: $x_i^* \in \xi_i(x^*)$ for every $i \in N$.

Denote by $E(\Gamma_c)$ the set of equilibria of game Γ_c, and by $\mathrm{LCE}(\Gamma)$ the set of 1-LCEs of game Γ.

PROPOSITION 1: Under A.1 and A.2., LCE $(\Gamma) \neq \emptyset$.

Proposition 1 shows that game Γ has a 1-LCE under weaker assumptions than those adopted by BZ. The proof of Proposition is based upon the following Lemma which is interesting in itself because it makes clear that every 1-LCE of game Γ can be rationalized in terms of the simpler behaviour described by problems (P_i), $i \in N$.

LEMMA: $E(\Gamma_c) = \mathrm{LCE}(\Gamma)$.

Proof: First, suppose that $x^* \in E(\Gamma_c)$. It is sufficient to show that x^* satisfies the following conditions:

(i) if $x_i^* \in X_i \backslash \partial X_i$, then $D_{x_i}\Pi_{i(x^*)} = 0$;

(ii) if $x_i^* \in \partial X_i$ then $D_{x_i}\Pi_{i(x^*)}(x_i - x_i^*) \leq 0$, for every $x_i \in X_i$.

To this end, suppose that $x_i^* \in X_i \backslash \partial X_i$ but $D_{x_i}\Pi_{i(x^*)} \neq 0$. Since x_i^* is an interior point of X_i, then the linearity of Φ_i implies that there exists a point $x_i^\dagger \in \partial X_i$ such that $\Phi_i(x^*, x_i^\dagger) > \Phi_i(x^*, x_i^*)$, which is a contradiction. Suppose now that $x_i^* \in \partial X_i$ and $D_{x_i}\Pi_{i(x^*)}(x_i - x_i^*) > 0$, for some $x_i \in X_i$. Clearly, x_i^* does not solve problem (P_i), a contradiction. Summarizing, $x^* \in \mathrm{LCE}(\Gamma)$.

Finally, suppose that $x^* \in \mathrm{LCE}(\Gamma)$. Then, x^* satisfies conditions (i) and (ii) in Definition 1. If $x_i^* \in X_i \backslash \partial X_i$, then $D_{x_i}\Pi_{i(x^*)} = 0$; therefore, $\Phi_i(x^*, x_i) = \Pi_i(x^*)$ for every $x_i \in X_i$. It follows that x_i^* solves problem (P_i). Consider now the case $x_i^* \in \partial X_i$ with $D_{x_i}\Pi_{i(x^*)} \cdot (x_i - x_i^*) \leq 0$ for every x_i in some neighbourhood of x_i^*, $N(x_i^*)$. By linearity, one obtains that $D_{x_i}\Pi_{i(x^*)} \cdot (x_i - x_i^*) \leq 0$ for every $x_i \in X_i$. Thus, also in this case x_i^* solves problem (P_i). Therefore, $x^* \in E(\Gamma_c)$. $\square$

Proof of Proposition 1: By taking into account the Lemma, it is sufficient to show that $E(\Gamma_c) \neq \emptyset$. By A.2 it follows that the function

$\Phi_i : X \times X_i \to \Re$ is continuous. Thus, by Berge's Maximum Theorem the correspondence $\xi_i : X \to X_i$ is upper hemi-continuous. It is also convex-valued because of linearity of Φ_i. Define the correspondence: $\xi : X \to X$ as follows: $\xi = \Pi_{i \in N} \xi_i$. Because of A.1., Bohnenblust and Karlin's fixed point theorem (see, for example, Smart [1974, Chapter 9]) ensures that there exists $x^* \in X$ such that $x^* \in \xi(x^*)$. Thus, $x^* \in E(\Gamma_c)$. □

3 Two Examples of Application

Example 1.

This example employs the existence result obtained in the preceding section to prove the existence of a 1-LCEE in a model of monopolistic competition similar to the BZ's one. In our model, however, the "boundary conditions" can be violated, thus BZ′s existence result cannot be applied.

Consider a monopolistic competitive market with n price-making firms. Firms are labelled by $i = 1, 2, ..., n$. Set $N = \{1, 2, ..., n\}$. The cost function of firm i is $C_i(q_i) = c_i q_i$, where q_i is the level of output of firm i and c_i is a positive number. For the sake of simplicity, we assume that firm i $(i \in N)$ chooses any price in the interval $[c_i, \infty)$, which is denoted by I_i. Set $I = \Pi_{I \in N} I_i$ and $I_{-i} = \Pi_{j \neq i} I_j$. The price set by firm i is denoted by p_i. Denote by p_{-i} the n–1-dimensional vector whose elements are the prices set by all firms except the i-th one. Set $p = (p_i, p_{-i})$. The function $D_i : I \to \Re$ is the demand function of firm i, and it is indicated by $D_i(p)$. The "true" profits of firm i are: $\Pi_i(p) = D_i(p) \cdot (p_i - c_i)$.

A.3. For every $i \in N$: function D_i is continuous on I. Moreover, the derivative $\partial D_i / \partial p_i : I \to \Re$ exists and is continuous.

A.4. For every $i \in N$: there exists $P_i < \infty$ such that $D_i(p_i, p_{-i}) = 0$ and $(\partial D_i / \partial p_i)(p_i, p_{-i}) = 0$ for every $p_i \geq P_i$ and $p_{-i} \in I_{-i}$.

Because of A.4 we can choose the interval $Z_i = [c_i, P_i]$ as the strategy set of firm i. Set $Z = \Pi_{i \in N} Z_i$ and $Z_{-i} = \Pi_{j \neq i} Z_i$.

A.5. For every $p_{-i} \in I_{-i}$, if $D_i(p'_i, p_{-i}) = 0$, $p'_i \in Z_i \backslash \{P_i\}$, then $(\partial D_i / \partial p_i)(p'_i, p_{-i}) \leq 0$ and $D_i(p''_i, p_{-i}) = 0$, for every $p''_i \geq p'_i$ [2].

Assumptions A.3, A.4 and A.5 are similar to those employed by BZ. Like them, we do not assume that D_i is differentiable with respect to p_j, $j \in N$, $j \neq i$. Unlike them, we do not assume that whatever the strategy price of the other firms, each firm is always able to earn positive profits (see BZ's Assumption 1 (b)); in fact, here it is possible that for every price in I_i, firm i's market demand is zero.

Since in this section we are concerned with 1-LCEEs, we shall assume that firms maximize their conjectural profit function calculated by

2. The derivative $(\partial D_i / \partial p_i)$ should be interpreted as left-hand derivative.

taking into account the linear approximation of their demand function. Given the *status quo* $p^0 \in Z$, the conjectural demand of firm i is: $\Delta_i(p_i, p^0) \equiv D_i(p^0) + (\partial D/\partial p_i)(p^0) \cdot (p_i - p_i^0)$, and the conjectural profit is: $\Omega_i(p_i, p^0) \equiv \Delta_i(p_i, p^0) \cdot (p_i - c_i)$.

DEFINITION 3 : A 1-LCEE is a vector $p^* \in Z$ such that for every $i \in N$:

(*) $\Omega_i(p_i^*, p^*) \geq \Omega_i(p_i, p^*)$ for every $p_i \in Z_i$.

Definition 3 means that at equilibrium firms are maximizing their conjectural profit function. It is easily seen that if p^* is a 1-LCEE, then: (i) $\Delta_i(p_i^*, p^*) = D(p^*)$, and (ii) $(\partial\Delta_i/\partial p_i)(p^*) = (\partial D_i/\partial p_i)(p^*)$. Condition (i) means that at equilibrium the conjectural demand must be equal to the true demand. Condition (ii) means that at equilibrium the slope of the true demand function is equal to the slope of the conjectural demand. Therefore, Definition 3 is substantially the concept of equilibrium employed by BZ in their application to monopolistic competition (BONANNO and ZEEMAN [1985, p. 281]) [3].

PROPOSITION 2 : Under A.3, A.4 and A.5 there exists a 1-LCEE
Note that BZ's existence result cannot applied here since, as said before, our assumptions are compatible with zero demand for firm i, whatever the price p_i in the interval Z_i. If this happens, then firm's i profit function is constant over this interval and equal to zero; therefore, BZ's "boundary conditions" are violated (BONANNO and ZEEMAN [1985, p. 278]).

Proof of Proposition 2: By setting $X_i = Z_i$ and $x_i = p_i$, $i \in N$, the industry we are considering reduces to the game Γ considered in the preceding section. Under A.3., A.4. and A.5. the game has clearly a 1-LCE, $x^* = (x_i^*)_{i \in N}$. Set $p_i^* = x_i^*$, $i \in N$. Thus, to prove Proposition 2 it is sufficient to prove that if $(p_i^*)_{i \in N}$ is a 1-LCE, then it satisfies condition (*) in Definition 3. To this end, we have to consider three possible cases: (a) $p_i^* = P_i$, (b) $p_i^* = c_i$, (c) $p_i^* \in Z_i \backslash \partial Z_i$, $i \in N$.

Case (a): $p_i^* = P_i$. Assumption A.4. ensures that $D_i(p^*) = (\partial D_i/\partial p_i)(p^*) = 0$. It follows that $\Delta_i(p_i, p^*) = 0$, $p_i \in Z_i$. Therefore, $\Omega_i(p_i^*, p^*) = \Omega_i(p_i, p^*) = 0$, $p_i \in Z_i$. Thus, condition (*) in Definition 3 is satisfied.

Case (b): $p_i^* = c_i$. Two cases can occur: (b1) $(\partial\Pi_i/\partial p_i)\ (p^*) = 0$; (b2) $(\partial\Pi_i/\partial p_i)(p^*) \neq 0$. In case (b1), it is not possible that $D_i(p^*) > 0$. In fact, if it is so, one has that $(\partial\Pi_i/\partial p_i)(p^*) = D_i(p^*) > 0$, which is a contradiction. If $D_i(p^*) = 0$, then $\Omega_i(p_i^*, p^*) \geq \Omega_i(p_i, p^*)$ for $p_i \in Z_i$ since $\Omega_i(p_i^*, p^*) = 0$ and $\Omega_i(p_i, p^*) = ((\partial D_i/\partial p_i)(p^*) \cdot (p_i - p_i^*)) \cdot (p_i - c_i) \leq 0$ because $p_i^* = c_i$ and $(\partial D_i/\partial p_i)(p^*) \leq 0$ from Assumption A.5. In case (b2), then by the fact that p^* is a 1-LCE, it must

3. Note that like in BZ's example, it is not possible to apply in a straightforward way a fixed point argument to prove the existence of a 1-LCE because function $\Omega_i(p_i, p^*)$ is not necessarily a concave function.

satisfy the condition: $(D_i(p^*)+(\partial D_i/\partial p_i)(p^*)\cdot(p_i^*-c_i))\cdot(p_i-p_i^*)\leq 0$, $p_i\in N(c_i)$, where $N(c_i)$ is a right neighbourhood of c_i. Because $p_i^*=c_i$, one has: $D_i(p^*)\cdot(p_i-p_i^*)\leq 0$, $p_i\in Z_i$. This implies that $D_i(p^*)=0$, and, therefore, by A.5, that $(\partial D_i/\partial p_i)(p^*)\leq 0$ and that $D_i(p_i, p_{-i}^*)=0$ for every $p_i\in Z_i\backslash\{c_i\}$. We shall prove that $\Omega_i(p_i^*, p^*)\geq\Omega_i(p_i, p^*)$ for $p_i\in Z_i$. In fact, $\Omega_i(p_i^*, p^*)=0$, while $\Omega_i(p_i, p^*)=(D_i(p^*)+(\partial D_i/\partial p_i)(p^*)\cdot(p_i-p_i^*))\cdot(p_i-c_i)=(\partial D_i/\partial p_i)(p^*)\cdot(p_i-c_i)^2\leq 0$ for every $p_i\in Z_i\backslash\{c_i\}$, from the above argument. Thus, also in this case condition (*) in Definition 3 is satisfied.

Case (c): $p_i^*\in Z_i\backslash\partial Z_i$. By definition of 1-LCE, one must have: $(\partial\Pi_i/\partial p_i)(p^*)=0$. Two cases can occur: (c1) $D_i(p^*)>0$, and (c2) $D_i(p^*)=0$. In case (c1) by noticing that $(\partial\Pi_i/\partial p_i)(p^*)=0$ implies $(\partial D_i/\partial p_i)(p^*)<0$ and that $(\partial^2\Omega_i/\partial p_i^2)(p_i^*, p^*)=2(\partial D_i/\partial p_i)(p^*)$, one can conclude that $(\partial\Pi_i/\partial p_i)(p^*)=0$ implies $(\partial^2\Omega_i/\partial p_i^2)(p_i^*, p^*)<0$. Thus condition (*) in Definition 3 is satisfied. In case (c2), if we prove that $(\partial D_i/\partial p_i)(p^*)=0$, we have completed the proof since in this case $\Omega_i(p_i^*, p^*)=\Omega_i(p_i, p^*)=0$, $p_i\in Z_i$. Suppose, on the contrary, that $(\partial D_i/\partial p_i)(p^*)<0$ (see A.5), then $(\partial\Pi_i/\partial p_i)(p^*)\cdot(p_i-p_i^*)=(D_i(p^*)+(\partial D_i/\partial p_i)(p^*)\cdot(p_i^*-c_i))\cdot(p_i-p_i^*)=(\partial D_i/\partial p_i)(p^*)\cdot(p_i^*-c_i)\cdot(p_i-p_i^*)>0$ for $p_i<p_i^*$, contradicting the hypothesis that p^* is a 1-LCE. Thus, also in this last case condition (*) in Definition 3 is satisfied. □

Example 2: This example shows that the existence results obtained in the preceding section can be employed to prove in an extremely easy way the existence of a 1-LCEE in a general equilibrium framework with imperfectly-competitive quantity-setting firms (see GARY-BOBO [1988], [1989a], [1989b]).

Consider an economy with production with m firms and g goods: $\mathcal{E}(G, I, J, (u_i)_{i\in I}, (X_i)_{i\in I}, (\omega_i)_{i\in I}, (\theta_i)_{i\in I}, (Y_j)_{j\in J})$, where G (resp. I, resp. J) is the index set of goods (resp. household, resp. firms); for the meaning of the remaining symbols–which are standard–and for a more detailed description of the model, see GARY-BOBO [1989a]. Given the production profile $y=(y_1, y_2, ..., y_m)\in Y\equiv\Pi_{j\in J}Y_j$, the intermediate endowment of consumer i is $\omega_i^0(y)=\omega_i+\Sigma_j\,\theta_{ij}\,y_j$. Denote by $\xi_i(p, y)$ the individual demand mapping, and by $z(p, y)$ the aggregate excess demand mapping of the economy at price $p\in\Delta\subseteq\Re_+^g$, given the production profile y. The symbol $W(y)$ indicates the set of Walrasian prices associated with production profile y; *i.e.* $W(y)=\{p\in\Delta|z(p, y)=0\}$. Finally, set: $V=\{y\in\Re^{mg}|\omega_i^0(y)\gg 0,\ i\in I\}$.

The following assumptions have been adopted by GARY-BOBO [1989a], [1989b].

A.6. For all i, u_i is such that $\xi_i(p, y)$ is single-valued, strictly positive and of class C^∞ in $\Re_{++}^g\times V$.

A.7. $V\supset Y$. Moreover, Y is compact, and Y_j is a convex set, $j\in J$.

A.8. If $W(y)$ is non-empty, then $W(y)$ is singleton.

Denote by z^{-} function z without the last element, by $p(y)$ the element of set $W(y)$, and by $D_{p_{-g}}$ the derivative with respect to the $g-1$ first components of p.

A.9. For all $y \in V$: rank $D_{p_{-g}}\, z^{-}[p(y), y] = g-1$.

Producer j calculates his/her profits on the basis of the linear approximation of the effective demand function: $p_j^{\#}(y_j, y^0) = p(y^0) + (y_j - y_j^0) \cdot D_{yj}\, p(y^0)^{\mathrm{T}}$, where y^0 is the *status quo* and D_{y_j} indicates the derivative with respect to y_j; symbol $^{\mathrm{T}}$ indicates the operation of transposition, and will be used only for matrices.

DEFINITION 4 : An 1-LCEE for economy $\mathcal{E}$ is a configuration $(p^*, (y_j^*)_{j\in J}) \in \Delta \times Y$ such that:

(**) $$p_j^{\#}(y_j^*, y^*) \cdot y_j^* \geq p_j^{\#}(y_j, y^*) \cdot y_j,\ y_j \in Y_j,\ j \in J.$$

Definition 4 means that at a 1-LCEE firms are maximizing their profits according their perceived demand functions. It is easily seen that if $(p^*, (y_j^*)_{j\in J})$ is a 1-LCEE, then: (i) $p_j^{\#}(y_j^*, y^*) = p(y^*),\ j \in J$, and (ii) $D_{y_j}\, p_j^{\#}(y_j^*, y^*) = D_{y_j}\, p(y^*),\ j \in J$. Condition (i) means that at 1-LCEEs perceived prices are equal to the true ones, while condition (ii) means that the slopes of the perceived demand curves are equal to the slopes of the true demand curves.

PROPOSITION 3: If assumptions A.6-A.9 hold true and if $\zeta \cdot D_{y_j}\, p(y) \cdot \zeta \leq 0$ for every $y \in Y$ and for every $\zeta \in \Re^g$, then economy $\mathcal{E}$ has a 1-LCEE.

Proof: By setting $X_j = Y_j$, $x_j = y_j$ and $\Pi_j(y_j, y) = p_j^{\#}(y_j, y) \cdot y_j$, $j \in J$, the economy $\mathcal{E}$ reduces to the game Γ introduced in the preceding section. Under Assumptions A.6-A.9, $p(y)$ is C^1 and this game has clearly a 1-LCE, $(x_j^*)_{j\in J}$. Set $y_j^* = x_j^*$, $j \in J$. In order to prove Proposition 3 it is sufficient to prove that $y^* = (y_j^*)_{j\in J}$ satisfies condition (**) in Definition 4. To this end, note that since y^* is a 1-LCE, then it must satisfy the condition:

$$p(y^*) \cdot y_j^* \geq p(y^*) \cdot y_j^* + [p(y^*) + y_j^* \cdot D_{y_j}\, p(y^*)] \cdot (y_j - y_j^*),$$

or

(1) $$p(y^*) \cdot (y_j - y_j^*) \leq y_j^* \cdot D_{y_j}\, p(y^*) \cdot (y_j^* - y_j), \quad y_j \in Y_j.$$

We prove the assertion if we show that y^* satisfies the following condition:

$$p(y^*) \cdot y_j^* \geq p(y^*) \cdot y_j + (y_j - y_j^*) \cdot D_{y_j}\, p(y^*)^{\mathrm{T}} \cdot y_j, \quad y_j \in Y_j;$$

that is:

(2) $$[p(y^*) + y_j \cdot D_{y_j}\, p(y^*)] \cdot (y_j - y_j^*) \leq 0,\ y_j \in Y_j$$

From the first member of (2) and by taking into account (1), one obtains: $p(y^*) \cdot (y_j - y_j^*) + y_j \cdot D_{y_j}\, p(y^*) \cdot (y_j - y_j^*) \leq y_j^* \cdot D_{y_j}\, p(y^*) \cdot (y_j^* - y_j) - y_j \cdot D_{y_j}\, p(y^*) \cdot (y_j^* - y_j) = (y_j^* - y_j) \cdot D_{y_j}\, p(y^*) \cdot (y_j^* - y_j) \leq 0$, $y_j \in Y_j$ by assumption. □

• References

BONANNO, G., ZEEMAN, C. E. (1985). – "Limited Knowledge of Demand and Oligopoly Equilibria", *Journal of Economic Theory*, 35, pp. 276-283.

GARY-BOBO, R. J. (1987). – "Locally Consistent Oligopolistic Equilibria are Cournot-Walras Equilibria", *Economic Letters*, 23, pp. 217-221.

GARY-BOBO, R. J. (1988). – "Équilibre général et concurrence imparfaite : un tour d'horizon", *Recherches Économiques de Louvain*, 54, pp. 53-86.

GARY-BOBO, R. J. (1989a). – "Cournot-Walras and Locally Consistent Equilibria", *Journal of Economic Theory*, 49, pp. 10-32.

GARY-BOBO, R. J. (1989b). – *Équilibre général et concurrence imparfaite*, Paris, Éditions du CNRS.

SILVESTRE, J. (1977). – "A Model of General Equilibrium with Monopolistic Behaviour", *Journal of Economic Theory*, 16, pp. 425-442.

SMART, D. R. (1974). – *Fixed Points Theorems*, Cambridge, Cambridge University Press.