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A Non-Walrasian General Equilibrium Model with Monopolistic Competition and Wage Bargaining

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ABSTRACT. – In a general equilibrium framework, this paper tries to reproduce an important stylized fact of real economies: firms set prices under demand uncertainty while consumption decisions are taken when prices are already known. Under these assumptions, there is place for a quantity rationing equilibrium since preferences are revealed when prices are already set and market-clearing can not be attained through changes in prices. "Demand heterogeneity" is introduced in the model and related to "demand uncertainty:" when firms set prices, their own market shares are not known with certainty, even if aggregate demand and the distribution of market shares are common knowledge. The main properties of the aggregate equilibrium are: a) some markets are demand constrained while other markets are supply constrained, b) aggregate production is smaller than aggregate demand and full-employment output, c) there is (involuntary) unemployment, and d) there is a positive spill-over effect from constrained to unconstrained demands.

Un modèle non-walrasien d'équilibre général en concurrence monopolistique et négociation salariale

RÉSUMÉ. – Dans le cadre d'un modèle d'équilibre général, ce papier cherche à reproduire un important fait stylisé des économies réelles : les entreprises fixent leurs prix sous incertitude de demande, tandis que les décisions de consommations sont prises lorsque les prix sont déjà connus. Sous cette hypothèse, il est possible de définir un équilibre avec contraintes quantitatives, parce ce que les préférences sont révélées lorsque les prix sont établis et l'équilibre du marché ne peut se réaliser au moyen d'un changement dans les prix. « Hétéroggénéité de la demande » et « incertidude de la demande » sont étroitement liées : les firmes établissent leurs prix sans connaître avec certitude leurs parts de marché, même si la demande agrégée et la distribution des parts de marché sont connus. Les propriétés principales de l'équilibre agrégé sont : a) un certain nombre de marchés sont contraints par la demande tandis que les autres le sont par l'offre, b) la production agrégée est inférieur à la demande agrégée et à la production de plein emploi, c) il y a du chômage (involontaire), et d) il y a des effets de reports positifs des demandes contraintes vers les demandes non-contraintes.

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1 Introduction

During the seventies the so-called "disequilibrium theory" shifted the economic adjustments from prices to quantities. The fundamental criticism against Walrasian models was that prices in many real markets do not adjust instantaneously to clear the market. Transactions are thus made at non-Walrasian prices and the economic agents may face some type of quantity rationing." [1] MALINVAUD's [1977] lectures, based on BARRO and GROSSMAN [1971], apply the notion of a fixed price equilibrium to macroeconomic problems, integrating in a unified framework both Keynesian and Classical macroeconomic policies. Disequilibrium models have yielded interesting empirical results, in particular under the "aggregation over micromarkets in disequilibrium" hypothesis [2]. This literature sees the economy as the aggregation of a large number of small heterogeneous markets, each one being constrained by demand or supply. Under some plausible assumptions on the distribution of supplies and demands over micromarkets, aggregate production is a function of aggregate demand and aggregate supply.

During the eighties a considerable effort was devoted to the search for an endogenous explanation of the price formation process in macroeconomic models, originating what is known as the "New Keynesian" approach. In this approach, endogenous price rigidities essentially depend on market imperfections and "menu costs" (the costs of changing prices) [3]. Integrating both theoretical schools in order to look for the microfoundations of price rigidities in disequilibrium models seems to be a very promising task. An effort in this direction was made by SNEESSENS [1987], who assumes that firms operate in a monopolistically competitive economy and that they preset prices before knowing with certainty demand, capacities and labor supply constraints. Imposing the same conditions as LAMBERT on the joint distribution of the stochastic shocks, SNEESSENS shows that the markup rate charged by firms depends, in addition to the price elasticity of demand, on "demand pressures" [4].

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1. Even though this literature was initially concerned with price formation in imperfectly competitive economies, "disequilibrium theory" is generallly associated with "fix-price equilibrium models." See BENASSY [1976] in this respect.
2. This point was initially stressed by MUELLBAUER [1978] and MALINVAUD [1980]. KOOIMAN [1984] and LAMBERT [1988], using somewhat different approaches, were the first to give empirically estimable forms to this idea. SNEESSENS and DRÈZE [1986] build a macroeconometric model based on LAMBERT's approach. Empirical applications to Europe indicate that demand and supply constraints are both relevant in explaining economic fluctuations, implying that a mix of Keynesian and Classical policies is required. See DRÈZE and BEAN [1990] in this respect.
3. AKERLOFF and YELLEN [1985], BLANCHARD and KIYOTAKI [1987] and MANKIW [1985] are the main references. See BLANCHARD and FISCHER[1989] for a survey of this literature.
4. In the same field, LICANDRO [1992a] and [1992b] analyses the investment process and ARNSPERGER and de la CROIX [1993] analyze wage bargaining in a general equilibrium perspective. A general equilibrium analysis of monopolistic competition and quantity rationing is also in BENASSY [1991].

Building on this theoretical framework, this paper tries to reproduce an important stilized fact of real economies: *firms set prices under demand uncertainty, while consumption decisions are taken when prices are already known.* Under these assumptions, there is room for a quantity rationing equilibrium since preferences are revealed when prices are already set and market-clearing can not be attained through changes in prices. "Demand heterogeneity" is introduced in the model to reproduce another stilized fact, namely, that market conditions are different among firms. The CES utility function is a useful instrument to introduce demand heterogeneity in a very simple way: firm's market shares depend linearly on some parameter of the utility function. The model is closed by a direct relation between demand heterogeneity and "demand uncertainty": when firms set prices, their own market shares are not known with certainty, even if aggregate demand and the distribution of market shares are common knowledge.

The main characteristics of this economy are described in Section 2. In Section 3 consumer behavior is formulated as an extension of the DIXIT and STIGLITZ [1977] model. Since markets do not necessarily clear, optimal consumption decisions are derived under quantity constraints. As it is standard in disequilibrium theory, "effective demand" is distinguished from "notional demand", which yields "spill-over effects": if some quantity constraints are binding, consumers spill their unsatisfied demands over to other markets.

The supply side of the economy is developed in Section 4. Monopolistic competition and efficient bargaining allow for endogenous prices and wages. Demand uncertainty modifies the price behavior of firms allowing for expected demand pressures to increase the monopoly power of the firm, as in SNEESSENS [1987]. The bargaining outcome determines the labor share as a function of union power, and expected employment as a function of utility parameters.

The equilibrium of the aggregate economy is studied in Section 5. Since firms and unions are ex-ante (before the revelation of preferences) identical, prices and wages are the same for each firm and the aggregation problem over prices and wages is avoided. However, employment and production are decided ex-post (when preferences are already revealed) and both differ among firms. To allow for aggregation, some particular assumptions about preferences are imposed such that the "aggregation over micro markets in disequilibrium" hypothesis holds. The main properties of the aggregate equilibium are analyzed: (a) some markets are demand constrained while other markets are supply constrained, (b) aggregate production is smaller than aggregate demand and full-employment output, (c) there is (involuntary) unemployment, and (d) there is a positive spill-over effect from constrained to unconstrained demands.

2 The Economy

There are three types of economic agents: households, unions and firms. Each household supplies a given quantity of labor to a particular firm and demands goods. Households are represented by unions, which are organized at the firm level. Firms hire labor from households and produce differentiated goods. Firms and unions bargain at the firm level.

A particular information structure is assumed: there are two periods in the model, ex-ante (before the revelation of individual preferences over goods) and ex-post (when all relevant information is public). Households supply labor and firms and unions decide (at the firm level) wages and prices without knowing with certainty the demand for the good produced by the firm. When prices and wages are public information, households demand goods and firms hire workers and produce.

3 The Demand Side

This section develops the demand side of the economy. As stated in the previous section, consumption decisions are taken ex-post when all relevant information is common knowledge. Households behave as in DIXIT and STIGLITZ [1977] but, as it will be showed later, markets do not necessary clear, implying that they must take into account quantity rationing constraints.

To better understand household behavior in the goods market, I use the distinction, proposed by CLOWER [1965], between "notional demand" and "effective demand". "Notional demand" for a particular good is defined as the demand function when all rationing constraints are not binding. In the same sense, "effective demand" is the demand function when at least some rationing constraints are binding. This distinction allows us to introduce the idea of "spill-over effect", *i.e.*, the amount by which the unsatisfied demands are transfered from the constrained goods to the unconstrained ones.

All of the househods have the same utility function and offer the same given quantity of labor. However, in the labor market workers must be treated asymmetrically: some may find a job while others may be unemployed. Revenues of households are different, even if all have the same non-labor income. In order to analyze the behavior of a representative consumer facing rationing constraints, it is convenient to assume that differences in revenues do not affect optimal consumption rules. To this end, let us assume that "rationing schemes" are uniform and non-stochastic, *i.e.*, all households are rationed in each market proportionally to their revenues [5].

This section is mainly concerned with the derivation of "effective demand" functions for a set of differentiated goods. It will be shown that the distribution of aggregate demand among goods depends, in addition to good's prices, on some parameters of the utility function. This property

5. "Rationing schemes" in disequilibrium markets are analyzed in BENASSY [1982].

allows us to introduce in a endogenous way the "aggregation over micromarkets" assumptions proposed by LAMBERT [1988].

3.1. The Representative Consumer

The representative consumer optimization problem is

$$\max_{\{c_i\}_{i=1}^{n}} C = \left\{ \sum_{i=1}^{n} \left(\frac{u_i}{n}\right)^{\frac{1}{\theta}} c_i^{\frac{\theta-1}{\theta}} \right\}^{\frac{\theta}{\theta-1}} \tag{1}$$

subject to

$$\sum_{i=1}^{n} p_i\, c_i = I$$

and

$$c_i \leq s_i \qquad \forall\, i;$$

where

$$\theta > 1 \quad \text{and} \quad \sum_{i=1}^{n} u_i = n.$$

C represents the consumption of a composite good, which plays the role of numeraire. c_i and p_i are, respectively, the consumption and the price of the good i, n is the number of differentiated goods, s_i represents the quantity rationing constraint associated to the good i and I represents revenues of the representative consumer. Both prices and quantity constraints are taken as given by the consumer. As in monopolistically competitive models, n is assumed large enough. The parameters θ and $u_i \,\forall i \in \{1, 2, \ldots, n\}$ are given.

As in the Dixit-Stiglitz framework, the utility function of the representative consumer depends on the consumption of a composite good. The main differences between this framework and Dixit-Stiglitz are: (a) consumers are not necessarily able to buy all they want at the given prices and (b) the weights u_i are not necessary unity, even if they add up to n.

The optimality condition for c_i is [6]

$$c_i = \begin{cases} s_i & \text{if} \quad \lambda_i > 0 \\ (\lambda\, p_i)^{-\theta}\, C_{\frac{u_i}{n}} & \text{if} \quad \lambda_i = 0 \end{cases} \tag{2}$$

where λ is the marginal value associated to the budget constraint and λ_i is the marginal value associated to the ith supply constraint.

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6. Since the utility function is concave in its arguments and the budget and the quantity constraints are linear, the Kuhn-Tucker conditions are necessary and sufficient for a maximum.

3.1.1. ***Notional Demands***

If the consumer does not face any rationing constraint (*i.e.*, when $\lambda_i = 0 \; \forall i$) the solution of the optimality conditions is equivalent to that of Dixit-Stiglitz. From equation (2), since $\lambda_i = 0 \; \forall i$, the "notional demand" for good i, denoted by d_i^n, can be written as

$$d_i^n = \left(\frac{p_i}{P_n}\right)^{-\theta} \frac{I}{P_n} \frac{u_i}{n} .$$

where

$$P_n = \left(\sum_{i=1}^{n} \left(\frac{u_i}{n}\right) p_i^{1-\theta}\right)^{\frac{1}{1-\theta}},$$

is the "notional price index" (the "true price index" when all constraints are not binding) [7]. The demand for the composite good is equal to I/P_n and is distributed among the differentiated goods depending on relative prices and the u_i/n weights.

3.1.2. ***Effective Demands***

As in BENASSY [1982], let us define "effective demand" for good i in the solution to problem (1) when the ith constraint is not taken into account. Since n is "large enough", eliminating only one constraint must not affect the aggregates, allowing us to use equation (2) to define the effective demand for all goods i,

$$d_i = \Lambda \, d_i^n, \tag{3}$$

where

$$\Lambda = (\lambda \, P_n)^{-\theta} \, \frac{P_n \, C}{I}$$

represents the "spill-over effect". Since n is large, changes in d_i or p_i do not have significant effects on Λ. When none of the quantity constraints is binding, or equivalently when $\lambda_i = 0 \, \forall \, i$, the spill-over effect is equal to one, implying that effective demand is identical to notional demand. From the budget constaint and (2), it is easy to show that $\Lambda \geq 1$. When some rationing constraints are binding the consumer optimally decides to spill unsatisfied demands over to unconstrained goods.

From the previous equation we know that total demand is distributed among goods following the distribution of the u_i parameters and relative prices. Uncertainty comes into the model because firms and unions do not know with certainty the specific u_i assigned to each particular good.

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7. See DEATON and MUELLBAUER [1980].

Let us denote by D at the "aggregate effective demand". From the definition of the composite good in problem (1) and the definition of the ith effective demand on (3), it is easy to show that

$$D = \Lambda \, \frac{I}{P_n} \, .$$

The definition of "effective demand" allows us to write the optimality condition (2) in an equivalent way [8]

$$c_i = \min \{s_i, \, d_i\}. \tag{4}$$

Optimal consumption c_i is equal to the quantity constraint s_i when $\lambda_i > 0$ or equal to the effective demand d_i when $\lambda_i = 0$.

4 The Supply Side

As stated in the Introduction, the aim of this paper is to integrate "quantity rationing" and "monopolistic competitive" models. For this purpose, we impose two assumptions that avoid market clearing. The first assumption is the existence of "nominal rigidities": prices and wages are decided under demand uncertainty, implying that when the stochastic demand takes place, prices and wages are already set and no price adjustment allows market-clearing. Secondly, labor market are supposed to be segmented implying that firms face full-employment constraints as an upper-bound on production.

On the supply side of the economy there are n monopolistically competitive firms, each of them producing a variety of a unique good. Workers offer l_s units of labor to one specific firm [9]. There are also n unions, each of them represents the workers offering their labor to a particular firm. Let us assume the following institutional arrangement: at the firm level, firms and unions bargain over wages and prices under

8. It is well-known that effective demands are not well-defined once the constraint is binding. Alternatively, we could adopt the DRÈZE [1975] definition of effective demand, corresponding to variable c_i in our model. However, as the lector can see in what follows, all our results do not depend on any particular definition of effective demand. BENASSY's definition allows us to give a more interesting interpretation to some of our results.

9. The representative household solve the problem (1) in Section 3.1. Because the marginal desutility of labor is zero, he is optimally working the maximum feasible time l_s.

demand uncertainty. Assuming that the bargaining process is at the same time over wages and prices is equivalent to assuming that firms and unions bargain over wages and expected employment (efficient bargaining) [10]. This assumption is not essential, but the results are easier to obtain than under the alternative assumption that both parties bargain only over wages and prices are set by the firm alone [11].

Additionally, labor markets are supposed to be segmented: each worker is offering his labor to a specific firm and, if a firm decides not to hire a worker, this worker is unable to offer his labor to another firm. Under these assumptions each firm faces an upper-bound on production given by the "full-employment output", defined as

$$y_f = l_s,$$

where the technical coefficient for labor is assumed given and equal to one for simplicity.

4.1. Firm's Objective

Let us assume that the "representative firm" forms its expectations rationally. It knows the "effective demand function" assigned to its particular variety. Aggregate effective demand and the distribution of the u_i's are common knowledge, but the representative firm does not know its own u_i with certainty.

Nominal rigidities and labor markets segmentation imply that ex-post optimal production is given by the minimum of the two constraints, those of effective demand and full-employment output:

$$y_i = \min\{d_i,\, y_f\}. \tag{5}$$

Ex-post, when demand is revealed, production is given by equation (5). If the demand shock is "good" the ith firm produces at the full-employment level. If the demand shock is "bad" the ith firm is not able to hire all the labor supply in the ith segment of the labor market and some workers are unemployed. Notice that the appropriate supply constraints for the representative consumer are $s_i = y_f \,\forall i$ and that the equilibrium in the market for variety i is defined by the condition $c_i = y_i$.

Expected profits are therefore

$$E(\pi_i) = (p_i - w_i)\, E(y_i). \tag{6}$$

The next important point is the determination of $E(y_i)$. The full-employment constraint is given and the effective demand function d_i comes from equation (3). Since n is large, let us assume that the histogram of the

10. The macroeconomic implications of wage bargaining is analyzed in McDonald and Solow [1981] and Nickell [1990].
11. See Arnsperger and de la Croix [1993].

parameters u_i can be approximated by a lognormal density function. This additional restriction on preferences allows us to employ some useful results from "quantity rationing theory". Under the assumption that u_i follows a lognormal distribution, expected production can be approximated by a CES function of expected demand and the full-employment constraint

$$E(y_i) = (E(d_i)^{-\rho} + y_f^{-\rho})^{-\frac{1}{\rho}}, \tag{7}$$

where ρ depends on the variance of the distribution of the u_i parameters and is positive for plausible values of this variance [12]. In particular, when there is no uncertainty, *i.e.*, when the variance goes to zero, the parameter $\rho \to \infty$, implying that expected production is equal to the minimum of expected demand and full-employment output. This is equivalent to the standard monopolistic competitive model, where $u_i = 1 \; \forall \, i$.

From equation (7) we can calculate the elasticity of expected production to expected demand, denoted by Φ_i, as

$$\Phi_i = \left(\frac{E(y_i)}{E(d_i)}\right)^{\rho} \leq 1. \tag{8}$$

This elasticity is smaller than one and decreases when expected demand increases. Φ_i is also a measure of the probability of excess supply (or demand constraint) in the ith market.

4.2. **Union's Objective**

In each market, a trade union represents the workers offering their labor to the firm producing the corresponding variety. Let us assume that the "representative union" has the same information as the "representative firm" and forms its expectations rationally. It knows the ith "effective demand function", but does not know with certainty the corresponding u_i.

Total revenues of the representative member of the union i are

$$I_i = w_i \, l_i + \sum_{j=1}^{n} \theta_j \, \pi_j,$$

where l_i is employment and θ_j represents the shares of firm j held by the ith worker. Since the number of households is large, θ_j is small, implying that the ith union does not care about ith firm's profits. Let us assume that the objective function of the union is

$$E(V) = w_i \, E(y_i), \tag{9}$$

where V represents the expected income of the risk-neutral representative member after the deduction of the fall-back level

$$\sum_{j=1}^{n} \theta_j \, \pi_j,$$

i.e., the non-labor income.

12. See the Appendix for a proof.

4.3. **The Bargaining Outcome**

The institutional arrangement is that the representative firm and the representative union bargain, at the level of the firm, over both prices and wages, or equivalently over expected employment and wages. The outcome of this bargaining process is the soluion of the Nash product

$$\max_{\{p_i,\, w_i\}} \mathcal{N} = (w_i\, E\,(y_i))^{\beta}\, ((p_i - w_i)\, E\,(y_i))^{1-\beta},$$

where $1 > \beta > 0$ represents the "union power" and $1 - \beta$ the "firm power" in the labor market. $E\,(y_i)$ is given by equation (7).

Optimality conditions for prices and wages are therefore

$$p_i = \left(\frac{1}{1 - \frac{(1-\beta)}{\theta \Phi_i}} \right) w_i \tag{10}$$

and

$$w_i = \beta\, p_i. \tag{11}$$

Equation (10) states that the firm (in agreement with the union) sets a markup over marginal costs. The monopoly power in the goods market is given by the inverse of the demand elasticity θ weighted by the elasticity of expected production to expected demand Φ_i. Since the latter elasticity is smaller than one, it reinforces the power of the firm in the goods market: as stated by equation (8), Φ_i could be interpreted as a measure of "demand pressures". However, since the union cares about expected employment, the monopoly power of the firm is weighted by its power in the labor market. When the union has no power $(\beta = 0)$, the "Lerner index" is given by the monopoly power only.

Equation (11) could be interpreted as the union and the firm bargaining over the labor share. At the optimum, this share is equal to union power, implying that the greater this power is, the greater the share of total revenues appropriated by workers will be.

Equations (10) and (11) are linearly dependent on prices and wages, implying that the labor share must be equal in both equations. This is possible because the monopoly power in equation (10) adjusts until the labor share becomes equal to β. Equalizing the labor share from both equations gives

$$\Phi_i = \frac{1}{\theta}\,. \tag{12}$$

From equations (7), (8) and (12) the expected unemployment rate, denoted by $E\,(v_i)$, can be determined as

$$E\,(v_i) = 1 - \left(\frac{\theta - 1}{\theta} \right)^{\frac{1}{\rho}}. \tag{13}$$

In the negotiation process both parties are mainly interested in their own share of total revenues and in expected (un)employment.

From equations (3), (7), (8), (11) and (12) the negotiated price and wage can be solved as functions of total revenues I, the spill-over effect Λ and full-employment output y_f. They depend on the parameters θ and ρ also.

5 The Aggregate Economy and Equilibrium Conditions

The assumptions on a representative consumer, a representative firm and a representative union allow us to define some aggregates in a very straightforward manner. The only difference among agents comes from the distribution of demand among goods, *i.e.*, the distribution of the u_i parameters.

Since prices and wages are set before the firm (and the union) knows its own position in the effective demand distribution, all firms and unions agree on the same prices and wages. Formally, $p_i = p$ and $w_i = w\,\forall\, i$. From equations (10) and (11)

$$p = \left(1 - \frac{1-\beta}{\theta\Phi}\right)^{-1} w,$$

and

$$w = \beta\, p.$$

As it was shown in the previous section, the negotiation determines the equilibrium value of the elasticity of expected production to expected demand: $\Phi = 1/\theta$. From equations (3), (7) and (8),

$$(\lambda\, p)^{-\theta}\left(\frac{Y}{Y_f}\right) = (\theta - 1)^{\frac{1}{\rho}}, \tag{14}$$

under the equilibrium condition that $C = Y$, where Y and Y_f represent aggregate production and aggregate capacities, respectively.

As stated before, the equilibrium in the ith market is defined by the condition: $c_i = y_i$, where $y_i = \min\{d_i,\, y_f\}$. From the representative household utility function, the equilibrium value of aggregate production Y can be approximated by

$$(Y)^{\frac{1-\theta}{\theta}\,\bar{\rho}} = [(\lambda\, p)^{-\theta}\, Y]^{\frac{1-\theta}{\theta}\,\bar{\rho}} + \alpha\,(Y_f)^{\frac{1-\theta}{\theta}\,\bar{\rho}}, \tag{15}$$

where the parameters $\bar{\rho} \geq \rho$ and $\alpha \geq 1$ (see the Appendix). Aggregate production is a CES function of aggregate effective demand and full-employment output.

The equilibrium for this economy, summarized in equations (14) and (15), shows some interesting properties.

PROPERTY 1: *The economy is unable to attain the full-employment equilibrium.*

From equations (14) and (15), the equilibrium value of aggregate production is

$$Y = \left(\alpha + (\theta - 1)^{-\frac{\theta-1}{\theta}\frac{\bar{\rho}}{\rho}}\right)^{-\frac{\theta}{\theta-1}\frac{1}{\rho}} \quad Y_f \leq Y_f,$$

Aggregate production is generally smaller than full-employment output, since some markets are in excess supply while other markets are in excess demand.

Note that when there is no uncertainty, *i.e.*, when $\rho \to \infty$, $\bar{\rho} \to \infty$ and $\alpha \to 1$, the economy is at the full-employment equilibrium. Uncertainty is essential for our result, because when uncertainty is removed "nominal rigidities" disappear and "labor market segmentation" does not play any role.

PROPERTY 2: *The unemployment rate is generally positive and equal to the expected unemployment rate.*

The "battle of the mark-ups" implicit in equations (10) and (11) determines the equilibrium value for $\Phi = 1/\theta$. Since aggregate employment l is given by

$$l \equiv \sum_{i=1}^{n} l_i = E(y_i),$$

from equations (7) and (8) the unemployment rate at the equilibrium is

$$v = 1 - \left(\frac{\theta - 1}{\theta}\right)^{\frac{1}{\rho}},$$

which is in general positive. Note that aggregate unemployment is equal to expected unemployment, given by equation (13).

There is full employment only in two extreme cases: if there is perfect competition, *i.e.*, if $\theta \to \infty$; or if there is no uncertainty, *i.e.*, if $\rho \to \infty$. In both cases "heterogeneity" is removed and "labort markets segmentation" disappears.

PROPERTY 3: *The representative consumer "spills-over" from the constrained demands to the non-constrained ones.*

The equilibrium value for the spill-over effect is

$$\Lambda = \theta^{\frac{1}{\rho}} \geq 1.$$

6 Conclusions and Remarks

The behavior of households under monopolistic competition in the Dixit-Stiglitz framework is extended to a situation in which quantity constraints could be binding. Under these conditions, the distinction between "notional demand" and "effective demand" is shown to be relevant to the analyses of consumer behavior and market equilibrium. "Demand heterogeneity" is introduced through weights in a CES utility function, and is related to "demand uncertainty". Since households do not reveal their preferences before prices are public information, from the firm point of view demand uncertainty is the consequence of demand heterogeneity. In an imperfectly competitive economy, with nominal rigidities and labor markets segmentation, demand uncertainty is the main cause of quantity rationing in the goods market and unemployment.

Given the difficulties of working with quantity constraints, the model presented in this paper introduced some important simplifying assumptions. First, since the model is static, firms are inhibited to learn about their own demand. To build a dynamic model of this type is not a very easy task, since households must take into account all future quantity constraints. However, it seems possible to reproduce the main results of this paper in an Overlapping Generation Model, where agents live for two periods. This will yield a better understanding of how households transfer constrained demands over time and how firms learn about their own demand. Secondly, the results in this paper are closely related to the assumption of "real rigidities", mainly caused by "labor market segmentation". Modelling the obstacles to labor mobility seems to be an important improvement.

Quantity Aggregators

This proof follows a similar proof from LAMBERT [1988]. The quantity index associated with preferences in problem (1), can be written in the following way

$$Y^{\frac{\theta-1}{\theta}} = D^{\frac{\theta-1}{\theta}} \int_{d(i)\leq Y_f} u(i)\, di + Y_f^{\frac{\theta-1}{\theta}} \int_{d(i)\geq Y_f} u(i)^{\frac{1}{\theta}}\, di,$$

where D represents the aggregate effective demand, $d(i) = D\,u(i)$ and Y_f the aggregate full-employment output.

Let us define $\bar{u}$, such that $D\bar{u} = Y_f$.

Let us assume that u follows a lognormal distribution, *i.e.*,

$$\log(u) = \lambda + \varepsilon,$$

where

$$\varepsilon \sim N(0,\, \sigma^2).$$

Since $E(u) = 1$,

$$\lambda = -\frac{\sigma^2}{2}\,.$$

We can now rewrite the aggregate quantity index as

$$\begin{aligned} Y^{\frac{\theta-1}{\theta}} &= D^{\frac{\theta-1}{\theta}} \int_{-\infty}^{\bar{\varepsilon}} \exp\left\{-\frac{\sigma^2}{2} + \varepsilon\right\} dF(\varepsilon) \\ &\quad + Y_f^{\frac{\theta-1}{\theta}} \int_{\bar{\varepsilon}}^{+\infty} \exp\left\{\frac{1}{\theta}\left(-\frac{\sigma^2}{2} + \varepsilon\right)\right\} dF(\varepsilon) \end{aligned}$$

where

$$\bar{\varepsilon} = \log\left(\frac{Y_f}{D}\right) + \frac{\sigma^2}{2}\,.$$

Let us define Π_1 and Π_2, such that,

$$Y^{\frac{\theta-1}{\theta}} = D^{\frac{\theta-1}{\theta}}\,\Pi_1 + Y_f^{\frac{\theta-1}{\theta}}\,\Pi_2.$$

Look first at Π_1

$$\begin{aligned} \Pi_1 &= \exp\left\{-\frac{\sigma^2}{2}\right\} \int_{-\infty}^{\bar{\varepsilon}} \frac{1}{\sqrt{2\pi}\,\sigma} \exp\left\{-\frac{\varepsilon}{2\sigma^2} + \varepsilon\right\} d\varepsilon \\ &= \int_{-\infty}^{\bar{\varepsilon}} \frac{1}{\sqrt{2\pi}\,\sigma} \exp\left\{-\frac{(\varepsilon-\sigma^2)^2}{2\sigma^2}\right\} d\varepsilon. \end{aligned}$$

Changing ε by $\eta = \dfrac{\varepsilon - \sigma^2}{\sigma}$, Π_1 becomes

$$\Pi_1 = \int_{-\infty}^{\bar{\varepsilon}} \frac{1}{\sqrt{2\pi}} \exp\left\{-\frac{\eta^2}{2}\right\} d\eta = F\left(\frac{1}{\sigma}\log\left(\frac{Y_f}{D}\right) - \frac{\sigma}{2}\right).$$

where $F(\cdot)$ is the standard normal distribution function. For the same argument,

$$\Pi_2 = \exp\left\{ \frac{\sigma^2}{2\,\theta}\left(\frac{1}{\theta} - 1\right)\right\} F\left(-\frac{1}{\sigma}\log\left(\frac{Y_f}{D}\right) - \frac{\sigma}{2} + \frac{\sigma}{\theta}\right).$$

Let us define

$$\mathcal{F}_1 \equiv \left(\frac{D}{Y}\right)^{\frac{\theta-1}{\theta}} \Pi_1 = \frac{D^{\frac{\theta-1}{\theta}}\,\Pi_1}{D^{\frac{\theta-1}{\theta}}\,\Pi_1 + Y_f^{\frac{\theta-1}{\theta}}\,\Pi_2} = \frac{1}{1 + \left(\frac{Y_f}{D}\right)^{\frac{\theta-1}{\theta}} \frac{\Pi_2}{\Pi_1}}\,.$$

Introducing the following change of variables

$$\exp\{a\} = \left(\frac{Y_f}{D}\right)^{\frac{\theta-1}{\theta}}$$

we can rewrite $\mathcal{F}_1$ as a function of a :

$$\mathcal{F}_1(a) = \frac{1}{1 + \exp\{a\}\dfrac{\pi_2(a)}{\pi_1(a)}}\,.$$

where

$$\pi_1(a) = F\left[\frac{\theta}{\theta-1}\,\frac{1}{\sigma}\,a - \frac{\sigma}{2}\right],$$

$$\pi_2(a) = F\left[-\frac{\theta}{\theta-1}\,\frac{1}{\sigma}\,a - \frac{\sigma}{2} + \frac{\sigma}{\theta}\right]\exp\left\{\frac{\sigma^2}{2\,\theta}\left(\frac{1}{\theta} - 1\right)\right\}.$$

Let us approximate the $\mathcal{F}_1(a)$ function by the exponential function $\Phi_1(a)$

$$\Phi_1(a) = \frac{1}{1 + \exp\{-\rho\,(a-\beta)\}}\,,$$

around the value $a = 1$, by equalizing $\mathcal{F}_1(1) = \Phi_1(1)$ and $\mathcal{F}_1'(1) = \Phi_1'(1)$. The values for ρ and β verify

$$\rho = -1 + \frac{\theta}{\theta-1}\,\frac{1}{\sigma}\left(\frac{f\left(-\frac{\sigma}{2} + \frac{\sigma}{\theta}\right)}{F\left(-\frac{\sigma}{2} + \frac{\sigma}{\theta}\right)} + \frac{f\left(-\frac{\sigma}{2}\right)}{F\left(-\frac{\sigma}{2}\right)}\right)$$

and

$$\alpha \equiv \exp\{\rho\beta\} = \exp\left\{\frac{\sigma^2}{2\,\theta}\left(\frac{1}{\theta} - 1\right)\right\}\frac{F\left(-\frac{\sigma}{2} + \frac{\sigma}{\theta}\right)}{F\left(-\frac{\sigma}{2}\right)}$$

where $f(\cdot)$ is the standard normal density function.

Since $\mathcal{F}_1 = \dfrac{\partial\log(Y)}{\partial\log(D)}$, by integration we have

$$Y^{\frac{1-\theta}{\theta}\rho} = D^{\frac{1-\theta}{\theta}\rho} + \alpha\,Y_f^{\frac{1-\theta}{\theta}\rho}. \quad \square$$

LAMBERT [1988] corresponds to $\theta \to \infty$, in which case

$$\rho = -1 + \frac{2}{\sigma}\ \frac{f\left(-\frac{\sigma}{2}\right)}{F\left(-\frac{\sigma}{2}\right)} \qquad \text{and} \qquad \alpha = 1.$$

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