

# A Dynamic Model of Bargaining in a Unionized Firm with Irreversible Investment

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**ABSTRACT.** – This paper develops a dynamic model of bargaining between a firm and a union. Capital is assumed to be firm-specific, so only nonnegative investments are possible. Collective bargaining contracts specify the level of the wage rate that will prevail for a fixed contract length, while the firm unilaterally chooses employment. Two types of equilibria are considered. In one equilibrium, the wage-employment outcomes lie on the marginal revenue product of labor curve and the wage is determined by a generalized Nash bargain. This equilibrium is Pareto inefficient. In another equilibrium, wage-employment pairs lie on the contract curve and wages are set above the marginal product of labor. In this equilibrium, the firm's desire to reduce employment is offset by punishment strategies in which the union bargains tougher in the future and the Pareto inefficient equilibrium prevails. Existence results are established and the equilibria are characterized for a particular specification of the firm's revenue function and the union's temporal utility function, using recursive methods. The model is calibrated on stylized facts from the U.S. economy. It turns out that the calibrated model can account for several other stylized facts; in particular, the relatively low variability of the wage rate and the countercyclicality of the union wage premium. Moreover, it is found that irreversibilities are crucial in this respect, in the ergodic as well as the nonergodic states.

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## Un modèle dynamique de négociations syndicales en présence d'investissement irréversible

**RÉSUMÉ.** – Les modèles de cycles économiques ont pu jusqu'à maintenant difficilement expliquer la faible variabilité des salaires réels relativement à celles de la productivité et de l'emploi. Ce document tente d'expliquer ce phénomène en utilisant un modèle où une entreprise et un syndicat négocient les salaires d'une manière continue. La dynamique des cycles économiques est introduite grâce à des chocs stochastiques de productivité, un comportement stratégique des agents ainsi que l'irréversibilité de l'investissement. Une fois le modèle calibré, les simulations générées montrent que le modèle peut expliquer le phénomène de la faible variabilité des salaires réels.

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# 1 Introduction

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This paper presents some initial attempts to analyze bargaining between firms and unions when there is uncertainty about the state of the economy and capital is firm-specific, so that negative investment is not feasible. One of the questions that we would like to address in this research is how irreversibility might affect bargaining outcomes and the cyclical behavior of variables such as wages, employment, and investment in an infinite-horizon bargaining game.

Two types of equilibria are considered below. The first is an "inefficient" equilibrium in which the firm sets employment along the marginal revenue product of labor curve after the wage rate has been set by contract for that period. As is well known in the theoretical literature on unionized firms, this type of solution will generally be Pareto inefficient (see OSWALD [1985]). In the second equilibrium, the firm and union cooperate to achieve a Pareto superior point, but it is required that the firm voluntarily sets employment at a Pareto efficient level (which will not, in the models presented below, be the level of employment that maximizes current profits). The firm cooperates because a failure to do so would cause a break-down of cooperation in future periods where the inefficient equilibrium prevails (ESPINOSA and RHEE [1989], STRAND [1989]). It is here that irreversibility may be very important. If the firm chooses a greater capacity in the efficient solution, a break-down of cooperation could cause the firm to "get stuck" with too large a capital stock, so irreversibility may make the union more powerful in the bargaining process. In this case wages may be high when output is low. This is consistent with the stylized facts of union wage behavior (LEWIS [1986], JARRELL and STANLEY [1990]). Further, it shows how wages can be countercyclical and have relatively low variability while productivity is procyclical in an environment where competitive behavior would have resulted in both variables being much more variable and strongly procyclical, as in standard real business cycles models (KYDLAND and PRESCOTT [1982], HANSEN [1985], and PRESCOTT [1986] <sup>1</sup>).

The main problem with irreversibilities in the investment process and/or incentive compatibility constraints is that they make it extremely difficult, if not impossible, to solve infinite-horizon models explicitly. In this paper we avoid this difficulty by specifying particular functional forms and using recursive methods to numerically approximate the model's solution.

Section 2 describes the general model. Section 3 develops the Nash solution, while Section 4 describes a contract curve solution. Section 5

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1. CHRISTIANO and EICHENBAUM [1992] account for the observed low correlation between real wages and hours worked in aggregate economy data by introducing government (demand) shocks in a real business cycles model. MORTENSEN [1991] abandons the Walrasian set up to account for some of these stylized facts. Our model could serve for one half of an alternative explanation for the low correlation between real wages and hours worked. That is, real wages will have low positive or negative correlation in an economy where the unionized firms behave as in our model and nonunionized firms behave as in a standard real business cycles model.

develops an algorithm for the later solution and calibrates the model on stylized facts from the U.S. economy. Both equilibria are simulated and their ergodic (invariant) distributions are computed so as to obtain various measures of volatility, persistence and comovement. Section 6 is the conclusion.

## 2 The Model

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We consider bargaining over time between a single firm and a single union. In each period  $t$ , the firm produces a single product,  $Y$ , using capital,  $K$ , and labor,  $L$ , according to a COBB-DOUGLAS production function of the form:

$$(1) \quad Y_t = f(K_t, L_t) = \theta_t K_t^\alpha L_t^{1-\alpha}; \quad \alpha \in (0, 1)$$

where:  $\theta_t$  is a nonnegative time dependent parameter that incorporates stochastic productivity shock (or Solow residual), in period  $t$ . We take  $\{\theta_t : t \in \mathbb{N}_+\}$  to be a Markov chain with finite support,  $\Theta$ , indexed by  $\mathbf{S} \subset \mathbb{N}_{++}$  and probability transition function given by <sup>2</sup>:

$$(2) \quad \text{prob} [\theta_{t+1} = \theta(s) | \theta_t = \theta(r)] = \lambda_{rs}; \quad \lambda_{rs} \in [0, 1), \quad \forall r, s \in \mathbf{S}$$

The capital market is competitive, but, once installed, capital becomes firm-specific and cannot be resold. That is, gross-investment is constrained to be nonnegative:

$$(3) \quad I_t = K_{t+1} - (1 - \delta) K_t \geq 0; \quad \delta \in (0, 1)$$

Labor services are supplied by a single union. The union's preferences are characterized by an infinite lifetime utility function of the form:

$$\sum_{t=0}^{\infty} \beta^t u(w_t, L_t)$$

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2. Actually, it makes no difference for most of our analysis whether  $\theta$  is interpreted as a demand shock or a total factor productivity shock.

where:  $\beta \in (0, 1)$  is a constant discount factor and  $u(w_t, L_t)$  is a temporal utility function, of the form:

$$(4) \quad u(w_t, L_t) = (w_t - \underline{w})^\varphi L_t; \quad \varphi \in (0, 1)$$

where  $w_t$  is the real wage rate in period  $t$ . This form of union preferences corresponds to the “utilitarian” model of McDONALD and SOLOW [1981] and OSWALD [1985], where the representative union member has a constant relative rate of risk aversion, provided that union membership is fixed<sup>3</sup>. The parameter  $\underline{w}$  is frequently interpreted as an “alternative wage” for union members, so that the ratio of  $w$  to  $\underline{w}$  may be interpreted as the union wage differential.

The demand for the firm’s product is characterized by a constant-price-elasticity inverse demand function of the form:

$$(5) \quad p_t = AY_t^{-\varepsilon}; \quad A > 0, \quad \varepsilon \in (0, 1)$$

where  $p_t$  is the relative price of the firm’s product.

The firm’s expected present value, as a function of the sequences of capital, labor, and wages, is given by:

$$(6) \quad \begin{aligned} & \Pi(\{K_{t+1}, L_t, w_t\}_{t=0}^\infty; (K_0, \theta_0)) \\ &= E_0 \sum_{t=0}^{\infty} \beta^t (p_t Y_t - w_t L_t - I_t) \\ &= E_0 \sum_{t=0}^{\infty} \beta^t \{A\theta_t^{\alpha_0} K_t^{\alpha_1} L_t^{\alpha_2} - w_t L_t - [K_{t+1} - (1 - \delta) K_t]\} \end{aligned}$$

where  $E_0$  is the expectations operator, conditional on information available at time zero,  $(K_0, \theta_0)$ <sup>4</sup>. The parameter  $\beta \in (0, 1)$  is the firm’s real discount factor in all periods  $a_0 = (1 - \varepsilon)$ ,  $a_1 = a(1 - \varepsilon)$ , and  $a_2 = (1 - a)(1 - \varepsilon)$ . Moreover, it is implicit in (6) that investment goods are taken as the numeraire and their price is normalized to one in all periods. In addition, that the firm and the union have the same discount rate  $\beta$ . Some additional technical restrictions on preferences and technologies will be imposed later, as required by theory.

In each period, there are three stages which describe the rules of behavior:

| STAGE 1 : At the beginning of period  $t$ , “Nature” chooses  $\theta_t$  according to the probability distribution described above.

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3. In this paper we abstract from union membership dynamics. This issue is taken up by BLANCHARD and SUMMERS [1986] and in explicit dynamic bargaining models by KIDD and OSWALD [1986] and SPINNEWYN and SVENJAR [1990a].

4. In order to avoid complicating the notation we do not make explicit the dependence of all  $t$ -period dated random variables in (6) on the realization of the state of nature up to this date,  $(\theta_0, \theta_1, \dots, \theta_t)$ , as would have been better for the designation of the probabilities needed for the evaluation of expectations.

STAGE 2 : The players bargain over the wage rate in period  $t$  having observed “Nature’s” choice in Stage 1 and all variables determined in past periods.

STAGE 3 : The firm chooses employment and investment for period  $t$ , having observed the moves in Stages 1 and 2 and all variables determined in past periods.

Clearly, the sort of questions we are interested in can only be addressed in a dynamic and stochastic set-up. Although this set-up is new in the literature of collective bargaining, our formulation draws heavily from that literature. Thus, ignoring capital, this formulation corresponds to what is called the “right to manage” model<sup>5</sup>. Manning [1987] has shown that this formulation is a special case of a two-stage sequential bargaining set-up, where the firm and the union bargain first over the wage rate and then over employment<sup>6, 7</sup>. Treating capital as a variable factor of production (*i.e.*, in the absence of irreversibility constraints, adjustment costs, time-to-build technologies, and the like) our set-up corresponds to what is referred to as a situation of “non-binding wage/employment contracts”, where contracts are signed after the firm commits to a specific investment level. This is the standard union labor market set-up<sup>8</sup>. GROUT [1984] showed that in this

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5. MANNING [1987] credits this model to NICKELL [1982]. Also, it should be noted that this model is a generalization of the so called “monopoly-union” model (DUNLOP [1944], OSWALD [1982]), where the firm chooses employment and the union chooses the wage rate.

6. In particular, MANNING [1987] argued that unions may have different bargaining powers over wages and employment. In his paper he developed a model in which wages and employment are determined sequentially using generalized Nash solutions in which the bargaining powers may differ at different stages. He showed that the “monopoly-union” model results if wages are determined first and the union has all the bargaining power over wages and the firm has all the bargaining power over employment. If this set up is amended so that both parties have positive bargaining power over wages, but the firm still has all the bargaining power over employment, the “right to manage” model results. If the bargaining powers over wages and employment are the same, the “efficient bargain” model results, where the firm and the union bargain over the wage rate and employment simultaneously, regardless of the order in which the variables are determined. Thus, the model nests the three main alternative models and also creates a new class of models, those in which both players have positive bargaining powers over both variables, but the powers over wages and employment differ.

7. Empirical comparison tests between the “right to manage” model and the “efficient bargain” model is not settled. DOIRON [1990] uses Manning’s framework to test the alternatives using data from the wood products industry in British Columbia, but the results are mixed on whether the “efficient bargain” or “right to manage” model is best. MACHIN, MANNING, and MEGHIR [1991] also use this framework in a dynamic model and provide estimates using panel data from the United Kingdom. Their results indicate a dynamic version of the “right-to-manage” model describes the data better. But taking into account the fact that the “right to manage” model implies that wage-employment pairs must lie on the labor demand curve, negative results for this model have been obtained by several authors that find evidence against the labor demand hypothesis. In particular, SVEJNAR [1986] estimated an “efficient bargains” model using data from U.S. industries and found that the model performed better than labor demand models. BROWN and ASHENFELTER [1986] found some support for the “efficient bargains” model using data from the U.S. printing industry. MACURDY and PENCAVEL [1986] also found some weak support for the “efficient bargains” model using data from the U.S. printing industry. Finally, CARD [1986] used data from the U.S. airline industry and estimated a model that incorporated adjustment costs in an intertemporal contracting model. While the properties of the “efficient bargain” model held in the results, the fit was poor, indicating only weak support for this model.

8. See, e.g., GROUT [1984], BEAN [1983], MANNING [1987], VAN DER PLOEG [1987], HOEL [1990].

case capital will be lower than in a situation of binding wage/employment contracts, where contracts are signed before the monopoly commits itself to a specific investment level, provided that the resale price of capital is less than its purchase price. The rationale for this is that without binding contracts a union cannot credibly commit to a wage that is conditional on the firm's investment<sup>9</sup>. Similar results have been obtained by Van der Ploeg [1987] in a dynamic, due to capital adjustment costs, but deterministic set-up, where only Nash behavior is considered.

### 3 Nash Equilibrium

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We begin by considering a stationary noncooperative equilibrium of this game when the bargains over wages are determined by a Nash solution in Stage 2 of each period (NASH [1950], ROTH [1979], and DEMENIL [1981]). As is well known, this solution picks the wage which will maximize a geometric average of the players' payoffs minus what they would get if there were disagreement and no trade took place. This, of course, must take as given the firm's optimal rule for choosing employment after the wage is set. By stationary, it is meant that the strategies in each period depend only on the current state. That is, the strategies depend only on the level of the capital stock and the current demand shock and not on the history of the game prior to that point, except in as much as this affects the level of the current capital stock. At this type of equilibrium, the firm's optimal choice of employment will maximize its current profit rate, so the optimal employment can be found by applying the usual marginal productivity condition. Further, since investment in the current period affects only future revenues, at a stationary equilibrium, so the optimal investment policy will not depend on the outcome of the current wage bargain. Thus, the firm's payoff minus what it gets if there is disagreement is just current revenue minus the wage bill. If there is disagreement, employment is zero, so the union's disagreement payoff is zero. Thus, the union's payoff minus its disagreement utility is just the union temporal utility itself,  $u(w, L)$ . Formally, then, we make the following:

DEFINITION : A **Nash stationary equilibrium** of the game defined in Section 2 is a sequence

$\{w_t, L_t, K_{t+1}\}_{t=0}^{\infty}$  such that:

(i)  $w_t = w(K_t, \theta_t)$ ,  $L_t = L(K_t, \theta_t)$ , and  $K_{t+1} = K(K_t, \theta_t)$ ;  $\forall t \in \mathbb{N}_+$  and for  $(K_0, \theta_0)$  given.

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9. The importance of this result hinges on the union's bargaining power. For that matter, Grout's underinvestment result may not hold in Manning's two-stage bargaining model where wages are negotiated first and employment is bargained out later. DEVERAUX and LOCKWOOD [1991], also, provide a counterexample to the underinvestment result in a more general equilibrium set-up.

(ii) Given  $K_t$  and  $\theta_t$ ,

$$(7) \quad w_t = \operatorname{argmax}_w \{u(w, L_t)^\mu [p_t Y_t - w L_t]\}$$

$$(8) \quad s.t. L_t = \operatorname{argmax}_L [p_t Y_t - w_t L]; \quad \forall t \in N_+$$

and

(iii)

$$(9) \quad \{K_{t+1}\}_{t=0}^\infty = \operatorname{argmax}_{\{x_{t+1}\}_{t=0}^\infty} \Pi(\{x_{t+1}, L_t, w_t\}_{t=0}^\infty; (x_0, \theta_0))$$

$$(10) \quad s.t. x_{t+1} \geq (1 - \delta) x_t \quad \text{and} \quad x_0 = K_0.$$

Equation (8) implies that the marginal revenue product of labor equals the wage rate. In (7) the parameter  $\mu \in \mathbb{R}_+$  is usually referred to as the union's "relative bargaining power" (See, for example, SVEJNAR [1986]).

It is straightforward to show that the wage which maximizes (7) subject to the marginal productivity condition (8) is given by:

$$(11) \quad w = \frac{\mu + \alpha_2}{[1 - (1 - \alpha_2) \phi] \mu + \alpha_2} \underline{w}$$

while, ignoring time subscripts, the firm's revenue minus the wage bill is given by:

$$(12) \quad pY - wL = (1 - \alpha_2) \left(\frac{\alpha_2}{w}\right)^{\frac{\alpha_2}{1-\alpha_2}} \theta^{\frac{\alpha_0}{1-\alpha_2}} K^{\frac{\alpha_1}{1-\alpha_2}}$$

Thus, given equations (1), (3), (5), (11) and (12), the firm's cash flow in period  $t$ , as a function of  $K$ ,  $K'$ , and  $\theta$ , can be written as:

$$(13) \quad F(K, K', \theta) = B \theta^{\frac{\alpha_0}{1-\alpha_2}} K^{\frac{\alpha_1}{1-\alpha_2}} + (1 - \delta) K - K'$$

where  $B > 0$  is constant over time. The properties of this function, which we shall need later, can be summarized as follows:

*Remark 1:* (a)  $F(K, K', \theta)$  is real valued, strictly increasing and strictly concave in  $K$ , strictly decreasing and affine in  $K'$ , and concave in  $(K, K')$ ,  $\forall \theta \in \Theta$ .

(b) There exists a  $\bar{K} \in (0, \infty)$  such that  $F(K, K', \theta) \leq 0, \forall K \geq \bar{K}$  &  $\theta \in \Theta$ .

*Proof:* See KOLLINTZAS and EBERWEIN [1993].

Clearly, any level of capital above  $\bar{K}$  would have resulted in revenues that would not have been enough to cover depreciation. Thus, an implication of Remark 1 is that for any initial capital stock,  $K$ , the firm's optimal choice of capital in the next period will lie in the compact set,  $\Gamma(K) = [(1 - \delta)K, \bar{K}]$ .

Because the choice of capital in any period will affect the set of feasible choices of capital in all future periods, it is difficult to use Euler-Lagrange equations to solve for the optimal investment policy<sup>10</sup>. Fortunately, there are easier methods for finding that policy.

Let  $V^n : [0, \bar{K}] X_\Theta \rightarrow \mathbb{R}$  be the firm's value along an optimal path for capital. Then  $V^n$  must satisfy:

$$(14) \quad V^n(K, \theta(r)) = \max_{K' \in \Gamma(K)} \{F[K, K', \theta(r)] + \beta \sum_{s \in \mathcal{S}} \lambda_{rs} V^n(K', \theta(s))\}$$

Let  $\Psi$  be the space of all bounded, continuous functions mapping from  $[0, \bar{K}] X_\Theta$  to  $\mathbb{R}$  with the sup-norm. Define  $T^n : \Psi \rightarrow \Psi$  as follows:

$$(15) \quad (T^n V)(K, \theta(r)) = \max_{K' \in \Gamma(K)} \{F[K, K', \theta(r)] + \beta \sum_{s \in \mathcal{S}} \lambda_{rs} V(K', \theta(s))\}$$

Then, it can be shown that:

*Remark 2:* (a)  $T^n$  is continuous in  $\Psi$  and satisfies Blackwell's conditions, so that  $T^n$  is a contraction mapping with modulus  $\beta$  in  $\Psi$ .  $V^n$  is a fixed point of  $T^n$  and, since  $T^n$  is a contraction mapping,  $V^n$  is the unique fixed point of  $T$ . Moreover, if we begin with an arbitrary function  $V_0$  in  $\Psi$  and define the sequence  $\{V_j\}$  by  $V_{j+1} = T^n V_j$  this sequence will converge monotonically to  $V^n$ .

(b)  $V^n$  is strictly concave so that the optimal capital stock at the beginning of next period,

$$(16) \quad \begin{aligned} K(K, \theta(r)) = \arg \max_{K'} \{ & F[K, K', \theta(r)] \\ & + \beta \sum_{s \in \mathcal{S}} \lambda_{rs} V^n[K', \theta(s)] \}, \\ & \forall (K, \theta) \in [0, \bar{K}] x \Theta \end{aligned}$$

is uniquely defined.

*Proof:* The proof of Part (a) follows from standard arguments. See, e.g., LUCAS and STOKEY ([1989], pp. 66-102). The proof of Part (b) can be found in EBERWEIN and KOLLINTZAS [1983].

Collecting results, we have established the following:

PROPOSITION 1 : **(Existence)** : There exists a unique Nash stationary equilibrium to the game of Section 2, which is characterized by (11), (8), and (16).

Since (14) cannot be solved analytically, the last point of Part (a) of the preceding remark suggests also a way of approximating  $V^n$  numerically and, hence, approximate the optimal investment policy in (16). We shall follow this path in Section 5.

10. A possible way out of these difficulties is the method suggested by MARCET and MARIMON [1992]. A comparison between this method and the methods developed in this paper should be an interesting research topic.

## 4 Contract Curve Equilibrium

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At the equilibrium discussed in Section 3, the wage-employment pairs lie on the marginal revenue product of labor curve. At these points, the firm's iso-profit curves have zero slope, while the union's indifference curves are downward sloping. Thus, these points are not Pareto efficient. In fact, the set of Pareto efficient points—the contract curve—will lie to the right of the marginal revenue product of labor curve (See, for example, McDONALD and SOLOW [1981], OSWALD [1985], and Figure 1, below). While both the union and the firm have an incentive to move to one of these points at which both could be made better off, there is a problem if only wages are set contractually in that the firm will have an incentive to set employment along the marginal revenue product of labor curve. The question, then, is how to make it incentive compatible for the firm to hire more workers than static profit maximization would dictate.

As in ESPINOZA and RHEE [1989], we use punishment strategies off the equilibrium path to support outcomes that Pareto dominate the equilibrium on the marginal revenue product of labor curve. To keep the model as simple as possible, we avoid issues related to renegotiation proofness and focus on the simplest kind of punishment strategies, namely, reversion to an equilibrium of the stage game. This, again, is the same as in ESPINOSA and RHEE.

Our equilibrium concept, is a stochastic dynamic game extension of that used in ESPINOSA and RHEE. The argument is that if the union bargains as tough as it can, it is capable of forcing the contract wage up to the level given by the Nash solution discussed in Section 3. At the Pareto superior solution, the union relaxes its demands and allows the contract wage to be lower. It does so, however, expecting the firm to respond by setting employment along the contract curve. Should the firm behave like expected, the union continues to the same behavior in the future. Should the firm renege on this implicit agreement, the union forces the wage as high as it can. The firm thus has a choice between renegeing and taking a higher current profit or cooperating and ensuring lower wages in the future. For the firm to cooperate, the union must set the wage so that the firm's value from cooperating is at least as great as its value if it reneges. It is presumed that the union's choice in any period will be the highest wage at which the firm is willing to cooperate. That is, the firm should be indifferent between cooperating and renegeing. This point is chosen since it is the union that concedes, so, by deciding how far to concede, it is essentially picking the wage. The union picks the wage that maximizes its temporal utility because it does not have a commitment technology to bind its choices of wages in the future (VAN DER PLOEG [1987]).

In order to define the underlying equilibrium more rigorously, define the function  $L^n(K, \theta, w)$  as follows. For any wage  $w \geq \underline{w}$ ,  $L^n(K, \theta, w)$  gives the level of employment such that the marginal revenue product of

labor equals  $w$ , given that the capital stock is  $K$  and the current demand shock is  $\theta$ . Then, we make the following:

DEFINITION 1 : A **contract curve stationary equilibrium** of the game defined in Section 2 is a sequence  $\{w_t^c, L_t^c, K_{t+1}^c\}_{t=0}^\infty$  such that:

(i)  $w_t^c = w^c(K_t, \theta_t)$ ,  $L_t^c = L^c(K_t, \theta_t, w_t^c)$ , and  $K_{t+1}^c = K^c(K_t, \theta_t)$ ;  $\forall t \in \mathbb{N}_+$  and for  $(K_0, \theta_0)$  given.

(ii) Given  $K_t$  and  $\theta_t$ ,

$$(w_t, L_t) \in \mathbb{C} \equiv (w^c, L^c) \in [\underline{w}, \infty) \times [0, \infty) | \exists \bar{u} \geq 0 \ni (w^c, L^c) \\ = \operatorname{argmax}_{(w, L)} \{(pY - wL) \text{ s.t. } u(w, L) = \bar{u}\}.$$

$$(iii) \{K_{t+1}^c\}_{t=0}^\infty \\ = \operatorname{argmax}_{\{x_{t+1}\}_{t=0}^\infty} \Pi(\{x_{t+1}, L^c(x_t, \theta_t, w_t^c), w^c(x_t, \theta_t)\}_{t=0}^\infty; x_0, \theta_0)$$

s.t.

$$(17) \quad \Pi(\{K_{t+1}^c, L^c(K_t^c, \theta_t, w_t^c), w^c(K_t^c, \theta_t)\}_{t=0}^\infty; (K_0, \theta_0)) \\ = \Pi(\{K_{t+1}^d, L^d(K_t, \theta_t), w^d(K_t^d, \theta_t)\}_{t=0}^\infty; (K_0, \theta_0)) \\ x_{t+1} \geq (1 - \delta)x_t \quad \text{and for } x_0 = K_0;$$

where

$$(w_t^d, L_t^d, K_{t+1}^d) = \begin{cases} (w_t^c, L_t^n(K_t, \theta_t, w_t^c), K_{t+1}), & t = 0 \\ (w_t, L_t, K_{t+1}), & t \geq 1 \end{cases}$$

Note that  $L^c(K, \theta, w)$  gives the level of employment along the contract curve when the wage is  $w$ , the capital stock is  $K$  and the current demand shock is  $\theta$ <sup>11</sup>. Also, note that the defection path  $\{K_{t+1}^d, L_t^d, w_t^d\}_{t=0}^\infty$  is the same with the Nash path  $\{K_{t+1}, L_t, w_t\}_{t=0}^\infty$  for all periods other than period 0.

In period 0,  $(K_1^d, L_0^d, w_0^d)$  corresponds to a hypothetical situation where in Stage 2 of the game, the union accepts the contract curve wage rate,  $w^c(K_0, \theta)$ , anticipating the firm to choose the cooperative employment in period 0 and capital stock at the beginning of period 1, in Stage 3 of the game. But, under this defection path the firm chooses the Nash levels of these variables. The Nash employment in this case (Point  $D$  in Figure 1) is different from the one that would have prevailed under the Nash solution (*i.e.*, point  $N$ ). However, the capital stock at the beginning of period 1 would be the same along the two paths. For, as already explained, the capital stock at the beginning of next period is independent of the current employment and wage rate choices, in the Nash solution. The problem with

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11. Both of these functions can be found explicitly and it is easy to show that  $L^n$  is strictly decreasing in  $w$  while  $L^c$  is strictly increasing in  $w$ .  $L^c$ , however, is only defined over the interval  $[\underline{w}, (1 - \varphi)^{-1} \underline{w})$  and tends to infinity as  $w$  tends to  $(1 - \varphi)^{-1} \underline{w}$ .

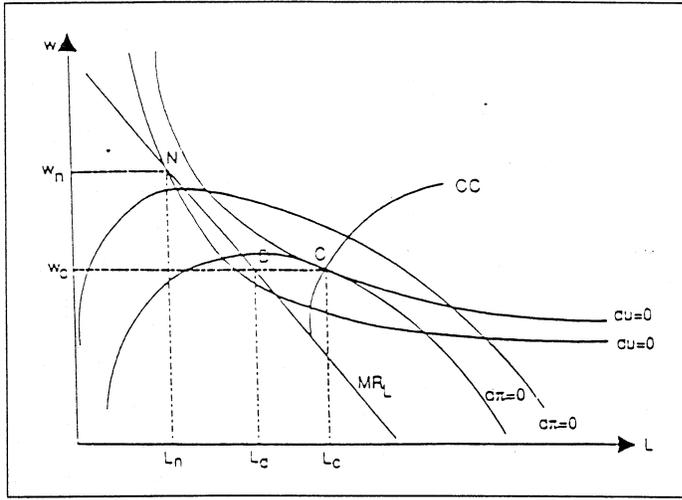


FIGURE 1

***Static Efficient and Inefficient Equilibria.***

this equilibrium, however, is that we cannot characterize the wage policy function before the investment policy function.

Our model relates to several models in the recent literature on other topics. First, it shares the set up of a stochastic, recursive problem with non-recursive constraints in GREEN [1987], PHELAN and TOWNSEND [1991], GREEN and OH [1991], ATKESON [1991], and ATKESON and LUCAS [1991]. In one sense, our model is easier to handle since there is no private information. However, since the probability distribution of the shocks depend on past realizations of the shock and the capital stock is irreversible, our model has dynamic elements that make applying the techniques developed in these papers difficult. A possibility is to follow fully recursive methods in solving our model may be provided by MARCET and MARIMON [1992] who introduce Lagrange multipliers, as mentioned earlier.

Let  $g(k, \theta)$  be some function which determines a wage rate for any combination of the capital stock and demand/productivity shock. Thus, initially, we make the following:

ASSUMPTION 1 :  $g(\cdot, \cdot) : [0, \bar{K}] \times \Theta \rightarrow [\underline{w}, \infty)$  is continuous in  $K$ ,  $\forall \theta \in \Theta$ .

$$\text{Let : } H^d(K, \theta; g) = A\theta^{\alpha_0} K^{\alpha_1} L^n [K, \theta; g(K, \theta)]^{\alpha_2} - g(K, \theta) L^n [K, \theta; g(K, \theta)]$$

(18) and

$$H^c(K, \theta; g) = A\theta^{\alpha_0} K^{\alpha_1} L^c [K, \theta; g(K, \theta)]^{\alpha_2} - g(K, \theta) L^c [K, \theta; g(K, \theta)]$$

Clearly,  $H^d(K, \theta; g)$  and  $H^c(K, \theta; g)$  give the firm's quasi-rent (revenue minus wage bill) as a function of  $(K, \theta)$  if it defects and sets employment along the marginal revenue product of labor curve and if it sets employment along the contract curve, respectively, when wages are determined by the function  $g(\cdot, \cdot)$ . Let  $V^d(K, \theta; g)$  and  $V^c(K, \theta; g)$  be the firm's value if it defects and if it always cooperates when wages are determined by  $g(\cdot, \cdot)$ . Then, the latter function must satisfy:

$$(19) \quad V^c(K, \theta(r); g) = \max_{K' \in \Gamma(K)} \{H^c(K, \theta(r); g) + (1 - \delta)K - K' \\ + \beta \sum_{s \in \mathcal{S}} \lambda_{rs} V^c(K', \theta(s); g)\}$$

As before, for all  $K > \bar{K}^c$ , where  $\bar{K}^c$  is defined analogously as  $\bar{K}$ , revenue will not even cover depreciation, so it will never pay to invest beyond  $\bar{K}^c$ . Thus, the return function,  $H^c(\cdot, \cdot; g) + (1 - \delta)K - K'$ , will be bounded above in  $[0, \bar{K}^c]$ <sup>12</sup>. We define the mapping  $T^c : \Psi \rightarrow \Psi$  as follows:

$$(20) \quad (T^c V)(K, \theta(r); g) \\ = \max_{K' \geq (1-\delta)K} \{H^c(K, \theta(r); g) \\ + (1 - \delta)K - K' + \beta \sum_{s \in \mathcal{S}} \lambda_{rs} V(K', \theta(r); g)\}$$

It can be shown as in Remark 2, above, that:

*Remark 3:* Given Assumption 1,  $T^c$  is continuous in  $\Psi$  and satisfies Blackwell's conditions, so that  $T^c$  is a contraction mapping with modulus  $\beta$  in  $\Psi$ .  $V^c$  is a fixed point of  $T^c$ , and since  $T^c$  is a contraction mapping,  $V^c$  is the unique fixed point of  $T^c$ . Moreover, if we begin with an arbitrary function  $V_0$  in  $\Psi$  and define the sequence  $\{V_j\}$  by  $V_{j+1} = T^c V_j$ , this sequence will converge monotonically to  $V^c$ .

Obviously, this result applies to any continuous function  $g(\cdot, \cdot)$ . Further, for any continuous  $g(\cdot, \cdot)$  the equilibrium policy may not be uniquely defined. For, we know nothing about the curvature properties of this function as well as those of the functional  $V^c$ . That is, we cannot establish the contract curve equilibrium counterpart of Part (b) of Remark 2. Instead, we let the underlying economic forces guide us in searching for a continuous wage function that satisfies the equilibrium conditions.

We search for a wage function that makes the firm indifferent between cooperating and renegeing at any combination  $(K, \theta)$ . Define  $x^n(K, \theta)$  to be the policy function associated with the Nash equilibrium defined earlier. That is,  $x^n(K, \theta)$  gives the firm's optimal choice of  $K_{t+1}$  given that  $(K_t, \theta_t) = (K, \theta)$ . Similarly, define  $x^c(K, \theta; g)$  to be a policy

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12.  $\bar{K}^c$  can be derived the same way as  $\bar{K}$  with  $\underline{w}$  replacing  $w$ . Since we only need assure that  $\bar{K}^c$  is finite, we will hereinafter ignore the distinction between  $\bar{K}$  and  $\bar{K}^c$ .

function associated with the contract curve value function when wages are determined by the function  $g(\cdot, \cdot)$ . Note that we do not know if  $x^c(\cdot, \cdot; g)$  is unique. If the firm reneges, it will get:

$$(21) \quad V^d(K, \theta(r); g) = H^d[K, \theta(r); g(K, \theta(r))] \\ + (1 - \delta)K - x^n(K, \theta(r)) \\ + \beta \sum_{s \in \mathcal{S}} \lambda_{rs} V^n[x^n(K, \theta(s)), \theta(s)]$$

Thus, for the firm to be indifferent between cooperating and reneging it must be that:

$$(22) \quad V^c(\cdot, \cdot; g) = V^d(\cdot, \cdot; g)$$

Suppose we have an initial guess,  $g_0$ , for the union's wage function. From this we can calculate the firm's cooperative value function,  $V^c(\cdot, \cdot; g_0)$ . Now, suppose the firm expects wages in all future periods to be determined by  $g_0$ . Then, for each  $(K, \theta)$ , we can calculate what the firm's value will be if it cooperates and what its value will be if it reneges, as a function of the current wage. We can then find the wage at which the firm is just indifferent between cooperating and reneging at each  $(K, \theta)$ . This gives us a new function, call it  $\hat{g}_0(K, \theta)$ . This introduces an operator, say,  $T^*$  such that  $\hat{g}_0(K, \theta) = T^* g_0(K, \theta)$ . In view of (15), (18), and (21):

$$(23) \quad H^d(K, \theta(r); T^* g(K, \theta(r))) + (1 - \delta)K - x^n(K, \theta(r)) \\ + \beta \sum_{s \in \mathcal{S}} \lambda_{rs} V^n[x^n(K, \theta(r)), \theta(r)] \\ = H^c(K, \theta(r); T^* g(K, \theta(r))) \\ + (1 - \delta)K - x^c(K, \theta(r); g(k, \theta(r))) + \beta \sum_{s \in \mathcal{S}} \lambda_{rs} V^c \\ \times [x^c[K, \theta(r); g(K, \theta(r))], \theta(s); g(k, \theta(r))]$$

The term on the l.h.s. of (23) is the firm's value if it reneges and the term on the r.h.s. of (23) is its value if it cooperates. Thus, we are looking for a fixed point of  $T^*$ .

Next, we show that under some mild technical assumptions,  $T^*$  has a fixed point in  $\Psi$ . First, we impose a "free disposal" type assumption, whereby at the beginning of any given period the firm can discard costlessly some of its existing capital stock. Then, if the firm at the beginning of a period has available capital  $K$  it may use  $x_1 \leq K$  capital during this period. However, we require that there exists an arbitrarily small lower bound on the stock of usable capital  $\underline{K}$ , beyond which production is not possible. Formally, if  $x_1$  stands for capital used in any given period and  $x_2$  stands for capital at the

beginning of the next this period, we restrict the feasibility correspondence and the quasi-rent function of the firm as follows:

ASSUMPTION 2 :

$$\begin{aligned} & \exists \underline{K} > 0 \ni \\ & \Gamma : [\underline{K}, \overline{K}] \rightarrow [\underline{K}, \overline{K}] \times [\underline{K}, \overline{K}] \ni \\ \Gamma(K) = & \{(x_1, x_2) | \underline{K} \leq x_1 \leq K, \max\{\underline{K}, (1-\delta)x_1\} \leq x_2 \leq \overline{K}\} \end{aligned}$$

is the feasibility correspondence and

$$\begin{aligned} H(x_1, \theta, w) : & [\underline{K}, \overline{K}] \times \Theta \times [\underline{w}, \infty) \rightarrow \mathbb{R} \ni \\ H(x_1, \theta, w) = & \theta x_1^{\alpha_1} L(x_1, \theta, w)^{\alpha_2} - wL(x_1, \theta, w) \end{aligned}$$

is the quasi-rent of the firm, in all periods.

Also, we restrict the range of  $g$  so that  $g(K, \theta)$  is bounded away from  $\underline{w}$ .

ASSUMPTION 3 :

$$\exists \Delta > 0 \ni g(\cdot, \cdot) : [\underline{K}, \overline{K}] \times \Theta \rightarrow [\underline{w} + \Delta, (1-\varphi)^{-1}\underline{w}]$$

If  $\underline{w}$  is the reservation wage of a union member, it is not very restrictive to focus attention on contract curve wages that are higher but arbitrarily close to  $\underline{w}$ . In the preceding assumption the upper bound on the range of  $g$  involves no loses of generality as:

*Remark 4* : The difference between the quasirents of the firm under defection and cooperation:

$$\begin{aligned} & H^d[K, \theta, w] - H^c[K, \theta, w] \equiv \\ & \{R[K, L^n(K, \theta, w)] - wL^n(K, \theta, w)\} - \{R[K, L^c(K, \theta, w)] - wL^c(K, \theta, w)\} \end{aligned}$$

is strictly increasing in  $w$  over  $[\underline{w}, (1-\varphi)^{-1}\underline{w}]$  and approaches plus infinity, as  $w$  approaches  $(1-\varphi)^{-1}\underline{w}$  from below. Here,  $H^c$  refers to quasi-rents along the contract curve and  $H^d$  refers to quasi-rents on the marginal revenue product curve.

*Proof* : See EBERWEIN and KOLLINTZAS [1993]. In view of (19) and (22), the last part of the preceeding remark implies:

COROLLARY 2 :  $\lim_{w \rightarrow (1-\varphi)^{-1}\underline{w}} V^d - V^c = +\infty$ . Hence,  $(1-\phi)^{-1}\underline{w}$  will never be part of a contract curve equilibrium.

We may now state the main result of this paper. It can be shown that  $T^*$ , defined in this way, maps from a compact convex subset of the space of bounded, continuous functions (with the sup-norm) back into itself and therefore that  $T^*$  has at least one fixed point. Moreover, this fixed point will satisfy the equilibrium condition set out at the beginning of this section.

**PROPOSITION 3 (Existence)** : Let  $\Psi$  be the space of bounded and continuous functions from  $[\underline{K}, \overline{K}] \times \Theta \rightarrow [\underline{w} + \Delta, (1 - \phi)^{-1} \underline{w}]$ . Given Assumptions 2 and 3 there exists a  $g^* \in \Psi$  such that  $g^* = T^*g^*$ .

*Proof* : See EBERWEIN and KOLLINTZAS [1993].

That is, Assumptions 2 and 3 imply Assumption 1. Hence,

**COROLLARY 4** : Given Assumptions 2 and 3, provided the union is at least as well off cooperating at a fixed point of  $T^*$  as at the Nash equilibrium, there will exist at least one contract curve equilibrium to the game of Section 2.

## 5 Quantitative Theory

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### 5.1. General

As usual with stochastic dynamic models, the nature of our problem precludes us from being able to derive analytical results about the invariant distribution of the underlying equilibria. Thus, we cannot obtain analytic answers to the questions motivating this paper. For example, we cannot provide an analytic answer to whether the union wage rate differential ( $w/\underline{w}$ ) behaves procyclically or countercyclically over the business cycle. And, in particular, what is the role of investment irreversibilities in this behavior? Some light to the answer of this type of questions can be shed, however, by the use of the computational experiment. Following recent developments in quantitative theory we calibrate our model on stylized facts from the U.S. Economy. Before doing so, however, we need an algorithm for computing  $g^*$ . Such an algorithm has been developed in EBERWEIN and KOLLINTZAS [1993] and is available on request.

### 5.2. Calibration

For the purposes of this section we view our model as characterizing behavior in the unionized sector of an economy consisting of unionized and nonunionized sectors. For this matter we shall think of  $\underline{w}$  as the wage rate in the nonunionized sector. Moreover, we shall assume that preferences and technology are the same in both sectors. These assumptions have two major implications. First, we can borrow parameter values from the Real Business Cycles literature (RBC); and, second, we can attribute differences in the behavioral patterns of the endogenous variables of the model to differences

in the nature of the labor market rather than differences in parameter values or preferences and technology. In addition we assumed that the productivity shock  $\theta$  may take a different value at the beginning of each month but that wages, employment, and investment are chosen at the beginning of each year, after the state of nature of the last month of the preview year has been observed and before the state of nature of the first month of the current year has been observed; and are constant over the year. Thus, the only variables that are permitted to change over the year are output and the price level. The reason for introducing this complication is a technical one. Namely, we could not make the nonnegativity constraint on investment ever binding in the ergodic distribution for “reasonable” differences in  $\theta$  between high and low productivity while still trying to match productivity transition probabilities with average duration of recessions and expansions in the post-World War II US data. More on this issue later, when the pertinent transition probabilities are set. Thus, we assigned values to the parameters of our model as follows:

$A = 1$ : This is a scale parameter and does not affect the variability or comovement properties of the equilibria.

$\delta = .1$ : This is the standard annual physical capital depreciation rate assumed in RBC models (KYDLAND and PRESCOTT [1982]). The preceding value assignment implies, as already noted, that we consider the time period of the model to be one year. Thus, the contract period between the union and the firm is also a year. The average contract length is probably longer than a year. The model can easily be extended to account for this but we chose not to do so here in order to avoid additional complications.

$\beta = .96$ : This corresponds to the standard annual pure rate of time preferences used in RBC models (KYDLAND and PRESCOTT [1982]).

$\alpha = .36$ : This parameter is the standard capital elasticity of output used in RBC models (KYDLAND and PRESCOTT [1982]).

$\varepsilon = .5$ : This corresponds to the typical price elasticity ( $1/\varepsilon$ ) of 2.0 found in the empirical monopoly, oligopoly, and monopolistic competition literature. See, e.g. HALL [1988].

$\mu = 1$  &  $\varphi = .5546218$ :  $\mu$  is the relative bargaining power of the union coefficient. It corresponds to an absolute bargaining power,  $\lambda$ , of the union coefficient of .5. That is, if  $V = U_U^\lambda U_F^{1-\lambda}$ ,  $\lambda \in (0, 1)$ , where  $U_U$ ,  $U_F$ , is the utility of the union and the firm, respectively. Then, maximizing  $V$  is equivalent to maximizing  $V^{\frac{1}{1-\lambda}} = U_U^{\frac{\lambda}{1-\lambda}} U_F = U_U^\mu U_F$ . SVENJAR [1986] in a deterministic and static version of our model could not reject the hypothesis of  $\lambda = .5$  for a number of U.S. firms with a unionized labor force. His results were obtained for various values of  $\varphi$ , that whenever estimated were done so rather imprecisely. So, we take  $\mu$  to be one while we let  $\varphi$  to be determined by the fact that a 1.4 wage differential is a reasonable upper bound to this differential according to the empirical labor literature LEWIS [1986] and the metaanalysis work of JARRELL and STANLEY [1990]. Thus, using (11) for  $w/\underline{w} = 1.4$ , yields  $\varphi = .5546418$ .

$$\theta^{\alpha_0} = \begin{cases} 3.2 & \text{Low productivity state} \\ 4.8 & \text{High productivity state} \end{cases}$$

$\theta^{\alpha_0}$  is also a scale parameter. So, its absolute value is not of any consequence. The ratio between high and low value of  $\theta$ , however, is important for whether the irreversibility constraint becomes binding at some point or not. We have chosen the low value of  $\theta$  to be 33% smaller than the high value so that the irreversibilities are indeed relevant in equilibria. Similar results were obtained letting the low value of  $\theta$  to be 26% smaller than the high value of this parameter.

$$\text{Prob}(\theta_{j+1} = 3.2 | \theta_j = 3.2) = .944444$$

$$\text{Prob}(\theta_{j+1} = 4.8 | \theta_j = 4.8) = .969697$$

These transition probabilities are set in such a way so as to match the average expected duration of post World War II US expansions and contractions. According to ZANROWITZ [1985] these average expected durations are 18 months for contractions and 33 months for expansions.

The monthly transition matrix of  $\begin{vmatrix} 0.944444 & 0.055556 \\ 0.030303 & 0.0969697 \end{vmatrix}$  corresponds to an

annual transition matrix of  $\begin{vmatrix} 0.944444 & 0.055556 \\ 0.030303 & 0.0969697 \end{vmatrix}^{12} = \begin{vmatrix} 0.5733 & 0.4267 \\ 0.2328 & .7672 \end{vmatrix}$ .

In computing the approximate solutions, we can replace  $\theta_L^{\alpha_0} = 3.2$  and  $\theta_H^{\alpha_0} = 4.8$  with their “certainty equivalent” values (3.6273536 and 4.5668556) values and solve the problem for annual outputs. That is, the “certainty equivalent” values yield the same expected revenue for the firm if  $\theta$  were fixed over the course of the year.

For these parameters, the operator  $T^n$  was applied iteratively starting with an initial function which is identically zero. The Nash union wage, for these parameters, is  $w = 1.4$ . The firm’s optimal policy has it investing up to  $K = 10.125$  when productivity is high ( $\theta = 4.5668566$ ). If productivity is low a sufficient number of periods in succession, the optimal policy has the capital stock falling to  $K = 8.8$ . When there is a transition from high to low productivity, the irreversibility constraint will be binding and the firm allows the capital to wear out through depreciation to 9.125. If productivity remains low, the firm chooses 0.587 investment and  $K$  falls to 8.8. There-after, the firm maintains the capital stock at 8.8 until there is a transition back to high productivity. There are thus, six possible states that occur along the equilibrium path corresponding to the three possible levels of the capital stock (8.8, 9.125, and 10.125) and the two possible equivalence levels of the productivity shock ( $\theta = 3.6273536$  or 4.5668556). Table 1 gives the ergodic distributions of the pertinent variables. That is, the values of the relevant variables across these six states along with the long-run relative frequencies with which each state occurs.

For the contract curve equilibrium  $T^c$  and  $T^*$  were jointly applied iteratively and converged for  $\sigma = .96$ . As in the Nash equilibrium, in the contract curve equilibrium there are also six possible states which occur along the equilibrium path, corresponding to three possible levels of the capital stock for each of the two possible levels of the productivity shock (at this equilibrium, the union is strictly better off than along the noncooperative path; also, the lower bounds on capital and wages never bind and the firm would never make use of its ability to freely dispose of some of its capital). In the contract curve solution, the firm invests up to  $K = 11.250$  following a period with high productivity (as compared to

TABLE 1

**Ergodic Distributions: Nash Equilibrium**

$s$	$K$	$\theta$	$K'$	$I$	$L$	$w$	$Y$	$p$	LRF
1	8.800	3.627	8.800	0.8800	1.350	1.400	34.88	.169	.116020
2	9.125	3.627	8.800	0.5875	1.363	1.400	35.56	.168	.086352
3	10.125	3.627	9.125	0.0125	1.401	1.400	37.57	.163	.150623
4	8.800	4.567	10.125	2.2050	1.894	1.400	68.68	.121	.086352
5	9.125	4.567	10.125	1.9125	1.912	1.400	70.01	.120	.064271
6	10.125	4.567	10.125	1.0125	1.966	1.400	73.97	.116	.496382

LRF = Long run frequency of state,  $s$ .

10.125 in the Nash equilibrium). If low productivity persists long enough (two or more periods) the capital stock falls to 10.000, after which the firm maintains this level until there is a transition to a high-productivity state (this is compared to  $K = 8.8$  in the Nash model). As expected, therefore, the firm increases its investment when on the contract curve with the union. The ergodic distribution of the contract curve equilibrium shows up in Table 2.

Wages are lower and employment is higher in the Nash, as compared to the contract curve, solution. (See, also Tables 3 and 4.) Again, this is as expected. One counterintuitive result is that wages are higher during periods of low productivity than in periods of high productivity.

In fact, in Table 2 the wage is decreasing in the capital stock when productivity is high (and more or less constant when productivity is low). For values of the capital stock outside the range shown in Table 2, wages are generally decreasing in the capital stock. The reason why this general

TABLE 2

**Ergodic Distributions: Contract Curve Equilibrium**

$s$	$K$	$\theta$	$K'$	$I$	$L$	$w$	$Y$	$p$	LRF
1	10.000	3.627	10.000	1.0000	3.058	1.222	61.65	.127	.116020
2	10.125	3.627	10.000	0.8875	3.068	1.222	62.04	.127	.086352
3	11.250	3.627	10.125	0.0000	3.154	1.222	65.60	.123	.150623
4	10.000	4.567	11.250	2.2500	4.202	1.207	119.75	.091	.086352
5	10.125	4.567	11.250	2.1375	4.214	1.207	120.51	.091	.064271
6	11.250	4.567	11.250	1.1250	4.317	1.205	127.11	.089	.496382

LRF = Long run frequency of state,  $s$ .

TABLE 3

**Ergodic Distribution Statistics: Nash Solution**

Statistic	$K$	$\theta$	$I$	$L$	$w$	$Y$	$p$
Mean	9.71	4.24	0.97	1.74	1.40	61.13	0.138
Standard dev.	0.57	0.20	0.58	0.28	.000	27.99	0.040
Coef. of var. (%)	5.92	4.76	60.2	15.8	0.00	45.79	28.77

trend changes during low-productivity conditions over the range of values shown in Table 2 appears to be due to the irreversibility of investment. The firm maintains a higher capital stock in the contract curve, as opposed to the Nash, solution. When there is a transition from high to low productivity, the firm is initially caught with a higher capital stock than it would like to maintain. It takes up to two periods for the firm to get back to the level it would like to maintain in the long run. If the firm were to choose a moment like this to renege, it could be stuck with too much capital even longer, since the long-run desired level of the capital stock is even smaller in the Nash solution. This appears to make the threat of Nash more severe, so the union need not concede as much to convince the firm to cooperate. Tables 3 and 4 point out that the model is consistent with broad stylized facts of relative variabilities. For example, investment fluctuates the most. Output fluctuates more than employment and capital while wages fluctuate the least of all variables.

TABLE 4

***Ergodic Distribution Statistics: Contract Curve Solution***

Statistic	$K$	$\theta$	$I$	$L$	$w$	$Y$	$p$
Mean	10.83	4.24	1.08	3.87	1.21	105.67	0.104
Standard dev.	0.57	0.20	0.61	0.57	.008	47.97	0.030
Coef. of var. (%)	5.26	4.76	56.2	14.7	0.67	45.40	28.48

Tables 5 and 6 give the correlations between several of the key variables of the model along the Nash and contract curve equilibrium paths. These tables contain the correlations between these variables and lagged values of the same variables for one, two, and three lags, respectively. For example, in Table 6, the element in the twentieth row and the first column states that the correlation between the capital stock and wages three periods earlier is  $-0.128$ , while the element in the nineteenth row and the third column shows that the correlation between the wage rate and the capital stock three periods earlier is  $-0.018$ .

Thus, all variables exhibit significant persistence (*i.e.*, significantly positive autocorrelations) except investment. The latter seems to be a drawback of the model that should be attributed to the lack of a smoothing propagation mechanism for investment (*i.e.*, adjustment costs). The cycles of all variables are nearly synchronous (*i.e.*, the cross correlations with the highest absolute values are those at the zero lag). Capital stock lags the cycles of all other variables by about one period. Capital, labor, investment are procyclical (*i.e.*, their cross correlations with output at zero lag are positive) while wages and prices are countercyclical. Since wages here should be interpreted as union wage premiums, this behavior accounts for the empirical stylized fact of countercyclical union wage premiums. Also, it should be noted that the countercyclicality of prices is likewise consistent with the stylized facts of many aggregate economies (FIORITO and KOLLINTZAS [1994]).

Incidentally, the preceding illustration of the table entries points out what appears to be one of the more remarkable features that arises from these

TABLE 5

*Ergodic Distribution Cross Correlations: Nash Solution*

	$K(t)$	$L(t)$	$W(t)$	$V(t)$	$I(t)$	$Y(t)$	$P(t)$	$\pi(t)$
$K(t)$	<b>1.000</b>	0.421	0.000	0.668	-.461	0.267	-.262	0.169
$L(t)$	0.421	<b>1.000</b>	0.000	0.957	0.601	0.625	-.629	0.467
$W(t)$	0.000	0.000	<b>0.000</b>	0.000	0.000	0.000	0.000	0.000
$V(t)$	0.668	0.957	0.000	<b>1.000</b>	0.346	0.599	-.601	0.437
$I(t)$	-.461	0.601	0.000	0.346	<b>1.000</b>	0.371	-.384	0.304
$Y(t)$	0.267	0.625	0.000	0.599	0.371	<b>1.000</b>	-.990	0.980
$P(t)$	-.262	-.629	0.000	-.601	-.384	-.990	<b>1.000</b>	-.976
$\pi(t)$	0.169	0.467	0.000	0.437	0.304	0.980	-.976	<b>1.000</b>
$K(t-1)$	<b>0.432</b>	0.152	0.000	0.264	-.238	0.097	-.095	0.059
$L(t-1)$	0.990	<b>0.424</b>	0.000	0.666	-.448	0.269	-.263	0.170
$W(t-1)$	0.000	0.000	<b>0.000</b>	0.000	0.000	0.000	0.000	0.000
$V(t-1)$	0.952	0.397	0.000	<b>0.632</b>	-.444	0.252	-.247	0.159
$I(t-1)$	0.602	0.280	0.000	0.423	<b>-.243</b>	0.177	-.173	0.114
$Y(t-1)$	0.617	0.556	0.000	0.655	-.000	<b>0.349</b>	-.348	0.247
$P(t-1)$	-.625	-.565	0.000	-.666	-.010	-.355	<b>0.355</b>	-.251
$\pi(t-1)$	0.461	0.533	0.000	0.586	0.115	0.334	-.334	<b>0.241</b>
$K(t-2)$	<b>0.147</b>	0.052	0.000	0.090	-.081	0.033	-.032	0.020
$L(t-2)$	0.436	<b>0.154</b>	0.000	0.267	-.240	0.098	-.096	0.059
$W(t-2)$	0.000	0.000	<b>0.000</b>	0.000	0.000	0.000	0.000	0.000
$V(t-2)$	0.405	0.143	0.000	<b>0.248</b>	-.223	0.091	-.089	0.055
$I(t-2)$	0.295	0.104	0.000	0.180	<b>-.162</b>	-.066	-.065	0.040
$Y(t-2)$	0.556	0.223	0.000	0.362	-.270	<b>0.141</b>	-.139	0.089
$P(t-2)$	-.573	-.228	0.000	-.371	0.281	-.145	<b>0.142</b>	-.090
$\pi(t-2)$	0.532	0.218	0.000	0.350	-.253	0.138	-.136	<b>0.087</b>
$K(t-3)$	<b>0.050</b>	0.018	0.000	0.031	-.028	0.011	-.011	0.007
$L(t-3)$	0.149	<b>0.052</b>	0.000	0.91	-.082	0.033	-.033	0.020
$W(t-3)$	0.000	0.000	<b>0.000</b>	0.000	0.000	0.000	0.000	0.000
$V(t-3)$	0.138	0.049	0.000	<b>0.084</b>	-.076	0.031	-.030	0.019
$I(t-3)$	0.100	0.035	0.000	0.061	<b>-.055</b>	0.023	-.022	0.014
$Y(t-3)$	0.225	0.080	0.000	0.138	-.124	<b>0.050</b>	-.050	0.031
$P(t-3)$	-.230	-.081	0.000	-.140	0.126	-.051	<b>0.051</b>	-.031
$\pi(t-3)$	0.222	0.078	0.000	0.136	-.122	0.050	-.049	<b>0.030</b>

tables. The capital stock is strongly correlated with recent past values of wages and employment, while the wages and employment have much weaker correlations with past values of the capital stock. This seems to be because irreversibilities make the capital stock more difficult to change, so its current level says less about the direction the firm will be going in the near future.

Sensitivity analysis of the solutions to the model with respect to the elasticity of demand revealed that irreversibilities become more crucial, the greater is this elasticity. For example, with an elasticity of 10, instead of 2, we have an increase in the spread between the largest and smallest levels of capital. Before, we had a total of three levels of capital along the equilibrium path. Now there are four levels of capital along the equilibrium path in the contract curve solution and seven in the punishment phase

equilibrium. Thus, irreversibility in capital binds more often and for longer periods of time when there is more elastic demand. One result of this is that the autocorrelations are now larger than before. Also, the wage in the contract-curve solution is now lower than before (from an average of 1.21 to an average of 1.166). This is because the labor demand curve is now more elastic (flatter) meaning that defection from the contract curve solution involves a larger decline in employment and thus a greater one-shot gain for the firm (this forces the union to concede more). The coefficients of variation for the most part increase, with the exception of output and prices. Most of these increase because of the greater spread in the scale of operations. Prices don't vary as much because demand is more elastic. So changes in output have smaller effects on prices.

TABLE 6

*Ergodic Distribution Cross Correlations: Contract Curve Solution*

	$K(t)$	$L(t)$	$W(t)$	$V(t)$	$I(t)$	$Y(t)$	$P(t)$	$\pi(t)$
$K(t)$	<b>1.000</b>	0.415	-.426	0.651	-.491	0.259	-.255	0.145
$L(t)$	0.415	<b>1.000</b>	-.998	0.961	0.588	0.616	-.620	0.399
$W(t)$	-.426	-.998	<b>1.000</b>	-.962	-.572	-.615	0.617	-.398
$V(t)$	0.651	0.961	-.962	<b>1.000</b>	0.340	0.593	-.594	0.377
$I(t)$	-.491	0.588	-.572	0.340	<b>1.000</b>	0.358	-.367	0.253
$Y(t)$	0.259	0.616	-.615	0.593	0.358	<b>1.000</b>	-.992	0.967
$P(t)$	-.255	-.620	0.617	-.594	-.367	-.992	<b>1.000</b>	-.962
$\pi(t)$	0.145	0.399	-.398	0.377	0.253	0.967	-.962	<b>1.000</b>
$K(t-1)$	<b>0.378</b>	0.144	-.151	0.236	-.199	0.090	-.089	0.050
$L(t-1)$	0.998	<b>0.414</b>	-.425	0.650	-.489	0.259	-.254	0.145
$W(t-1)$	-.991	-.412	<b>0.423</b>	-.646	0.485	-.258	0.253	-.144
$V(t-1)$	0.947	0.389	-.401	<b>0.614</b>	-.469	0.244	-.239	0.136
$I(t-1)$	0.621	0.268	-.273	0.413	-. <b>293</b>	0.168	-.164	0.094
$Y(t-1)$	0.613	0.551	-.557	0.647	-.019	<b>0.341</b>	-.340	0.210
$P(t-1)$	-.619	-.560	0.562	-.656	0.012	-.346	<b>0.346</b>	-.213
$\pi(t-1)$	0.399	0.514	-.516	0.551	0.138	0.138	-.318	<b>0.201</b>
$K(t-2)$	<b>0.129</b>	0.049	-.051	0.080	-.068	0.031	-.030	0.017
$L(t-2)$	0.378	<b>0.144</b>	-.150	0.236	-.199	0.090	-.089	0.050
$W(t-2)$	-.377	-.144	<b>0.150</b>	-.235	0.198	-.090	0.088	-.049
$V(t-2)$	0.355	0.135	-.141	<b>0.221</b>	-.187	0.085	-.083	0.047
$I(t-2)$	0.247	0.094	-.098	0.154	-. <b>130</b>	0.059	-.058	0.032
$Y(t-2)$	0.532	0.215	-.222	0.342	-.267	<b>0.135</b>	-.132	0.075
$P(t-2)$	-.544	-.219	0.226	-.349	0.274	-.137	<b>0.135</b>	-.076
$\pi(t-2)$	0.505	0.207	-.213	0.327	-.251	0.129	-.127	<b>0.072</b>
$K(t-3)$	<b>0.044</b>	0.017	-.018	0.027	-.023	0.011	-.010	0.006
$L(t-3)$	0.129	<b>0.049</b>	-.051	0.080	-.068	0.031	-.030	0.017
$W(t-3)$	-.128	-.049	<b>0.051</b>	-.080	0.068	-.031	0.030	-.017
$V(t-3)$	0.121	0.046	-.048	<b>0.075</b>	-.064	0.029	-.028	0.016
$I(t-3)$	0.084	0.032	-.034	0.053	-. <b>044</b>	0.020	-.020	0.011
$Y(t-3)$	0.195	0.074	-.078	0.122	-.103	<b>0.047</b>	-.046	0.026
$P(t-3)$	-.199	-.076	0.079	-.124	0.105	-.048	<b>0.047</b>	-.026
$\pi(t-3)$	0.188	0.072	-.075	0.117	-.099	0.045	-.044	<b>0.025</b>

## 6 Conclusion

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In this paper we look at the quantitative properties of a labor market consisting of a union and a firm. We introduce physical capital and investment irreversibilities in a model that may be thought of as a stochastic dynamic extension of the repeated bargaining game developed by ESPINOSA and RHEE [1989]. It turns out that the ergodic distribution of the contract curve equilibrium exhibits many of the stylized facts of labor markets with significant union presence. In particular the model can account for the low variability of wages as well as the counter-cyclical union wage differentials (and total average wages in some countries). More significantly, these results are driven by an interaction of dynamic bargaining and investment irreversibilities.

There are two extensions of our work that seem desirable. First, we would like to consider the sensitivity of our findings vis a vis the nature of the solution concept. In our model the solution is the highest wage at which the firm would be willing to cooperate. This is only one of many possible incentive-compatible equilibria and we would like to investigate the properties of this equilibria in a manner analogous to ATKESON [1991].

Second, we want to integrate the labor market developed here into a more general equilibrium model (that would include a competitive sector) in the spirit of KYDLAND and PRESCOTT [1982]. We hope that this generalization will permit us to compare the labor markets of countries with different union densities.

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