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A Microeconomic Model of Female Labour Supply in the Presence of Unemployment and Underemployment

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ABSTRACT. – In this paper we model female labor supply within a discrete choice framework. Two different rationing mechanisms are incorporated into the model: involuntary unemployment and underemployment. The model is set-up as a generalization of the Double-Hurdle model of CRAGG [1971]. Our estimation strategy consists in estimating the model first without taking into account the rationing mechanisms (Model 1). Next we introduce these mechanisms but do not use the information on sample separation (Model 2). Finally, the information is used and the likelihood altered accordingly (Model 3). It turns out the information on rationing makes a dramatic difference on the preference parameters and hence, on desired hours of work. This suggest that the labor supply models that incorporate unobserved constraints are potentially biasing the estimation of preference parameters. Our results show that the uncompensated wage and income elasticities are slightly overestimated in Model 1 and largely underestimated in Model 2 relative to Model 3.

Un modèle microéconomique de l'offre de travail des femmes en présence de sous-emploi et de chômage involontaire

RÉSUMÉ. – Dans ce travail nous modélisons l'offre de travail des femmes à l'intérieur d'un cadre de choix discrets. Deux mécanismes de rationnement sont introduits pour tenir compte de la probabilité de chômage involontaire et de sous-emploi. Nous estimons le modèle dans un premier temps sans tenir compte de ces mécanismes de rationnement. Dans un deuxième temps, ces mécanismes sont introduits sans utiliser toutefois l'information relative au rationnement. Enfin, cette information est introduite dans le modèle et la fonction de vraisemblance est modifiée de façon appropriée. Les paramètres de la fonction d'utilité, et donc les heures désirées, sont très sensibles à l'introduction de l'information sur le rationnement. En conséquence, les modèles qui incorporent des contraintes non-observées sur le marché du travail sont susceptibles de biaiser les estimations paramètres de la fonction d'utilité. Enfin nos résultats empiriques montrent que les élasticités salaire et revenu non compensées sont légèrement sur-estimées lorsque les contraintes de rationnement ne sont pas introduites dans le modèle et sensiblement sous-estimées lorsque l'information sur les contraintes n'est pas explicitement incorporée dans le modèle.

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1 Introduction

The traditional approach to modeling labor supply assumes that an individual can work as many hours as desired. Hence, in the majority of empirical studies it is customarily assumed that the observed hours of work correspond to desired hours. HAM [1982] first pointed out that some workers may be involuntarily unemployed or underemployed for a variety of reasons. To the extent that the incidence of these constraints is correlated to individual characteristics, omitting constrained individuals from the sample may bias the structural parameters of the labor supply model. In the same vein, KAHN and LANG [1991], using Canadian data on desired hours of work, found that using observed rather than desired hours of work causes wages elasticities in simple labor supply models to be slightly upward biased.

The work of HAM [1982] has prompted many authors to investigate the nature of the constraints on the labor market. Thus, BLUNDELL and LAISNEY [1988] considered the consequences of misclassifying job seekers as voluntarily unemployed individuals within a discrete choice framework. BLUNDELL, HAM and MEGHIR [1987] proposed a model along the same line that allowed unemployed individuals to have positive desired hours with a certain probability. However, for those observed to work, desired and observed hours were assumed to coincide. ILMAKUNNAS and PUDNEY [1990] modeled female labor supply within a discrete choice framework and allowed for both underemployment and overemployment.

A number of authors have questioned the validity of the standard labor supply model in view of its poor fit to cross sectional data on hours of work. Their efforts have thus been directed towards demand-side constraints that limit the number of available hours on the market. Thus, DICKENS and LUNDBERG [1993] and TUMMERS and WOITTIEZ [1991] have proposed a model in which an individual receives a random number of job offers, each associated with weekly hours drawn from a discrete distribution common to all individuals. Involuntary unemployment is akin to drawing zero job offers or to drawing a job offer with zero hours. Underemployment arises whenever the best "draw" does not correspond to desired hours.

There are two difficulties with most of these studies. First, they are based on data sets that contain no information on perceived constraints (one notable exception is ILMAKUNNAS and PUDNEY [1990]). Hence, one is asking a cross section on the distribution of hours to identify simultaneously the preference parameters and the rationing mechanism. Second, the probability of being rationed is not related to individual characteristics. This is a source of potential bias if such a link exists, as found in HAM [1982] and ILMAKUNNAS and PUDNEY [1990].

In this paper we attempt to avoid these problems by using a data set that contains information on perceived constraints for workers and non workers alike. We thus model labor supply as a discrete choice between a finite number of weekly hours, taking into account two different rationing mechanisms. The first of these concerns involuntary unemployment and follows the work of BLUNDELL *et al.* [1987]. The second addresses demand-side constraints that translate into underemployment. By jointly estimating the preference parameters and the rationing mechanisms, our model generalizes the Double-Hurdle model of CRAGG [1971].

The empirical strategy consists in estimating the model first without taking into account the existence of constraints on the market. Next, we build these constraints into the model but do not use the information on perceived constraints. Only individual characteristics are used to predict both forms of rationing. This is a common procedure used when, as is usually the case, no information on rationing is available. Finally, we estimate the model using the information. The purpose of this is to see whether the preference parameters, and thus the predicted desired hours of work, differ significantly in the two cases. As it turns out, predicted desired hours are wildly different.

2 The Model

Consider an individual who maximizes a utility function $U(C, h; X, \varepsilon, \theta)$ by choosing weekly hours of work, h, and consumption, C, where X is a vector of observable characteristics, ε is a random component and θ is a vector of parameters of appropriate dimension. The utility function is assumed continuous and quasi-concave. We assume that every individual faces a finite set of job opportunities denoted $H = h_i, i = 0, 1, 2,$ where h_0 represents non-participation, h_1 is part-time work and h_2 is full-time work. The consumption level associated with h_i is given by $C_i = y + \omega h_i - T(\omega h_i, X, y)$ where y is non-labor income, ω is the gross wage rate and $T(\cdot)$ is the tax-transfer system which is a function of labor and non-labor income as well as X.[1] In this simple model, the probability that an individual is observed at h_i corresponds to

$$(1) \qquad P[U(C_i, h_i; X, \varepsilon, \theta) > U(C_j, h_j; X, \varepsilon, \theta)] \quad \forall \quad i \neq j$$

Many authors have studied labor supply within this framework (BLUNDELL and LAISNEY [1988], FRAKER and MOFFITT [1988], ILMAKUNNAS and PUDNEY [1990], MOFFITT [1984] and ZABALZA, PISSARIDES and BARTON [1980]). In every

1. Some authors allow the wage to depend on hours of work (MOFFITT [1984], TUMMERS and WOITTIEZ [1991]). In our case, doing so would make the computation of the likelihood function cumbersome. Indeed, the likelihood function would have to be expressed conditionally on the wage rate, and the error term of the wage equation integrated out (see FRAKER and MOFFITT [1988]). Since the budget constraint is a function of the wage rate, it would have to be generated at each single point of the integration, making the computation of the likelihood function prohibitively time consuming.

case, convexity of the budget set has been assumed.[2] It has been reckoned that casting the labor supply model within a discrete choice framework is appropriate for studying the labor supply of women since, as found in most studies, the major source of supply response is usually between regimes rather than along a continuous set of hours.

2.1. Rationing Mechanisms

The model so far assumes individuals are free to choose from the weekly hours in H the h_i that maximizes their utility. Yet, many factors may prevent them from doing so. For instance, some individuals may be constrained to work less than desired (ILMAKUNNAS and PUDNEY [1990], DICKENS and LUNDBERG [1993], KAHN and LANG [1991]) while others may be involuntarily unemployed (BLUNDELL *et al.* [1987], HAM [1982], PHIPPS [1990]). In this paper we shall assume that the two underlying mechanisms are different.[3] More precisely, we define two index functions that determine whether an individual is rationed or not:

$$R_0^* = Z_0\gamma_0 + \nu_0; \qquad R_1^* = Z_1\gamma_1 + \nu_1, \tag{2}$$

where R_i^* is a latent variable, Z_i is a vector of individual characteristics, γ_i is an appropriately dimensioned vector of parameters and ν_i is an error term assumed normally distributed with mean equal to zero and unit variance, $i = 0, 1$. The observable counterpart to R_i^* is

$$R_i = \begin{cases} 1 \text{ iff } R_i^* \geq 0 \\ 0 \text{ otherwise} \end{cases}. \tag{3}$$

R_0^* determines whether an individual who wishes to work is able to find work. Similarly, R_1^* determines whether a working individual is underemployed, *i.e.* constrained to work part-time when full-time is desired. Hence, the probability of observing $h = 0$ is given by:

$$\begin{aligned} P(h=0) = & P[U(C_0, 0; \cdot) \geq \max_{j>0} U(C_j, \cdot)] + \\ & P[U(C_0, 0; \cdot) < \max_{j \neq 0} U(C_j, h_j; \cdot, \nu_0 \geq -Z_0\gamma_0]. \end{aligned} \tag{4}$$

This equation simply says that an individual observed to work zero hours may be doing so for two reasons: (i) zero hours is optimal; (ii) zero hours

2. FRAKER and MOFFITT [1988] allow non-convexities but implicitly assume that the budget set is not "too non-convex".

3. The model could be written to encompass the possibility that some women are overemployed, *i.e.* that they would prefer to work less weekly hours. In our sample, only 21 women reported being overemployed. Hence it was not deemed worthwhile including additional regimes in the model.

is not optimal but no work is available. This is the so-called Double-Hurdle model of CRAGG [1971]. It has been used in the empirical labor literature by BLUNDELL *et al.* [1987] among others. The probability of observing an individual working part-time is given by:

$$
\begin{aligned}
P(h = h_1) =& P[U(C_1, h_1; \cdot) \geq \max_{j \neq 1} U(C_j, h_j; \cdot), \nu_0 < -Z_0\gamma_0] + \\
& P[U(C_1, h_1; \cdot) < U(C_2, h_2; \cdot), \nu_0 < -Z_0\gamma_0, \nu_1 \geq -Z_1\gamma_1].
\end{aligned} \tag{5}
$$

The first term on the right hand side says that h_1 is optimal and work is available. The second member says that h_1 is not optimal, that work is available but the individual is constrained to work less than desired. Finally, the probability of observing h_2 is:

$$
\begin{aligned}
& P(h = h_2) \\
& \quad = P[U(C_2, h_2; \cdot) > \max_{j \neq 2} U(C_j, h_j; \cdot), \nu_0 < -Z_0\gamma_0, \nu_1 < -Z_1\gamma_1].
\end{aligned} \tag{6}
$$

This simply says that someone working h_2 hours cannot be constrained in any way.

2.2. **Preference Specification**

In order to estimate the model, we must specify a functional form for the utility function. Following HAUSMAN [1981], we use the following direct utility function:

$$
\log U(C, h; \alpha, \beta, \gamma) = -\log(\gamma - \beta h) - \frac{\beta(h - \alpha - \beta C)}{\gamma - \beta h}. \tag{7}
$$

This functional form has been used extensively in the empirical literature on labor supply. Maximizing this utility function with respect to a linear budget constraint yields a linear labor supply function:

$$
h = \alpha + \beta y + \gamma \omega. \tag{8}
$$

Slutsky conditions are satisfied whenever $\gamma - \beta h \geq 0$ or more generally when $(\gamma - \beta\alpha) - \gamma\beta\omega - \beta^2 y \geq 0$. The main advantage of this functional form is that it is relatively easy to estimate. Its main drawback is that it is monotonic in ω (see HAUSMAN [1981] and STERN [1986] for a detailed discussion).

To introduce individual heterogeneity we write α as $\alpha = X\psi + \varepsilon$, where X and ε have been defined previously and ψ is an appropriately dimensioned vector of parameters. Since we are considering a finite number of weekly hours of work, we can divide the domain of ε into intervals that correspond to each h_i. For example, by equating $U(c_1, h_1; \cdot)$ and $U(c_2, h_2; \cdot)$ we can compute a value of ε that makes an individual indifferent between h_1 and h_2. If we repeat this procedure for all $h_i \in H$, each h_i will correspond to a given interval on the domain of ε. Simple algebra shows that the value of ε that makes h_i and h_j indifferent is given by

$$\varepsilon_{i,j} = \frac{(\gamma - \beta h_j)(\gamma - \beta h_i)}{\beta^2 (h_j - h_i)} \log\left(\frac{\gamma - \beta h_j}{\gamma - \beta h_i}\right) + \frac{\gamma}{\beta} - \gamma\left(\frac{c_j - c_i}{h_j - h_i}\right) - \beta\left(\frac{h_j c_i - h_i c_j}{h_j - h_i}\right) - X\psi. \tag{9}$$

If we ignore rationing for the moment, the probability that an individual is observed at zero hours of work is given by

$$P(h = h_0) = P(\varepsilon < \min_{j \neq 0} \varepsilon_{0,j}), \tag{10}$$

where $\varepsilon_{0,j}$ is the value of ε that makes an individual indifferent between not working and working h_j hours. Similarly, for an interior solution the probability is:

$$P(h = h_1) = P(\varepsilon_{0,1} < \varepsilon \leq \varepsilon_{1,2}). \tag{11}$$

Finally, for an individual working h_2 hours, this probability is:

$$P(h = h_2) = P(\varepsilon \geq \max_{j \neq 2} \varepsilon_{2,j}). \tag{12}$$

2.3. Likelihood Functions

The estimation strategy that we follow is first to ignore the existence of rationing. The model in this case boils down to a discrete choice between participation, part-time work and full-time work. Such a model is very common in the empirical literature on female labour supply. In what follows, it will be referred to as "Model 1". Next, we incorporate into the model the possibility that an individual can be constrained on the market. As shown previously, the probability of observing an individual working h_i is the sum of two distinct probabilities. When information on rationing is not used, we have three regimes as in the previous model since we do not know which individual is constrained. This is similar to a switching regression model with unobserved sample separation (MADDALA [1983]). This version of the model will be referred to as "Model 2". Finally, we exploit the information on rationing. This allows us to decompose the probabilities associated with non-participation and part-time work into two parts. We will refer to this model as "Model 3".

Model 1 From (10)-(12) the likelihood function is:

$$
\begin{aligned}
L_1 = & \prod_{\Omega_0} P(\varepsilon \leq \min_{j \neq 0} \varepsilon_{0,j}) \\
& \times \prod_{\Omega_1} P(\varepsilon_{0,1} \leq \varepsilon < \varepsilon_{1,2}) \times \prod_{\Omega_2} P(\varepsilon \geq \max_{j<2} \varepsilon_{2,j}),
\end{aligned}
\tag{13}
$$

where Ω_0 is the sub-sample of individuals observed as non-participants, Ω_1 is the sub-sample of those observed at h_1, and Ω_2 is the sub-sample of those observed working full-time.

Model 2 From (10)-(12) and (5)-(6), the relevant likelihood when rationing is incorporated into the model is:

$$
\begin{aligned}
L_2 = & \prod_{\Omega_0} [P(\varepsilon \leq \min_{j \neq 0} \varepsilon_{0,j}) + P(\varepsilon > \min_{j \neq 0} \varepsilon_{0,j}, \nu_0 \geq -Z_0\gamma_0)] \\
& \times \prod_{\Omega_1} [P(\varepsilon_{0,1} \leq \varepsilon_{0,j} < \varepsilon_{1,2}) \\
& + P(\varepsilon \geq \varepsilon_{1,2}, \nu_0 < -Z_0\gamma_0, \nu_1 \geq -Z_1\gamma_1)] \\
& \times \prod_{\Omega_2} [P(\varepsilon \geq \max_{j<2} \varepsilon_{2,j}, \nu_0 < -Z_0\gamma_0, \nu_1 < -Z_1\gamma_1)].
\end{aligned}
\tag{14}
$$

Model 3 When information on rationing is used, we know in what regime the individual belongs. The likelihood function in this case is given by:

$$
\begin{aligned}
L = & \prod_{\Omega_0^-} [P(\varepsilon \leq \min_{j \neq 0} \varepsilon_{0,j})] \times \prod_{\Omega_0^+} [P(\varepsilon > \min_{j \neq 0} \varepsilon_{0,j}, \nu_0 \geq -Z_0\gamma_0)] \\
& \times \prod_{\Omega_1^-} [P(\varepsilon_{0,1} \leq \varepsilon_{0,j} < \varepsilon_{1,2})] \\
& \times \prod_{\Omega_1^+} [P(\varepsilon \geq \varepsilon_{1,2}, \nu_0 < -Z_0\gamma_0, \nu_1 \geq -Z_1\gamma_1)] \\
& \times \prod_{\Omega_2} [P(\varepsilon \geq \max_{j<2} \varepsilon_{2,j}, \nu_0 < -Z_0\gamma_0, \nu_1 < -Z_1\gamma_1)],
\end{aligned}
\tag{15}
$$

where Ω_0^- is the sub-sample of individuals observed as non-participants who are not constrained, and Ω_0^+ is the sub-sample of involuntarily unemployed individuals. Ω_1^- and Ω_1^+ are similarly defined as the sub-sample of part-time unconstrained and part-time constrained workers, respectively.

The purpose of estimating the model twice is to verify to what extent the structural (utility) parameters can be estimated using a purely probabilistic approach to rationing and using as regressors exogenous individual characteristics. Since information on rationing is seldom provided in empirical surveys, this approach has used been extensively in the literature (PHIPPS [1990], BLUNDELL *et al.* [1987], BLUNDELL and LAISNEY [1988]) without ever being formally tested.[4] In a different context, LACROIX and FORTIN [1992] have found the structural parameters of a simultaneous labor supply model to be sensitive to the treatment of the information on rationing.

There are three error terms in the model, ε, ν_0, ν_1. Since an individual cannot be simultaneously observed constrained not to work and constrained to work fewer hours than desired, $\rho_{0,1}$ cannot be estimated. Furthermore, a frequent problem with switching regression models when the sample separation is unknown is the possibility of convergence to a point where the correlation between the residuals is either +1 or –1 (*e.g.* GOLDFELD and QUANDT [1978]). This problem can be exacerbated if, as in our case, some regimes have few observations (KAPTEYN, KOOREMAN and VAN SOEST [1990]). We encountered this problem for each different specification that was estimated. Hence we imposed $\rho_{\varepsilon,0} = \rho_{\varepsilon,1} = 0$ to allow estimation of the model.

3 Data and Budget Constraint

3.1. The Data

The data we use come from a survey that was conducted in 1986 in the metropolitan area of Quebec City, Canada. The survey technique was based on the methodology used by Statistics Canada in its Labor Force Survey (see LACROIX and FORTIN [1992] and LEMIEUX, FORTIN and FRÉCHETTE [1994] for details). Once we eliminate women aged over 65 and students, we are left with 927 women in our sample, of whom 712 are married. Table I presents summary statistics for the sample. As expected, single women are younger on average, have fewer children in both age groups (0-6 and 7-17) and have more schooling. Part of the difference in schooling may partly reflect cohort effects. Also, the proportion of bilingual women (english as a second language) is considerably higher among single women. The dichotomous variable HEALTH takes the value one if the individual considers herself as having a good or very good health. Similarly, the dichotomous variable IMMIGRANT is equal to one if the individual was born outside Canada.[5] Note also that the distribution of single and married women amongst broad

4. Monte Carlo evidence suggests there is a considerable loss of information if sample separation is not known (see GOLDFELD and QUANDT [1975]).

5. The reason for including this variable is to see whether being an immigrant has an impact on the labor market outcome. The metropolitan area of Quebec City has historically attracted few immigrants. In 1985, 95% of the population was white, francophone and catholic. It is thus considered the most homogeneous sizable city of the province.

TABLE 1

Sample Statistics

Variable	Single	Married	Total
Bilingual	0.321	0.264	0.277
Immigrant	0.023	0.039	0.036
Health	0.879	0.935	0.922
Schooling			
Primary	0.121	0.149	0.142
Secondary	0.428	0.484	0.471
College	0.256	0.232	0.237
University	0.195	0.135	0.149
Manufacturing	0.051	0.041	0.043
Transport	0.037	0.032	0.033
Finance	0.183	0.074	0.076
Government	0.098	0.059	0.068
Services	0.413	0.355	0.369
$R_0^{\dagger}$	0.147	0.056	0.077
$R_1^{\ddagger}$	0.301	0.103	0.149
Age	35.172	39.081	38.175
	(11.840)	(9.836)	(10.459)
Child6	0.126	0.349	0.298
	(0.419)	(0.642)	(0.606)
Child17	0.469	1.028	0.986
	(1.008)	(1.081)	(1.090)
Experience	10.460	10.828	10.673
	(8.899)	(7.505)	(7.871)
Weekly Hours[1]	32.942	31.870	32.163
	(7.739)	(8.587)	(8.371)
Wage[2]	8.697	11.055	10.486
	(4.822)	(5.749)	(5.759)
Non Labor Income	1690.293	1163.762	1285.871
	(4781.111)	(5579.902)	(5407.264)
Observations	215	712	927

† Conditional on not working.
‡ Conditional on working less than 35 hours per week
[1] Conditional on being greater than zero.
[2] Reported gross wage rate.

industrial categories is quite similar. On the whole, service industries and the public service make up as much as 75% of total employment.

The two dummy variables R_0 and R_1 were constructed using several questions asked in the survey. First, for those not working, the survey asked whether a disease or an illness prevented them from working. Those reporting an illness were not considered involuntarily unemployed. The others were then asked whether they had searched for a job during the year. If so, they had to report the number of weeks and the weekly hours spent looking for a job. In order to qualify as involuntarily unemployed, individuals had to file positive numbers for both variables.[6] Overall, 14.7%

6. Five women reported looking for a job but failed to provide all the required information. They were thus not considered involuntarily unemployed.

of single women and 5.6% of married women who were not working at the time of the survey were seeking work and are thus considered involuntarily unemployed. Second, conditional on working, individuals were asked if they were satisfied with their current weekly hours of work or whether they would prefer to work more or less hours at the same wage rate. It turns out all the women who wanted to work more were working less than 35 hours per week. It thus seems natural to define part-time work as work involving 34 hours of work per week or less.[7] Only 21 women reported wanting to work less weekly hours of work. The average desired decrease was 1.9 hours/week with a standard deviation of 0.86 (min = 1, max = 4). This response was deemed insufficient to warrant including it in the model. There are thus 364 women not working in our sample, of whom 26 reported being constrained not to work, and there are 204 women working part-time, of whom 56 reported being underemployed. Finally, there are 359 women working full-time.

3.2. Budget Constraint

In this paper, the tax laws and most transfer programs in place in 1985 are taken into account in computing the net weekly earnings of every women. On the tax side, there are as many as 25 provincial and 15 federal tax brackets. Federal and provincial brackets overlap and have different tax rates. The highest (combined) marginal tax rate in 1985 was 62% for earnings above $71 395 (in 1985 dollars). In computing net weekly earnings we also include U.I. contributions, pension plan contributions, married couple deductions, child tax credit and property tax credit. These various programs are susceptible to create non-convexities at low hours of work or for low-income workers. The government transfers are made up of essentially four programs: (1) federal and provincial family allowances (not means-tested); (2) provincial welfare program[8]; (3) WIS (Work Income Supplement), a negative income tax program which adds between 18% to 62% of labor earnings for earnings ranging from $3 500 to $10 332; (4) Availability Allowance, which is a lump-sum transfer commensurate to

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7. Two additional reasons prompted us to define part-time work this way. First, Statistics Canada defines full-time work as work involving 35 hours/week or more. Hence, most studies utilizing canadian data use this definition. Secondly, and most importantly, the typical work-week in the civil service in Québec is 35 1/2 hours/week. Since this probably constitutes a lower bound, it seems reasonable to define part-time work the way we did.

8. In 1985, a single mother with one child was entitled to $7 128. This program is means-tested with an implicit marginal tax rate of approximately 90%. The transfer is also tested against assets. In our sample, as many as 204 single women out of 215 and 12 married women turned out to be eligible to welfare payments. It is assumed to simplify that none of them own assets beyond allowable levels.

the number of children below the age of six to compensate daycare expenses or to compensate individuals who decide not to work to look after their children.[9]

In order to generate the budget constraint of each married women, we first compute their husband's net income at reported hours of work.[10] Next, this income is added to each married woman's non-labor income. For single women, the imputed income at zero hours of work is calculated as the social welfare payment they are entitled to. Naturally, we do not observe a wage rate of women that are not working. The procedure we follow consists in predicting a wage rate based on an auxiliary (gross) wage equation estimated from the sample of working women.[11] Then for each woman, we computed the expected net income at $h = 0$, $h = 20$ and $h = 40$ taking into account the tax-transfer programs discussed above (C_0, C_1 and C_2 in the text). According to the tax-transfer provisions, and using household information, only 12 married women were entitled to welfare payments at $h = 0$. Single women, naturally, were all eligible to welfare payments.

4 Main Findings

4.1. Parameter Estimates

Table 2 presents the parameter estimates of the different versions of the model. Recall that Model 1 refers to the case where no account is made of the rationing mechanisms. This is the approach most often encountered in the empirical literature. Model 2 corresponds to the case where information on sample separation between hurdles is not know. The parameter estimates are obtained by maximizing (14). This approach is used more and more frequently to ascertain the impact of different policies in the presence of potential (*i.e.* unobserved) rationing. Finally, Model 3 refers to the case where information on sample separation is available. This version refers (15).

9. The yearly payment for the first child is \$300, and \$500 for each additional child.
10. It must be noted that for all practical purposes, the taxation system in Canada treats husbands and wives separately.
11. There was no evidence of sample selectivity in our parameter estimates. We also investigated the possibility that the wage profile may depict a concave relationship with hours of work (see MOFFITT [1984]). We could not reject the null assumption of a flat profile. For the sake of brevity, we do not report these results here but they are available on request.

TABLE 2

Estimation of Preference Parameters and Rationing Mechanisms*

Preference Parameters	Model 1	Model 2	Model 3
Intercept	0.357 (4.5)	0.507 (5.0)	0.611 (5.6)
Age	-0.011 (6.7)	-0.007 (3.4)	-0.016 (6.9)
Child6	-0.070 (3.0)	-0.027 (1.1)	-0.093 (3.0)
Child17	-0.050 (4.1)	-0.044 (3.2)	-0.053 (3.3)
Health	0.220 (4.1)	0.182 (4.4)	0.319 (4.4)
Coll-Univ	0.039 (1.3)	-0.023 (0.7)	0.083 (2.0)
Married	-0.050 (1.3)	0.022 (0.6)	-0.115 (2.3)
β**	-0.036 (0.9)	-0.022 (0.6)	-0.069 (1.4)
γ***	0.139 (3.8)	0.030 (1.1)	0.140 (2.6)
Unemployment Parameters			
Intercept		-1.497 (4.8)	-1.858 (3.8)
Bilingual		-0.154 (1.0)	0.150 (0.7)
Immigrant		0.487 (1.5)	0.128 (0.2)
Age		0.063 (7.8)	0.029 (2.1)
Child6		0.302 (2.7)	0.097 (0.5)
Child17		0.059 (0.9)	0.034 (0.2)
Coll-Univ		-0.627 (3.9)	-0.147 (0.6)
Married		0.348 (1.8)	-0.177 (0.6)
Experience		-0.165 (9.6)	-0.077 (3.9)
Underemployment Parameters			
Intercept		1.055 (0.9)	-0.240 (0.4)
Bilingual		-0.430 (1.2)	-0.074 (0.4)
Immigrant		-6.078 (0.0)	0.256 (0.6)
Age		-0.102 (2.0)	-0.005 (0.3)
Coll-Univ		0.738 (1.9)	-0.065 (0.3)
Married		0.398 (1.2)	-0.086 (0.4)
Experience		0.013 (0.3)	-0.038 (1.7)
Union		0.153 (0.5)	-0.456 (2.1)
Weeks 84		-0.014 (1.9)	-0.012 (2.4)
Transport		0.128 (0.1)	0.462 (1.1)
Finance		1.418 (2.1)	0.636 (1.7)
Government		0.802 (1.1)	0.048 (0.1)
Services		0.554 (0.9)	0.445 (1.3)
$\sigma_\varepsilon^\dagger$	0.335 (0.021)	0.193 (0.014)	0.430 (0.042)
Log-likelihood	-908.3	-766.3	-1072.3

* Unsigned t-statistics in parentheses
** Multiplied by 1000
*** Multiplied by 100
† Standard error in parentheses

The different versions of the model were estimated using the BHHH algorithm. Since the likelihood function is a complex function of the parameters, the score and the Hessian were computed numerically. The results that are reported here concern only three alternatives; non-participation, part-time work and full-time work. Many women faced mild non-convexities in their budget constraint but only three had severe non-convexities (*i.e.* $C_i > C_j, j > i$). They were dropped from the sample to avoid unnecessary complications.[12]

The Slutsky condition is satisfied at each sample point in all versions. Also, the labor supply curve is forward bending in each case ($\gamma > 0$) although it is not significant in Model 2. As expected, nonlabor income has a negative effect but is not statistically significant. Also, the labor supply is negatively and significantly related to the AGE variable and to the number of preschool children (CHILD6) and school age children (CHILD17). As expected, good health (HEALTH) is associated with more hours of work and being married (MARRIED) with lower hours of work. A college or university degree is associated with more hours of work in Model 1 and Model 3, but only significant in the latter model. In general, the parameters of the utility function are less precisely estimated in Model 2.

The variables included in the vectors Z_0 and Z_1 reflect the differential incidence of constraints across different types of individuals. Because our data come from a narrowly defined geographical area, we cannot control for macroeconomic variables that may have an influence on labor market tightness or demand-side constraints (*i.e.* industry specific unemployment rates, percentage shares of manufacturing and services in the locality, etc.). Hence, our parameter estimates of the rationing mechanisms have to be interpreted conditionally on the structure of the economy in Québec City in 1985.

In Model 2, where sample separation information is not used, most parameters of the unemployment mechanism are statistically significant. The probability of being unemployed increases with IMMIGRANT (=1 if born outside Canada), AGE, CHILD6 and MARRIED. On the other hand, having a college or university degree decreases it. The probability of being underemployed is seen to increase with a college or university degree and if employed in the finance sector. The probability decreases with AGE, EXPERIENCE and with WEEKS84. This latter variable was included to proxy the ability of the individual to substitute weekly hours of work by weeks of work. It thus seems individuals who worked many weeks in the previous year do not feel as constrained in their weekly hours of work in the current year.

As mentioned earlier, Model 3 uses the information on sample separation. Notice that the preference parameters of this version are closer to those of Model 1 than those of Model 2. A striking feature here is that most unemployment parameters are not statistically significant anymore. Only AGE and EXPERIENCE seem to have any effect on the probability of being unemployed. On the other hand, several parameters of the underemployment mechanism remain significant. For instance, the probability of being underemployed decreases with EXPERIENCE, UNION (=1 if member of a union) and WEEKS84, and increases if employed in the FINANCE sector.

12. The non-convexities are mild in the sense that $\varepsilon_{1,2} > \varepsilon_{0,1}$, *i.e.* the probability of choosing part-time work is always nonzero. If the budget set is very non-convex (*e.g.* if there were a 90-degree angle in the constraint at the part-time point), utility maximisation at the part-time point would be impossible for any individual, regardless of his indifference map. Our likelihood function is misspecified in this case. Naturally, for those observed to be working part-time, we assume that utility maximisation occurs there and that therefore the probability of part-time work is indeed nonzero.

Contrary to Model 2, AGE and POST-SEC do not seem to matter anymore.

4.2. Model Comparison

The top panel of Table 3 gives an indication of the fit of the model by comparing actual frequencies and mean probabilities of the expressed *ex ante* choices and also the observed *ex post* outcomes. *Ex ante* predictions use only the preference parameters, whereas *ex post* outcomes use the underemployment and the unemployment parameters as well. Model 1 reproduces the main characteristics of the sample fairly well. This is not surprising since, in this case, the preference parameters are capturing the effects of both types of constraints on top of individual preferences. To the extent that the constraints are statistically significant, these parameters are biased estimates of the true preference parameters.

Ex ante predictions from Model 2 indicate that as many as 61% of women would like to work full-time. *Ex post* outcomes are much closer to actual frequencies. The mean probabilities of non-participation and part-time work are decomposed into their two hurdles (optimal and constrained). As shown, Model 2 predicts that out of 39.0% of women who do not work, 11.9% choose freely so while 27.1% are constrained not to work.[13] The model also predicts that part-time work is optimal for 17.0% of individuals out of 21.6%, while only 4.6% are constrained to work part-time rather than full-time. Hence, according to Model 2, many women are constrained not to work and few are constrained to work part-time when full-time is desired.

Ex ante predictions from Model 3 show that 36.6% of women voluntarily choose not to work, that 16.7% of women would like to work part-time and 46.8% would like full-time work. *Ex post* outcomes replicate the data extremely well. According to these, only 4.2% of women are constrained not to work while 36.6% choose not to work, and 7.0% do not work as many hours as desired. These figures are close to the sample means of 2.8% and 6.0%. Notice how Model 2 overestimates the extent of unemployment and underestimates the extent of underemployment.

In the middle panel of Table 3, we report the results of simulating a 10% increase in the net wage rate of every woman. By comparing the first and second panels, it can be seen that proportionately more women leave non-participation and part-time work to enter full-time work in Model 3 than in Model 2. Consequently, the uncompensated wage elasticity is lower in the latter version. As expected, *ex post* elasticities are somewhat smaller than *ex ante* elasticities. Nevertheless, the figures we obtain fall well within the range reported in the literature (see KILLINGSWORTH and HECKMAN [1986]). Note however that the elasticity of Model 1 is somewhat higher since it does not account for constraints on the market. This result is consistent with

13. This result is similar to those obtained by PHIPPS [1990] in a different context. In her model, individuals choose the number of weeks of work in a given year in the presence of unemployment insurance. Since it is not known whether unemployment is the result of individual choice or demand-side constraints, the probability of being constrained must be estimated. Her results show that 82% of the women observed to be unemployed at one time during the year are constrained not to work.

TABLE 3

Actual and Predicted Frequencies of Ex ante and Ex Post Work Status (%)

	Non-Participation	Part-Time Work	Full-Time Work	Expected Hours of Work	Elasticity
Actual	39.3	22.0	38.7	19.9	
Model 1	39.4	22.1	38.5	19.8	
Model 2					
Ex Ante	11.9	26.6	61.4	29.9	
Ex Post	39.0 (11.9+27.1)	21.6 (17.0+4.6)	39.2	20.0	
Model 3					
Ex Ante	36.6	16.7	46.8	22.1	
Ex Post	40.8 (36.6+4.2)	22.4 (15.4+7.0)	36.8	19.2	
Predicted Effect of a 10% Increase in the Net Wage Rate					
Model 1	34.7	24.4	40.9	21.2	0.707
Model 2					
Ex Ante	11.0	27.2	61.7	30.1	0.067
Ex Post	38.5 (11.0+27.5)	22.0 (17.4+4.6)	39.5	20.2	0.100
Model 3					
Ex Ante	32.8	19.6	47.6	22.9	0.362
Ex Post	37.3 (32.8+4.5)	25.3 (18.1+7.2)	37.5	20.1	0.469
Predicted Effect of a $1000 Increase in the Welfare Payments (Married and Single Women that are Eligible)					
Actual	28.4	23.3	48.4	24.0	
Model 1					
Baseline	29.9	20.8	49.3	23.9	
Response	31.2	19.5	49.3	23.6	-0.318
Model 2					
Ex Ante - Baseline	10.9	23.2	65.8	30.9	
Ex Ante - Response .	11.2	23.0	65.8	30.9	-0.030
Ex Post - Baseline .	27.4 (10.9+16.5)	24.5 (17.9+6.6)	48.1	24.1	
Ex Post - Response .	27.6 (11.2+16.4)	24.4 (17.8+6.6)	48.1	24.1	-0.021
Model 3					
Ex Ante - Baseline .	24.2	14.2	61.6	27.5	
Ex Ante - Response .	25.1	13.3	61.6	27.3	-0.164
Ex Post - Baseline .	29.5 (24.2+5.3)	23.6 (13.0+10.6)	46.9	23.5	
Ex Post - Response .	30.3 (25.1+5.2)	22.8 (12.2+10.6)	46.9	23.3	-0.170

Note:(1) *Ex ante* predictions are based on preference parameters only. *Ex post* predictions are based on preference and rationing mechanisms.
(2) The first figure in parentheses represents the proportion of unconstrained individuals and the second represents the proportion of constrained individuals.

those of KAHN and LANG [1991] and ILMAKUNNAS and PUDNEY [1990] who find that the labor supply response to increases in the wage rates is smaller when constraints on the market are accounted for in estimation.

Finally, in the bottom panel of Table 3, we simulate the impact of increasing the welfare payment by $1 000 (approximately 12% increase). This simulation is performed using the sub-sample of women who qualified for welfare payments (204 single women and 12 married women) according to the parameters of the tax-transfer system. The first row indicates the distribution of this sub-sample amongst the three work statuses. The participation rate is higher than for the whole sample; nearly half the women work full-time. The second row ("Baseline") presents the predictions of Model 1 prior to the welfare increase. The predictions are very close to the actual distribution. The next row ("Response") presents the predictions following the increase in the welfare payment. Full-time workers do not react to the increase. Only part-time workers leave the market. As a result, the expected hours of work decrease. The estimated elasticity is -0.318, again well within the usual range.

We repeat the same exercise with Model 2 and Model 3, and differentiate between *ex ante* and *ex post* responses in the following lines. For this sub-sample, 4.2% of women reported being unemployed while 10.2% reported being underemployed. As before, the predictions from Model 3 fit these quite well, whereas Model 2 still overestimates the extent of unemployment and underestimates the extent of underemployement. And as before, women are predicted to react very little to the increase in welfare payments in Model 2. As a consequence, the estimated elasticity is only -0.021. The reactions predicted by Model 3 yield an estimated elasticity of -0.170.

In a shell, Table 3 reveals the following results: (1) models that do not take into account constrained behaviour on the labour market (Model 1) tend to overestimate the wage and income elasticities somewhat; (2) Models that account for *potential* constraints (Model 2) underestimate greatly individual responses; (3) Information on sample separation (Model 3) affects significantly the estimated labour supply responses.

4.3. Diagnostic Tests

The above discussion highlighted the fact that the predicted desired hours of work differ significantly between models. Although informative, these comparisons do not allow to statistically discriminate between models. Furthermore, since the models are not nested within each other, we can not turn to likelihood-based tests to perform any kind of comparison. Nevertheless, various goodness-of-fit tests have been proposed in the literature that can be used to test the null hypothesis that the various parametric models are correct (see *e.g.* ANDREWS [1988 *a*, 1988 *b*] and HECKMAN [1984]).[14] On the other hand, Model 2 is a marginal model of Model 3. Indeed, it is sufficient to integrate out reported rationing in Model 3 to derive Model 2. If the model is properly specified, estimators in both versions should converge to the same value. We can thus use standard specification tests to verify this.

Following ANDREWS [1988 *a*] and HECKMAN [1984], we partition the sample between non-participants, part-time workers and full-time workers. We then compute a quadratic form based on the difference between the number of observed outcomes in each cell and the conditionally expected number in each cell given the observed covariates. More specifically, let $P_0^{(i)} = 1$ if the

14. This section owes much to the suggestions of an anonymous referee.

individual is a non-participant and $P_0^{(i)} = 0$ otherwise. Similarly define $P_1^{(i)}$ and $P_2^{(i)}$ for part-time and full-time workers, respectively. Let $F^{(i)}(j|X_i, \hat{\theta})$ be the probability that the individual i is in state $j (j = 0, 1, 2)$, conditional on the vector of covariates X_i and parameter estimates $\hat{\theta}$. Array $P_j^{(i)}$ into a 3×1 vector $\mathbf{P}^{(i)}$, and $F^{(i)}(j|X_i, \hat{\theta})$ into vector $\mathbf{F}^{(i)}(\mathbf{J}, |\mathbf{X}_i, \hat{\theta})$. Under certain assumptions (see ANDREWS [1988 *a*]), it can be shown that

$$(16) \qquad \frac{1}{\sqrt{N}} \sum_{i=1}^{N} (\mathbf{P}^{(i)} - \mathbf{F}^{(i)}(\mathbf{J}|\mathbf{X}_i, \hat{\theta}))$$

has an asymptotic normal distribution with covariance matrix Σ. Furthermore, under the null assumption that the parametric model is not misspecified, it can be shown that the statistic

$$(17) \qquad \begin{aligned} G = & \frac{1}{N} \left\{ \sum_{i=1}^{N} (\mathbf{P}^{(i)} - \mathbf{F}^{(i)}(\mathbf{J}|\mathbf{X}_i, \hat{\theta})) \right\}' \\ & \Sigma^{-1} \left\{ \sum_{i=1}^{N} (\mathbf{P}^{(i)} - \mathbf{F}^{(i)}(\mathbf{J}|\mathbf{X}_i, \hat{\theta})) \right\} \end{aligned}$$

is asymptotically χ^2 distributed with degrees of freedom given by the rank of Σ^-, and where Σ^- is a Moore-Penrose generalized inverse of Σ.

This test can be carried out using the parameter values for the three versions of the model. The chi-square statistic associated with each model is: $\chi_1^2 = 0.205$, $\chi_2^2 = 0.180$, $\chi_3^2 = 1.890$. These values must be compared with $\chi_{0.95}^2(2) = 5.99$. On the basis of this test, each version of the model seems well specified. This result is not very surprising since, as mentionned previously, the predictions from Model 1 are relatively accurate, and so are the *ex post* predictions from Models 2 and 3. It thus appears that the models can not be distinguished on the basis of their overall goodness-of-fit. We can nevertheless investigate whether we can discriminate between Models 2 and 3 on the basis of another test.

As mentionned previously, Model 2 constitutes a marginal model of Model 3. Hence, if the models are well specified, the parameter estimates should converge to the same value. Let $\hat{\gamma}_2$ and $\hat{\gamma}_3$ represent the parameter estimates of Model 2 and Model 3, respectively. Under the null assumption, it follows that $\partial L_2(\hat{\gamma}_3)/\partial\gamma = 0$, *i.e.* the score of Model 2 evaluated at the parameter estimates of Model 3 should be zero. Following RUUD [1984], we can define the following statistic:

$$(18) \qquad M = N \frac{\partial L_2(\hat{\gamma}_3)'}{\partial \gamma} V_2^{-1}(\hat{\gamma}_3)[V_2(\hat{\gamma}_3) - V_3(\hat{\gamma}_3)]^{-1} V_2^{-1}(\hat{\gamma}_3) \frac{\partial L_2(\hat{\gamma}_3)}{\partial \gamma},$$

where $V_2^{-1}(\hat{\gamma}_3)$ is the covariance matrix of Model 2 evaluated at the parameter estimates of Model 3. It can be shown that M is distributed χ^2 with as many degrees of freedom as there are parameters (=31). Our parameter estimates yield $M = 224$, which implies that the null assumption according to which $\hat{\gamma}_2$ and $\hat{\gamma}_3$ converge to the same value must be rejected. This result is not very surprising. Indeed, the comparison between the parameter estimates depends crucially on the relative efficiency of each model. Yet, GOLDFELD and QUANDT [1975] have shown that the loss of efficiency of the marginal model in small samples can be considerable. KIEFER [1979] has shown this to be the case even in very large samples. Hence, although the null assumption is rejected, both models can not be rejected on the basis of a goodness-of-fit test.

5 Conclusion

In this paper we model female labor supply within a discrete choice framework. A detailed account is made of the tax-transfer system that prevailed in the Province of Quebec in 1985. Two different rationing mechanisms are incorporated into the model: involuntary unemployment and underemployment.

Our estimation strategy consists in estimating the model first without taking into account the rationing mechanisms (Model 1). Next we introduce these mechanisms but do not use the information on sample separation (Model 2). Finally, that information is used and the likelihood altered accordingly (Model 3). It turns out the information on rationing makes a dramatic difference on the preference parameters and hence, on desired hours of work. When rationing is observed, the incidence of unemployment is not correlated to individual characteristics whereas the incidence of underemployment is. When rationing is not observed, the incidence of both unemployment and underemployment varies with these characteristics. This suggest that the labor supply models that incorporate unobserved constraints are potentially biasing the preference parameters.

Our results show that the uncompensated wage and income elasticities are usually smaller when account is made of the constraints in the labor market than when these constraints are ignored. This is not surprising since by omitting the constraints it is implicitly assumed that the observed hours of work correspond to desired hours. Also, we find that the elasticities of Model 2 are systematically smaller than those of Model 3. This is because the probability of being involuntarily unemployed in Model 2 is overestimated and also because the predicted hours of work are seriously overestimated.

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