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Information Structures and the Delegation of Monitoring

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ABSTRACT. – If the owner of a firm cannot commit ex-ante to monitor his workers and the monitoring technology may accuse a diligent worker of shirking (produce a "false positive"), the lack of commitment problem is extremely severe: the worker may shirk even if monitoring is costless. However, the same presence of false positives is necessary for this commitment problem to be fully resolved either through delegation of monitoring or through contracting with a passive third party.

Structures d'information et délégation du contrôle

RÉSUMÉ. – Quand le propriétaire d'une entreprise est incapable de contrôler ex-ante ses travailleurs, et que la technologie de contrôle peut accuser un travailleur diligent (produire des faux positifs) le manque d'engagement pose un problème extrêmement sevère : le travailleur peut fainéanter même si le contrôle est gratuit. Néanmoins, les faux positifs sont nécessaires pour résoudre le problème de manque d'engagement, soit par la délégation de la tâche de contrôle à un superviseur, soit à travers un contract avec une tierce partie.

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1 Introduction

We explore a situation in which the (risk neutral) owner of a firm cannot commit ex-ante to monitor his (risk averse) workers. Of course, this brings about the well-known problem that, once the worker is assumed to be diligent, the owner has no incentive to monitor the worker, which the worker himself anticipates. Consequently, the worker shirks in equilibrium (with some probability), *i.e.*, the owner is unable to implement the full commitment actions [1].

We show that the particular information structure induced by the monitoring technology at work has an important impact on

1) how severe the lack of commitment problem is

and

2) whether this problem can be fully solved.

In particular, we focus our analysis on the mistake that is made when the monitoring signal suggests that a worker has been shirking when in fact he was diligent. We call this mistake a *false positive* (FP). Conversely, we call *false negative* (FN) the error made when the signal suggests that the worker was diligent when in fact he was shirking.

To this effect, we study a family of monitoring technologies (MT henceforth) parameterized by an unidimensional variable $\lambda \in [0, 1]$ producing a monitoring signal Y that can only take up two values, high (H) or low (L). Hence, we say that a false positive is produced if $Y = L$ when the worker is diligent and that a false negative is produced if $Y = H$ when the worker shirks. The family of technologies is modeled so that, when $\lambda = 0$, false positives are impossible (even if monitoring effort is low), and only false negatives may occur. Conversely, when $\lambda = 1$, the only error that the MT can produce is a false positive, and false negatives are impossible. Note that λ is an exogenous variable, while monitoring intensity is a choice variable to whoever monitors.

Here is a real world example of a MT with $\lambda > 0$. Suppose that in a public school (firm) a teacher's (worker's) effort cannot be observed directly. The teacher's effort is evaluated by interviewing his students (monitoring technology). These interviews yield an unidimensional signal that could be interpreted as how close the teacher comes to some teaching standard. If the interviews are performed unthoroughly (with low monitoring effort), a diligent teacher could be accused of shirking (false positive).

On the other hand, a MT with $\lambda = 0$ could be illustrated by the case of a secretary who is fully responsible for his typos and who cannot be accused of shirking if no typos are detected (no false positives can be produced).

1. Thi issue has been extensively studied both in the context of tax evasion (*see*, for instance, Melumad and Mookherjee (1989) or Reinganum and Wilde (1986)) and in the context of law enforcement (*see* Besanko and Spulber (1989)).

The more carefully one checks his manuscripts (monitoring intensity), the more typos can be found if the secretary shirked.

One of our main results is that, if the owner is unable to commit to any monitoring strategy, the worker shirks in equilibrium with probability equal to λ even if monitoring costs are zero.

The intuition is simple. Suppose first that $\lambda > 0$. The owner likes false positives to occur because they give him an excuse to pay a low wage. Suppose the owner believes that the worker is being diligent. Then the owner's best response is to not monitor, thereby maximizing the probability that a false positive will occur. Note that this argument is independent of how costly monitoring is. Moreover, the larger λ is, the stronger the incentives are for the owner not to monitor.

On the other hand, if $\lambda = 0$, this argument falls apart, as false positives are impossible. In fact, the perfect information outcome is attained if monitoring costs are zero under this MT. Hence, the lack of commitment problem is more severe if FP are present.

In the contract theory literature, there have been two approaches in the design of mechanisms that alleviate a lack of commitment problem. To set some terminology, call the player that is unable to commit "the principal". In ROGOFF [1985], FERSHTMAN and JUDD [1987], SKLIVAS [1987], MELUMAD and MOOKHERJEE [1989], BESANKO and SPULBER [1989], and MACHO and PÉREZ [1991], the principal delegates the choice of the action that cannot be committed in advance to a third individual. In HART [1983], HOLMSTROM [1982], BROWN and WOLFSTETTER [1989], and GREEN [1990], the principal brings in a (passive) third party with whom a contract is signed. This third party does not participate in the game at all [2].

We explore both possibilities, and refer to them as "delegation", and "third party contracting", respectively.

We show that, if players are financially constrained, both delegation and third party contracting allow the owner to implement the full commitment actions if λ is large enough, whereas, if λ is small or zero, the full commitment actions cannot be even approximated, no matter which mechanism is used. Moreover, if λ is zero, the full commitment actions cannot be implemented even if the players are not financially constrained (although these actions can be approximated arbitrarily closely).

To tie it up, the same presence of false positives, which is so pernicious if the commitment problem is left unattended, turns out to be necessary for a full solution to that problem.

What makes the presence of false positives so crucial? Consider what happens if λ is large and the monitoring task has been delegated to a supervisor. Suppose that the owner chooses to reward both the worker and the supervisor whenever the monitoring signal suggests that the worker was diligent (that is, when $Y = H$). Then, if the worker is diligent, the supervisor's optimal strategy is to monitor the worker intensely, *so as to*

2. These two approaches are for static games. If the game is repeated, commitment can be also a result of reputation. See for instance, ALLEN (1984), MASSÓ (1995), or SHAPIRO (1983).

avoid false positives. Given that the supervisor is monitoring the worker intensely, the worker's best reply is to be diligent.

On the other hand, this scheme is useless if $\lambda = 0$. Indeed, if the owner rewards the supervisor for a high signal, the supervisor does not monitor so that the probability that a high signal is obtained is maximized if the worker shirks. If the worker is diligent, the signal *will* be high anyway, since false positives are impossible. This forces the owner to reward the supervisor for a low signal.

The last two paragraphs contain the main empirical implication of our model: in the delegation game, the supervisor should be rewarded if the signal is high when false positives are possible, whereas he should be rewarded if the signal is low if false positives are impossible.

A similar intuition is given in the text for the case when the owner brings in a passive third party.

This can be readily applied to the real world examples introduced earlier. In the teacher's example ($\lambda > 0$), society (owner) delegates the evaluation process to the school superintendent (supervisor). The superintendent should be rewarded when the student's interviews suggest that the teacher was diligent because, supposing the teacher is diligent, such a reward scheme would motivate the school superintendent to conduct the interviews thoroughly enough to reveal that diligence. The teacher would be diligent because he would know that the interviews were going to be conducted thoroughly.

On the other hand, in the case of the secretary ($\lambda = 0$), a supervisor should be rewarded for finding typos, since otherwise the supervisor would shirk. However, this brings back the commitment problem. Once the secretary is diligent, there are no incentives for the supervisor to inspect the secretary's work.

These results support the following suggestion by Melumad and Mookherjee [1987, 2] [3]:

The non-coincidence of principal [in our model, owner] and agent's [supervisor's] interests may in fact be beneficial in some contexts. Specifically, the principal may have difficulty in committing to desirable long run policies with respect a second set of agents [the worker in our model], owing to its own opportunistic propensities to deviate ex post from past promises... Faced with a problem of commitment, the principal may more credibly delegate responsibility to an agent [supervisor] with private preferences different from its own.

It turns out that, in our model, only if $\lambda > 0$ is the owner able to introduce such a divergence of interests between himself and the supervisor: the owner likes false positives whereas the supervisor tries to avoid them.

If all players are financially constrained our results are reinforced. The full-commitment actions cannot be approximated even if λ is positive, if λ is small enough. The reason is that if λ is close to zero (*i.e.*, if false

3. This idea can be traced back to Thomas SCHELLING (1960). See also FERSHTMAN and JUDD (1987).

positives are almost impossible even if monitoring is low) the owner must give the supervisor a huge incentive to monitor *via* the wage contract, which may be unfeasible given the financial constraints of both players.

These findings add to the growing list of apparently paradoxical results in which some MT may be better at providing incentives than another MT that, from a purely statistical point of view, is more efficient. For instance, CRÉMER [1993] shows that access to an improved information structure may undermine the incentives of an agent whose abilities are unknown. Indeed, suppose that the principal wants to commit to firing the agent if a bad result obtains. If the principal learns, thanks to the improved information structure, that a bad result is due to bad luck rather than to an inadequacy of the worker, the principal will prefer to keep the agent rather than firing her. This fact undermines the provision of incentives for the worker to exert effort in her task. In a nutshell, the principal has an ex-post incentive to manipulate information for his own benefit. This weakens the principal's commitment power.

The intuition in our model is slightly different. The inefficiency in the MT (the presence of false positives) has to be compensated for by exerting extra effort, which makes the announcement of intense monitoring credible. However, since the owner likes false positives, he has to either a) delegate monitoring and reverse this preference on the part of the supervisor, or b) reverse his own preference by contracting with a passive third party.

Indeed, the owner shapes the monitoring player's objectives by means of a contract. In this respect, it is very important that all contracts be publicly observable and non-renegotiable, which also rules out private recontracting between the players (and the formation of coalitions) [4]. For instance, consider what would happen if we relax this assumption when $\lambda > 0$ and the owner rewards the supervisor whenever the signal is high. Once the worker is assumed to be diligent, the owner could bribe the supervisor not to monitor, offering him a higher utility payment no matter how the signal comes out. This would generate monitoring cost savings, plus an increase in the probability of a false positive, which the owner likes since he has to reward both the worker and the supervisor when the signal is high.

In this connection, KATZ [1992] studies a model where one of two competing principals may delegate her actions to an agent, let us call him "delegate", with whom an unobservable or renegotiable contract is signed. The delegate and the other principal play what is called "the second stage game" after contracts have been signed. In terms of our model, the principals would be the owner and the worker, and the delegate would be the supervisor. His main result can be summarized as follows. If contracts are unobservable and the principal and delegate have the same preferences over income and effort, then such delegation does not affect the second stage play. In other words, delegation is, in this case, a useless commitment device.

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4. In the case of contracting with a third party, it is possible to find a contract that is renegotiation-proof if there exists an interim information asymmetry, as DEWATRIPONT (1988) and DE BIJL (1994) show. Note that no such information asymmetry exists in our model.

This is what we illustrate in our example of bribes above. Since contracts (bribes) are unobservable, the owner (once the worker is assumed to be diligent) re-aligns the supervisor's incentives with his own. The worker, anticipating this, will not be diligent. Consequently, the owner must set a contract up with the supervisor such that neither of them will want to renegotiate it privately (e.g., through bribes). Such a contract will run into the same commitment problems as the ones encountered when the owner doe not delegate [5]. Our contribution is that, *even if contracts are publicly observable and non-renegotiable*, delegation may sometimes be useless as a commitment device.

It is also important to note that, since contracts must be based (in our static model) on publicly observable information, the assumption that the monitoring signal be publicly observable is therefore crucial in our analysis. There would be at least two ways to relax this assumption. One would be to assume that the monitoring signal is publicly observable if communicated, but that it can be concealed. Another way would be to assume that the signal is observable only to whoever monitors. In the first case, our model would have to incorporate the wage that is paid when no signal is communicated. Note that the owner could commit beforehand to disclosing the monitoring signal by setting up a contract that pays the worker the same wage when the signal is high as when there is no signal at all (in the spirit of the principle that "anyone must be assumed innocent unless proven otherwise"). Checking whether it would be in the interest of the owner to commit to disclose the realization of the monitoring signal is beyond the scope of this paper.

In the second case, a reporting problem would arise between the supervisor and the owner. This would destroy the effectiveness of the commitment device consisting of rewarding the supervisor when the signal is high, since then the supervisor would always report a high signal. Contracts would have to satisfy the additional constraint that contracts induce truthful reporting. Note, however, that if the signal is only observable to whoever monitors, explicit contracts could not be written when the owner does not delegate. The owner would always report to the court that the signal is low so as to pay the worker the smallest wage.

We outline the model in Section 2. Section 3 is devoted to exploring a situation where commitment is feasible (monitoring intensity is chosen ex-ante and is observable). We call this model the full commitment (FC) game, which we use as a benchmark for comparison. Section 4 is devoted to the no commitment (NC) game, where the owner cannot commit to any monitoring strategy and the lack of commitment problem is left unattended. In Section 5 we study the delegation game (DG), where the owner delegates monitoring to another agent. In Section 6 we discuss third party contracting.

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5. Some other authors have also addressed this problem. In a similar setting as this paper, *see* Tirole (1986), Macho and Pérez (1993), and Kofman and Lawarree (1990). In a bargaining situation, *see* Green (1990). These authors do not explore the impact of different information structures on the equilibrium contracts.

2 The Model

The firm is composed of a risk neutral owner and a risk averse worker. The owner is the residual claimant. The worker exerts unobservable effort $a \in \{s, d\}$ where s stands for "shirk" and d for "diligent". This effort generates cash flow $\pi > 0$ when $a = d$ and zero when $a = s$. The worker has a Von Neumann-Morgenstern utility function that is additively separable in money income and effort: $U(m, a) = u(m) - V(a)$, where

ASSUMPTION 1 : The function $u(\cdot)$ satisfies: $u(0) = 0$, $u(x) \to -\infty$ as $x \to -\infty$; $u'(\cdot) > 0$ and $u''(\cdot) \leq 0$. V is the disutility of effort. Let $V(d) = v > 0$ and $V(s) = 0$. The worker obtains utility $T \geq 0$ (for sure) if he leaves the firm.

Contracts on cash flow cannot be signed because cash flow is unobservable. However, a monitoring technology (MT) that provides society with a publicly observable and verifiable signal Y is available. Contracts contingent on this signal, and on this signal alone, can be written. The owner spends resources on monitoring according to a cost function $C(\cdot) : [0, 1] \to \mathbb{R}_+$ where θ is a measure of monitoring intensity belonging to the interval [0, 1], and $C(\cdot)$ satisfies

ASSUMPTION 2 : $C'(\theta) > 0$ for all $\theta > 0$, $C'(0) = C(0) = 0$, $C'(1) = \infty$, and $C''(\cdot) > 0$.

For analytical simplicity, assume the signal can take up only two values: H (for "high") or L (for "low"). The probabilities of these outcomes depend on the monitoring intensity, θ, and the effort exerted by the worker, a. The realization of H and L has the natural interpretation, H indicating that the worker was diligent, and L indicating that the worker shirked, because the MTs specified below satisfy the Monotone Likelihood Ratio Condition (MLRC) [6].

We consider a family of monitoring technologies parameterized by a one-dimensional variable $\lambda \in [0, 1]$. The distribution of probability of the monitoring signal Y is given by

$$\Pr(\text{false positive}) = \Pr(Y = L | a = d) = (1 - \theta)\lambda$$

and

$$\Pr(\text{false negative}) = \Pr(Y = H | a = s) = (1 - \theta)(1 - \lambda).$$

Thus, as θ increases, both the probability of false positives and the probability of false negatives decreases, that is, the signal becomes more informative. The role of λ is to measure the likelihood of false positives. For instance, if $\lambda = 0$, false positives are impossible no matter how low the monitoring intensity θ is (and false negatives are also impossible if $\theta = 1$).

6. *See* MILGROM (1981).

monitoring signal (Y)	worker's effort (a): s	worker's effort (a): d
L	$1 - (1-\theta)(1-\lambda)$	$\lambda(1-\theta)$
H	$(1-\theta)(1-\lambda)$	$1 - \lambda(1-\theta)$

FIGURE 1

The Monitoring Technology.

In the other extreme ($\lambda = 1$) the *only* type of error that the monitoring technology can produce is a false positive. See also Figure 1. It is easy to check that the MLRC holds for all λ and θ in [0, 1].

Three different games are studied. The first game is referred to as FC for "full commitment" game. In it, the owner's choice of θ is observed by the worker before he chooses his effort level. The owner can therefore commit to a level of monitoring intensity before production starts. The order of moves is the following: at the beginning of the game, the owner offers the worker a contract $w = (h, \ell)$ that stipulates the wage h that the worker receives if the realized signal is H, and the wage ℓ that the worker receives if the realized signal is L. An element of the pair w is denoted by w_i. Then the worker decides whether to accept or reject the contract. If the worker rejects the contract, the game ends and the owner receives zero. If the worker accepts the contract, then the owner chooses θ in [0, 1]. Next, the worker observes θ and then chooses whether to shirk, $a = s$, or to be diligent, $a = d$. Finally, chance chooses the signal according to the probabilities specified by the MT given θ and a, and wage is paid. The extensive form of this game is depicted in Figure 2a.

In the second game, referred to as NC for "no commitment" game, the true monitoring intensity cannot be observed by anyone. This makes it impossible for the owner to commit to any level of θ. As before, at the beginning of the period the owner offers a contract to the worker, which he accepts or rejects. If the worker accepts, the worker's effort decision and the owner's monitoring intensity decision are made, in effect, simultaneously [7]. These simultaneous decisions are made in the "second

7. Note that in the FC game we assumed that the owner chooses θ after the worker accepts the contract (rather than choosing θ and w simultaneously). This makes the strategy sets in the FC and NC games as similar as possible. This is done to highlight the fact that the differences between the equilibria in the FC and NC games come, exclusively, from the owner not being able to commit to θ before the worker chooses his strategy.

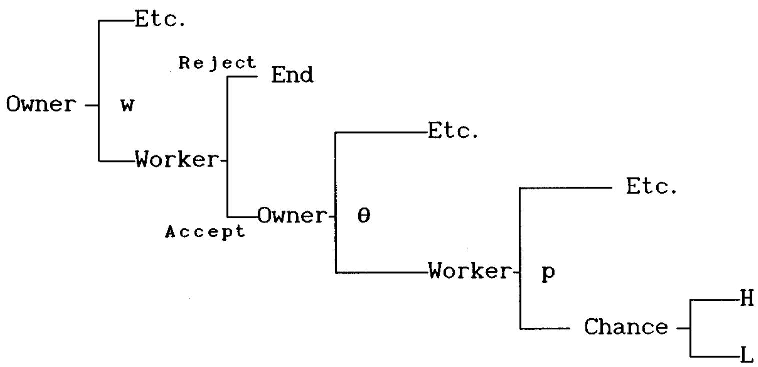

FIGURE 2a

The Full Commitment Game in Extensive Form.

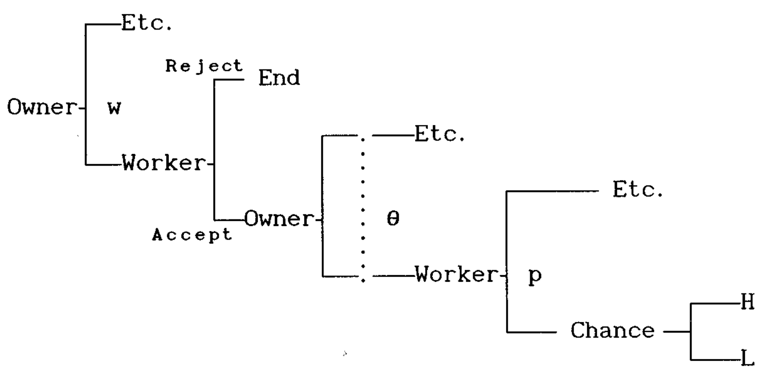

FIGURE 2b

The No Commitment Game in Extensive Form. The dotted line represents a single information set for the agent.

stage of the game". Denote by p the probability that the worker is diligent *i.e.*, p is the probability that the worker sets $a = d$. The extensive form of this game is depicted in Figure 2b.

The *payoffs* of the players in the FC and NC games are the following: provided that the worker accepts the contract, the owner's payoff is

$$U^0 (w, \theta, p; \lambda) = p\pi - E (w_i | \theta, p; \lambda) - C (\theta), \tag{1}$$

where $E (\cdot | \theta, p; \lambda)$ denotes expectation conditional on p and θ, given λ, and is given by

$$\begin{aligned} E (g (w_i) | \theta, p; \lambda) = p & [[1 - (1 - \theta) \lambda] g (h) + (1 - \theta) \lambda g (\ell)] \\ & + (1 - p) [(1 - \theta) (1 - \lambda) g (h) \\ & \qquad + [1 - (1 - \theta) (1 - \lambda)] g (\ell)] \end{aligned}$$

for any function $g (\cdot)$.

The worker's is

$$U^w(w, \theta, p; \lambda) = E(u(w_i)|\theta, p; \lambda) - pv. \tag{2}$$

The following is assumed throughout:

ASSUMPTION 3 : $u(\pi) > v + T$.

This assumption implies that in the first best world without moral hazard, the equilibrium would entail hiring the worker, having him be diligent with probability one, and paying him a wage $u^{-1}(v+T)$. This is a necessary but not sufficient condition for the existence of equilibria in which the worker is hired in the games studied, as we will see.

Using the concavity of $u(\cdot)$, and the fact that $u(0) = 0$ and $C(\theta) \geq 0$, it is easy to show the following Lemma, which will be often used in the remainder.

LEMMA 1 : If $T > 0$ and $p = 0$ in the second stage equilibrium of either the FG game or the NC game, then the owner's payoff, $U^o(w, \theta, p; \lambda)$, would be negative if the worker was hired.

In the third game, referred to as DG for "delegation game", the owner delegates monitoring to a third player, the supervisor. The supervisor has a Von Neumann-Morgenstern utility function that is additively separable in money income and monitoring intensity θ, and given by $U^s(m, \theta) = u_s(m) - C(\theta)$ [8], where $u_s(\cdot)$ satisfies

ASSUMPTION 4 : $u_s'(\cdot) > 0$, $u_s''(\cdot) < 0$, and $u_s(x) \to -\infty$ as $x \to -\infty$.

The order of moves in the DG game is the following: first, the owner writes a contract $W = (w, w_s)$ that stipulates a wage schedule for the worker, $w = (h, \ell)$, and a wage schedule for the supervisor, $w_s = (h_s, \ell_s)$. Wage h_s (respectively, ℓ_s) is paid to the supervisor if the realized signal is H (respectively, L). An element of w_s is denoted by w_{si}. Second, the worker decides whether to accept or reject the contract. Third, if the worker decides to accept, the supervisor has to decide whether to accept, and if so, choose also a monitoring intensity θ in [0, 1]. If either the worker or the supervisor reject their contracts, the game ends, the worker receives $T \geq 0$, the supervisor receives T_s, and the owner receives 0. Assumptions on T_s are given below. If the supervisor accepts the contract, the worker chooses $a \in \{d, s\}$, in the ignorance of the monitoring intensity chosen by the supervisor. (Recall that p denotes the probability that the worker chooses $a = d$). Finally, the signal is realized and the wage is paid. The extensive form of this game is depicted in Figure 2c. The subgame that starts when it is the supervisor's turn is referred to as subgame Γ^s.

8. Thus, the supervisor has the same disutility of θ function as the owner. This captures the idea that owner and supervisor have access to the same MT and have the same preferences over monitoring effort.

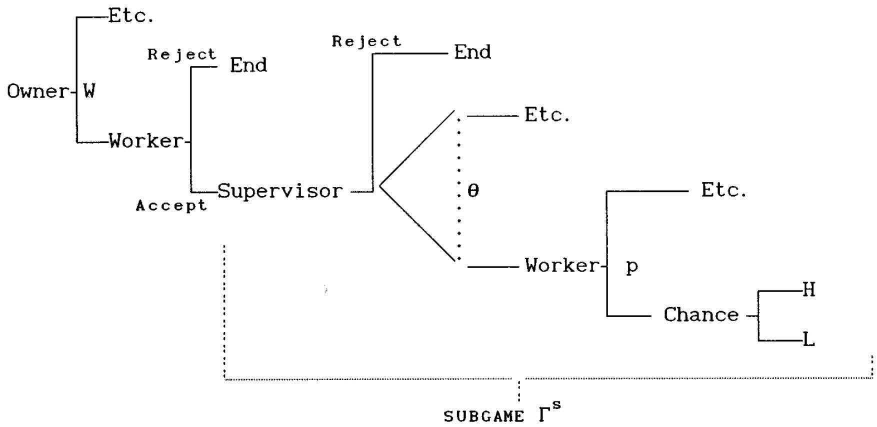

FIGURE 2c

The Delegation Game in Extensive Form. The dotted line represents a single information set for the agent.

The players' *payoffs* in the DG game if contracts are accepted are given next. The worker's payoff is given in (2). The supervisor's payoff is

(3) $$U^s(w_s, \theta, p; \lambda) = E[u_s(w_{si})|\theta, p; \lambda] - C(\theta).$$

The owner's payoff is

(4) $$U^0(W, \theta, p; \lambda) = p\pi - E(w_i + w_{si}|\theta, p; \lambda).$$

So that the DG game is non-trivial, we assume the following.

ASSUMPTION 5 : There exists x such that $u_s(x) > T_s$.

To study delegation as a commitment device, we have to make sure that delegation is not advantageous to the owner when commitment to θ is feasible. For this we need another assumption.

ASSUMPTION 6 : $u_S^{-1}[T_S + C(\theta)] \geq C(\theta)$ for all θ [9],

We prove in the Appendix that the following lemma follows Assumption 6.

LEMMA 2 : Suppose that commitment to θ is feasible no matter who is monitoring. Then, delegation is not beneficial to the owner under Assumption 6.

9. For instance, suppose the supervisor is risk neutral, *i.e.*, $u_s(x) \equiv x$. Then Assumption 6 requires $T_s \geq 0$.

The following Lemma is analogous to Lemma 1, and it is also easily proven using the concavity of $u_s(\cdot)$, Assumption 6, and the assumptions used to prove Lemma 1.

LEMMA 3 : If $T > 0$ and $p = 0$ in the second stage equilibrium of the DG game, then the owner's payoff would be negative if the worker was hired.

We assume throughout that both the worker and the supervisor, if hired, are subject to a financial constraint.

ASSUMPTION 7 : $w_i \geq A$ for all $w_i \in \{\ell, h)$ and

$$w_{is} \geq A_s \text{ for all } w_{is} \in \{\ell_s, h_s).$$

For the sake of notation, we denote the difference $u(h) - u(\ell)$ by Δ, and the difference $u_s(h_s) - u_s(\ell_s)$ by Δ_s.

3 The Full Commitment Game

To find an equilibrium, solve the problem recursively, starting from the last move. Thus, the worker solves

$$\underset{p \in [0,1]}{\text{Max}}\ p\left[\left[1-(1-\theta)\lambda\right]u(h) + (1-\theta)\lambda u(\ell)\right] \\ + (1-p)\left[(1-\theta)(1-\lambda)u(h) + \left[1-(1-\theta)(1-\lambda)\right]u(\ell)\right] - pv,$$

which yields a reaction function

$$p^r(w, \theta; \lambda) = \begin{cases} 0 & \text{if} \quad \theta\Delta < v \\ [0, 1] & \text{if} \quad \theta\Delta = v \\ 1 & \text{if} \quad \theta\Delta > v. \end{cases} \tag{5}$$

From (5) we see that, first, this reaction function does not depend on λ, and, secondly, for any w satisfying $\Delta \geq v$, the worker is indifferent between shirking and not shirking if θ is set equal to

$$\theta^c(w) \equiv v/\Delta. \tag{6}$$

Since it is feasible for the owner to commit to the monitoring intensity $\theta^c(w)$, for large enough π (given λ, $u(\cdot)$, $C(\cdot)$, T, and v) the equilibrium involves the worker setting $p = 1$ and the owner setting $\theta = \theta^c(w)$.

PROPOSITION 1 : In a FC equilibrium, for π large enough, $\theta = \theta^c(w)$ and $p = 1$.

The issue of existence is addressed in next section.

4 The No-Commitment Game

The owner's and the worker's payoffs are the same as in Section 3 (FC), and given by (1) and (2) , respectively. The equilibrium is found recursively, starting from the last (simultaneous) moves.

The worker's reaction function, $p^r(w, \theta; \lambda)$, is the same as in the previous section and given in (5). The owner's reaction function $\theta^r(w, p; \lambda)$ is found by solving

$$\max_{\theta\in[0,1]} p\pi - \{p[[1-(1-\theta)\lambda]h + (1-\theta)\lambda\ell] + (1-p)[(1-\theta)(1-\lambda)h + [1-(1-\theta)(1-\lambda)]\ell]\} - C(\theta).$$

The first order conditions are

$$(7) \qquad (1-\lambda-p)(h-\ell) - C'(\theta)\begin{cases} >0 & \text{then} \quad \theta = 1 \\ =0 & \text{then} \quad \theta\in[0,1] \\ <0 & \text{then} \quad \theta = 0. \end{cases}$$

Hence, using Assumption 2,

$$(8) \qquad \theta^r(w, p; \lambda) = \begin{cases} 0 & \text{if} \quad (1-p-\lambda)(h-\ell)\leq 0 \\ [C']^{-1}[(1-p-\lambda)(h-\ell)] & \\ \text{if} \quad (1-p-\lambda)(h-\ell) > 0. & \end{cases}$$

The most important result of this section is given next.

> Proposition 2 : 1) For any subgame perfect equilibrium of the entire NC game, the probability of shirking, $1-p^*$, is larger than or equal to λ, even if $C'(\theta) = 0$ for all θ. 2) Moreover, if $C'(\theta) > 0$ for all $\theta > 0$, and the owner is also subject to a financial constraint, say $h \leq B$, then, letting $\overline{\Delta} = u(B) - u(A)$, the probability of shirking is larger than or equal to $\text{Min}\left\{\lambda + \dfrac{C'(v/\overline{\Delta})}{B-A}, 1\right\}$.

This is obviously a very negative result. For instance, if false positives are the only type of mistake that the m.t. can produce (that is, if $\lambda = 1$), then $p^* = 0$, and the worker is never hired since the owner obtains zero by not hiring the agent, whereas $p^* = 0$ implies a negative payoff to the owner if $T > 0$ (by Lemma 1). In fact, this will also be true, by continuity, if λ is close to 1. Intuitively, if false positives exist, the owner finds it beneficial to lower θ when $a = d$, since the probability of having to pay $(h-\ell)$ decreases as θ decreases. Lowering θ harms the owner when at $a = s$, since he has to pay $(h-\ell)$ with a higher probability. If $p > 1-\lambda$ (*i.e.*, if $a = d$ is relatively likely) the benefits of lowering θ outweigh its costs. Thus, the owner lowers θ down to zero if $p > 1-\lambda$, to which the best response of the worker is to set $p = 0$. On the other hand, if $\lambda = 0$, the owner is able to implement p^* close to 1 if C' is close to zero and/or the financial constraint in either side is sufficiently weak ($-A$ and B sufficiently large) [10]. Note, finally, that none of these results depend on the curvature of $u(\cdot)$. In particular, the full commitment actions cannot be approximated, at any cost, even if the worker is also risk neutral. The probability of shirking is always greater than or equal to λ. This is in contrast with many works in which the full commitment (or even the first best) outcomes can be approximated arbitrarily closely or even obtained if the agent is risk neutral [11].

Existence

We show now that, although the owner may not be financially constrained, it is not in his interest to set h arbitrarily large.

PROPOSITION 3 : 1) Suppose that $\lambda < 1$. Then, it is not in the interest of the owner to set h arbitrarily large in either the FC or NC games.
2) Suppose that $\lambda = 1$. Then, the owner does not hire the worker in the NC game. In the FC game, it is not in the interest of the owner to set h arbitrarily large if we assume that the worker is risk averse even asymptotically, that is, if $\lim_{x \to \infty} u(x)/x = 0$.

Intuitively, as long as $\lambda < 1$, false negatives are possible, and therefore the owner may have to pay h with some positive probability. Hence, setting h arbitrarily large implies that the expected wage outlay is also arbitrarily large. However, when $\lambda = 1$ and $p^* = 1$, the probability of having to pay h is θ, which goes to zero as $h \to \infty$, since $\theta^* = \theta^c(w) \to 0$ as $h \to \infty$. We need to make sure then that θ goes to zero slower than h goes to infinity, which is accomplished by assuming that the worker is risk averse even asymptotically.

5 The Delegation Game

The owner delegates monitoring to a supervisor. The worker's payoff is given in (2), the supervisor's payoff is given in (3), and the owner's payoff is given in (4).

As before, solve the model recursively. The last subgame is Γ^s, which starts once the worker has accepted the contract. The reaction function of the worker is, once more, the same as in the previous games, and given in (5).

10. It is easy to see that, if $\lambda = 0$, setting $-\ell$ arbitrarily large implements $p^* = 1$ arbitrarily closely at no additional cost to the owner (in the spirit of "setting up arbitrarily large fines with an arbitrarily small probability"). This is unfeasible in our model, due to the worker's financial constraint. However, although setting h arbitrarily large also implements $p^* = 1$ arbitrarily closely, this is at an arbitrarily large cost to the owner (since $\Pr(Y = H) \to 1$ as $p^* \to 1$).

11. See, for instance, NALEBUFF and SHERFSTEIN (1987) or CRÈMER and MC LEAN (1985).

If the supervisor accepts the contract, the supervisor's reaction function is found by solving

$$\max_{\theta\in[0,\,1]} p\,[[1-(1-\theta)\,\lambda]\,u_s(h_s)+(1-\theta)\,\lambda u_s\,(\ell_s)]$$
$$+\,(1-p)\,[(1-\theta)\,(1-\lambda)\,u_s\,(h_s)+[1-(1-\theta)\,(1-\lambda)]\,u_s\,(\ell_s)]-C\,(\theta).$$

The first order conditions are

$$(1-\lambda-p)\,(-\Delta_s)-C'\,(\theta)\begin{cases}>0 & \text{then} \quad \theta=1\\ =0 & \text{then} \quad \theta\in[0,\,1]\\ <0 & \text{then} \quad \theta=0.\end{cases} \tag{9}$$

Hence, using Assumption 2,

$$\theta^r\,(w_s,\,p;\,\lambda)=\begin{cases}0 \quad \text{if} \quad (1-p-\lambda)\,(-\Delta_s)\le 0\\ [C']^{-1}\,[(1-p-\lambda)\,(-\Delta_s)]\\ \text{if} \quad (1-p-\lambda)\,(-\Delta_s)>0.\end{cases} \tag{10}$$

Technically, this is the supervisor's reaction function whenever $U^s\,(w_s,\,\theta^r\,(w_s,\,p;\,\lambda),\,p)\ \ge\ T_s$, otherwise he rejects the contract and no θ is chosen.

Now, the owner has two options:

a) reward the supervisor when the supervisor "catches" the worker shirking, that is, set $\ell_s > h_s$, or

b) reward the supervisor when the supervisor "shows" that the worker is diligent, that is, set $h_s > \ell_s$.

Under the first option, we have the following Proposition:

PROPOSITION 4 : If $\Delta_s < 0$, for any subgame perfect equilibrium of the entire game, the equilibrium probability of shirking, $1-p^*$, is larger than or equal to λ, even if $C'\,(\theta)=0$ for all θ. Moreover, if $C'\,(\theta)>0$ for all $\theta>0$, and the owner is subject to a financial constraint, say $w_i,\,w_{is}\le B$, then, letting $\overline{\Delta}_s=u_s\,(B)-u_s\,(A_s)$, the probability of shirking is larger than or equal to $\text{Min}\left\{\lambda+\dfrac{C'\,(v/\overline{\Delta})}{-\overline{\Delta}_s},\,1\right\}$.

Note that in this case we obtain the same very negative result as in the previous section. The reason is simple: by setting $h_s < \ell_s$, the owner has reproduced on the supervisor his same ill incentives: the supervisor also likes false positives, and will monitor lightly if he assumes that the worker is diligent.

Alternatively, the owner can follow option (b), that is, reward the supervisor when he produces evidence showing that the worker is diligent.

PROPOSITION 5 : Suppose that (i) $\lambda > 0$, and (ii) $\Delta > 0$. Then,

a) If (iiia) $\Delta_s=\dfrac{1}{\lambda}\,C'\,(v/\Delta)>0$, then $p^*=1$ and $\theta^*=v/\Delta$ constitute an equilibrium of subgame Γ^s.

b) If (iiib) $0\le\Delta_s<\dfrac{1}{\lambda}\,C'\,(v/\Delta)>0$, then $p^*=0$.

Part (a) of this proposition reflects one of the main results in the paper. The owner rewards both the worker and the supervisor whenever the signal is high so that, if the worker is diligent with probability one, the supervisor's optimal strategy is to monitor the worker intensely, in order to avoid false positives. Given that the supervisor is monitoring the worker intensely, the worker's best reply is to be diligent. By rewarding the supervisor when the signal is high the owner is able to introduce a divergence of interests between himself and the supervisor: the owner likes false positives whereas the supervisor tries to avoid them. This is what makes delegation so effective.

Of course, this argument crucially depends on the presence of false positives ($\lambda > 0$). Let us now compare Propositions 4 and 5 to clarify the role of false positives in our model.

The heart of the intuition behind these two propositions lies in the basic incentive/risk-sharing trade-off that characterizes most moral hazard problems. An easy way to understand this trade-off is to look at the wage spread, that is, the difference between the wage paid when $Y = H$ and the wage paid when $Y = L$. The wider this spread is, the more incentives are provided, but the worse risk sharing becomes (recall that the owner is risk neutral). To this simple intuition, our model adds a new ingredient. The fear of making a mistake discourages the supervisor from monitoring lightly. When this fear is intense because the probability of such mistakes is large, the owner does not need to set a large wage spread to induce intense monitoring.

Recall that Pr(false positive)$=(1-\theta)\,\lambda$ whereas Pr(false negative)$=(1-\theta)\,(1-\lambda)$. Now suppose that $h_s > \ell_s$. Then the supervisor prefers $Y = H$, and therefore he fears false positives ($Y = L$). But if λ is small, this fear is dim and the owner needs to set a large wage spread to induce intense monitoring. This is reflected in condition (iiia) of Proposition 5. In this case it may be better for the owner to set $h_s < \ell_s$ instead, so that now the supervisor prefers $Y = L$ and therefore he fears false negatives ($Y = H$). The fear of false negatives (relatively more likely when λ is small) becomes the engine for incentives. However, note that for false negatives to be the engine of incentives, the worker must shirk with some probability. Otherwise, false negatives are impossible. This explains why $p^* = 1 - \lambda$ when $h_s < \ell_s$. In other words, if $h_s < \ell_s$, the supervisor looses his commitment power. The costs of this lack of commitment are measured by λ, as $1 - \lambda$ is an upper bound on p^*, by Proposition 4. However, if λ is small, this cost may be small.

On the other hand, if λ is large, then it is worthwhile for the owner to set $h_s > \ell_s$, for two reasons. First, the supervisor regains his commitment power, so $p^* = 1$ is implementable, and second, the wage spread does not need to be large since the fear of false positives is intense enough to induce intense monitoring.

We thus obtain an extremely sharp empirical implication: if false positives are relatively likely, the owner should reward the supervisor when the signal is high, whereas the supervisor should be rewarded when the signal is low if false positives are (almost) impossible.

The discussion on the desirability of implementing $p = 1$ or p arbitrarily close to 1 is very convoluted in the absence of a wealth constraint for the owner. We thus relegate it to an informal discussion in the Appendix.

It is to avoid this cumbersome discussion that we introduce the financial constraint on the part of the owner. Indeed, using wealth constraints to bind both agents' wage spreads is just a way to avoid discussing the risk sharing/incentive trade-off. This reduces the discussion to checking whether $p^* = 1$ (or p close to 1) is implementable or not.

Note that an alternative way to simplify the analysis in the same manner is to assume that both the worker's and the supervisor's utility functions are bounded above. (We give an example of this also in the Appendix.)

• Financially Constrained Owner

Consider now the case that the owner is also financially constrained. Recall that $u_s(B) - u_s(A_s) \equiv \overline{\Delta}_s$ and $u(B) - u(A) \equiv \overline{\Delta}$, are the maximum utility spreads for the supervisor and the worker, respectively. Now, suppose that $\lambda < \frac{1}{\overline{\Delta}_s} C'(v/\overline{\Delta})$. This implies that condition (iiia) of Proposition 5 cannot be satisfied, since wage spreads are bounded. Hence, either $0 \leq \Delta_s < \frac{1}{\lambda} C'(v/\Delta)$ and (by part (b)) $p^* = 0$, or $\Delta_s < 0$ and (by Proposition 4), $p^* \leq 1 - \lambda - C'(v/\overline{\Delta})/(-\overline{\Delta}_s)$, which is bounded away from 1 *even if* $\lambda = 0$.

This reflects one of our main ideas. The same presence of false positives (so pernicious when the commitment problem is left unattended) facilitates the solving of this problem by means of delegation. Given the same restrictions on wages, the owner can implement $p^* = 1$ by delegating the monitoring task. But for this to be so, λ must be bounded away from zero. Otherwise, $p^* = 1$ cannot be even approximated no matter which contracts the owner chooses.

The intuition behind this result is simple. Recall that when λ is small, the incentive to increase θ (so as to avoid false positives) is very dim. Now, since wage spreads are bounded, the owner is unable to compensate this effect by increasing the wage spread arbitrarily. Given a maximum wage spread, we have found a lower bound for λ such that any λ below this lower bound will make it impossible for the owner to provide sufficient incentives to the supervisor. The only thing the owner can do is to turn around the wage schedule by setting $h_s < \ell_s$ (so as to make the avoidance of false negatives the engine for incentives), thereby sacrificing the beneficial conflict of interest between himself and the supervisor. This brings about the original commitment problem, which of course is worsened by the presence of the boundaries on the wage spreads [12].

• Multiplicity of Equilibria

Once we know that, according to Proposition 5, $(p^*, \theta^*) = (1, v/\Delta)$ is an equilibrium of subgame Γ^s as long as conditions (i), (ii) and (iiia) hold, the next question is whether other equilibria exist in subgame Γ^s. Of course, once $h_s > \ell_s$, one may think that the worker and the supervisor shirking may be another equilibrium of this subgame: once the worker shirks, the best way to avoid low signals is to monitor lightly. Once monitoring is light, the worker shirks. We avoid this possibility by letting the supervisor reject the contract as one possible alternative to setting any monitoring intensity at all, and ensuring that the supervisor receives his opportunity utility T_s whenever the equilibrium $(p^*, \theta^*) = (1, v/\Delta)$ is played. Basically, we show that if $p < 1$, the supervisor's best reply is always to reject the contract.

This, however, is not totally satisfactory, since now the supervisor rejecting a contract is another equilibrium of subgame Γ^s. We introduce below an argument that justifies the choice of the equilibrium where the supervisor accepts the contract. The argument is based on an iterated elimination of dominated strategies. Formally:

> PROPOSITION 6 : Suppose that conditions (i), (ii) and (iii) of Proposition 5 hold. Suppose, moreover, that w_s is set so that $U^s(w_s, v/\Delta, 1; \lambda) = T_s$. Then, $(p^*, \theta^*) = (1, v/\Delta)$ is the only equilibrium in which the supervisor accepts the contract.

As stated above, the supervisor rejecting the contract is another equilibrium outcome of subgame Γ^s given a contract W satisfying conditions (ii) and (iii). For instance, pick any $0 \le p_0 \le 1 - \lambda$. Then, given such p_0, the supervisor's best strategy is to reject the contract: if he decided to accept, he would choose $\theta^r(w_s, p_0; \lambda) = 0$, since $(p_0 + \lambda - 1)\Delta_s \le 0$ (*see* (10)). The supervisor would obtain $U^s(w_s, 0, p_0; \lambda)$. However $[\partial U^s/\partial p](w_s, 0, p; \lambda) = 0$ for all p, so we have $U^s(w_s, 0, p_0; \lambda) = U^s(w_s, 0, 0; \lambda)$, which we showed in the proof of Proposition 6 to be less than T_s.

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12. As an example, suppose that $\lambda = 0$. Then, the NC equilibrium of Section 4 involves $p^* \le \text{Max}\left\{1 - \frac{C'(v/\overline{\Delta})}{B - A}, 0\right\} < 1$, by Proposition 2. Does delegation solve this commitment problem? Note that $\lambda = 0$ implies that $\lambda < C'(v/\overline{\Delta})/\overline{\Delta}_s$. Therefore, $\overline{\Delta}_s < 0$ and (by Proposition 4) $p^* \le \text{Max}\left\{1 - \frac{C'(v/\overline{\Delta})}{u_s(B) - u_s(A_s)}, 0\right\}$, which is also less than 1. In fact, if the supervisor is risk neutral and $A_s = A$, the upper bounds on p^* are the same in the DG and the NC games. If, on the other hand, $u_s(\cdot)$ and A_s are such that $u_S(B) - u_s(A_S) < B - A$, then delegating the monitoring task may worsen the lack of commitment problem.

This equilibrium cannot be eliminated by arguing that the owner could always raise both the worker's and the supervisor's payoff by some arbitrarily small $\varepsilon > 0$. Even if $U^s(w'_s, \theta^c(w'), 1) = T_s + \varepsilon$, the worker setting $p = 0$ (or any $p \leq 1 - \lambda$) and the supervisor rejecting the contract is still an equilibrium of Γ^s, because once the supervisor has rejected the contract, the worker's information set is never reached [13].

Nevertheless, since our emphasis is on the issue of implementing the full-commitment level of worker's diligence ($p = 1$), we show in the Appendix that, if $\lambda > 0$, some $W' \in \mathbb{R}^4$ exists such that, in the only outcome of the induced subgame Γ^s that subsists after iterated elimination of dominated strategies [14], the supervisor accepts the contract and picks some $\theta > \theta^c(w')$, and the worker sets $p = 1$.

Intuitively, some $W' = (w', w'_s)$ is chosen so that (a) the supervisor rejecting the contract dominates any strategy $\theta \leq \theta^c(w')$, (b) the supervisor's best response to $p = 1$ is to set $\theta = \hat{\theta} > \theta^c(w')$, and (c) the supervisor prefers *strictly* to accept the contract when $w = w'$, $p = 1$ and $\theta = \hat{\theta}$. Once the strategies in the interval $[0, \theta^c(w')]$ have been eliminated, θ must be larger than $\theta^c(w')$, and the worker sets $p = 1$ (as $\theta^c(w')$ is the monitoring intensity that leaves the worker indifferent between shirking and not shirking). Given $p = 1$, the supervisor accepts the contract and sets $\theta = \hat{\theta}$ by virtue of (b) and (c) [15].

The problem is that the owner's payoff is larger under outcome $(W, \theta^c(w), 1)$ of Proposition 5 than under outcome $(W', \hat{\theta}, 1)$, but W induces a subgame Γ^s in which (reject, $p \leq 1 - \lambda$) is an equilibrium that cannot be eliminated using a dominated-strategy argument.

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13. Altering the structure of the game, e.g., letting the worker decide whether to reject the contract after the supervisor's has decided to accept the contract but before the supervisor has chosen θ – thus creating an additional subgame – does not solve this problem. It is easy to check that the following pair of strategies is an equilibrium of the subgame that starts once the supervisor and the worker have accepted the contracts w_s and w: $(\theta, p) = (0, 0)$ (which is even worse for the owner).
Consider also the following forward induction argument: once it is the worker's turn to move, he knows that the supervisor will be playing the strategy $\theta = \theta^c(w)$, because by rejecting the contract he could always have gotten utility T_s. This argument is not adequate to eliminate the undesired equilibrium, since the worker, by accepting the contract, may be also signaling that he is playing according to the equilibrium $(\theta, p) = (0, 0)$, which gives him a utility higher than T, as can be readily checked.

14. In this two-player game, we say that player 1's strategy $s \in S_1$ is dominated by another strategy $s' \in S_1$ if his payoff when choosing s is lower or equal to his payoff when choosing s' no matter what the other player's strategy is, and there exists some strategy $t \in S_2$ for player 2 such that, given that player 2 chose t, strategy s gives player 1 a strictly lower payoff than strategy s'. We say that a strategy s is undominated if there does not exist another strategy s' that dominates s.

15. This is in the spirit of DEMSKY, SAPPINGTON and SPILLER (1988), where the equilibrium notion is strengthened to solve the problem of existence of multiple equilibria.

Existence

As we did for the FC and NC games, we show now that, even though the owner may not be financially constrained, it is not in his interest to set either h, h_s or ℓ_s arbitrarily large.

PROPOSITION 7 : In the DG game,
1) If $\lambda < 1$, it is not in the interest of the owner to set h or h_s arbitrarily large.
2) If $\lambda = 1$, it is not in the interest of the owner to set h or h_s arbitrarily large as long as the worker is risk averse even asymptotically, that is, as long as $\lim_{x\to\infty} u(x)/x = 0$.
3) If $\lambda > 0$, it is not in the interest of the owner to set ℓ_s arbitrarily large.
4) If $\lambda = 0$, it is not in the interest of the owner to set ℓ_s arbitrarily large as long as the supervisor is risk averse even asymptotically, that is, as long as $\lim_{x\to\infty} \frac{u_s(x)}{x} = 0$.

6 Third Party Contracting

The main results obtained for delegation as a commitment device are:

1) the presence of false positives is a necessary condition to implement the full commitment actions, and

2) if false positives are relatively unlikely (for all θ), or equivalently, if λ is close or equal to zero, the FC actions cannot be even approximated if the players are financially constrained.

We show now that these results also hold when the commitment device used is third-party contracting instead of delegation.

Let w, w_t, and w_n (all in $\mathbb{R}^2$) represent, respectively, the contracts specifying the transfers from owner to worker, from worker to the third party, and from the third party to the owner, contingent on the realization of the signal. Let $W \equiv (w, w_t, w_n)$.

The worker's reaction function is now given by

$$p^r(w, \theta; \lambda) = \begin{cases} 1 & \text{if} \quad \theta\Delta > v \\ [0,1] & \text{if} \quad \theta\Delta = v \\ 0 & \text{if} \quad \theta\Delta < v, \end{cases}$$

where $\Delta = u(h - h_t) - u(\ell - \ell_t)$ has the same interpretation as in the previous sections.

The owner's reaction function is given by

$$\theta^r(w, p; \lambda) = \begin{cases} 0 & \text{if} \quad (1-p-\lambda)\Delta_0 \leq 0 \\ [C']^{-1}[(1-p-\lambda)\Delta_0] & \text{if} \quad (1-p-\lambda)\Delta_0 > 0, \end{cases}$$

where $\Delta_0 = [h - h_n] - [\ell - \ell_n]$.

Now, if $\lambda = 0$, then $(1 - p - \lambda)\,\Delta_0 = (1 - p)\,\Delta_0$. If $p = 1$, then $(1-p)\,\Delta_0 = 0$ and $\theta = 0$, so $p = 0$, contradiction. Therefore, $p = 1$ cannot be implemented if $\lambda = 0$. Also, note that if no TP is contracted with, we obtain the same reaction function as in the NC game of Section 4. All our results in this section come from the interaction between these reaction functions, so none of the results is altered by introducing announcements by the owner (or any other player).

If $\lambda > 0$ but small, the owner can implement $p = 1$, but, as we discussed in the previous section and in the Appendix, doing so may be very costly to the owner. The same is true if $\lambda = 0$ and one wants to approach $p = 1$ arbitrarily closely. Again, it all depends on the curvature of the third party's utility function [16]. The same intuitions apply in this case.

As in the previous section, much more clear cut results are derived when we assume that all players are financially constrained. Suppose that, for the owner,

(i) $h - h_n \leq B$ and (ii) $\ell - \ell_n \leq B$;

for the worker,

(iii) $h_t - h \leq A$ and (iv) $\ell_t - \ell \leq A$;

and for the third party,

(v) $h_n - h_t \leq D$ and (vi) $\ell_n - \ell_t \leq D$.

Before we go on, note that these inequalities imply

$-[\ell - \ell_n] \leq D + A$ (adding (iv) and (vi));

$h - h_t \leq B + D$ (adding (i) and (v)); and

$h - h_n \geq -(D + A)$ (adding (v) and (iii)).

Therefore,

$$\Delta_0 = [h - h_n] - [\ell - \ell_n] \leq B + D + A \equiv \overline{\Delta}_0.$$

And also,

$$\Delta_0 = [h - h_n] - [\ell - \ell_n] \geq -(D + A) - B \equiv -\overline{\Delta}_0.$$

Similarly,

$$\Delta = u\,[h - h_t] - u\,[\ell - \ell_t] \leq u\,(B + D) - u\,(-A) \equiv \overline{\Delta}.$$

We have the following Proposition.

16. Indeed, if one assumes that the TP is risk neutral, the cost of setting large wage spreads is going to be nil and it may be the case that it is beneficial to the owner to implement either π equal to 1 or p approaching 1, even if false positives are almost impossible or simply impossible.

PROPOSITION 8 : Suppose that all players are financially constrained. Let A, B, and D be, respectively, the maximum amount that the worker, owner, and third party are able to pay. Then

1) If Δ_0 is positive, then $p^* \leq \text{Max}\left\{1 - \lambda - \dfrac{C'\left(v/\overline{\Delta}\right)}{\overline{\Delta}_0}, 0\right\}$.

2) Suppose that $0 \leq \lambda < C'\left(v/\overline{\Delta}\right)/\overline{\Delta}_0$. Then, if Δ_0 is negative, $p^* = 0$.

3) Suppose $C'\left(v/\overline{\Delta}\right)/\overline{\Delta}_0 \leq \lambda \leq 1$. Then, if $\Delta > 0$ and $\Delta_0 = -C'\left(v/\Delta\right)/\lambda < 0$, $p^* = 1$ and $\theta^* = v/\Delta$ constitute an equilibrium given $W(\hat{\theta})$.

Remarks: 1) This proposition tells us that the existence of false positives is a necessary condition for $p^* = 1$ to be implementable, since λ must be at least as large as $C'\left(v/\overline{\Delta}\right)/\overline{\Delta}_0$ (part 3). Moreover, if $\lambda < C'\left(v/\overline{\Delta}\right)/\overline{\Delta}_0$, then $p = 1$ cannot be even approximated, since, if $\lambda < C'\left(v/\overline{\Delta}\right)/\overline{\Delta}_0$, then Δ_0 has to be positive (otherwise $p = 0$) and therefore p is bounded away from 1 (parts (1) and (2)). Thus, we have the same results as the ones obtained for delegation.

2) Note that the way to implement $p = 1$ (when possible) is to set $\Delta_0 < 0$. The owner thus gives himself the appropriate incentives to monitor when the worker is diligent, since $\Delta_0 < 0$ implies that the realization of a low signal (a false positive if $a = d$) is bad for him.

3) Note that $\Delta_0 < 0$ and $\Delta > 0$ imply that $(h_t - h_n) < (\ell_t - \ell_n)$, so the third party prefers $Y = H$, that is, he likes false positives.

4) Finally, note that is is crucial here also that contracts be publicly observable and non-renegotiable. Otherwise, suppose that, as in part 3, $C'\left(v/\overline{\Delta}\right)/\overline{\Delta}_0 \leq \lambda \leq 1$, $\Delta > 0$ and $\Delta_0 = -C'\left(v/\Delta\right)/\lambda$, so that $p^* = 1$ and $\theta^* = v/\Delta$. The owner could then bribe the third party as follows: "I will not monitor, so that a FP occurs with maximum probability. Thus, your expected payment from the worker will increase, I will save on monitoring costs, and we can share these rents."

Proof of Lemma 2

Let (w_f, θ_f, p_f) be the owner's choice when he does not delegate the monitoring task; and let $U^0(w_f, \theta_f, p_f; \lambda|\mathrm{no\,dg})$ be his payoff in this case. Similarly, let (w_s, θ_s, p_s) be the owner's choice when he *does* delegate the monitoring task, and let $U^0(w_s, \theta_s, p_s; \lambda|\mathrm{dg})$ be his payoff. If the owner delegates monitoring to a supervisor, and the supervisor can also commit to θ, the optimal contract consists of a fixed wage $h_s = \ell_s = w'$ that covers his opportunity utility and monitoring disutility. In other words, $w' = u_s^{-1}[T_s + C(\theta_s)]$, and $U^0(w_s, \theta_s, p_s; \lambda|\mathrm{dg}) = p_s\pi - E(w_{is}|\theta_s, p_s; \lambda) - u_s^{-1}[T_s + C(\theta_s)]$. Assumption 6 is as sufficient condition to ensure that $U^0(w_f, \theta_f, p_f; \lambda|\mathrm{no\,dg}) \geq U^0(w_s, \theta_s, p_s; \lambda|\mathrm{dg})$.

To see this, suppose by contradiction that

$$U^0(w_f, \theta_f, p_f; \lambda|\mathrm{no\,dg}) < U^0(w_s, \theta_s, p_s; \lambda|\mathrm{dg})$$
$$= p_s\pi - E(w_{is}|\theta_s, p_s; \lambda) - u_s^{-1}[T_s + C(\theta_s)].$$

Be revealed preference, we know that

$$U^0(w_f, \theta_f, p_f; \lambda|\mathrm{no\,dg}) \geq U^0(w_s, \theta_s, p_s; \lambda|\mathrm{dg})$$
$$= p_s\pi - E(w_{is}|\theta_s, p_s; \lambda) - C(\theta_s).$$

The last two inequalities imply $u_s^{-1}[T_s + C(\theta_s)] < C(\theta_s)$, which contradicts Assumption 6.

Proof of Proposition 1

Once the owner and the worker have signed the contract w, if, by contradiction, $p[w, \theta^c(w); \lambda]$ is less than 1 (say, $p[w, \theta^c(w); \lambda] = 1-\delta,\ \delta > 0$), the owner gains by deviating to $\theta^c(w)+\mu$ where μ is arbitrarily small but positive. The gains from such deviation are $\delta\pi$, a fixed positive number, whereas the costs are $C[\theta^c(w)] - C[\theta^c(w) + \mu]$ (which can be made arbitrarily small) plus $E(w_i|\theta^c(w)+\mu, 1; \lambda) - E(w_i|\theta^c(w), 1-\delta; \lambda)$. The gains are larger than the costs for π large enough.

Also by contradiction, if $\theta > \theta^c(w)$, say $\theta = \theta^c(w) + \mu$, the owner gains by deviating to $\theta' = \theta^c(w) + \mu/2$, since the worker still sets $p = 1$ whereas both monitoring cost and expected wage outlay are smaller. Finally, if $\theta < \theta^c(w)$, then the worker sets $p = 0$ and the owner gains π by increasing θ to $\theta^c(w) + \mu$, at a cost $C(\theta^c(w) + \mu) - C(\theta) + E(w_i|\theta^c(w) + \mu, 1; \lambda) - E(w_i|\theta, 0; \lambda)$, which is less than π for π large enough.

Proof of Proposition 2

Suppose, by contradiction, that $1 - p < \mathrm{Min}\left\{\lambda + \dfrac{C'(v/\overline{\Delta})}{B - A}, 1\right\}$. Then $p > 0$ and (5) imply that $h > \ell$. Now, since $h - \ell \leq B - A$ and also $\Delta \leq \overline{\Delta}$, we can write $(1-p-\lambda)(h-\ell) < \dfrac{C'(v/\overline{\Delta})}{B-A}(h-\ell) \leq C'(v/\overline{\Delta}) \leq C'(v/\Delta)$. By (8), this implies that $\theta = \theta^r(w, p; \lambda) < [C']^{-1}[C'(v/\Delta)] = v/\Delta$. Using again (5), this implies that $p = 0$, contradiction. This proves the second part of the proposition. The first part follows an analogous argument.

Proof of Proposition 3

Suppose first that $p^* \to 0$ when $h \to \infty$. Then $U^0 \to U < 0$ (by Lemma 1), but the owner can always obtain zero offering a wage contract that the worker would reject. Suppose, therefore, that $p^* \to \overline{p} > 0$. We known that the expected wage oultay is $E(w_i|\theta, p; \lambda) = Pr(Y = H)h + Pr(Y = L)\ell$, where $Pr(Y = L)\ell$ is bounded below. Therefore, it suffices to show that $Pr(Y = L)h \to \infty$ when $h \to \infty$. Now $Pr(Y = H) = p[1-(1-\theta)\lambda] + (1-p)(1-\theta)(1-\lambda) \geq p[1-(1-\theta)\lambda]$. Since $\overline{p} > 0$, it suffices to show that $[1-(1-\theta)\lambda]h \to \infty$ as $h \to \infty$. This is trivially true if $\lambda < 1$, since in this case $1-(1-\theta)\lambda$ is bounded away from zero. This shows part (1) of the proposition.

Suppose now that $\lambda = 1$. Obviously, since $p^* = 0$ for all w in the NC game, the owner does not hire the worker. On the other hand, $p^* = 1$ and $\theta^* = v/\Delta$ in the FC game, by proposition 1. Hence $Pr(Y = H) = \theta = v/\Delta$. Therefore,

$$\lim_{h\to\infty} Pr(Y = H)h = \lim_{h\to\infty}(v/\Delta)h = \lim_{h\to\infty} \frac{v}{\dfrac{u(h)}{h} - \dfrac{u(\ell)}{h}} = \infty,$$

since $u(x)/x \to 0$ as $x \to \infty$. This shows part (2) of the proposition.

Proof of Proposition 4

Suppose, on the contrary, that $1 - p < \mathrm{Min}\left\{\lambda + \dfrac{C'(v/\overline{\Delta})}{-\overline{\Delta}_s}, 1\right\}$. Then, as $\Delta_s < 0$, we can write $(1-p-\lambda)(-\Delta_s) < \dfrac{C'(v/\overline{\Delta})}{\overline{\Delta}_s}(-\Delta_s)$, where $(-\Delta_s) = u_s(\ell_s) - u_s(h_s) \leq u_s(B) - u_s(A_s) \equiv \overline{\Delta}_s$, and therefore $\dfrac{C'(v/\overline{\Delta})}{\overline{\Delta}_s}\Delta_s \leq C'(v/\overline{\Delta})$. We can write, since $\overline{\Delta} \geq \Delta$, $\theta = \theta^r(w, p; \lambda) = [C']^{-1}[(1-p-\lambda)(-\Delta_s)] < [C']^{-1}[C'(v/\Delta)] = v/\Delta$. Therefore $p = 0$, contradiction.

Proof of Proposition 5

Part (a). Suppose that $p = 1$. Then

$$\begin{aligned}\theta^r(w_s, p; \lambda) &= [C']^{-1}[(1-p-\lambda)(-\Delta_s)] = [C']^{-1}[\lambda\Delta_s] \\ &= [C']^{-1}[C'(v/\Delta)] = v/\Delta.\end{aligned}$$

Now suppose that $\theta = v/\Delta$. The $p^r(w_s, v/\Delta; \lambda) = [0, 1]$, which contains 1.

Part (b). We have

$$\theta = \theta^r(w_s, p; \lambda) = [C']^{-1}[(1-p-\lambda)(-\Delta_s)]$$
$$= [C']^{-1}[(\lambda+p-1)\Delta_s] \leq [C']^{-1}[\lambda\Delta_s],$$

since $p \leq 1$ and $\Delta_s \geq 0$. But, by (iiib), $[C']^{-1}[\lambda\Delta_s] < [C']^{-1}\left[\lambda \frac{1}{\lambda} C'(v/\Delta)\right] = v/\Delta$. Hence $\theta < v/\Delta$, and therefore $p = 0$.

The role of false positives when the owner is not financially constrained

Consider what happens when λ is close to zero. We showed that the owner needs to set Δ very large (h very large) or to set Δ_s very large (h_s very large) to obtain $p^* = 1$. This would not be a problem if $Pr(Y = H)$ was low, but note that $p = 1$ and λ close to 0 imply that $Pr(Y = H)$ is large. Therefore, this solution would be very costly to the owner. So, although it is feasible to implement $p = 1$, it is at a large cost to the principal. So, he may choose to renounce to obtaining $p^* = 1$ and just try to approach it, by setting $h_s < \ell_s$. But *then p is bounded away* from 1 if $\lambda > 0$.

In the limit, however, when $\lambda = 0$ and $h_s < \ell_s$ (*i.e.*, $\Delta_s < 0$), p^* is no longer bounded away from 1. In fact, it is easy to see, using (5) and (10), that the second stage equilibrium is given by

$$p^* = 1 - C'(v/\Delta)/(-\Delta_s)$$
$$= 1 - \frac{C'(v/[u(h) - u(\ell)])}{u_s(\ell_s) - u_s(h_s)}, \qquad \text{and } \theta^* = v/\Delta.$$

Therefore, by setting ℓ_s arbitrarily large or h arbitrarily large, the owner can approximate $p^* = 1$ arbitrarily closely. However, since $Pr(Y = H)$ tends to 1 as p tends to 1 (recall that $\lambda = 0$), setting h arbitrarily large is clearly not beneficial to the owner. On the other hand, since $Pr(Y = L)$ tends to 0 as p tends to 1, one should consider the possibility of setting ℓ_s arbitrarily large. However, if $u_s(\ell_s)$ tends to infinity relatively slowly (as ℓ_s tends to infinity), then also $Pr(Y = L)$ will tend to 0 slowly. It may then be the case that $\ell_s\, Pr(Y = L)$ tends to infinity as ℓ_s tends to infinity. In this case it is not beneficial to the owner to set ℓ_s arbitrarily large. It all depends on the curvature of the function $u_s(\cdot)$ (*see* part (4) of Proposition 7).

In conclusion, although false positives ($\lambda > 0$) are not necessary to implement (or to approach) $p^* = 1$, it may be undesirable to do so when λ is very small or zero.

As an extreme case, consider what happens when both the worker's and the supervisor's utility functions are bounded above, say $u(\cdot) < \overline{u}$ and $u_s(\cdot) < \overline{u}_s$. Then p will be bounded away from 1 even if the owner is not financially constrainted. Indeed,

$$1 - \frac{C'(v/[\overline{u} - u(A)])}{\overline{u}_s - u_s(A_s)}$$

is an upper bound on p. Consequently, $Pr(Y = L) = (1-p)\,\theta$ is bounded away from zero (as $\theta > v/[\overline{u} - u(A)]$) and therefore $\ell_s\, Pr(Y = L)$ tends to infinity when ℓ_s tends to infinity. As stated in Section 5, to set upper bounds to the utility functions of worker and supervisor leads to the same results as assuming that the owner is financially constrained.

Proof of Proposition 6

Suppose, on the contrary, that there exists another equilibrium (θ', p') in which the supervisor accepts the contract. Then (θ', p') has to satisfy $U^s(w_s, \theta', p'; \lambda) \geq T_s = U^s(w_s, \theta, p; \lambda)$, $\theta' \in \theta^r(w_s, p'; \lambda)$ and $p' \in p^r(w, \theta'; \lambda)$.

Step 1:

Show that θ' must be larger than 0.

If $\theta' = 0$, then $p^r(w, 0; \lambda) = 0$. Then $\theta^r(w_s, 0; \lambda) = 0$. Hence, if $\theta' = 0$, then $p' = 0$. Now, since $\theta^r(w_s, p; \lambda)$ is singleton for all $(w_s, p; \lambda)$, we have $U^s(w_s, \theta^*, p^*; \lambda) > U^s(w_s, \theta, p^*; \lambda)$. Now, when $\theta = 0$, then $[\partial U^s/\partial p](w_s, 0, p; \lambda) = 0$ for all p, so $U^s(w_s, \theta, p^*; \lambda) = U^s(w_s, 0, 0; \lambda)$.

Therefore, $T_s = U^s(w_s, \theta^*, p^*; \lambda) > U^s(w_s, 0, 0; \lambda) = U^s(w_s, \theta', p'; \lambda)$ contradiction.

Step 2:

Show that $p' = 1$.

Since $\theta^* = \theta^r(w_s, 1; \lambda)$, we have $U^s(w_s, \theta^*, 1; \lambda) \geq U^s(w_s, \theta', 1; \lambda)$. Note that $[\partial U^s/\partial p](w_s, \theta, p; \lambda) = \theta\Delta_s > 0$ for all p and $\theta > 0$. Now, suppose, by contradiction, that $p' < 1$. Then $U^s(w_s, \theta', 1; \lambda) > U^s(w_s, \theta', p'; \lambda)$, since $\theta' > 0$. We can write $T_s = U^s(w_s, \theta^*, 1; \lambda) \geq U^s(w_s, \theta', 1; \lambda) > U^s(w_s, \theta', p'; \lambda)$ contradiction.

Step 3:

Show $(p', \theta') = (p^*, \theta^*)$.

Given $p' = 1 = p^*$, and substituting into (10), we have

$$\theta' = [C']^{-1}[\lambda\Delta_s] = v/\Delta = \theta^*.$$

Existence of a wage contract $W \in \mathbb{R}^4$ that implements $p = 1$ as the unique outcome that survives elimination of dominated strategies

To ease notation, let $u_s(h_s) \equiv H_s$, $u_s(\ell_s) \equiv L_s$, $u(h) \equiv H$, and $u(\ell) \equiv L$.

The supervisor's option of rejecting the contract is denoted by R. We write $\theta = \theta_0$ to represent the strategy of accepting the contract and setting $\theta = \theta_0$.

Pick $0 < \hat{\theta} < 1$, and for some arbitrary $\varepsilon > 0$ and $\delta > 0$, let

$$\begin{aligned} H &= T + v + \varepsilon + (1 - \hat{\theta})\,\lambda v/(\hat{\theta} - \delta), \\ L &= T + v + \varepsilon - [1 - (1 - \hat{\theta})\,\lambda]\,v/(\hat{\theta} - \delta), \\ H_s &= T_s + C(\hat{\theta}) + \varepsilon + (1 - \hat{\theta})\,C'(\hat{\theta}), \\ L_s &= T_s + C(\hat{\theta}) + \varepsilon - \left[\frac{1-\lambda}{\lambda} + \hat{\theta}\right] C'(\hat{\theta}). \end{aligned}$$

It is easy to check that, (L, H, L_s, H_s) as defined above, together with $p = 1$, and $\theta = \hat{\theta}$ satisfy:

a) $U^s(w_s, \hat{\theta}, 1; \lambda) = T_s + \varepsilon > T_s$ and $U^w(w, \hat{\theta}, 1; \lambda) = T + \varepsilon > T$ (*i.e.*, these contracts are accepted given $(p, \theta) = (1, \hat{\theta})$),

b) $p = 1 \in p^r(w, \hat{\theta}; \lambda)$ and $\hat{\theta} \in \theta^r(w_s, 1; \lambda)$ (*i.e.*, $(p, \theta) = (1, \hat{\theta})$ is an equilibrium of Γ^s).

c) for T_s or T large enough (given $\hat{\theta}$ and λ), we have that $\ell \geq A$ and $\ell_s \geq A_s$ (*i.e.*, the financial constraints are satisfied).

We will use a graphical argument to show our point. In Figure 3 we depict the supervisor's payoff given $p = 1$ and given $p = 0$. A previous step is to show that $\hat{\theta} > \theta^c(w)$. By (6),

$$\begin{aligned} \theta^c(w) &= v/(H - L) = v/[(1 - \hat{\theta})\,\lambda v/(\hat{\theta} - \delta) + [1 - (1 - \hat{\theta})\,\lambda]\,v/(\hat{\theta} - \delta)] \\ &= \hat{\theta} - \delta < \hat{\theta}. \end{aligned}$$

Note that $\theta^c(w)$ is independent of ε.

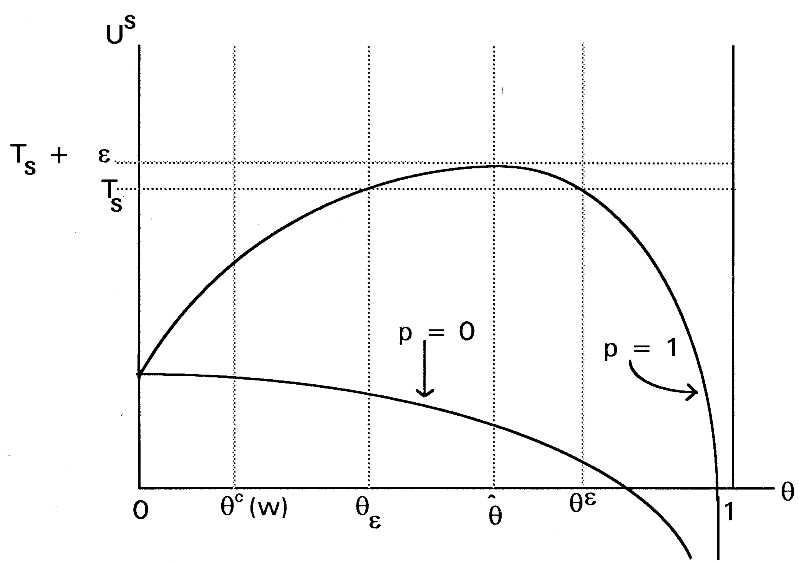

FIGURE 3

If the supervisor rejects the contract, he obtains T_s. Any strategy $\theta \leq \theta^c(w)$ is domminated by the strategy of rejecting the contract.

Note that

$$\begin{aligned} U^s(w_s,\theta,1;\lambda) &= \lambda(1-\theta)L_s + (1-(1-\theta)\lambda)H_s - C(\theta) \\ &= T_s + \varepsilon + [C(\hat\theta) - C(\theta) - C'(\hat\theta)(\hat\theta - \theta)] \\ &= T_s + \varepsilon + G(\hat\theta,\theta), \end{aligned}$$

where $G(\hat\theta,\theta)$ is independent of ε, $G(\hat\theta,\hat\theta)=0$, and $G(\hat\theta,\theta)$ attains a (strict) maximum at $\theta=\hat\theta$, since $\partial G(\hat\theta,\theta)/\partial\theta = C'(\hat\theta) - C'(\theta)$ (recall that C' is increasing). Note also that $G(\hat\theta,\theta)$ is strictly concave on θ.

Therefore, letting ε be small enough, there exist θ_ε and θ^ε such that $\theta^c(w) < \theta_\varepsilon < \hat\theta < \theta_\varepsilon$ and

$$U^s(w_s,\theta,1;\lambda) = T_s + \varepsilon + G(\theta,\hat\theta)\begin{cases} > T_s & \text{if} \quad \theta\in(\theta_\varepsilon,\theta^\varepsilon) \\ = T_s & \text{if} \quad \theta\in\{\theta_\varepsilon,\theta^\varepsilon\} \\ < T_s & \text{otherwise.}\end{cases}$$

We can now drawn $U^s(w_s,\theta,1;\lambda)$.

Note that

$$\begin{aligned} U^s(w_s,\theta,0;\lambda) &= (1-\theta)(1-\lambda)H_s + [1-(1-\theta)(1-\lambda)]L_s - C(\theta) \\ &= T + C(\hat\theta) + \varepsilon - C(\theta) + (1-\theta)(1-\lambda)(1-\hat\theta)C'(\hat\theta) \\ &\quad - [1-(1-\theta)(1-\lambda)]\left[\frac{1-\lambda}{\lambda} + \hat\theta\right]C'(\hat\theta), \end{aligned}$$

where the first three terms remain constant when θ increases, $-C(\theta)$ decreases with θ, also $(1-\theta)(1-\lambda)(1-\hat\theta)C'(\hat\theta)$ decreases with θ; and $[1-(1-\theta)(1-\lambda)]\left[\frac{1-\lambda}{\lambda}+\hat\theta\right]C'(\hat\theta)$ increases with θ. Therefore, $U^s(w_s,\theta,0;\lambda)$ decreases with θ. Finally, note that, when $\theta=0$, $U^s(w_s,0,0;\lambda) = U^s(w_s,0,1;\lambda)$. We can now draw $U^s(w_s,\theta,0;\lambda)$.

From the picture it is easy to see that R dominates all θ outside $(\theta_\varepsilon,\theta^\varepsilon)$, since choosing R gives the supervisor a payoff T_s, whereas both $U^s(w_s,\theta,1;\lambda)$ and $U^s(w_s,\theta,0;\lambda)$ are below T_s (strictly, for $p=0$) given any θ outside $(\theta_\varepsilon,\theta^\varepsilon)$. (Recall that each p in (0, 1) represents the worker randomizing between the options of shirking, *i.e.* setting $p=0$, and being diligent, *i.e.* setting $p=1$.) Obviously, $\hat\theta$ is not dominated, since $\theta^r(w,1;\lambda)=\hat\theta$ and $U^s(w_s,\hat\theta,1;\lambda) > T_s$ by construction.

Now we prove that we cannot start by eliminating worker strategies. This is important since we want to ensure that there is no other procedure of˜ elimination that would eliminate the equilibrium $(\hat\theta,1)$. (To prove this it suffices to study pure strategies, since the worker has only two pure strategies available: $p=1$ and $p=0$.)

Since $p^r(w,\theta;\lambda)=1$ if $\theta>\theta^c(w)$ and $p^r(w,\theta;\lambda)=0$ if $\theta<\theta^c(w)$, neither $p=0$ nor $p=1$ are dominated.

All θ outside $(\theta_\varepsilon,\theta^\varepsilon)$ (and maybe some others too, but not $\hat\theta$) can be eliminated in the first iteration. All θ that are left satisfy $\theta>\theta^c(w)$. Given this, $p = 1$ dominates any other strategy for the worker, as $p^r(w, \theta; \lambda) = 1$ if $\theta > \theta^c(w)$.

Finally, given that all $p < 1$ have been eliminated, and that only $p = 1$ remains, the strategy $\hat{\theta}$ dominates all other θ in $(\theta_\varepsilon, \theta^\varepsilon)$ (because $\hat{\theta} = \theta^r(w_s, 1; \lambda))$ and also dominates R (because $U^s(w_s, \hat{\theta}, 1; \lambda) > T_s$). Thus, the only outcome left is $(\theta, p) = (\hat{\theta}, 1)$.

Proof of Proposition 7

Suppose that $p^* \to 0$ as either $h \to \infty$, $h_s \to \infty$ or $\ell_s \to \infty$. Then $U^0 \to U < 0$ (by Lemma 3). Suppose, therefore, that $p^* \to \overline{p} > 0$ whenever either h, h_s, or ℓ_s tend to ∞. The expected wage outlay is $E(w_i + w_{is}) = Pr(Y = H)(h + h_s) + Pr(Y = L)(\ell + \ell_s)$, where $Pr(Y = L)\,\ell$ is bounded below. Therefore, it suffices to show that (1) $Pr(Y = H)\,h \to \infty$ as $h \to \infty$; (2) $Pr(Y = H)\,h_s \to \infty$ as $h_s \to \infty$; and (3) $Pr(Y = L)\,\ell_s \to \infty$ as $\ell_s \to \infty$.

Show first (1) and (2). Now,

$$Pr(Y = H) = p[1 - (1 - \theta)\lambda] + (1 - p)(1 - \theta)(1 - \lambda) \geq p[1 - (1 - \theta)\lambda].$$

Since $\overline{p} > 0$, it suffices to show that $[1 - (1 - \theta)\lambda] \to \infty$ as $h \to \infty$, and that $[1 - (1 - \theta)\lambda]\,h_s \to \infty$ as $h_s \to \infty$. This is trivially true if $\lambda < 1$, since in the case $1 - (1 - \theta)\lambda$ is bounded away from zero. This shows part (1) of the Proposition.

Suppose, alternatively, that $\lambda = 1$. Recall that $p > 0$ only if $\theta \geq v/\Delta$. Also, $\theta = [C']^{-1}[(1 - p - \lambda)(-\Delta_s)] = [C']^{-1}[p\Delta_s]$. Therefore, $\theta \geq v/\Delta$ implies $p\Delta_s \geq C'(v/\Delta)$, which in turn implies $\Delta_s \geq C'(v/\Delta)$, since $p \leq 1$. Now $\Delta_s \geq C'(v/\Delta)$ implies that $p^* = 1$ and $\theta^* \geq v/\Delta$ constitute an equilibrium, since $1 \in p^r(w, v/\Delta; \lambda)$, and $\theta^* = \theta^r(w, 1; \lambda) = [C']^{-1}[\Delta_s] \geq v/\Delta$. Hence, as $\lambda = 1$ and $p = 1$, we have $Pr(Y = H) = \theta$, and

$$\begin{aligned}\lim_{h\to\infty} Pr(Y = H)\,h &= \lim_{h\to\infty} \theta h \geq \lim_{h\to\infty} (v/\Delta)\,h \\ &= \lim_{h\to\infty} \frac{v}{\dfrac{u(h)}{h} - \dfrac{u(\ell)}{h}} = \infty,\end{aligned}$$

since $u(x)/x \to 0$ as $x \to \infty$. Similarly,

$$\begin{aligned}\lim_{h_s\to\infty} Pr(Y = H)\,h_s &= \lim_{h_s\to\infty} \theta h_s \geq \lim_{h_s\to\infty} (v/\Delta)\,h_s \\ &= \lim_{h_s\to\infty} \frac{v}{\dfrac{u(h)}{h_s} - \dfrac{u(\ell)}{h_s}} = \infty.\end{aligned}$$

This shows part (2) of the proposition.

Finally, we need to show that $\ell_s \to \infty$ implies $Pr(Y = L)\,\ell_s \to \infty$. Since it cannot be beneficial to set both h_s and ℓ_s arbitrarily large (otherwise $E(w_{is}|\cdot)$ would to tend to ∞), we are assuming that $\ell_s > h_s$, or equivalently, $\Delta_s < 0$. Recall also that we are assuming that $p^* \to \overline{p} > 0$. Also, if $\ell_s \to \infty$ implies that $\theta \to 1$ then $C(\theta) \to \infty$. Therefore, assume $\theta \to \overline{\theta} < 1$ as $\ell_s \to \infty$. Taking account all this, and since $Pr\,(Y = L) = p\,(1-\theta)\,\lambda + (1-p)\,[1-(1-\theta)\,(1-\lambda)] \geq p\,(1-\theta)\,\lambda$, if $\lambda > 0$ we are done, since $Pr\,(Y = L)\,\ell_s \to \overline{p}\,(1-\overline{\theta})\,\lambda\infty = \infty$ as $\ell_s \to \infty$. This shows part (3) of the proposition.

Alternatively, suppose $\lambda = 0$. Then $Pr\,(Y = L) = (1-p)\,\theta$. Now, $p > 0$ implies $\theta > 0$ and $\theta = C'^{-1}\,[(1-p-\lambda)\,(-\Delta_s)]$ and therefore $1-p-\lambda = C'\,(\theta)/-\Delta_s$. Also, $p > 0$ implies $\theta\Delta \geq v$, so $\theta \geq v/\Delta$. Therefore, $Pr\,(Y = L)\,\ell_s = (1-p)\,\theta\ell_s \geq \dfrac{C'\,(v/\Delta)}{(-\Delta_s)}\,\dfrac{v}{\Delta}\,\ell_s$ (recall that we are assuming that $-\Delta_s > 0$). Let $D = C'\,(v/\Delta)\,\dfrac{v}{\Delta}$ (a fixed quantity when $\ell_s \to \infty$). Then

$$\begin{aligned}\lim_{\ell_s\to\infty} Pr\,(Y = L)\,\ell_s &\geq \lim_{\ell_s\to\infty} D\,\frac{\ell_s}{-\Delta_s}\\ &= D\,\frac{1}{\displaystyle\lim_{\ell_s\to\infty}\frac{u_s\,(\ell_s)}{\ell_s} - \lim_{\ell_s\to\infty}\frac{u_s\,(\ell_s)}{\ell_s}} = \infty,\end{aligned}$$

since $\displaystyle\lim_{x\to\infty}\frac{u_s\,(x)}{x} = 0$. This shows part (4) of the proposition.

Proof of Proposition 8

(Part (1)). Suppose, on the contrary, that

$$p^* > \mathrm{Max}\left\{1-\lambda-\frac{C'\,(v/\overline{\Delta})}{\overline{\Delta}_0},\,0\right\}.$$

Then, since Δ_0 is positive,

$$\theta = [C']^{-1}\,[(1-p-\lambda)\,\Delta_0] < [C']^{-1}\left[\frac{C'\,(v/\overline{\Delta})}{\overline{\Delta}_0}\,\Delta_0\right] \leq [C']^{-1}\,[C'\,(v/\Delta)]$$
$$= v/\Delta,$$

and therefore $p = 0$, contradiction.

(Part (2)). Suppose $0 \leq \lambda < C'\,(v/\overline{\Delta})/\overline{\Delta}_0$ and $\Delta_0 < 0$. Then, $(1-p-\lambda)\,\Delta_0 = (1-p)\,\Delta_0 - \lambda\Delta_0 < C'\,(v/\overline{\Delta})\,(-\Delta_0)/\overline{\Delta}_0$, since $(1-p) \geq 0$, $\Delta_0 < 0$, and $\lambda < C'\,(v/\overline{\Delta})/\overline{\Delta}_0$. Now $C'\,(v/\overline{\Delta})\,(-\Delta_0)/\overline{\Delta}_0 \leq C'\,(v/\overline{\Delta}) \leq C'\,(v/\Delta)$, since $\Delta_0 \geq -\overline{\Delta}_0$ and $\Delta \leq \overline{\Delta}$. Therefore, $\theta = [C']^{-1}\,[(1-p-\lambda)\,\Delta_0] < [C']^{-1}\,[C'\,(v/\Delta)] = v/\Delta$, which implies that $p = 0$.

(Part (3)). Since $C'\,(v/\overline{\Delta})/\overline{\Delta}_0 \leq \lambda$, this means that $-\overline{\Delta}_0 \leq -C'\,(v/\overline{\Delta})/\lambda$. We can then pick $\Delta \leq \overline{\Delta}$ and $\Delta_0 \geq -\overline{\Delta}_0$ such that $\Delta_0 = -C'\,(v/\overline{\Delta})/\lambda$. Suppose now that $p = 1$. Then

$$\theta = [C']^{-1}\,[(1-p-\lambda)\,\Delta_0] = [C']^{-1}\,[(-\lambda)\,\Delta_0] = [C']^{-1}\,[C'\,(v/\Delta)] = v/\Delta,$$

so $p = 1 \in p^r\,(w\,(\hat{\theta}),\,\hat{\theta},\,v/\Delta;\,\lambda) = [0,\,1]$.

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