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A Simple Macroeconomic Model with Endogenous Credit Rationing

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ABSTRACT. – In this paper, a two-period macroeconomic model, in which output is demand determined, is constructed. In the first period firms may borrow to finance investment, which reduces their marginal costs in the second period; however, since default by borrowers is possible there is an incentive compatibility constraint which may or may not bind. If it binds, there is endogenous credit rationing. Three regimes are possible; in Regime I there is no investment; in Regime II there is positive investment without the ICC holding, whereas in the third the condition binds and there is credit rationing. The behaviour of the economy in each of these regimes, and the circumstances under which each of these regimes obtains, are analyzed and some conclusions drawn. The possibility that the model possesses multiple equilibria is considered.

Un simple modèle macroéconomique avec rationnement endogène du crédit

RÉSUMÉ. – Nous construisons un modèle macroéconomique à deux périodes. Pendant la première période, les entreprises peuvent emprunter pour financer leur investissement, réduisant leurs coûts marginaux dans la seconde période. Comme le risque de défaut des emprunteurs existe, une contrainte incitative de comptabilité peut lui être associée. Si tel est le cas, il y a rationnement endogène du crédit. Plusieurs régimes apparaissent : nous les étudions, analysons le comportement de l'économie dans chaque cas et tirons des conclusions.

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1 Introduction

The idea that changes in credit conditions may be of particular importance for explaining changes in levels of economic activity has had a long history and has received considerable support in recent years (e.g., Bernanke, 1983). Also, there has been extensive discussion of credit rationing, by which is meant a situation in which there is excess demand for credit. There is some evidence that credit rationing does exist (see below), and much effort has been devoted to explaining its existence theoretically – the basic problem being why, in a situation of excess demand for credit, interest rates on loans do not rise equilibrate supply and demand. The typical explanation is that lenders do not raise interest rates because it is not in their interest to do so; an increase in interest rates may reduce the quality of loans (*i.e.*, raise default rates) for at least one of two reasons: (i) adverse selection effects, whereby higher interest rates reduce the average quality of loan applicants; (ii) incentive effects, whereby higher interest rates induce borrowers to act in such a way as to increase their probability of default (e.g., Stiglitz and Weiss, 1981).

There is some empirical work on the existence of credit rationing. For example, Zeldes [1989] and Japelli [1990] produce evidence that a fair number of American consumers are credit constrained. Hoshi, Kashyap and Scharfstein [1991] study the investment behaviour of a number of Japanese firms and conclude that their results support the view that "capital-market imperfections contribute to excessive output fluctuations" (op. cit., p. 57). There is also work on the importance of credit in explaining macroeconomic fluctuations (e.g., Bernake, 1983). A sceptical piece is by Berger and Udell [1992], who use information on a large number of commercial bank loans and conclude that "equilibrium [credit] rationing is not a significant macroeconomic phenomenon" (p. 1047). This conclusion is based on study of the ratio of commitment to non-commitment loans under certain circumstances and a crucial part of their argument is that commitment borrowers are not subject to credit rationing. This contention, however, is not discussed and would seem dubious – whether commitment borrowers are, or may be, subject to credit rationing depends, of course, on the nature of the commitment. Unless lenders are committed to lending as much as is demanded under all circumstances – which is absurd – there must be circumstances under which commitment borrowers are subject to rationing. For further discussion of empirical evidence on credit rationing, see Jaffee and Stiglitz [1990].

Despite the apparent empirical importance of credit rationing and the numerous partial equilibrium attempts to explain it, there is, however, little work at the moment which attempts to construct macroeconomic models with endogenous credit rationing, yet this is something which it would seem desirable to do if we are to obtain an adequate account of the role of credit in the economy. One would like to know, for example, how changes in the macroeconomy affect the degree of credit rationing and how changes in this react back on the economy, and so forth. Macroeconomic models with endogenous credit rationing include Azariadis and Smith [1993], Kiyotaki and Moore [1993], Tsiddon [1992] and Zeira [1991]. All of these models differ substantially, though, from that presented here, which constructs a macroeconomic model where output is demand determined. Azariadis and Smith construct an overlapping generations model in which there is private information about endowments received by old agents and this gives rise to adverse selection and credit rationing. They consider a number of issues, such as the implications of credit rationing for interest rates; it would seem to reduce them, as greater credit rationing presumably raises saving. Kiyotaki and Moore [1993] develop a model in which lending depends on collateral; credit restrictions reduce the value of collateral and this reduces lending still further, and so on. Tsiddon [1992] considers a growth model in which agents" investment in human capital is subject to a moral hazard problem. Zeira [1991] examines fiscal policy in a open economy overlapping generations framework.

Another area of active current research in macroeconomics (e.g., Cooper and John, 1988) concerns the existence and implications of multiple equilibria. It is perhaps surprising that, as far as we are able to ascertain, virtually none of the macroeconomic literature on credit rationing has incorporated this possibility [1]; after all, it is possible to tell the following story, which seems intuitively plausible: "lending may be high in certain circumstances because banks expect output to be high and therefore are prepared to lend extensively (*i.e.*, high expected output leads to a relaxation of the credit rationing constraint); however the fact that lending is high means that both current and expected future economic activity is high, and so expectations are fulfilled. Similarly, pessimistic expectations may be self-fulfilling". One of the aims of this paper is to formalise this story and investigate the circumstances under which multiple expectational equilibria of this kind can indeed occur. Moreover, models with multiple equilibria can provide a plausible account of how a collapse in economic activity may occur (and certain collapses in economic activity, such as the Great Depression, do seem to be accompanied by considerable turmoil in credit markets). Consider Figure 1, which as drawn shows an economy with multiple equilibria. Suppose the economy is initially at the equilibrium denoted by D, and something happens which shifts the $f\,(\,.\,)$ locus (which we describe as the reaction function) in a downward direction. Then, if it shifts downwards far enough, two of the equilibria will disappear and there will be a sudden downward movement in economic activity until the one remaining equilibrium is reached. This account is suggestive, but little more; what we attempt to do in this paper is present a more rigorous treatment of the multiple equilibria issue in a model with endogenous credit rationing.

Accordingly, we assume a two-period economy populated by an infinite number (continuum) of industries, each of which produces a differentiated product. Each industry has one large firm with some monopoly power;

1. Tsiddon [1992], in which multiple steady states can exist, is the only exception I am aware of. The equilibria in our model, which might be described as expectational equilibria (*i.e.*, which one of a number of equilibria happens to obtain depends on expectations) are somewhat different.

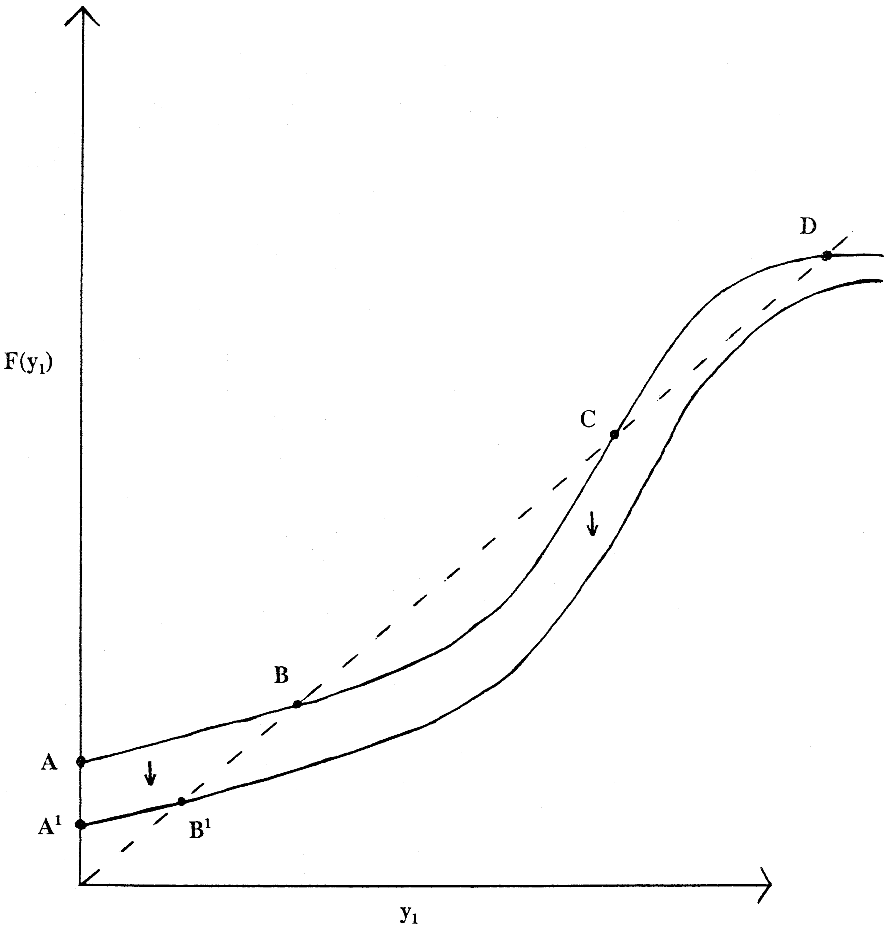

FIGURE 1

Multiple Equilibria and the Possibility of a Collapse in Economic Activity.

however there is also a competitive fringe. Our assumptions ensure that the firm acts as a price taker and produces what is demanded at a given price. The firm can undertake an investment in the first period which will (with a certain probability) reduce its marginal costs in the second. Its incentive to invest depends, at least in part, on its expectations about the second period. The typical firm cannot finance its investment entirely by retained earnings in the first period but needs to borrow from a financial intermediary. The lender cannot be certain that the borrower will invest the money it lends; instead, the firm (or perhaps its owners) has the option of taking the money and running (e.g., absconding abroad), in which case the lender would receive nothing. Accordingly, the lender will lend money only if he believes that the borrower has an incentive not to do this; that is, the expected return to the borrower from investing the money must exceed the gain from taking the money and running; this gives rise to an incentive compatibility constraint. If the constraint is binding, then we have credit rationing; firms would borrow more than they are allowed to if they could commit to investing the money which, however, they are incapable of doing.

So investing firms may or may not find themselves credit rationed; it turns out that the relevant investment functions depend very much on whether firms are credit rationed. We then embed this model of possibly credit rationed firms in a macroeconomic framework where output is demand determined.

The paper is organised as follows: Section 2 describes the model, the following Section explores the equilibrium features of the model, including the possibility of multiple equilibria. and there is a brief concluding Section.

2 The Model

There are two time periods, 1 and 2, within which economic activity takes place. Agents have rational expectations – a strong, although standard assumption. A defence of the assumption is that we would not want any of the results derived to depend on expectational errors.

- **Firms**

In our assumptions about firms and market structure we follows Murphy, Shleifer and Vishny [1989]. We assume a continuum of goods, indexed by $q \in [0, 1]$; each can be produced by a large firm. There is also a competitive fringe which has constant marginal costs, converting one unit of labour (the only factor of production) into one unit of output (this is just a normalisation). It is necessary to make some sort of assumption about the labour market; given our main focus, an assumption of market clearing would not be particularly appropriate. Accordingly, we make what is probably the simplest assumption compatible with the notion that output is demand determined, namely that real wages are rigid and labour is supplied elastically at this real wage; labour can hence be used as the numeraire. There are a number of ways in which the rigid real wage assumption can be justified. One possibility would be an efficiency wage, another trade union behaviour and a third would be to make assumptions about agents' preferences (as in Kiyotaki, 1988) which generate this [2].There is a maximum

2. Obviously this assumption of an exogenous real wage is not satisfactory. It would be desirable, and in the spirit of the paper, to endogenise it, using perhaps one of the many efficiency wage models (see section V of Mankiw & Romer, 1991, for example). In defence, I would plead that it is impossible to do everything at once and that in order to make progress in studying one phenomenon, it is often defensible to employ simplifying assumptions about other aspects of the model which are not one's main focus of attention.

amount of labour which can be supplied in the economy; however, in the analysis we assume that this constraint is not binding.

In each period the large firm faces constant marginal costs and fixed costs F, and has to decide how much output to produce. In the first period marginal costs are α_0, whereas in the second they are $\alpha(I)$, where I is the amount of investment undertaken by the firm in the first period, providing the investment is successful (more on this below). We also assume, for simplicity, although this is not strictly necessary, that $\alpha(0) = \alpha_0$. So the firm, in choosing its investment level in the first period is also choosing its marginal costs in the second; however, once the second period has arrived, the firm faces constant marginal costs. Market demand for the good sold by each large firm is iso-elastic (a consequence of consumers having logarithmic preferences). The competitive fringe will always price at unity, so the large firm cannot raise its price above unity (if it wants to sell anything). However, the firm has no incentive to cut its price as, with iso-elastic demand, doing so will not increase its revenue but will increase its (total) costs. The upshot is that the large firm sells what is demanded at a price of unity, providing it is profitable for it to operate. The condition for this is that the PDV of (gross) profits exceeds the cost of investment, where investment is chosen optimally. If this condition is not met, then output is produced by the competitive fringe and the monopoly does not operate. So there will be a set of values of present and future output $(y_1,\ y_2(y_1))$ at which it is just profitable for the monopoly to operate. For values of present and future output above (below) these values, the monopoly (competitive fringe) will operate. The large firm may operate even with negative current profits if these are offset by sufficiently high expected future profits. However, if maximum future profits from operating are negative, the firm can just cease operation, so there is a non-negativity constraint on future profits. What these assumptions about firms, market structure and the labour market do is enable us to consider a framework where output is demand determined with the minimum of complications (e.g., prices and wages do not change as output changes), in a way which is still rigorous.

• Investment and Credit Rationing

Firms finance investment first of all by using retained profits. If profits are insufficient to finance the investment firms wish to do, then they borrow the remainder, so we have

$$I = \Pi_1 + L, \tag{1}$$

where Π_i represents the firm's profits in period i and L the amounts the firm borrows. If profits exceed firms' investment, then the remainder is distributed to consumers; in this case (1) still holds, but L is negative. It will be optimal, of course, for a firm to use retained earnings to finance investment before borrowing if the cost of borrowing is higher than the (opportunity) cost of using internal funds; even if the costs are the same it is compatible with profit maximisation to assume that firms use internal funds first, so (1) does not embody any questionable assumptions. Given the focus of the paper, we will usually consider the case where the firm needs to borrow to finance investment.

Investment of an amount I means that the firm's marginal costs in period 2 are $\alpha(I)$ (with $\alpha'(I) < 0$) with probability θ. With probability $(1-\theta)$, the investment fails, in which case its marginal costs in the second period stay at $\alpha_0 (\geq \alpha(I)$ for all $I)$. So:

$$\begin{aligned}
&\Pi_1 = (1-\alpha_0)\, y_1 - F. \\
(2) \qquad &\Pi_2 = \max[0,\ (1-\alpha(I))\, y_2 - F], \quad \text{with probability } \theta \\
&\quad\ = \max[0,\ (1-\alpha_0)\, y_2 - F], \quad \text{with probability } 1-\theta.
\end{aligned}$$

Here, y_i is the typical firm's output in period i and F represents its fixed costs – which it must incur if it is to produce anything – in each period. It may be noted that profits as defined are gross (*i.e.*, before loan repayments). The loan contract is such that the firm repays to the lender in the second period the amount:

$$R = \min[(1+r)\, L,\ \Pi_2]$$

So the firm repays the loan if it has sufficient profits to do so; if not, then the lender claims the entire future profits of the firm [3]. It can be shown that this is an optimal loan contract (we do not present a formal demonstration of this here, though). It is straightforward to show that it is not optimal to condition repayments on future profits as this will distort the investment decision. So a repayment in each state which is independent of the actions of the firm is called for (subject to a feasibility requirement). There are many such loan contracts; the one postulated is the one that imposes the maximum penalty in the bad state. This ensures that the firm is always better off in the good state than in the bad state, so it never has an incentive to claim falsely that the bad state has occurred, or to try to persuade the lender that the bad state has occurred by generating the level of profits that would occur in the bad state (assuming profits, but not the state, can be observed by the lender). The notion that the maximum penalty possible should be imposed in the bad state is standard (see Townsend, 1979).

In the absence of credit rationing, the firm chooses investment to maximise expected profits net of loan repayments (it is risk neutral):

$$(3) \qquad E(\Pi_2 - R) = \theta\,[y_2\,(1-\alpha(I)) - F - (I - \Pi_1)(1+r)]$$

Here it is assumed that in the bad state, the firm's profits are entirely absorbed in loan repayment, so the net return to the firm in this state is zero (so there is no term involving $1-\theta$ in (3)). It is straightforward to derive the condition for the case where the firm can repay the full value of the loan in the bad state, but do not consider it here to prevent the analysis from becoming too complex. The first-order condition for maximisation of (3) is

3. It might be asked why the firm bothers to produce anything in the second period if it is incapable of repaying its loan in full, since it will end up with zero profits. Strictly, the loan contract must allow the firm an infinitesimal amount in this state, but it makes no analytical difference if we assume the lender is left with nothing in this state but continues to produce.

(4) $$y_2\,\alpha'(I) = -(1+r)$$

and it is easily checked that the second-order condition is that $\alpha(I)$ is convex (*i.e.*, $\alpha''(I) > 0$). We can hence write the following equation for the optimal level of investment:

(5) $$I^* = I^*(r,\, y_2)$$

with the following partial derivatives:

(5 a) $$\frac{\partial I^*}{\partial r} = -[y_2\,\alpha''(I^*)]^{-1} < 0.$$

(5 b) $$\frac{\partial I^*}{\partial y_2} = \frac{-\alpha'(I^*)}{\alpha''(I^*)\,y_2} > 0.$$

It may be noted that investment depends just on the (rationally expected) level of future output and on the interest rate on loans; this is a very standard type of investment function. It is also useful to compute the second derivative of investment with respect to future output:

(5 c) $$\frac{\partial^2 I^*}{\partial y_2^2} = \frac{\left[-(\alpha'')^2\; y_2\,\dfrac{\partial I^*}{\partial y_2} + \alpha'\left(\alpha'' + y_2\,\alpha'''\,\dfrac{\partial I^*}{\partial y_2}\right)\right]}{(\alpha''\; y_2)^2}$$

It is apparent that a (very weak) sufficient condition for negativity of this expression is that $\alpha''' \geq 0$; this would be the case, for example, with a quadratic function. We will henceforth assume that the $\alpha(\,.\,)$ function is such that (5 c) is negative, and hence that investment is a concave function of future output. This, again, seems a very reasonable property for an investment function to have; it means that successive increments to future output induce diminishing amounts of investment. Also, we shall suppose that there is a level of investment at which α' becomes zero – that is, further increments of invesment do not reduce costs at all. At this (and higher) levels of investment, $\partial I/\partial y_2$ is also zero.

There is also, obviously, a non-negativity constraint on investment so investment in fact will be the maximum of (5) and zero.

In order to generate the possibility of credit rationing, we assume that the lender cannot ensure that the funds lent are actually invested; instead, the borrower has the option of “taking the money and running”. The firm does this if the return on investing (expected future profits less loan repayment) is less than the value of the funds it would otherwise invest. The lender will obviously not lend if it believes the borrower has an incentive to take the money and run, so we have the following incentive compatibility constraint (ICC) which must be satisfied if lending is to take place:

(6) $$\theta\,[\Pi_2 - (I - \Pi_1)\,(1+r)] \geq I\,(1+r^*)$$

where r^* is the safe rate of interest (assumed to be an exogenous world rate); for simplicity we assume there are no costs associated with “taking the money and running", such as expected costs of being prosecuted, etc., although of course it would be possible to introduce them. If the ICC is satisfied when investment is at its optimal level, then no credit rationing occurs. However, if it is not satisfied, then the lender will restrict his lending until the ICC is satisfied with equality, so that investment is then determined by the following equation [4]:

$$I = X\,\theta\,[\Pi_1\,(1+r) + \Pi_2] \tag{7}$$

where $X \equiv [1 + r^* + \theta\,(1+r)]^{-1}$.

The firm's investment decision in the absence of credit rationing and in its presence, respectively, is illustrated in Figures 2 and 3.

Substituting for Π_1 and Π_2, and confining our attention to the case where the large firm operates in both periods, we obtain the following expression for investment under credit rationing:

$$I = X\,\theta\,[y_1\,(1-\alpha_0)\,(1+r) + y_2\,(1-\alpha\,(I)) - (2+r)\,F] \tag{8}$$

It is straightforward to calculate the relevant partial derivatives. These are:

$$\frac{\partial I}{\partial y_1} = \frac{X\,\theta\,(1+r)\,(1-\alpha_0)}{S} > 0 \tag{9}$$

where $S \equiv 1 + X\,\theta\,y_2\alpha'\,(I)$,

$$\frac{\partial I}{\partial y_2} = \frac{(1-\alpha\,(I))\,X\,\theta}{S} \tag{10}$$

$$\frac{\partial I}{\partial r} = \frac{[(y_1\,(1-\alpha_0) - F)\,X - I]\,\theta}{S} < 0 \tag{11}$$

$$\frac{\partial I}{\partial r^*} = \frac{-1}{S} < 0. \tag{12}$$

$$\frac{\partial I}{\partial F} = \frac{-(2+r)\,X\,\theta}{S} < 0. \tag{13}$$

In signing the denominator, we use the fact that invesment is less than it would be in absence of credit rationing, and therefore, from equation (4), $y_2\,\alpha'\,(I) < -(1+r)$; it follows that the denominator is positive. Also using this inequality we can show that (9) is greater than $\theta\,(1-\alpha_0)$. Investment appears on both sides of equation (8) and this gives rise to what might

4. Strictly, it must be slightly more preferable for the firm to invest than take the money and run, but since the difference can be arbitrarily small, it makes no effective difference to assume equality.

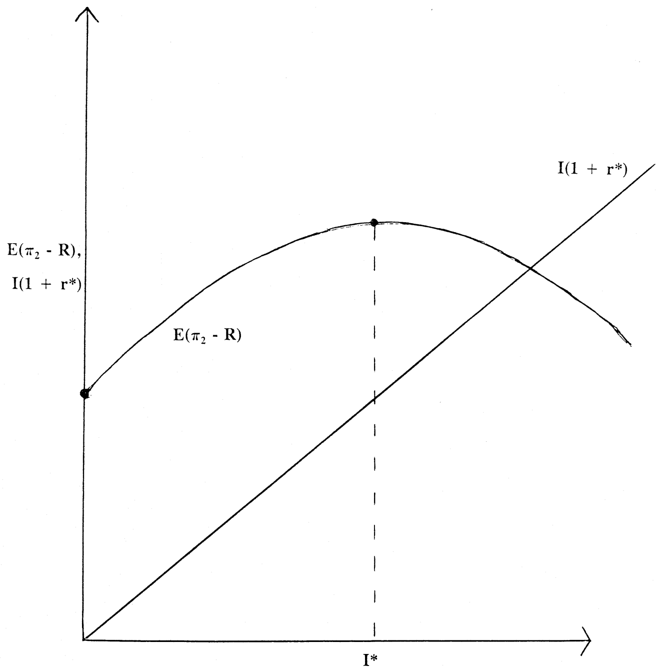

FIGURE 2

Investment in the Absence of Credit Rationing.

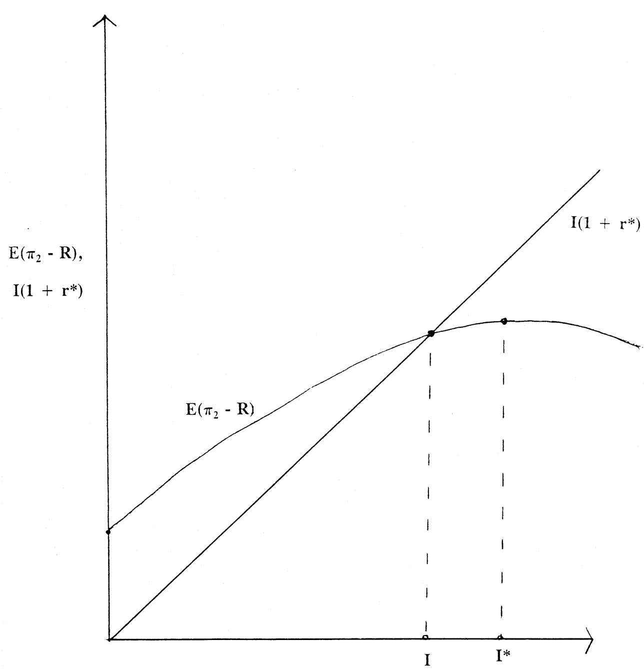

FIGURE 3

Investment under Credit Rationing.

be described as multiplier effects on investment; any change which relaxes the ICC and raises investment under credit rationing raises future profits (because of the fall in future costs), hence relaxing the ICC further and generating more investment, and so on. 1/S might hence be described as the "credit rationed investment multiplier".

It is apparent that the credit rationing investment function (8) is very different from the one which operates in its absence (5). First, current output, the safe rate of interest and fixed costs all affect investment under credit rationing whereas they have no effect in its absence. Secondly, some of the effects may be very powerful. If θ is 0.9 and α_0 0.5, then (9) must be greater than 0.45; *i.e.*, a one unit increase in current output raises investment by at least 0.45 units. If $\theta = 1$, which we assume later in the paper, investment rises by more than profits. This means that lending rises as well. An effect of current profits on investment, which is what our model of investment under credit rationing predicts, is consistent with empirical evidence (e.g., Abel and Blanchard, 1986, Fazzari, Hubbard and Petersen, 1988). Higher future output raises investment in the presence of credit rationing as it does in its absence, although the magnitude may be different. A higher risky rate of interest reduces investment under credit rationing, but again the magnitude of the effect may be different from its effect in the absence of credit rationing. Increases in both the safe rate of interest and fixed costs reduce investment under credit rationing whereas they have no impact in its absence.

• Lenders

The typical lender obtain deposits at the safe rate of interest (or, equivalently, can borrow on the world money markets at the safe rate of interest) and lends the money to firms to invest at the risky interest rate r. In equilibrium the incentive compatibility constraint is always satisfied, so no owners of firms will actually "take the money and run". Instead, risk to lenders comes from the possibility that the investment fails, and the borrower does not have enough resources to repay the loan (in which case the borrower receives the full value of the lender's second-period profits). If this is the case, the profits of a typical lender per loan are given by:

(14) $$\theta(1+r)L + (1-\theta)(y_2(1-\alpha_0) - F) - L(1+r^*)$$

We assume competition among lenders, so this will be equated to zero. The relationship between the risky and the risk-free interest rates is hence given by:

(15) $$r = (1/\theta)\left[\frac{L(1+r^*) - (1-\theta)(y_2(1-\alpha_0) - F)}{L}\right] - 1$$

It can be shown from (15) that increases in either current or future economic activity reduce the risky rate of interest vis-à-vis the safe rate. This provides a sharp contrast with standard IS/LM analysis, where an increase in economic activity tends to raise interest rates, and this tends to dampen the rise in economic activity [5]. The explanation is that the value of the resources the lender can claim when the borrower defaults is greater in good economic times, hence reducing the risk involved in lending, and, with competition, the risky rate of interest. This gives us a mechanism which should reinforce, rather than dampen, cyclical movements in economic activity and way well contribute to the model possessing interesting macroeconomic properties [6]. (Note that the probability of a bad outcome to the investment, which in this model might be regarded as a technological parameter, is exogenous and hence independent of the level of economic activity. It might be more plausible for this probability to decrease in good economic times, which should further reinforce the effect described above.)

• Consumers

It remains to specify the behaviour of consumers. We assume one representative consumer, whose preferences within a period can be represented by a logarithmic utility function, so that he consumes an equal quantity of all goods within the period. We can hence consider his behaviour in terms of his consumption of a typical good in each period, and write his budget constraint as:

$$c_1 + c_2/(1 + r^*) = \Omega \tag{16}$$

where Ω represents the DPV of the resources at the consumer's disposal. Largely for technical reasons, we assume the consumer receives a constant fraction β of output in each period. This can be rationalised as follows: profits accruing to firms do not affect consumption (this may be the case if the firms are foreign-owned). Costs incurred by firms $(\alpha(I)\,y_i + F)$ can be divided into two components; the first, $(\gamma(I)\,y_i + F)$ constitutes payments to firms outside the economy which do not affect consumers; the second $(\beta\, y_i)$ constitutes payments to consumers (workers). We therefore have $\alpha(I) = \beta + \gamma(I)$ and in particular investment does not affect β. We also assume that when the competitive fringe operates, a fraction β of its wage payments enters into the consumer's budget constraint. The reason for this assumption is that it enables us to avoid the considerable (technical) complications there

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5. However, in this model what happens is that the risky rate of interest falls relative to the exogenous risk-free interest rate. A more satisfactory comparison might be provided by an extension of the model which endogenised the risk-free rate of interest, but this is beyond the scope of the paper.
6. However, one would expect this mechanism, whereby increases in economic activity reduce the borrower's rate of interest, would operate only for lower levels of economic activity. Once activity (in the second period) has risen so as to enable borrowers to repay the full value of loans in the bad state, there is no reason why the borrower's rate of interest should decline as activity rises further.

would otherwise be in having the level of investment affecting the marginal propensity to consume in the second period. We therefore have:

$$\Omega = \beta\,[y_1 + y_2/(1+r^*)] - T \tag{16 a}$$

where T is the present value of taxation imposed on the consumer. The consumer maximises an intertemporal utility function subject to this constraint; the simplest assumption about preferences, which we will adopt here, is that they are Cobb-Douglas, so the utility function is $U = A\,(c_1)^k\,(c_2)^{1-k}$ where A and k are constants; it follows that the consumer consumes a constant fraction of wealth in the first period [7]. We therefore have:

$$c_1 = k\,\Omega \tag{17}$$

and

$$c_2 = (1-k)\,\Omega\,(1+r^*). \tag{18}$$

• Market Equilibrium Conditions

Finally, we need to specify that demand for goods in each period equals the amount produced in each period:

$$c_1 + I + g_1 = y_1 \tag{19}$$

$$c_2 + g_2 = y_2 \tag{20}$$

Here, g_i is government spending on goods in the *i*th period. If investment takes place it takes place only in the first period and is given by either equation (5) or equation (8), depending upon whether the ICC is satisfied when investment is given by equation (5). Since the two investment functions are very different, we might conjecture that the equilibrium properties of the model depend very much on whether credit rationing happens to obtain, something which is investigated in the next section.

With appropriate substitution, equations (19) and (20) can be rearranged so that they depend on y_1, y_2 and r. We obviously need another equation, and this is given by (15), hence giving us three equations in three unknowns.

3 Equilibrium Properties of the Model

In this section, we investigate some of the properties of the model; in particular, we are interested in the possibility that there may be multiple equilibria. In this paper we investigate the special case where θ is unity;

7. However, since in our model the interest rate relevant for the consumer's decision remains unchanged, there are many other utility functions which would generate this result.

this implies that there is no risk involved in lending (since the ICC is always satisfied) so $r = r^*$ (an alternative way of justifying this assumption would be if in the bad state profits are always sufficiently high to repay the loan). We hence have two equations in two unknowns (y_1 and y_2).

Since one of our main concerns is the multiple equilibria issue, we use what is described as the "reaction function" approach. This involves reducing the model to one equilibrium condition: $f(y_1) = y_1$. There are multiple equilibria if this equation has more than one solution. There is also a neat graphical interpretation (see Figure 1 again). Equilibria occur where the reaction function crosses the 45% line. It is apparent that a necessary condition for multiple equilibria is that the reaction function have a slope (at an equilibrium) of greater than unity (we ignore the perserve possibility that the reaction function is tangential to the 45% line without actually crossing it a number of times), providing the reaction function is continuous (a condition which may not, in fact, be satisfied). However, deriving the reaction function for the model is not at all easy. The problem is that there will almost certainly be what might be called "regime switches" – at certain levels of y_1, the economy may shift from being credit rationed to not being credit rationed, or vice versa. There is also the possibility that the monopoly does not operate, in which case the competitive fringe operates. Further complications arise when it is recognized that investment may be either zero or positive. (It is possible to generate a fairly large taxonomy of cases!) We hence distinguish three regimes – Regime I, where there is no investment – either because the competitive fringe is operating or the monopoly is operating but does not invest; Regime II, where the monopoly operates (and invests) and the ICC is not binding, and Regime III (the credit rationing regime) where the monopoly operates and the ICC is binding. We proceed by first considering what will happen in the absence of an ICC, and then by seeing what difference imposing it makes.

3.1. **The Reaction Function in the Absence of an Incentive Compatibility Constraint**

This will contain two segments: for low levels of current output, the competitive fringe will operate; at a certain level of current output it will become profitable for the monopoly to operate, so for higher levels of output, just the monopoly will operate. First of all, we examine what happens when the competitive fringe operates. We adopt the following procedure for deriving the reaction function: we solve the future goods market equilibrium condition to obtain an expression for y_2 as a function of y_1, and we then substitute this into the (LHS of the) current goods market equilibrium condition to obtain the reaction function. We assume throughout this section that there is no taxation and no future government spending. By appropriate substitution we can derive the following expression for future output:

(21) $$y_2 = (1-k)(1+r^*)\beta y_1 / \{1-(1-k)\beta\}$$

This expression will always hold, regardless of the regime. We write this relationship as $y_2 = G(y_1)$. Substituting (21) into the budget constraint, using (17) and (19) and manipulating, we get the following expression for the reaction function when the fringe operates or when the fringe operates without investing (*i.e.*, for Regime I):

$$f(y_1) \equiv [k\beta/\{1-(1-k)\beta\}]\, y_1 + g_1 \tag{22}$$

The slope of this is given by the term in square brackets and, given that β is between zero and unity, is positive but less than one. Equating this to y_1, we obtain the following expression for the equilibrium level of output in this regime (this presupposes there is an equilibrium in this regime, an issue which be considered below):

$$y_1 = \{1 + k\beta/(1-\beta)\}\, g_1 \tag{22 a}$$

The "multiplier" (the term multiplying g_1) is greater than unity, but finite. It tends towards unity as either k or β tend towards zero and tends towards infinity as β tends towards one. This regime may be described (oxymoronically?) as a "Rational Expectations Keynesian Cross". It is effectively a two-period Keynesian cross model where consumption is determined by intertemporal optimisation (under rational expectations) and hence the savings propensity does not appear in the multiplier formula. An increase in current income is expected to lead to an increase in future expenditure and income, and this affects current expenditure, and so on.

However, all this presumes that the monopoly finds it unprofitable to operate. The (necessary and sufficient) condition for the monopoly to operate is that its PDV of profits exceeds the cost of any investment it may undertake, when investment is chosen optimally. These are in fact two possibilities, shown in Figure 4. The first is that when the monopoly starts to produce, it finds it optimal to invest a stricly positive amount, so there is a jump (from zero) in the level of investment undertaken in the economy at the critical level of output at which it is just profitable for the monopoly to start producing. In this case, there is an upward jump in the reaction function at this level of output, as well. The second possibility is that the optimal level of investment is zero at the critical level of output and the reaction function is the same as that in the previous regime, until output has risen to such a level that it is now rational to start investing a positive amount. In this case, there is no discontinuity in the reaction function. (There is, as well, the borderline case between these two, where investment becomes strictly positive at exactly the level of output at which it is just profitable for the monopoly to operate, but this will occur with probability zero).

It is relatively straightforward to derive an expression for the reaction function in the case where investment is positive. Again, equation (21) holds. Substituting this into (the LHS of the) first period goods market equilibrium condition we obtain:

$$f(y_1) = k\beta\,[y_1 + G(y_1)/(1+r^*)] + I^*(G(y_1)) + g_1 \tag{23}$$

where I^* represents the unconstrained investment function (5), the dependence of which on the interest rate is not shown. This is the reaction function in the no credit rationing case with positive investment. Equating

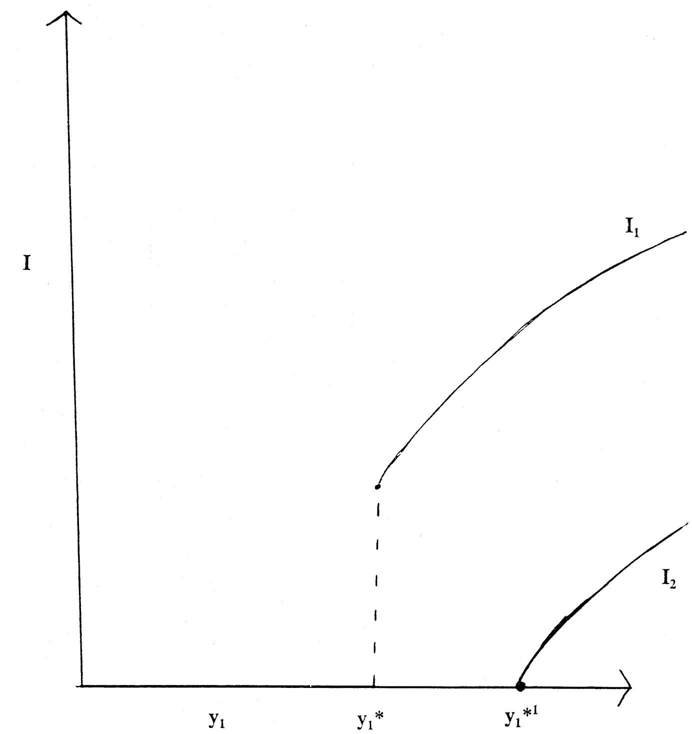

FIGURE 4

Possible Relationship between Current output and Investment in the Absence of Credit Rationing.

this to y_1 gives an equation for the equilibrium level of y_1. The slope of the reaction function is easily calculated to be:

(24) $$f'(y_1) = k\beta\{1-(1-k)\beta\} + I^{*\prime}(G(y_1)) + G'(y_1)$$

Differentiating this again, and remembering that investment is a concave function of future output, we derive the result that the reaction function is a concave function of current output. Also, since $G'(y_1) > 0$, (24) is bounded below by the first term on the RHS of (24), the multiplier of Regime I. The reaction function hence attains its greatest slope at the critical level of y_1 at which investment becomes positives (y_1^*); the slope declines continuously and approaches $k\beta/\{1-(1-k)\beta\}$ asymptotically (concavity is not sufficient to ensure this, however – in addition, we need the assumption that the effect of future output changes on investment eventually go to zero). There is nothing to prevent the slope of the reaction function being greater than unity initially, though.

It seems that in the absence of credit rationing, there can be anything from one to three equilibria. There are the following possibilities: (i) There is a Regime I equilibrium. The reaction function then jumps from below to above the 45% line at the level of current income which separates Regime I from Regime II. There is then a Regime II equilibrium where the reaction function crosses the 45% line from above (this must exist given the properties discussed above of the reaction function). So in this case there are two (stable?) equilibria. This is shown in Figure 5. (ii) There is a Regime I equilibrium. There is a jump in the reaction function, as in (i), but one which still leaves the economy below the 45% degree line. The Regime II reaction function then crosses the 45% line twice. This gives us three equilibria, two stable and one unstable. (iii) There is a Regime I equilibrium, but no further equilibria – the reaction function always lies below the 45% line (and may or may not jump). This leaves us with one equilibrium. (iv) There is no Regime I equilibrium. There is a Regime II equilibrium where the reaction function crosses the 45% degree line from above. Again, we have one equilibrium. It may be noted that there must always be at least one equilibrium, since the reaction function always starts off above the 45% degree line; given the fact that the Regime II reaction function will eventually have a slope less than unity (and also that jumps can only be in an upwards direction), it must cross the 45% line somewhere

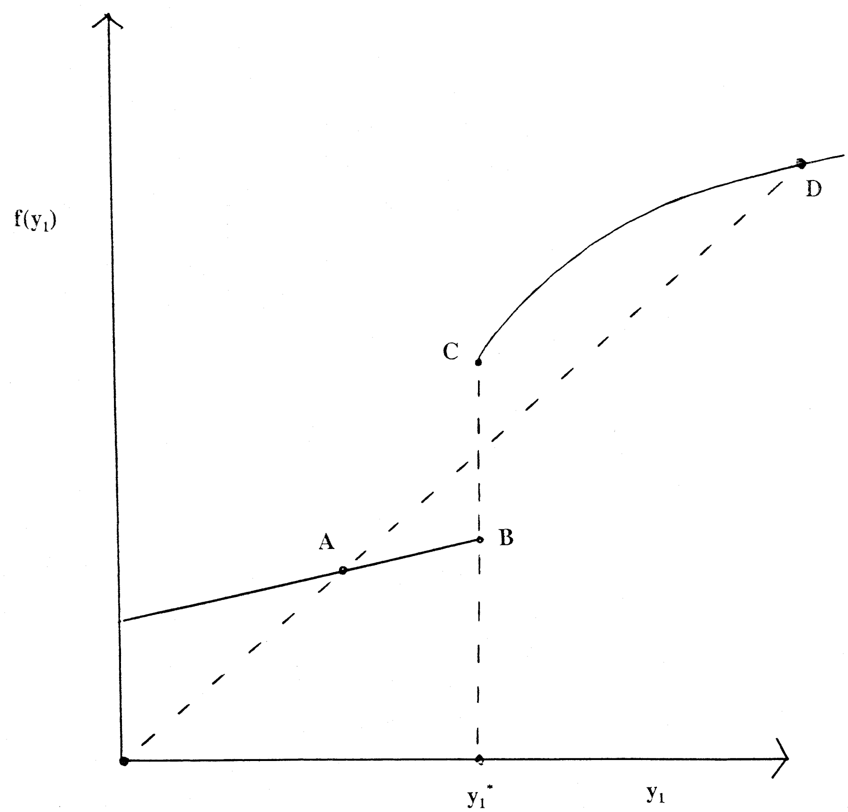

FIGURE 5

A Possible Reaction Function in the Absence of Credit Rationing.

if it starts above it (and if it starts below it, this means the Regime I reaction function must have crossed the 45% degree line somewhere).

3.2. The Reaction Function with a Binding Incentive Compatibility Constraint

We now consider the implications of imposing the Incentive Compatibility Constraint. Consider the situation where the monopoly finds it just worthwhile to produce, and this involves a positive level of investment, as in Figure 4. The condition for the firm to be indifferent between producing and not producing (in the absence of the ICC) is that

$$\Pi_1 + \Pi_2 (I^*)/(1 - r^*) - I^* = 0 \tag{25}$$

where $\Pi_2 (I^*)$ is the level of profits generated in the second period when investment in the first period is I^*, where I^* is chosen optimally. It is clear that the Incentive Comptability Constraint (6) is not satisfied here, so lending will not be forthcoming to finance this (or any) level of investment at this level of output. So what happens? If the monopoly produces and invests nothing, it will lose money, given that it would only just break even if it invests its optimal amount. It is clear that it will no longer be profitable for the monopoly to operate until output has risen until the following condition is satisfied:

$$\Pi_1 + \Pi_2 (0)/(1 + r^*) = 0. \tag{27}$$

This means that the monopoly is just breaking even intertemporally with zero investment. (It would, of course, like to invest more, but cannot, because of the Incentive Compatibility Constraint.) The ICC is just binding at this point, and further increases in current-period output relax the ICC and investment becomes positive (determined by equation (8)).

The reaction function in this case can be written:

$$f (y_1) = k\,\beta\,[y_1 + G (y_1)/(1 + r^*)] + I\;(y_1,\; G (y_1)) + g_1 \tag{28}$$

Its slope is calculated to be:

$$f' (y_1) = k\,\beta\,\{1 - (1 - k)\,\beta\} + I_1 (y_1, G (y_1)) + I_2 (y_1, G (y_1))\, G' (y_1) \tag{29}$$

where the subscripts on the investment function denote the relevant partial derivatives. This may be compared with the expression for the slope of the reaction function in the absence of the ICC (equation (24)). The difference concerns the terms involving investment. In the absence of credit rationing, an increase in current income raises investment only inasmuch as it raises expectations of future income. It may do this if the rise in current income is expected to mean higher future consumption spending and output. However, in the presence of credit rationing, there is a more direct mechanism; by raising current profits, the ICC is relaxed and this can lead to a powerful effect on investment. There is also an effect via expectations of future output. It seems then that a rise in current output can have a strong influence on aggregate demand under credit rationing. It is certainly possible to choose parameter values such that the slope of (29) is greater than unity.

It might be asked whether once the economy is in the credit rationing regime, it will stay there indefinitely. This seems unlikely. As current output rises, then both the constrained and the optimal levels of investment must rise. The optimal level of investment is above the constrained level in the credit rationing regime (by definition); provided that the constrained level of investment rises faster than the optimal level above a certain level of output, then the gap between the two levels of investment will decline and eventually go to zero and we will then switch to the unconstrained regime [8]. Under our assumption that the cost function $\alpha(I)$ becomes horizontal at a certain level of investment, then the unconstrained investment function will also become horizontal, whereas the constrained investment function always has a slope which is greater than $(1 - \alpha_0)$. (See the discussion following equation (13).) It follows that as current output rises, at some point Regime III ends and Regime II begins (this presupposes, of course, that capacity constraints do not bite until after this has happened).

The picture we thus have is as follows. Initially, for low levels of current output, we are in Regime I. When a certain level of current output is reached, the economy switches to the credit rationed regime. Investment becomes positive and rises in this regime as current output rises still further. However, the degree of credit rationing diminishes and eventually disappears, when the economy switches to Regime II. At the first switch (from Regime I to Regime III), the slope of the reaction function definitely increases; at the second switch, the slope of the reaction function decreases. The reaction function has its steepest slope in the credit rationing regime. With the ICC, there is no jump in the reaction function as there was in its absence.

The second case, where there is no jump in the reaction function in the unconstrained case, is less interesting. It seems probable that the ICC constraint will never bind, so that credit rationing does not occur at all for the economy with this type of reaction function. So it is not the case that credit rationing must always exists for some levels of current output.

4 Conclusion

The aim of this paper was to formulate a simple macroeconomic model with the possibility of endogenous credit rationing and to examine some of its properties. The model is in some respects extremely simple. All relative prices are held fixed and aggregate demand determines the level of output

8. Strictly, it is necessary that dI/dy_1 and dI^*/dy_1 always differ by more than any positive constant (this rules out the difference tending to zero).

of the one good produced in the economy. There are just two periods and a simple moral hazard mechanism is used to generate the possibility of credit rationing. In spite of the simplicity of the model, it is often difficult to derive unambiguous conclusions without imposing additional assumptions about parameter values, etc. Nevertheless, a number of conclusions do emerge. It is not the case that credit rationing is either necessary or sufficient for the existence of multiple equilibria. However, the model suggests that with credit rationing, investment may be highly responsive to changes in current output (which may well not be the case in the absence of credit rationing). So, it is likely the reaction function has a slope greater than unity under credit rationing hence satisfying a necessary condition for the existence of multiple equilibria. The model is consistent with the notion that, in practice, credit rationing may be important in generating multiple equilibria, but it is does not seem possible to make any stronger claim.

Also, the analysis suggests that the incentive compatibility constraint which may generate credit rationing may eliminate a discontinuity which may occur in its absence; that the slope of the reaction function may increase with current output over certain ranges and that credit rationing may be more likely to occur at lower levels of output.

It is not difficult to identify ways in which it might be desirable to extend the analysis. The model is extremely aggregated, with a large number of firms which all behave in exactly the same way. It might be interesting, but probably not too easy, to introduce differences between firms so that, for example, some are credit rationed whilst others are not. We believe that the way we have introduced the possibility of credit rationing into a macroeconomic model is quite novel and might be used in other types of models (e.g., market clearing models). Relaxing the assumption that partial default cannot take place (and hence endogenising the risky rate of interest) might be useful. Also, we have a model with credit rationing but without money. Repairing this deficiency might enable us to endogenise the safe rate of interest. But these are not extensions wa can pursue in this paper.

● References

ABEL, A., BLANCHARD, O., (1986). – “The Present Value of Profits and Cyclical Movements in Investment”, *Econometrica*, pp. 249-273.

AZARIADIS, C., SMITH, B., (1993). – “Adverse Selection in the Overlapping Generations Model: The Case of Pure Exchange”, *Journal of Economic Theory*, pp. 277-305.

BERGER, A., UDELL, G., (1992). – “Some Evidence on the Empirical Significance of Credit Rationing”, *Journal of Political Economy*, pp. 1047-1077.

BERNANKE, B., (1983). – “Nonmonetary Effects of the Financial Crisis in the Propagation of the Great Depression”, *American Economic Review*, pp. 257-276, and in Mankiw and Romer, (1991) vol. 2.

COOPER, R., JOHN, A., (1988). – “Coordinating Coordination Failures in Keynesian Models”, *Quarterly Journal of Economics*, pp. 441-463, and in Mankiw and Romer, (1991) vol. 2.

FAZZARI, S., HUBBARD, R., PETERSEN, B., (1988). – “Investment, Financing Decisions, and Tax Policy”, *American Economic Review* (papers and proceedings), pp. 200-205.

Hoshi, T., Kashyap, A., Scharfstein, D., (1991). – "Corporate Structure, Liquidity, and Investment: Evidence from Japanese Industrial Groups", *Quarterly Journal of Economics*, pp. 33-60.

Jaffee, D., Stiglitz, J., (1990). – "Credit Rationing", in B. Friedman and F. Hahn (eds.), *Handbook of Monetary Economics*, North Holland, vol. 2, pp. 837-888.

Japelli, T., (1990). – "Who is Credit Constrained in the U.S. Economy?", *Quarterly Journal of Economics*, pp. 219-234.

Kiyotaki, N., (1988). – "Multiple Expectational Equilibria under Monopolistic Competition", *Quarterly Journal of Economics*, pp. 695-714.

Kiyotaki, N., Moore, J., (1993). – "Credit Cycles", unpublished paper.

Mankiw, N. G., Romer, D., (1991). – *New Keynesian Economics*, MIT Press.

Murphy, K., Shleifer, A., Vishny, R., (1989). – "Industrialization and the Big Push", *Journal of Political Economy*, pp. 1003-1027.

Stiglitz, J., Weiss, A., (1981). – "Credit Rationing in Markets with Imperfect Information", *American Economic Review*, pp. 393-410, and in Mankiw and Romer, (1991) vol. 2.

Townsend, R., (1979). – "Optimal Contracts and Competitive Markets with Costly State Verification", *Journal of Economic Theory*, pp. 265-293.

Tsiddon, D., (1992). – "A Moral Hazard Trap to Growth", *International Economic Review*, pp. 299-321.

Zeira, J., (1991). – "Credit Rationing in an Open Economy", *International Economic Review*, pp. 959-972.

Zeldes, S., (1989). – "Consumption and Liquidity Constraints: An Empirical Investigation", *Journal of Political Economy*, pp. 305-46.