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Nominal Rigidities and Market Structure: some Macroeconomic Implications

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ABSTRACT. – This paper analyzes the macroeconomic implications of asymmetric price adjustment, under alternative assumptions regarding market structure and the nominal rigidity. It is shown that: a) aggregate output does not decreases (and sometimes increases) with the dispersion of shocks; b) with strategic price setting higher inflation rates are associated with higher output levels; c) sectorial shocks may create aggregate output fluctuations; d) the relationship between average inflation and aggregate output fluctuations is ambigous.

Rigidité nominale et structure du marché, quelques implications macroéconomiques

RÉSUMÉ. – Cet article analyse les implications macroéconomiques des prix d'ajustement asymmétrique, dans le cas des suppositions tenant compte du marché et de la rigidité nominale.

Il est montré que :

a) Les rérsultats globaux ne diminuent pas, voir parfois augmentent avec la dispersion des chocs.

b) Des prix augmentant les taux d'inflation engendrent des niveaux de résultats élevés.

c) Des chocs sectoriels peuvent engendrer des fluctuations générales.

d) La relation entre une inflation moyenne et les fluctuations globales qui en résultent sont ambigus.

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1 Introduction

Nominal rigidities are a crucial ingredient in some keynesian theories of aggregate fluctuations. Very often, these nominal rigidities are assumed to take an asymmetric form: nominal prices (or wages) are thought to be more rigid downwards than upwards.

This asymmetry raises at least three questions. First, is it true that in the real world nominal prices are more rigid downwards than upwards? Second, why is this so? (and thus what is the exact form this asymmetry takes). Third, what are their macroeconomic implications?

In this paper I focus on the interaction between the second and the third questions [1]. The aggregate implications may actually differ with the source of the relative downward price rigidity.

The macroeconomic effects of asymmetric price adjustment have been widely discussed. I focus here on a particular subset of beliefs, or conjectures. Conjecture number one: higher dispersion of aggregate shocks decreases average output. The reason is that if prices respond more to a positive shock than to a negative one, the expansion caused by a positive shock is likely to be smaller than the recession caused by a negative one of similar magnitude. If this is the case stabilizing aggregate shocks increases average output.

Conjecture number two: a positive rate of inflation may be optimal because it helps achieving relative price flexibility. In other words, if some real prices are too high due to a downward nominal rigidity, then inflation may succeed in reducing them, increasing average output.

Conjecture three: sectorial shocks may cause aggregate output fluctuations. The reason is that, when there is an economy-wide shock that affects positively some sectors and negatively some others (like a change in consumer preferences, a trade liberalization or an oil price change), asymmetric price adjustment implies that in average prices increase, and aggregate output falls.

Conjecture four: inflation may reduce aggregate output variability. The reason is that by increasing relative price flexibility inflation reduces the aggregate fluctuations caused by sectorial shocks.

Although these conjectures have been part of some keynesian thinking for a long time, it is not easy to associate particular names to them. Probably, the exception is Conjecture two, which seems to correspond to the message of Tobin's famous 1972 paper. In some sense Conjecture four can be though as a weaker version of Conjecture two.

1. Some recent empirical evidence, both at the micro level, about asymmetric price adjustment include Cover [1992], DeLong and Summers [1988] and Newmark and Sharpe [1992].

The goal of this paper is to examine these four conjectures under the light of some simple models, that differ only on the market structure and the specific form of the nominal rigidity at the individual firm level. The types of issues I raise as well as the modeling approach overlaps with a recent paper by Larry Ball and Greg Mankiw [1992]. The main difference is that they concern themselves only with the monopoly model with fixed price adjustment costs. In fact, one of the points of my paper is that worrying about strategic price interaction may payoff, i.e. the price dynamics arising from a monopoly or an oligopoly model can be substantially different. In this paper I also draw from Rotemberg and Saloner [1987], Caminal [1987] and, specially, Bernhardt [1993]. All these papers deal with the interaction between price adjustment costs and oligopolistic competition, but do not address the same issues.

Asymmetric price adjustment arises in a broad variety of situations. For instance, in the inventory literature (Reagan and Weitzman, 1982) prices will be raised to take advantage of a positive demand shock (along with a decrease in inventory holdings) but may not be lowered when the firm receives a negative demand shock. The reason is that, in the latter case, it is optimal for the firm to keep prices relatively high and store the excess supply.

A similar story follows in the context of search costs. (Stiglitz, 1987). If customers are better informed about the prices charged by the usual supplier than about the prices charged by other firms, then a particular firm may respond asymmetrically to shocks (firms face a kinked demand curve), since it anticipates that a price increase will encourage some customers to search, while a price decrease is not likely to attract many new customers.

One final example can be drawn from the contract literature (Holmstrom, 1983). If the two-period labor contract is only binding to the firm but not to the worker, the optimal contract between a risk-neutral firm and a risk-averse worker involves asymmetric price adjustment in the second period: if the worker's opportunity cost increases the firm raises her wage, but if it decreases the contract guarantees a minimum wage level.

All these stories are interesting and carry useful insights about macroeconomic adjustment. The problem is that all of them are stories about asymmetric real price adjustment, and they say nothing about the potential downwards rigidity of nominal prices. In fact, economists have been much more successful dealing with the determination of real rather than nominal prices. The reason is the difficulty of modeling the transactions role of money.

In order to explain how real (as well as nominal) shocks may affect the adjustment of nominal prices some economists have invented a shortcut: the existence of real costs of changing nominal prices. These costs are usually explained in terms of the real resources involved quoting a different price or, alternatively, as the result of boundedly rational behavior, or a combination of both. Although more work is needed on the foundations of these costs, and specially its relation with the monetary aspects of economic transactions, this is somehow a useful and plausible device.

With trend inflation and fixed costs of price adjustment monopolists adjust their prices upwards more easily than downwards, in the sense that the probability of an upwards adjustment is higher than in the opposite direction. See Tsiddon [1993] and Ball and Mankiw [1992]. The intuition is simple. If a firm receives a negative shock and does nothing the real price eventually falls to the desired level. If the shock is positive, the only choice is to adjust it upwards.

Representing the nominal rigidity at the level of the firm as a symmetric fixed price adjustment cost to study price dynamics in inflationary environments has brought about many useful insights. The interpretation of these fixed costs is much more controversial. If we interpret them literally, then these costs should be endogenized. For instance, a firm in the retail sector considering to change its price may simply change the labels of the goods in the shelves; but additionally may post a sign at the entrance of the store, mail new catalogs, or advertise the new price on national TV. Hence, the magnitude of these costs is usually an endogenous variable [2]. Instead, if we interpret them as a reduced form then we should worry about the robustness of results to changes in their specification.

Although more work is needed in order to improve our understanding of the barriers to price adjustment at the individual firm level, this goal is not pursued in this paper. However, in order not to restrict the analysis exclusively to the fixed price adjustment cost specification we also consider models where the downward rigidity of nominal prices is given exogenously.

Throughout the paper we consider situations where the individual firm's optimization problem has finite horizon, (two periods, for simplicity), although the economy goes on forever. This can be justified in terms of the product cycles. Suppose a variety is sold only for two periods. At the end of the second period the old variety becomes obsolete and a new one is introduced. In this context, the firm's pricing problem has a two-period horizon.

The plan of the paper is the following. In the next section, I examine the optimal pricing policy of a monopolist that faces an exogenous downward rigidity constraint during the product cycle. In Section 3 the analysis is extended to incorporate strategic price interaction. Section 4 deals with a monopoly pricing problem subject to fixed costs of price adjustment, similar to the one considered in Ball and Mankiw [1992]. Section 5 combines strategic price interaction with fixed costs of price adjustment in an analysis related to Rotemberg and Saloner [1986], Caminal [1987], and Benhardt [1993]. Some conclusions follow.

2. In a separate paper (CAMINAL, 1993) I construct a static model where a monopolist choose the intensity of prices advertising. In that particular context I show that in equilibrium price adjustment is asymmetric.

2 Monopoly Pricing with Exogenous Downward Price Rigidity

In this section I examine the optimal pricing policy of a monopolist in an stochastic environment with an exogenous downward price rigidity constraint. We assume that every two periods a new variety is introduced. In the first period of this cycle the firm can set the price freely. However, in the second period the firm can only raise its nominal price but not lower it. Here we do not justify such a constraint, we take it as given and examine its implications in a rational expectations framework.

The monopolists' profits in period t are given by:

$$\pi_t = (a_t - bp_t)\, p_t$$

where p_t is the log of the firm's real price, b is a positive parameter [3], and a_t is a random variable that, for simplicity, takes value $\hat{a}$ in the first period of the product cycle, but in the second period can take values in the interval $[\underline{a}, \overline{a}]$ according to the probability density function, $h\,(a)$, which is symmetric around its mean, $\hat{a}$.

In the second period of the product cycle, the monopolist can choose to raise its nominal price after learning the realization of the random variable. If the price is adjusted it will be set at the short-run profit maximizing level, since in the following period the firm will be allowed to set a new price free of restrictions. If the nominal price is not adjusted the resulting real price is $(p_1 - g)$, where p_1 is the real price prevailing in the first period and g is the inflation rate (also in logs), $g \geq 0$. Because of the downward nominal price rigidity $p_1 - g$ is the lower bound of second period real prices. Thus, the constrained optimal second period real price is:

$$\begin{array}{ll} \dfrac{a}{2\,b} & \text{if} \quad a \geq 2\,b\,(p_1 - g) \\ p_1 - g & \text{otherwise.} \end{array}$$

In the first period of the product cycle history does not matter and the firm sets the price that maximizes the present discounted value of profits (again, since two periods ahead the price will be set freely again, the optimization problem has a two-period horizon). Furthermore, we assume there is no

3. Postulating a quadratic profit function in the log of the real price greatly simplifies the computations. Moreover, this is the only case where deterministic inflation has no effect on the average real price, and hence we can focus on the real effects of inflation due exclusively to the asymmetric price adjustment behavior. This formulation is very standard in this literature and is equivalent to taking a second order approximation to a general profit function. See BALL and ROMER [1989] and, particularly, the Appendix of its working paper version.

discounting. Thus, in the first period of the product cycle the optimization problem can be stated as follows:

$$\text{Max}\left(\hat{a}-bp_1\right)p_1+\int_{\underline{a}}^{x}\left(a-b\left(p_1-g\right)\right)\left(p_1-g\right)dH\left(a\right)$$
$$+\int_{x}^{\overline{a}}\frac{a^2}{4\,b}\,dH\left(a\right)$$

where $x\;=\;2\;b\;(p_1\;-\;g)$.

The optimal price p_1 must satisfy the first order condition:

$$\hat{a}-2\,b\,p_1+\int_{\underline{a}}^{x}\left(a-2\,b\right)\left(p_1-g\right)dH\left(a\right)=0.$$

A closed form solution can only be computed if h(a) takes some specific functional form. In any case, rewriting the first order condition we can learn something about the properties of the solution:

$$p_1=\frac{\hat{a}+\displaystyle\int_{\underline{a}}^{x}a\,dH\left(a\right)+2\,bg\,H\left(x\right)}{2\,b\left(1+H\left(x\right)\right)}$$

Thus, the optimal price is the weighted average between the price that maximizes first period profits and the one that maximizes second period profits (given that with some probability the nominal price will not be adjusted). It can be checked that the solution satisfies $x<\hat{a}$. Thus. with low inflation, the downward price rigidity in the second period implies that the optimal price in the first period is below the level that maximizes first period profits.

We are now ready to compute the average real price, $\hat{p}$.

$$\hat{p}=\frac{1}{2}\left\{p_1+H\left(x\right)\left(p_1-g\right)+\int_{x}^{\overline{a}}\frac{a}{2\,b}\,dH\left(a\right)\right\}.$$

Using the first order condition of the first period optimization problem and the definition of x, we get:

$$\hat{p}=\frac{\hat{a}}{2\,b}.$$

The average real price (and thus the average level of output) is independent of both the distribution of shocks and the inflation rate. In the second period the downward rigidity implies higher average prices, but the anticipation of future asymmetric adjustment leads to lower first period prices. Similarly, inflation reduces the downward rigidity in the second period and thus lowers second period average real prices, but increases the first period price.

This extreme neutrality result is not robust to the non-discounting assumption or to the symmetry of the profit function. I have chosen these assumptions precisely to make more transparent the effects due to market structure and the specific form of the nominal rigidities.

Let us now turn to some other implications about aggregate fluctuations. Suppose the economy is populated by a large number of these monopolists, all of them *ex ante* identical, except that their product cycles overlap; *i.e.*

half of them can change their prices freely at odd periods, but the other half at even periods. If shocks are independent across firms the distribution of firms' real prices is constant over time and aggregate output is constant. However, if the dispersion of shocks varies over time then, aggregate output fluctuates.

To see this more clearly let us slightly change the structure of shocks. For each firm in the second period of their product cycle with probability μ $a_2 = \hat{a}$, and with probability $(1-\mu)$ a_2 is drawn from a probability density function $h(a)$ with the properties stated above. Suppose that whether a shock occurs or not is perfectly correlated across firms, *i.e.* there are two aggregate states, s_1 and s_2, with associated probabilities μ and $(1-\mu)$. In state s_1 all firms in their second period stick to $\hat{a}$, and in state s_2 each firm gets an independent draw from the probability density function $h\,(a)$ [4].

Since $x < \hat{a}$, in state s_1 the average real price of firms in the second period of their product cycle is:

$$\frac{\hat{a}}{2\,b}.$$

In state s_2 the average real price of those firms in their second period is:

$$\int_{\underline{a}}^{x} (p_1 - g)\, dH\,(a) + \int_{x}^{\overline{a}} \frac{a}{2\,b}\, dH\,(a).$$

Clearly, average second period prices are lower in state s_1 than in state s_2 (See Figure 1). In other words, aggregate output will be relatively lower in periods of higher dispersion of sectorial shocks.

What is the effect of inflation on these output fluctuations? Notice that inflation does not affect the real price of firms in their second period in state s_1, but reduces the real price of firms binded by the downward rigidity constraint, in state s_2, as $(p_1 - g)$ decreases with g. Hence, aggregate output fluctuations caused by sectorial shocks are reduced with inflation.

Summarizing:

Result 1:

In the monopoly model with exogenous downward nominal price rigidity:

1.1) The average output level is independent of the distribution of shocks

1.2) The average output level is independent of the inflation rate

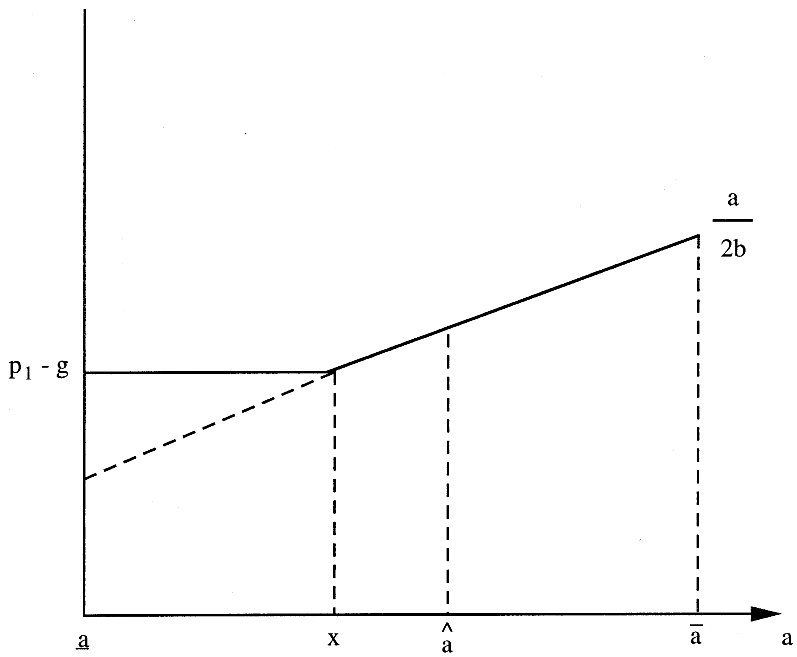
1.3) Sectorial shocks can create aggregate output fluctuations

1.4) Aggregate output fluctuations due to sectorial shocks are reduced with inflation.

Notice that in this simple model Conjectures 1 and 2 do not hold, but 3 and 4 do. What we learn is that firms take into account the posterior asymmetric price adjustment behavior in setting their initial prices. In the second period the downward price rigidity raises average real prices, but this is compensated by lower first period prices. Although average real

4. All the results stated above still hold but exact formulas change.

FIGURE 1



prices and output may be insensitive to the nominal rigidity, the dynamics of prices change in ways that are macroeconomically relevant.

3 Oligopoly Pricing with Exogenous Downward Price Rigidities

In this section we introduce strategic firm interaction. In each industry there are two firms, A and B, that sell imperfect substitutes. The profit function of firm i is given by:

$$\pi_t^i = (a_t - b\,p_t^i + c\,p_t^j)\,p_t^i \qquad i \neq j,\ i,\ j = A,\ B.$$

The parameters b and c are both positive with $b > c$. The structure of shocks is exactly the same as in the previous section, *i.e.* $a_1 = \hat{a}$, and a_2 is distributed according to $h\,(a)$ on $[\underline{a},\ \overline{a}\,]$. Also they face the same taboo: a nominal price can no be decreased in the second period of the product cycle. Notice that shocks are industry-wide and not firm-specific, and that the timing of product cycles is synchronized across firms [5].

We limit the possibilities of collusion by considering only Markov strategies, *i.e.* the price of a firm can only depend on current payoff-relevant variables. In our case this means that at the beginning of the product cycle prices do not depend on history (since it is irrelevant), and that players face a two-period horizon. One of the reasons to focus on these type of strategies is that if we allow too much collusion then the oligopoly will resemble the monopoly of the previous section. By making oligopolists as competitive as possible the collusive role of nominal rigidities appears much more transparent.

Thus, this is equivalent to solve for subgame perfect equilibria of the following two-period game. In the first period, firms simultaneously set first period prices p_1^A, p_1^B, in order to maximize the sum of first and second period profits. In the second period, after learning the realization of a_2, firms simultaneously set second period prices p_2^A, p_2^B, in order to maximize second period profits, subject to the constraint that nominal prices can not be decreased; *i.e.* $p_2^i \geq p_1^i - g$.

Without loss of generality suppose that $p_1^i \geq p_1^j$. In the second period, prices can fall in one of three regions (See Figure 2.1).

(I) If

$$\frac{a}{(2\,b-c)} \geq p_1^i - g$$

then the unique equilibrium consists of both firms raising their nominal prices to set $\dfrac{a}{2\,b-c}$ and make profits $\dfrac{b\,a^2}{(2\,b-c)^2}$.

(II) If

$$\frac{a}{(2\,b-c)} \leq p_1^i - g \qquad \text{and} \qquad p_1^j \leq \frac{(a + c\,(p_1^i - g))}{2\,b}$$

then the unique equilibrium consists of firm j raising its nominal price to set $\dfrac{a + c\,(p_1^i - g)}{2\,b}$.

(III) Finally, if

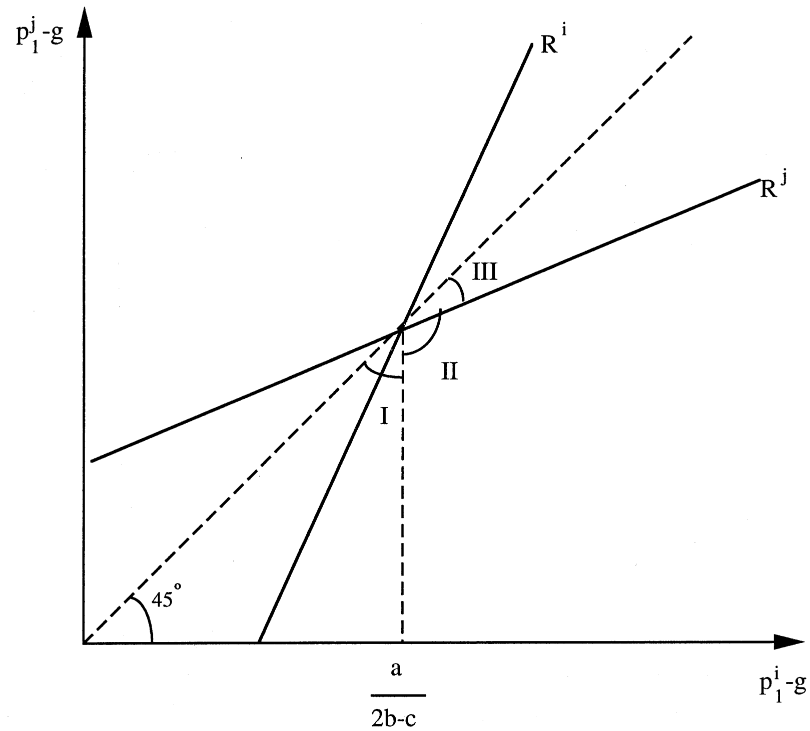
$$\frac{a}{(2\,b-c)} \leq p_1^i - g \qquad \text{and} \qquad p_1^j \geq \frac{(a + c\,(p_1^i - g))}{2\,b}$$

then in the unique equilibrium both prices are rigid.

In the first region initial prices are irrelevant. In the third region, any change in the initial price of an individual firm does not induce any reaction in its rival, as prices are rigid. However, in the second region the adjustment of firm j depends on firm i's initial price. In this region, firm i is acting as the Stackelberg leader.

5. With asynchronized product cycles the possibilities for collusion increase, reinforcing the results.

FIGURE 2.1



If we define the realizations of a_2 that separate these regions as follows:

$$y = (2\,b - c)\,(p_1^i - g)$$
$$x = 2\,b\,(p_1^j - g) - c\,(p_1^i - g)$$

we can write down the optimization problem of firm i in period 1:

$$\text{Max}\left\{\hat{a} - b\,p_1^i + c\,p_1^j)\,p_1^i + \int_{\underline{a}}^{x} \frac{ba^2}{(2\,b - c)^2}\,dH\,(a)\right.$$
$$+ \int_x^y \left(a - b\,(p_1^i - g) + c\left(a + \frac{c\,(p_1^i - g)}{2\,b}\right)\right)(p_1^i - g)\,dH\,(a)$$
$$\left.+ \int_y^{\overline{a}} b\,\frac{a^2}{(2\,b - c)^2}\,a^2\,dH\,(a)\right\}.$$

The first order conditions is:

$$\hat{a} - 2\,bp_1^i + cp_1^j + \int_{\underline{a}}^{x} [a - 2\,b\,(p_1^i - g) + c\,(p_1^j - g)]\,dH\,(a)$$
$$+ \int_x^y \left[a - 2\,b\,(p_1^i - g) + \frac{ca}{2\,b} + \frac{c^2}{b}\,(p_1^i - g)\right] dH\,(a) = 0.$$

If we compare this expression with the first order condition of the monopoly problem of the previous section, we notice that in the oligopoly case there is an additional term. If a firm increases its price above the level set by the other firm gets some extra profits coming from the fact that, with some probability, firm i acts as a Stackelberg leader; that is, because of the downward price rigidity firm i can not change the price set in the previous period but induces an upward adjustment in its rival.

Notice that a small deviation from a symmetric price vector is never profitable, since the probability that this deviation induces its rival to raise its price is zero. Thus, if a firm chooses to deviate it raises its price by a discrete amount.

The distribution of shocks matters a lot to determine the magnitude of the Stackelberg leader effect; if shocks are very concentrated a relatively small deviation will imply a relatively high probability of being a Stackelberg leader in the next period, which makes the deviation more attractive. Instead, if shocks are very dispersed a relatively large deviation is needed in order to achieve a significant probability of being a Stackelberg leader.

In order to clarify these issues I work out the uniform distribution case. I assume that $h(a) = \lambda$; thus, the higher the value of λ the more concentrated is the distribution of shocks around its mean.

LEMMA 1 : In the uniform distribution case, there exists a λ^* such that:

(i) if $\lambda < \lambda^*$ then the first period price of the unique symmetric equilibrium of the game is implicitly given by:

$$\hat{a} - (2\,b - c)\,p + \int_{\underline{a}}^{x} \left[a - (2\,b - c)\,(p - g)\right] dH\,(a) = 0$$

where $x = (2\,b - c)\,(p - g)$

(ii) If $\lambda > \lambda^*$ then the equilibrium is asymmetric, with the first period prices given by:

$$\hat{a} - 2\,bp_1^i + cp_1^j + \int_{\underline{a}}^{x} \left[a - 2\,b\,(p_1^i - g) + c\,(p_1^j - g)\right] dH\,(a)$$
$$+ \int_{x}^{y} \left[a - 2\,b\,(p_1^i - g) + \frac{ca}{2\,b} + \frac{c^2}{b}\,(p_1^i - g)\right] dH\,(a) = 0$$

and

$$\hat{a} - 2\,bp_1^j + cp_1^i + \int_{\underline{a}}^{x} \left[a - 2\,b\,(p_1^j - g) + c\,(p_1^i - g)\right] dH\,(a) = 0$$

(iii) λ^* decreases with g and decreases with $\hat{a}$ and c. If $\lambda > \lambda^*$ then first period prices increase with λ.

The Appendix contains the proofs of all the lemmas stated in the text.

We can now compute the average price, $\hat{p}$,

$$\hat{p} = \frac{1}{2}\left\{p_1 + H\,(x)\,(p_1 - g) + \int_{x}^{\overline{a}} \frac{a}{(2\,b - c)}\,dH(a)\right\}.$$

If $\lambda < \lambda^*$ then (as in the monopoly case) the average price is independent of both the distribution of shocks and the inflation rate. The reason is that in this case strategic firm interaction does not introduce any particular additional effect.

The more interesting case is $\lambda > \lambda^*$.

LEMMA 2 : If $\lambda > \lambda^*$ the average real price increases with λ and decreases with inflation.

The reason is that higher concentration of shocks makes more profitable for the Stackelberg leader to increase the initial price to induce its rival to raise its price next period. Inflation reduces this incentive, since real second period prices are reduced and so is the adjustment induced in its rival.

With a non-uniform probability density function more effects are captured, usually of the same sign. For instance inflation may reduce the probability that a price increase today induces its rival's adjustment tomorrow.

For future reference let us notice that in equilibrium x can be higher or lower than $\hat{a}$, depending on the parameter values.

Let us now turn to sectorial shocks. As in the monopoly case sectorial shocks may create aggregate output fluctuations. Suppose that in each sector there is a probability μ that $a_2 = \hat{a}$, and with probability $(1-\mu)$ a_2 is uniformly distributed with $h(a) = \lambda$. [6]

Like in the previous section suppose there are two aggregate states. In state s_1 all sectors have $a_2 = \hat{a}$, while in state s_2 each sector gets an independent draw from the uniform distribution parametrized by λ. Finally, the probability of state s_1 is μ.

The average second period price in state s_1 is the maximum of $(p_1 - g)$ and $\dfrac{\hat{a}}{(2b-c)}$. Clearly, the average price is state s_2 is higher. See Figure 2.2.

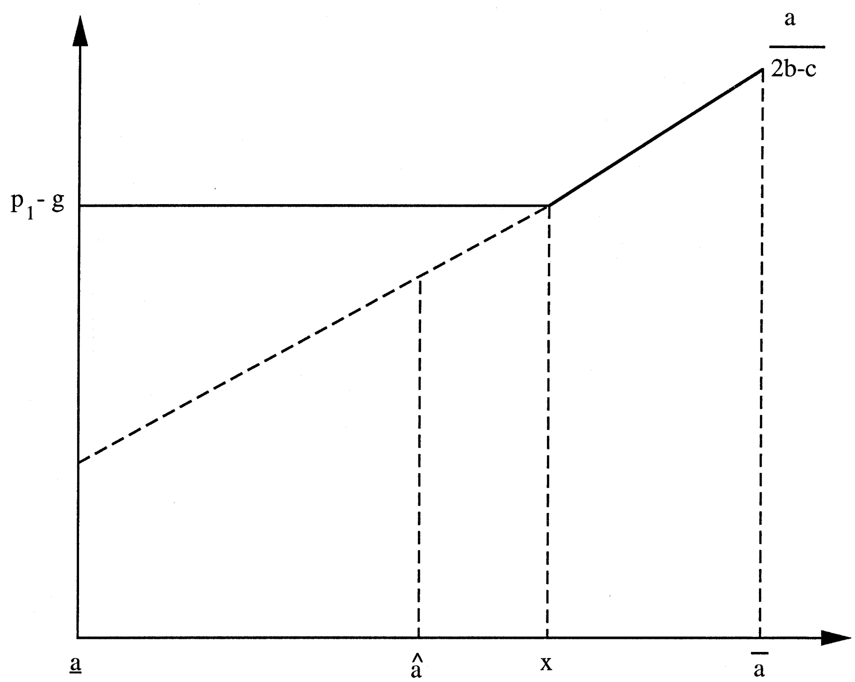
Suppose $p_1 - g > \dfrac{\hat{a}}{(2b-c)}$, then higher inflation reduces the average real price relatively more in state s_1. The reason is that in state s_1 inflation affects all the firms, while in state s_2 only affects the real price of those firms constrained by the downward nominal price rigidity. Thus, output fluctuations will be exacerbated by inflation.

However, if $p_1 - g < \dfrac{\hat{a}}{(2b-c)}$, in state s_1 prices are adjusted upwards and inflation is irrelevant. In this case inflation reduces output fluctuations. The first case is more likely as inflation is low, shocks are more concentrated and products are closer substitutes.

All these results can be summarized as follows:

6. This assumption introduces a kink in firm i's profit function and therefore arises the possibility of multiple equilibria, which is not our concern here. The comparative static results below implicitly assume that there are no jumps between equilibria caused by the marginal change in the inflation rate.

FIGURE 2.2



Result 2:

With strategic price interaction and exogenous downward nominal price rigidity:

2.1. Higher variability of shocks tends to increase average output.

2.2. Average output tends to increase with inflation

2.3. Sectorial shocks may cause aggregate output fluctuations

2.4. Aggregate output fluctuations caused by sectorial shocks may be reduced or exacerbated by inflation.

4 Monopoly Pricing with Fixed Price Adjustment Costs.

In this section I examine again a monopoly pricing problem, but now the second period nominal rigidity takes the form of a symmetric fixed cost of price adjustment [7]. Let us consider again the model of Section 1, the same profit function, product cycle and structure of shocks. Now in the second period any nominal price adjustment in any direction involves a real cost k. As we will see below, with positive inflation the second period real price is below the short-run profit maximizing level for $\hat{a}$. Thus, there is also asymmetric price adjustment and firms are more likely to raise than to lower their prices.

In the second period the monopolist adjusts its price if and only if:

$$a_2 \leq 2\,b\,(p_1 - g) - d_1$$
$$a_2 \geq 2\,b\,(p_1 - g) + d_1$$

where $d_1 = 2\,b^{.5}\,k^{.5}$.

If we define

$$x = 2\,b\,(p_1 - g) - d_1$$
$$y = 2\,b\,(p_1 - g) + d_1$$

the first period optimization problem can be written as follows:

$$\operatorname*{Max}_{p_1} \left\{ \begin{array}{l} (\hat{a} - bp_1)\,p_1 + \displaystyle\int_{\underline{a}}^{x} \left(\frac{a^2}{4\,b} - K\right) dH\,(a) \\ \quad + \displaystyle\int_{x}^{y} [a - b\,(p_1 - g)]\,(p_1 - g)\,dH\,(a) \\ \quad + \displaystyle\int_{x}^{y} \left(\frac{a^2}{4\,b} - K\right) dH\,(a) \end{array} \right\}$$

The solution is characterized by the first order condition:

$$\hat{a} - 2\,bp_1 + \int_{x}^{y} [a - 2\,b\,(p_1 - g)]\,dH\,(a) = 0.$$

The optimal first period price is again the weighted average of the price that maximizes first period profits and the one that maximizes second period expected profits, given that for some range of shocks nominal prices remain unchanged.

As in Section 1, we can compute the average real price from the first order condition and the definitions of x and y and check that it is independent of the distribution of shocks or the inflation rate.

It is easy to check that in the second period there is asymmetric price adjustment as long as $g > 0$, *i.e.* $y - \hat{a} < \hat{a} - x$. The reason is that with positive inflation the second period real price is below the one that maximizes second period expected profits. Thus, it is more likely that the firm adjusts its price upwards than downwards. An immediate implication is that, as long as inflation is positive, sectorial shocks can cause aggregate output fluctuations.

As in the previous sections, suppose that there are two aggregate states s_1 and s_2, with probabilities μ and $(1 - \mu)$. In state s_1, all firms in their second period get $a_2 = \hat{a}$. In state s_1 each firm gets an independent draw from a probability distribution $h\,(a)$.

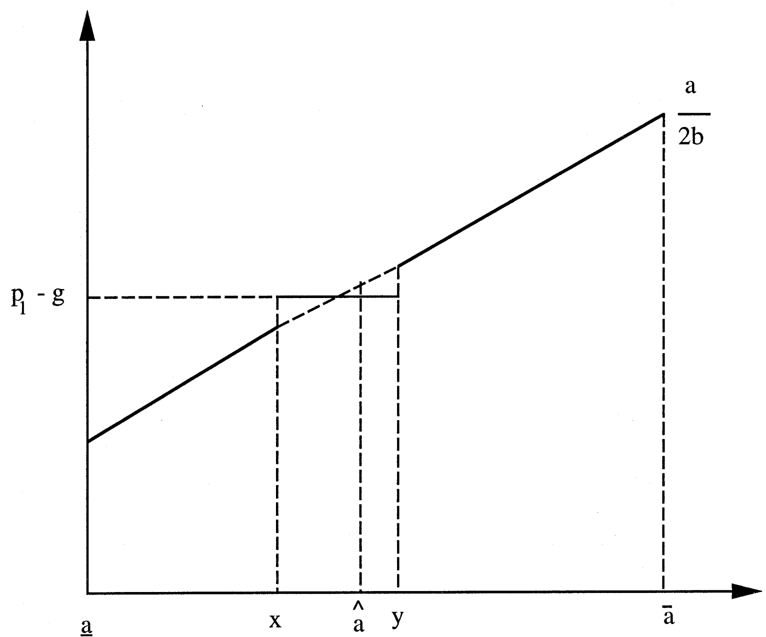
7. The model is very similar to the one in Ball and Mankiw (1992).

With positive inflation $2\,b\,(p_1 - g) < \hat{a}$. If inflation is moderate then $y \geq \hat{a}$ (in state s_1 no firm changes its price in the second period). In this case, the average real price set by firms in their second period in state s_1 is $(p_1 - g)$, and in state s_2 is:

$$\int_{\underline{a}}^{x} \frac{a}{2\,b}\, dH\,(a) + \int_{x}^{y} (p_1 - g)\, dH\,(a) + \int_{x}^{\overline{a}} \frac{a}{2\,b}\, dH\,(a)$$

which is higher than $(p_1 - g)$. See Figure 3. Hence, there will be aggregate output fluctuations. [8]

FIGURE 3



What is the effect of inflation on these output fluctuations? Clearly, inflation reduces the average second period price relatively more in state s_1 than in state s_2, exacerbating aggregate output fluctuations.

Result 3:

In the monopoly model with fixed costs of price adjustment:

3.1. The average level of output is independent of the distribution of shocks.

8. If inflation is large we have $y \leq \hat{a}$. In this case aggregate output is constant over time if and only if shocks are uniformly distributed . In general the result is ambiguous: output may be higher or lower in states of low dispersion of sectorial shocks.

3.2. The average level of output is independent of the inflation rate.

3.3. Sectorial shocks may cause aggregate output fluctuations as long as inflation is positive.

3.4. Inflation exacerbates aggregate fluctuations caused by sectorial shocks.

5 Oligopoly Pricing with Fixed Costs of Price Adjustment

In this section I combine the assumptions about the number of firms, the structure of demand and the realization of the random variables of Section 3 with the assumptions about the nominal rigidity of Section 4; namely, a symmetric fixed price adjustment cost. More specifically, in each sector there are two firms, A and B, that sell imperfect substitutes. The profit function of firm i is given by:

$$\pi_t^i = (a_t - b\,p_t^i + c\,p_t^j)\,p_t^i \qquad i \neq j, \quad i,\, j = A,\, B.$$

The parameters, b and c are positive, with $b > c$. The structure of shocks is the usual one, *i.e.* $a_1 = \hat{a}$, and a_2 is distributed according to $h\,(a)$ on $[\underline{a},\, \overline{a}]$. In period 1 both firms simultaneously quote the first period price. In the second period, nominal prices can be changed after learning the realization of the shock, provided firms pay a fixed real cost, k [9].

As in Section 2 we assume that in the second period of the product cycle firms only look at short-run profits, since next period prices can be adjusted freely, and thus there is no state variable. Similarly, in the first period of the product cycle firms have a two-period horizon because they realize that current prices affect next period profits.

As it is well known (Rotemberg and Saloner, 1986; Caminal, 1987; and Bernhardt, 1993), due to the combination of strategic complementarities and fixed price adjustment costs, the incentives for an individual firm to change its price depends crucially on whether it conjectures that its rival will do so. In the second period if the initial price is the same for both firms and they move simultaneously then there are multiple symmetric equilibria than can be Pareto ranked from the point of view of both firms profits (Caminal, 1987; and Bernhardt, 1993). More formally,

9. The timing of the second period subgame is discussed below.

LEMMA 3 : If $p_1^i = p_1^j = p_1$, then

a) Both firms adjusting its price to $\frac{a_2}{(2\,b-c)}$ is an equilibrium for the subgame if:

$$a_2 \geq (2\,b-c)\,(p_1-g)+d_2$$
$$\leq (2\,b-c)\,(p_1-g)-d_2$$

where $d_2 \equiv (2\,b-c)\,K^{1/2}\,b^{-1/2}$

b) Neither firm changing its price is an equilibrium for the subgame if:

$$a_2 \leq (2\,b-c)\,(p_1-g)+d_1$$
$$> (2\,b-c)\,(p_1-g)-d_1$$
$$d_1 \equiv 2\,b^{1/2}\,K^{1/2}$$

c) There is no equilibrium for the subgame where only one firm adjust its price.

d) If $a_2 \in [(2b-c)\,(p_1-g)-d_1, (2\,b-c)\,(p_1-g)-d_2]$ the equilibrium with unchanged prices leads to higher profits for both firms than the one with adjusted prices.

If $a_2 \in [(2\,b-c)\,(p_1-g)+d_2, (2\,b-c)\,(p_1-g)+d_1]$ the equilibrium with adjusted prices leads to higher profits for both firms than the one with unchanged prices [10].

If both firms adjust their prices, they quote the unique equilibrium price, $\frac{a}{2\,b-c}$ and make profits $\frac{ba^2}{(2\,b-c)^2}$.

If firms could communicate before deciding their second period price presumably they would coordinate on the Pareto superior equilibrium (in the sense of higher payoffs for both active players) [11]. If this is the case price adjustment in the second period is asymmetric with respect to the initial price.

In fact, multiple equilibria arise because of the strict simultaneity of moves. If one firm could anticipate the other and precommit to either leave the nominal price unchanged or to adjust it, without revealing the price set in the latter case, it will do so. Unlike the usual Stackelberg game, firms would be indifferent between being the leader or the follower since by staggering their moves they only choose between two symmetric, Pareto-ranked equilibria [12].Thus, an alternative timing for the second period subgame would be the following. After learning the realization of the shock firm i (selected randomly) sets its price. Next firm j chooses its price, but at this

10. These intervals are non-empty as $d_2 < d_1$.
11. Such a behavior in the second period does not select the Pareto optimal equilibrium of the two-period game, since the Pareto-dominated equilibrium behavior could be used as a credible threat to induce higher first period prices. See Bernhardt [1993]. However, assuming commitment not to renegotiate may be strong in this context.
12. In a dynamic model where a large firm competes with a competitive fringe such a multiplicity of equilibria desappairs as the large firm plays the role of the leader in both pricing and adjustment (Caminal, 1994). Also, eliminating the multiplicity of equilibria in the second period avoids the renegotiation issue discussed in the previous footnote.

point only knows whether firm i's nominal price is the same or is different from its first period price. Finally, both real prices become common knowledge and demand gets allocated. Clearly, in this subgame there is a unique equilibrium for each realization of the random variable, which corresponds to the Pareto superior equilibrium of the simultaneous moves game.

In any case we assume that the second period is characterized by the asymmetric behavior with respect to the initial second period price described above [13]. Let us turn to the determination of first period prices. For that we would actually need to compute the set of equilibria for the second period for an arbitrary initial price vector. In the Appendix, it is shown that if a large deviation increases expected profits in the second period so does a small one. Thus, to compute first period prices we need only to worry about small deviations from symmetric price vectors.

Provided initial prices are the same for both firms, the limits of the interval of shock realizations such that prices remain unchanged is given by:

$$x = (2\,b - c)\,p_1 - d_1$$
$$y = (2\,b - c)\,p_1 + d_2$$

Now we need to examine how x and y change with a unilateral deviation from the symmetric price vector. Without loss of generality suppose $p_1^i \geq p_1^j$. x will be determined by the incentives of firm i to lower its price, *i.e.*

$$\begin{aligned}&[x - b\,(p_1^i - g) + c(p_1^j - g)]\,(p_1^i - g) + K\\ &= \operatorname*{Max}_{p_2^i}\,\{[x - bp_2^i + c\,(p_1^j - g)]\,p_2^i\}\end{aligned}$$

and

$$\frac{dx}{dp_1^i} = -c \quad at \;\; p_1^i = p_1^j, \qquad p_1^i \leq p_1^j \;\; \text{(left derivative)}$$
$$\frac{dx}{dp_1^i} = 2\,b \qquad p_1^i = p_1^j, \qquad p_1^i \leq p_1^j \;\; \text{(right derivative).}$$

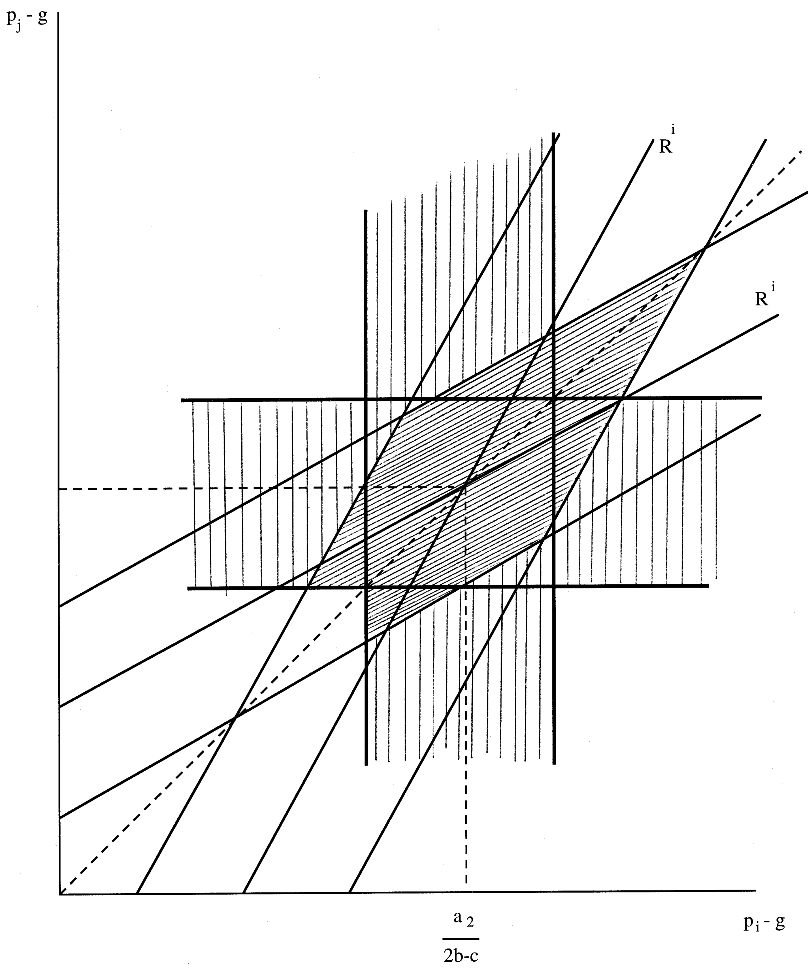
Similarly, y will be determined by the incentives of firm i to change its price:

$$\begin{aligned}&\left[y - b\,(p_1^i - g) + c\,\frac{y}{2\,b - c}\right](p^i - g) + K\\ &= \operatorname*{Max}_{p_2^i}\left\{\left[y - bp_2^i + c\,\frac{y}{2\,b - c}\right]p_2^i\right\}\\ &\frac{dy}{dp_1^i} = 0 \quad \text{at} \quad p_1^i = p_1^j, \qquad p_1^i \leq p_1^j \quad \text{(left derivative)}\\ &\frac{dy}{dp_1^i} = 2\,b - c \quad \text{at} \quad p_1^i = p_1^j, \qquad p_1^i \leq p_1^j \quad \text{(right derivative).}\end{aligned}$$

---

13. Figure 4 (see Appendix). describes the selected equilibria as a function of the second period real price vector. The darkest area represents the set of price vectors that imply no adjustment in the second period, and the area marked with vertical strips the set that imply only one firm adjusting its nominal price.

FIGURE 4



A corollary of Lemma 3 is that at x firms make higher profits by keeping prices unchanged, but at y firms make higher profits adjusting their prices. From the point of view of the first period, firms' profits increase as x and y fall.

The above expressions show that the profit function is not differentiable at the symmetric price vector and there are strong incentives to quote the same price as the rival firm.

The reasons for the kink are quite intuitive. If firm i unilaterally decreases its first period price it raises the incentives of its rival to do so in the second period when firm i's price remains unchanged. Thus, for that realization of the random variable that made firm j indifferent between lowering the price or keeping it unchanged, given that firm i's price is not adjusted, firm j strictly prefers to adjust it. On the other hand, the realization of the random variable that made firm j indifferent between increasing the price or keeping it unchanged, given that firm i's price is adjusted, is unaffected by the deviation in the first period price.

If firm i unilaterally increases its first period price the intuition is similar. It makes more likely to lower prices for relatively bad shocks, and makes adjustment upwards less likely for relatively good shocks. Both effects tend to decrease profits.

Firm i's first period optimization problem is:

$$\underset{p_1^i}{\operatorname{Max}} \left\{ \begin{array}{l} \left(\hat{a} - bp_1^i + cp_1^j\right) p_1^i + \displaystyle\int_{\underline{a}}^{x} \left[\frac{ba^2}{(2\,b-c)^2} - K\right] dH\,(a) \\ \quad + \displaystyle\int_{x}^{y} \left[a - b\,(p_1^i - g) + c\,(p_1^j - g)\right] (p_1^i - g)\, dH\,(a) \\ \quad + \displaystyle\int_{y}^{\overline{a}} \left[\frac{ba^2}{(2\,b-c)^2} - K\right] dH\,(a) \end{array} \right\}.$$

The first order condition for a downward deviation from a symmetric price vector is:

$$\begin{aligned} &\hat{a} - (2\,b - c)\,p + \int_{x}^{y} \left[a - (2\,b-c)\,(p-g)\right] dH\,(a) \\ &\quad + h\,(x)\,c\left[\left[x - (b-c)\,(p-g)\right](p-g) + K\,\frac{bx^2}{(2\,b-c)^2}\right] \geq 0. \end{aligned}$$

The first two terms are the ones analogous to the monopoly case. They are the sum of the marginal profits in the first period and in the states of the second period where nominal prices are unchanged. The third term is the interesting one, since it reflects the strategic interaction effect. The expression between squared brackets is the difference in profits between adjusting or non adjusting, evaluated at x. In other words, the third term captures the gains obtained from not lowering the price and expand the range of shocks for which prices are not adjusted downwards, and kept closer to the joint profit maximizing level.

Similarly, the first order condition for an upward deviation from a symmetric price vector is:

$$\begin{aligned} &\hat{a} - (2\,b - c)\,p + \int_{x}^{y} \left[a - (2\,b-c)\,(p-g)\right] dH\,(a) \\ &\quad - h\,(x)\,2\,b\left[\left[x - (b-c)\,(p-g)\right](p-g) + K - \frac{bx^2}{(2\,b-c)^2}\right] \\ &\quad - h\,(y)\,(2\,b-c)\left[yp - (b-c)\,p^2 + k - \frac{by^2}{(2\,b-c)^2}\right] \leq 0. \end{aligned}$$

In this case, the third term captures the losses incurred by unilaterally raising the first period price.

Since staying in line with the rival increases profits, first period prices are also subject to a coordination problem similar to the one of the second period, and multiple equilibria result. Again, if firms can communicate it is reasonable to assume that they would coordinate on the highest possible equilibrium price [14], which is given by the first order condition that sets the marginal profits of a price decrease equal to zero, *i.e.*

$$\hat{a} - (2\,b - c)\,p + \int_x^y [a - (2\,b - c)\,(p - g)]\,dH\,(a) \\ + h\,(x)\,c\left[[x - (b - c)\,(p - g)]\,(p - g) + K - \frac{bx^2}{(2\,b - c)^2}\right] = 0$$

Since the this third term is positive, this means that taking into account the effect of first period prices on x increases equilibrium profits.

Summarizing, fixed price adjustment costs combined with strategic firm interaction creates multiple equilibria. Even if we make collusion difficult by making firms move simultaneously, if firms can communicate at each subgame second period adjustment is asymmetric with respect to the initial second period real price, and taking this fact into account raises first period prices.

Let us now turn to average real prices, assuming that the first period price is determined by the above equation, and concerning ourselves with the uniform distribution case:

LEMMA 4 : If $h\,(a) = \lambda$, then the average real price increases with λ and decreases with inflation, as long as x and y are in the interior of $[\underline{a},\, \overline{a}\,]$.

As in Section 3 inflation interferes with the collusive mechanism. Since inflation raises the first period price but lowers the second period initial real price the gains from not lowering the nominal price are reduced.

The reasoning about sectorial shocks is analogous to the one in Section 4 and will not be repeated here. However, it is important to emphasize that asymmetric adjustment (and hence, aggregate shocks) do not depend on a positive inflation rate, but on strategic firm interaction. Also, the relation between inflation and output fluctuations is analogous, *i.e.* provided inflation is moderate, we have $y \geq \hat{a}$, and inflation exacerbates aggregate output fluctuations.

Result 4

With strategic firm interaction and fixed price adjustment costs:

4.1. Average output increases with the dispersion of shocks

4.2. Average output increases with inflation

4.3. Sectorial shocks may cause aggregate output fluctuations

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14. Multiple equilibria arise also with monopolistic competition and fixed price adjustment costs. See Caminal [1992]. In this paper it is showed that a fixed frequency of price adjustment (a time-contingent pricing rule) is not only compatible with equilibrium behavior but very plausible in that context.

4.4 Inflation exacerbates the aggregate fluctuations caused by sectorial shocks

6 Concluding Remarks

We can go back now to the four conjectures discussed in the introduction. First, these simple models provide very little support for stabilization policies. Independently of the form of the nominal rigidity a monopolist sets the initial price anticipating future asymmetric behavior. In the optimal solution the average real price and the average output level are independent of the distribution of shocks. With oligopolistic competition the result tends to be the opposite of what has been conjectured. Higher dispersion of shocks tends to make collusion more difficult. Therefore, the rationale for stabilization policies has to be looked for somewhere else. Asymmetric nominal price adjustment alone is not likely to do the job.

Second, the existence of a long-run Phillips curve due to asymmetric nominal price adjustment, also seems to depend on market structure, and not on the particular form of the nominal rigidity. Inflation does affect relative price flexibility, but this does not necessarily imply an increase in average output, as exemplified in the monopoly models. However, if nominal rigidities are used by oligopolists as a collusive device then inflation interferes with such a mechanism, reduces average real prices and increases output [15].

Third, any model of asymmetric price adjustment has the implication that sectorial shocks may create aggregate output fluctuations. This is quite obvious and does not deserve further discussion. The interesting part (fourth) is the interaction between inflation and aggregate output fluctuations caused by barriers to full relative price flexibility. This is where the form of the nominal rigidity plays a more crucial role. The monopoly model with exogenous downward rigidity is the only one compatible with Conjecture four. This is the only case where inflation, by enhancing relative price flexibility, reduces aggregate output fluctuations. This is not even robust to the assumption about market structure. If collusion is strong enough the result can be reversed, as it happens with fixed price adjustment costs, under both market structures.

It is probably redundant to mention that no policy implication should be derived from the analysis of the paper, since many of the crucial factors

15. Benabou [1991] presents evidence of the negative relationship between inflation and mark-ups. The negative effect of inflation on mark-ups is compatible with our collusion story, but also with the model in Benabou [1988 and 1992]. In this model, because of price adjustment costs, inflation increases relative price variability, which induces higher search efforts and reduces firms' market power.

linking money, prices and output have been left out of the models Here. I have just tried to make sense of some popular beliefs about the relationship between relative price flexibility, inflation and output, by examining some familiar models with (I believe) ambiguous success. On the one hand, the value of stabilization policies do not seem to be related to the relative downward rigidity of nominal prices. On the other hand, inflation may induce higher average output by making collusion more difficult, although it may exacerbate the fluctuations on economic activity caused by sectorial shocks.

An obvious shortcoming of the paper is that I have discussed macroeconomic issues without a general equilibrium model. For instance, inflation has been taken as exogenous but at the same time my simple models had implications for the aggregate price level. In fact by trend inflation I meant money growth rates. The difficulty here is not only to take into account all the general equilibrium effects when analyzing a single industry, but more importantly to deal simultaneously with the independent effects of the price level and the money stock on the individual firm's real profits (particularly, in the oligopoly case).

If incorporating the effects of nominal rigidities in general equilibrium models is important, in my opinion the most urging task in the research agenda is to improve our understanding of the nature of nominal rigidities at the individual firm level. Two channels look promising: to make price adjustment costs endogenous and relating these costs to the monetary aspects of the transaction process.

Lemma 1

Consider the symmetric price that solves the F.O.C. of firm is optimization problem evaluated at $p_1^i = p_1^j \equiv p_1^*$:

$$[\hat{a} - (2\,b - c)\,p_1^*] + \int_{\underline{a}}^{x} [a - (2\,b - c)\,(p_1^* - g)]\,dH\,(a) = 0$$

given that $x = (2\,b - c)\,(p_1^* - g)$ and $h\,(a) = \lambda$, we can solve for p_1^* explicitly:

$$p_1^* = \frac{\lambda\,\hat{a} + (2\,b - c)\,g\,\lambda - \dfrac{3}{2} + \sqrt{a\,[1 - (2\,b - c)\,g\,\lambda]}}{\lambda\,(2\,b - c)}$$

(The other solution of the F.O.C. can be ruled out using the S.O.C.)

The S.O.C. is:

$$\left.\frac{d^2\,\pi_1^i}{d\,(p_1^i)^2}\right|_{p_1^i = p_1^j \equiv p_1^*} = -2\,b\,[1 + H\,(x)] + \lambda\,c^2\,(p_1^* - g) < 0.$$

This condition guarantees that p_1^* is a local maximum. However, since the third derivative has the same sign everywhere this condition is sufficient for p_1^* to be a global maximum.

Let λ^* be the solution to:

$$-2\,b\,[1 + H\,(x)] + \lambda^*\,c^2\,(p_1^* - g) = 0$$

where $x = (2\,b - c)\,(p_1^* - g)$. We can solve for λ^* explicitly for $g = 0$:

$$\lambda^* = \frac{3\,b\,(2\,b - c)}{\hat{a}\,c^2} - \frac{4\,b^2 - 2\,bc - c^2}{ac^2}\left(\frac{3}{2} - \sqrt{2}\right).$$

Hence,

$$\frac{d\lambda^*}{dc} < 0 \qquad \frac{d\lambda^*}{d\hat{a}} < 0\,.$$

And using the implicit function theorem, for g small:

$$\frac{d\lambda^*}{dg} < 0.$$

If $\lambda > \lambda^*$ at the symmetric price that solves the F.O.C., S.O.C. is violated. Hence, the equilibrium must be asymmetric.

Without loss of generality, $p_1^i > p_1^j$. Then equilibrium the following conditions hold:

$$(1) \qquad \hat{a} - 2\,bp_1^j + cp_1^i + \int_{\underline{a}}^{x} [a - 2\,b\,(p_1^j - g) + c\,(p_1^i - g)]\,dH\,(a) = 0$$

$$(2) \qquad \hat{a} - 2\,bp_1^i + cp_1^j + \int_{\underline{a}}^{x} [a - 2\,b\,(p_1^i - g) + c\,(p_1^j - g)]\,dH\,(a)$$
$$+ \int_{x}^{y} \left[a - 2\,b\,(p_1^i - g) + \frac{ca}{2\,b} + \frac{c^2}{b}\,(p_1^i - g)\right] dH\,(a) = 0$$

where

$$x = 2\,b\,(p_1^j - g) - c\,(p_1^i - g)$$
$$y = (2\,b - c)\,(p_1^i - g).$$

The effect of λ on $(p_1^i,\, p_1^j)$ can be more easily seen by setting $g = 0$. These two conditions can be written:

$$(1') \qquad 2\,bp_1^j - cp_1^i = \hat{a} - \frac{\frac{3}{2} - \sqrt{2}}{\lambda} \equiv z$$

$$\begin{aligned}
(2') \quad & -\lambda\,\frac{(2\,b-c)\,(4\,b^2 - 3\,c^2)}{4\,b}\,(p_1^i)^2 \\
& - \left\{\frac{4\,b^2 - c^2}{2\,b}\left[1 + \lambda\left(z - \hat{a} + \frac{1}{2\,\lambda}\right)\right] - \lambda\,\frac{2\,b^2 - c^2}{b}\right\} p_1^i \\
& + \hat{a} + \frac{cz}{2\,b}\left[1 + \lambda\left(z - \hat{a} + \frac{1}{2\,\lambda}\right)\right] + \frac{\lambda}{2}\left[z^2 - \left(\hat{a} - \frac{1}{2\,\lambda}\right)^2\right] \\
& - \lambda\,\frac{2\,b + c}{4\,b}\,z^2 = 0.
\end{aligned}$$

It can be checked that $\dfrac{dp_1^i}{d\lambda},\ \dfrac{dp_1^j}{d\lambda} > 0$. $\square$

Lemma 2

From equations (1) and (2) if g increases then $(p_1^i - g) + (p_1^j - g)$ fall. If we compute average prices:

$$\begin{aligned}
2\,\hat{p}_1^i &= p_1^i + \int_{\underline{a}}^{y} (p_1^i - g)\,dH\,(a) + \int_{y}^{\overline{a}} \frac{a}{2\,b - c}\,dH\,(a) \\
2\,p_1^j &= p_1^j + \int_{\underline{a}}^{x} (p_1^j - g)\,dH\,(a) + \int_{x}^{y} \frac{a + c\,(p_1^i - g)}{2\,b}\,dH\,(a) \\
&\quad + \int_{y}^{\overline{a}} \frac{a}{2\,b - c}\,dH\,(a).
\end{aligned}$$

Using the F.O.C. and after some manipulations:

$$\begin{aligned}
2\,\hat{p}_1^i &= \frac{2\,\hat{a}}{2\,b - c} + \frac{c^2}{4\,b^2 - c^2} \int_x^y (p_1^i - g)\,\lambda\,da \\
2\,\hat{p}_1^j &= \frac{2\,\hat{a}}{2\,b - c} + \frac{c}{2\,b} \int_x^y \left[(p_1^i - g) - \frac{a}{2\,b - c}\right] \lambda\,da.
\end{aligned}$$

Clearly, $\hat{p}_1^i,\ \hat{p}_1^j > \dfrac{\hat{a}}{2\,b - c}$.

As λ increases the direct effect on $\hat{p}_1^i, \hat{p}_1^j$ is positive. Also, since $\dfrac{dp_1^i}{d\lambda} > 0$, the effect through p_1^i is also positive. In principle λ can affect the limits of the integral in such a way that $\hat{p}_1^i, \hat{p}_1^j$ end up falling with λ. Since $\hat{p}_1^i, \hat{p}_1^j$ can no be computed explicitly, this possibility can not be checked algebraically. Numerical simulations indicate that this possibility is not feasible and that $(\hat{p}_1^i,\, \hat{p}_1^j)$ always increase with λ.

Similarly, as g increases $(p_1^i - g)$ falls, and this $\hat{p}_1^i,\, \hat{p}_1^j$ also fall, except that the indirect effect through x and y could compensate this negative direct effect. Again, this can not be checked algebraically, but numerical simulations also indicate that $(\hat{p}_1^i,\, \hat{p}_1^j)$ always decrease with g. $\square$

Claim

In the two-period game of Section w, $(p_1^i - g)$, $(p_1^j - g)$ are above or below $\dfrac{\hat{a}}{2\,b-c}$ depending on the values of λ and g.

Discussion

If λ is low and $g = 0$, then

$$p_1^i = p_1^j = \frac{\hat{a}}{2\,b-c} - \frac{\frac{3}{2} - \sqrt{2}}{\lambda\,(2\,b-c)} < \frac{\hat{a}}{2\,b-c}.$$

If λ goes to infinity, $(\hat{p}_1^i,\, \hat{p}_1^j)$ are characterized by (again $g = 0$)

$$\left\{ \begin{array}{c} \hat{a} - 2\,bp_1^j + cp_1^i = 0 \\ \hat{a} - 2\,bp_1^i + cp_1^j + \hat{a}\,\frac{2\,b+c}{2\,b} - \left(2\,b - \frac{c^2}{b}\right)^{p_1^i = 0} \end{array} \right\}.$$

Hence,

$$p_1^i = \frac{2\,(2\,b+c)}{8\,b^2 - 3\,c^2} > \frac{\hat{a}}{2\,b-c}$$

which implies that $p_1^j > \dfrac{\hat{a}}{2\,b-c}$.

In any case, $(p_1^i - g)$, $(p_1^j - g)$ fall with g. $\square$

Lemma 3

a) If firm j adjusts its price to $\dfrac{a_2}{2\,b-c}$, it is optimal for firm i to do so if

$$\frac{ba_2^2}{(2\,b-c)^2} - K \geq \left[a_2 - b\,(p_1^i - g) + c\,\frac{a_2}{2\,b-c}\right](p_1^i - g)$$

i.e.

$$\begin{aligned} a_2 &\geq (2\,b-c)\,(p_1^i - g) + d_2 \\ &\leq (2\,b-c)\,(p_1^i - g) - d_2. \end{aligned}$$

b) If firm j does not adjust its price, it is optimal for firm i not to do it either if

$$\begin{aligned} &[a_2 - b\,(p_1^i - g) + c\,(p_1^j - g)]\,(p_1^i - g) \\ &\quad \leq \operatorname*{Max}_{p^i} \{[a_2 - bp^i + c\,(p_1^j - g)]\,p^i - K\} \end{aligned}$$

i.e.

$$
\begin{aligned}
a_2 &\leq (2\,b-c)\,(p_1-g)+d_1 \\
&\geq (2\,b-c)\,(p_1-g)-d_1.
\end{aligned}
$$

c) An asymmetric equilibrium involves:

$$
\begin{aligned}
&\left[a_2-b\,(p_1^i-g)+c\,\frac{a_2+c\,(p_1^i-g)}{2\,b}\right](p_1^i-g) \\
&\quad\geq \operatorname*{Max}_{p^i}\left\{\left[a_2-bp_i+c\,\frac{a_2+c\,(p_1^i-g)}{2\,b}\right]p_i-K\right\}
\end{aligned}
$$

and

$$
\begin{aligned}
&[a_2-b\,(p_1^j-g)+c\,(p_1^i-g)]\,(p_1^j-g) \\
&\quad\leq \operatorname*{Max}_{p^i}\{[a_2-bp^j+c\,(p_1^i-g)]\,p^j-K\}
\end{aligned}
$$

both inequalities can not hold simultaneously given identical initial prices.

d) If $a_2\in[(2\,b-c)\,(p_1-g)-d_1,\,(2\,b-c)\,(p_1-g)-d_2]$ higher prices and no adjustment costs clearly imply higher profits than in the case of adjustment.

If $a_2\in[(2\,b-c)\,(p_1-g)+d_2,\,(2\,b-c)\,(p_1-g)+d_1]$

$$
\begin{aligned}
&\frac{ba_2^2}{(2\,b-c)^2}-K-[a_2-(b-c)\,(p_1-g)]\,(p_i-g) \\
&\quad>\frac{b}{(2\,b-c)^2}\,[(2\,b-c)\,(p_1-g)+d_2]^2-K \\
&\quad-[(2\,b-c)\,(p_1-g)+d_2-(b-c)\,(p_1-g)]\,(p_1-g)>0. \quad\square
\end{aligned}
$$

Lemma 4

The F.O.C. that characterizes the Pareto Superior equilibrium is:

$$
\begin{aligned}
&\hat{a}-(2\,b-c)\,p+\int_x^y[a-(2\,b-c)\,(p-g)]\,dH\,(a) \\
&\quad+\lambda\,c\left[\frac{cd_1}{2\,b-c}\,(p_1-g)+K-\frac{bd_1^2}{(2\,b-c)^2}\right]=0.
\end{aligned}
$$

Solving for p:

$$
p=\frac{\left\{\begin{aligned}&\hat{a}+\frac{\lambda}{2}\,(d_2^2-d_1^2)+c\,\lambda\left(K-\frac{bd_1^2}{(2\,b-c)^2}\right)\\&+g\left[(2\,b-c)\,\lambda\,(d_1+d_2)-\frac{c^{2\,\lambda\,d_1}}{2\,b-c}\right]\end{aligned}\right\}}{(2\,b-c)\left[1-\frac{c^{2\,\lambda\,d_1}}{2\,b-c}\right]}.
$$

The average real prices, $\hat{p}$, is given by:

$$2\,\hat{p} = \frac{\left\{\begin{array}{l} \hat{a}\left(2 - \dfrac{c^2\,\lambda\,d}{2\,b-c}\right) - \dfrac{\lambda}{2}\,(d_1^2 - d_2^2)\,\dfrac{c^2\,\lambda\,d}{2\,b-c} \\ +\,c\,\lambda\left(K - \dfrac{bd_1^2}{(2\,b-c)^2}\right) \\ -\,g\,[1-(2\,b-c)\,(d_1+d_2)]\,\dfrac{c^2\,\lambda\,d}{2\,b-c} \end{array}\right\}}{(2\,b-c)\left[1 - \dfrac{c^2\,\lambda\,d}{2\,b-c}\right]}$$

$$\frac{d\hat{p}}{d\lambda} > 0 \qquad \frac{d\hat{p}}{dg} < 0.$$

Claim

In the two-period game of Section 4 we need to worry only about small deviations from a symmetric price vector.

Discussion

This can be seen by looking at Figure 4. A small deviation from a symmetric price vector only changes the realization of the random variable below which a downward adjustment takes place in the second period.

With a larger deviation, there will be realizations of the random variable for which only one firm adjusts its price in the second period. But such a pattern of adjustment can never be profitable.

If the deviation from the symmetric price vector is large enough there are realizations of the random variable for which in the second period the rival firm lowers its price (lowering second period profits). For other realizations it might be the deviating firm who is induced to adjust its price upwards. In this case second period profits are also lower since getting closer to its short-run reaction function does not compensate for the adjustment costs. □

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