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Non Verifiability, Costly Renegotiation and Efficiency

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ABSTRACT. – We study the implications of the non verifiability of information for the allocation of resources and the bearing of risk in a two party relationship. We consider a two step approach. In step one the two parties define a non contingent contract which will be executed when the non verifiable information will become common knowledge of the two parties. In step two a costly exogenous bargaining process takes place. The main result is that with risk neutrality it is possible to induce the first best as a Nash equilibrium of the contract without having to renegotiate. A counter-example shows that the result does not extend to risk averse parties for which non verifiability of information will impede in general risk sharing.

Non vérifiabilité, renégociation coûteuse et efficacité

RÉSUMÉ. – Nous étudions les implications de la non vérifiabilité de l'information pour l'allocation des ressources et le partage des risques dans une relation principal-agent. Nous considérons une approche en deux étapes. Dans la première, les deux parties définissent un contrat non contingent qui sera exécuté lorsque l'information non vérifiable deviendra connaissance commune des deux parties. Dans la deuxième étape, un processus de marchandage exogène et coûteux a lieu. Le résultat essentiel est, qu'avec neutralité envers le risque, il est possible d'induire une allocation efficace comme équilibre de Nash du contrat sans avoir à renégocier. Un contre exemple montre que le résultat ne s'étend pas à des parties adverses au risque pour lesquelles la non vérifiabilité de l'information gênera l'allocation des risques.

"Both buyer and seller have identical information and assume, furthermore, that this information is entirely sufficient for the transaction to be completed. Such exchanges might nevertheless experience difficulty if, despite identical information, one agent makes representations that the true state of the world is different than both parties know it to be and if in addition it is costly for an outside arbiter to determine what the true state of the world is"

WILLIAMSON [1975]

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1 Introduction

Considerable attention has been devoted recently to the implications of the non verifiability of information for the allocation of resources and the bearing of risk. In these models, two players engage in a potentially beneficial economic relationship and would ideally like to contract contingent on the state of nature that will become known to both of them in the future, before any payoff-relevant actions must be taken. The problem is that the state of nature is not verifiable by any third party. Thus, although it is assumed that there is a third party present to enforce the contract, this third party has less information than either of the contracting players and this fact may limit the ways in which the contract can function in the mutual interest of the players.

To have a complete description of the contractual possibilities we must specify precisely what role the third party can play. What powers of observation does the third party have? What powers of enforcement does it have? To what extent can it influence any extra-contractual bargaining that takes place after the state of nature becomes known? It is in these three respects that the various papers in this literature differ.

We assume that, after the state of nature becomes known, the players can select messages that are observable by the third party and can therefore be used to determine a tentative outcome. We will call this outcome the contractually determined outcome (CDO). The CDO serves as the status quo point in any subsequent bargaining. Thus, instead of agreeing on a fixed status quo point, the status quo point varies with the state of nature because it arises as the non-cooperative equilibrium of the game that the players play against each other by choosing their messages after the state of nature is known. Even though the game itself is independent of the state of nature it allows the contract to reflect to some extent the information shared ex post by the players.

The CDO may be inefficient. Whenever this is the case, we assume that some effort to reach a better allocation will ensue and that this is foreseen by the parties when they choose their actions. Our model of this improvement on the CDO is that there is a bargaining mechanism that maps any contractually specified agreement into one that Pareto dominates it. We assume that this mapping is exogenous and cannot be influenced by the contractual arrangements. It is a cooperative bargaining solution that depends on the state of nature, the status quo and the set of feasible ex post utility levels. We assume that this ex post bargaining leads to an ex post Pareto efficient point, except that the renegotiation process causes some welfare loss for both agents. Therefore, the feasible set of utility outcomes that can be reached in a given state of nature via renegotiation when the CDO is inefficient is smaller than the set of utilities that could have been reached with a CDO which is efficient.

Given any contract, and an equilibrium of the game that follows the revelation of the state of nature in which the players choose their observable messages, the renegotiation process results in an allocation of resources.

We say that this allocation, which is a function of the state, is *induced by the contract*. In the spirit of mechanism design theory, we ask whether an efficient allocation of resources can be induced by a well chosen contract, or is *inducible*.

First, we consider a model with risk neutrality, where monetary transfers enter linearly into utility functions. Efficient allocations are simply those in which the sum of the utilities of the agents is maximized ex post. In this model we show that the efficient allocation of resources is inducible. As renegotiation is always costly inducing efficiency entails that the outcome be achieved without effective renegotiation. This is done by a contract that divides the total utility available to the two players in a particular fashion. Other equally efficient divisions of utility are not achievable by any contract. The constraints imposed on the realized utilities cause it to be impossible, in general, to induce an ex post efficient allocation in any model without quasi-linear utility functions. Characterizing the set of inducible allocations in models without quasi-linearity is a question awaiting further research. It requires a precise specification of the renegotiation game and of its costs. Before presenting our analysis, we comment on some of the differences between our model and results and those of several other recent papers[1].

A major paper is HART and MOORE [1988]. They allow for a post-contractual bargaining phase by assuming an explicit structure for the exchange and verification of messages and for the enforcement of agreements. By virtue of the very fact that their renegotiation phase is modelled as a finite extensive form game, there are some terminal nodes which correspond to inefficient allocations serving as threats. The motivation for these threats is that after some elapsed length of time, trading has no value. Therefore, threats depend on a particular type of impatience induced by this terminal date. Although renegotiation is costless on the equilibrium path, the equilibrium is crucially dependent on this feature. Finally, their renegotiation phase specifies a complex allocation of bargaining powers as a function of the messages.

RUBINSTEIN and WOLINSKY [1992] use an explicit extensive form as a model of the post contractual renegotiation phase. This extensive form is played recursively until a solution is reached. It is not a finite game. In their model, the efficient collective decision is independent of the state: There is a buyer and a seller of an indivisible object, and the buyer's valuation

1. The non verifiability of information by third parties was viewed by WILLIAMSON [1975] as an alternative justification to incomplete contracting, instead of the traditional fixed costs of including contingencies in the contract. GROSSMAN and HART [1986] model this incomplete contractibility by assuming that nothing is contractible except for the ownership rights which define the bargaining powers in the ex post renegotiation games. However, agents can often contract on publicly observable variables such as price and quantity and will generally want to do so (as proved in BULL [1987] and GREEN and LAFFONT [1992]). Hostages or cancellation fees (WILLIAMSON [1983]), front end loading in contracts (HOLMSTROM [1983]) and contracts contingent on public ex post information (AGHION and BOLTON [1986]) may be introduced to improve the allocation of resources. The more complex these contracts are the heavier use of the third party is made, and the more the analysis depends on the availability of a perfectly benevolent and costless third party. The costs of using a court motivates a line of research where only self-enforcing contracts are considered (TELSER [1980], BULL [1983], GROUT [1984]) and may explain the assumption made by HART and MOORE [1988] of voluntary trade ex post.

is known to be higher than the seller's. There is always a surplus to be divided, and their focus is on whether an arbitrary (monotonic) division of this surplus can be realized. Thus, the primary difference between our analysis and theirs is that the main question is different. We want to know whether an efficient allocation can be achieved, but in their model this is taken as given. There is also a difference between this paper and RUBINSTEIN and WOLINSKY [1992] in the results concerning the division of the surplus. They show that when there is any bargaining cost, however small, any monotonic division of the surplus is possible. In our model, the division of the surplus is severely constrained by the requirement that an ex post efficient allocation of resources be selected. Finally, our papers differ in the way in which the contract specifies the outcomes. We assume that it can specify a CDO. They have a given extensive form in which there is one free parameter which can be contractually specified. This is, however, not a disagreement outcome to be later renegotiated. Rather it is the price at which trade will actually take place.

The paper of AGHION, DEWATRIPONT and REY [1994] is similar to our in spirit because the CDO is a status quo from which further bargaining begins and because the efficient collective decision varies over the states. The principal difference is that, as in CHUNG [1991], they allow the contract to specify one additional factor. Instead of taking the bargaining process as entirely exogenous, they assume that the contract can specify, as a function of the observable actions, which of the parties retains the right to act as a Stackelberg leader in these negotiations. Given the CDO, the state and the identity of the leader, a bargaining mechanism determines the final outcomes. Thus, in their model the bargaining is not totally exogenous; it is somewhat contractually determined. That means that the third party must be able to exercise control over the bargaining process, at least to the degree necessary to insure that the correct player has the leadership role. This is not possible without assuming that the players and the enforcement agency can commit to inefficient outcomes. While this model may be applicable in some cases, we feel that the *raison d'être* of models of incomplete contracts and their renegotiation is the impossibility for the third party to control the bargaining. Therefore we have taken the stance that to understand the implications of contractual limitations due to unverifiable information and incomplete contractual enforcement, it is better to study a third party limited to monitoring the CDO as a disagreement outcome. This two-step approach may look artificial, but we think that it is quite realistic. First, there is the unfolding of the contractual agreement that should not be problematic and is taken to be costless. Second, there is a bargaining phase, to realize a Pareto improvement from an inefficient status quo, which is costly because of opportunism and assumed inability of the third party to monitor the bargaining phase.

Some final comments on the relationship of our work with the Nash implementation literature will be useful. In Nash implementation it is assumed that the CDO is the final allocation. In this context MASKIN [1985], MOORE and REPULLO [1988] have shown that almost any allocation can be achieved. With costless but exogenous bargaining rules for renegotiation, MASKIN and MOORE [1988] have characterized what can be achieved with costless renegotiation in social choice contexts. The success of Nash implementation in achieving almost any outcome is due to the implicit assumption that renegotiation is perfectly controllable or infinitely costly. As the cost of renegotiation falls, two things happen. On the one hand, the "punishment" outcomes lose their bite, as they can be renegotiated. That makes it harder to control the outcome function. On the other hand less expensive renegotiation removes the possibility that very inefficient outcomes will arise.

In this paper we ask a question which can be viewed as a preliminary to the analysis of particular institutions for contract enforcement and renegotiation. Accepting the general idea that renegotiation is *not* perfectly controllable and is costly, we ask whether efficiency is achievable by contracts which are not based on any non verifiable information, but which may specify enforceable allocations depending on verifiable messages sent after the players have learned the state of nature.

We find that with risk neutrality, whatever the renegotiation rules and costs are, the first best is implementable. However, with risk aversion this is not always true. When renegotiation costs are low some efficient allocations cannot be induced by contracts of the type we study. The general characterization and second-best analysis of implementable allocations becomes very dependent on the institutions available for renegotiation in these cases.

2 The Model

The payoff relevant variables are an action $x \in X$, a monetary transfer from player 2 to player 1 $y \in \boldsymbol{R}$, and the state of nature $\theta \in \Theta$.

Player 1's utility function is

$$V = y - v(x, \theta)$$

and players 2's utility function is

$$U = u(x, \theta) - y.$$

For example, $u(x, \theta)$ can be thought of as the gross benefit to the purchase of a quantity x, and $v(x, \theta)$ could be the cost of producing x, each of which might depend on the state of nature. There is no structure imposed upon the state space Θ which is common knowledge.

The most important aspects of the model relate to the timing of the receipt of information, observability and verifiability of various variables, and the opportunities available to the players outside the relationship. If there is no relationship between the players, we normalize the action taken at $x_0 \in X$ and assume that there is no transfer, $y = 0$. If the players form a relationship a contract is signed by the players at a time when they do not know θ.

Let $\bar{U}$ and $\bar{V}$ be the levels of utilities that players 1 and 2 respectively can attain outside the relationship.

A contract specifies that the two players shall engage in an observable choice of messages and that their choice of messages determine a collective action (x, y). The messages and the collective action are observable by the players and the enforcement agency. A *contract* C consists of a pair of *message spaces* A_1, A_2 and an *outcome function* f mapping $A_1 \times A_2$ into an *agreement*, or *status quo*, (x, y). We write $f(a_1, a_2) = \Big(f_x(a_1, a_2), f_y(a_1, a_2)\Big)$ when it is necessary to distinguish the two coordinates of the status quo outcome.

After the contract has been signed both players learn the state of nature. If the players have agreed to $C = (A_1, A_2, f)$, they must then select messages $a_1 \in A_1$, $a_2 \in A_2$ non-cooperatively. The result, $f(a_1, a_2)$, is the agreement between them. This agreement is binding because the messages are observable by an enforcement agency which can mandate $f(a_1, a_2)$, unless a mutually acceptable modification of this agreement supersedes it. Note that the enforcement agency does not have independent knowledge of the state of nature, which is the reason why state contingent contracts are not possible. It is the role of the contract to get around this problem of non verifiability. This process of playing the game (A_1, A_2, f) defined by the contract and the state θ will be referred to as *actuating* the contract. We postpone the discussion of how the contract is actuated for a moment until we have described the renegotiation process through which the interim agreement results in a final outcome.

Finally, the agreement is renegotiated. Because the situation with an agreement $(\bar{x}, \bar{y}) = f(a_1, a_2)$ in place is one of complete information between the parties, we assume that the renegotiation depends only on the feasible set of outcomes and the utility levels provided by this status quo agreement. The rule that associates a final *allocation* to each situation is called the bargaining rule, and is denoted b. We now describe the operation of the bargaining rule.

We model bargaining in the utility space. Exclusive of any costs of renegotiation, the result of bargaining is assumed to be an ex post Pareto efficient allocation that weakly dominates the status quo.

Let $t(\theta)$ be the total utility available in state θ and let $s_i(\bar{x}, \bar{y}, \theta)$ be the status quo utility levels, $i = 1, 2$.

$$t(\theta) = u(x^*(\theta), \theta) - v(x^*(\theta), \theta)$$

where $x^*(\theta)$ maximizes $u(x, \theta) - v(x, \theta)$.

$$s_1(\bar{x}, \bar{y}, \theta) = \bar{y} - v(\bar{x}, \theta)$$
$$s_2(\bar{x}, \bar{y}, \theta) = u(\bar{x}, \theta) - \bar{y}.$$

Then, exclusive of bargaining costs, the *bargaining rule* gives the utilities of the players as a function of t, s_1 , s_2:

$$b \,:\, \boldsymbol{R}^3 \rightarrow \boldsymbol{R}^2.$$

Denote

$$b(t, s_1, s_2) = \Big(b_1(t, s_1, s_2), b_2(t, s_1, s_2)\Big).$$

Bargaining or renegotiation is assumed to result in a loss of utility that is related to the possible gain. Some, but not all, of any gain is dissipated in the renegotiation process.

Let $\ell(t, s_1, s_2)$ be the *utilities lost in the renegotiation process*, then

$$0 \leq \ell \leq b - (s_1, s_2)$$

with strict inequality unless $b = (s_1, s_2)$. The final utilities achieved are

$$b(t, s_1, s_2) - \ell(t, s_1, s_2).$$

When the actuation date arrives the players choose a_1, a_2 with full knowledge of b and ℓ and hence with full information about the resulting payoffs. Because θ is already known, there is a game determined by C, b, ℓ and θ in a natural way. The payoff function of the game is just the composition of the bargaining rule with the contract:

$$\begin{aligned} g(a_1, a_2, \theta) = b\Big(&t(\theta), u(f_x(a_1, a_2), \theta) \\ &- f_y(a_1, a_2), f_y(a_1, a_2) - v(f_x(a_1, a_2), \theta)\Big) \\ &- \ell\Big(t(\theta), u(f_x(a_1, a_2), \theta) - f_y(a_1, a_2), f_y(a_1, a_2) - v(f_x(a_1, a_2), \theta)\Big). \end{aligned}$$

The game determined by C, b, ℓ and θ is a game in normal form and we denote the set of its Nash equilibria by $E(C,\ b,\ \ell,\ \theta)$. Given the contract and the bargaining rule, let $a(\theta) \equiv \Big(a_1(\theta), a_2(\theta)\Big)$ be a selection from $E(C,\ b,\ \ell,\ \theta)$. We will say that a *utility allocation* $\Big(U(\theta), V(\theta)\Big)$ is *induced* by C if there exists a selection $a(.)$ from $E(C,\ b,\ \ell,\ \theta)$ such that

$$\Big(U(\theta), V(\theta)\Big) \equiv g(a(\theta), \theta).$$

Since the outcome function depends on both the announced characteristics and the true characteristics (through bargaining) we distinguish the notion of inducibility from the notion of implementability of a utility allocation.

The bargaining rule will in general modify the status quo agreement in order to improve efficiency. However, if the status quo agreement is itself a Pareto optimum, then no actual renegotiation will take place and hence no bargaining cost will be incurred. Such situations will be particularly important for the results of this paper. Thus, we shall say that when there exists a selection $a(.)$ of the Nash equilibria such that

$$f_x(a(\theta)) \equiv x^*(\theta) \quad \text{for any } \theta \in \Theta$$

the first best is *induced without renegotiation*.

3 Inducing the First Best

Due to the fact that $\ell > 0$ unless $t = s_1 + s_2$, it is desirable to induce an efficient decision directly, without renegotiation. In section 3.1 we give preliminary results concerning the limited power of delegation to achieve that result in certain circumstances. Then, in section 3.2 we provide a general possiblity theorem by constructing a contract in which both players are active and that always induces the first best without renegotiation when players are risk neutral. Section 3.3 extends the result to the case where the bargaining rule is uncertain at the time of contracting. Finally, section 3.4 provides a counterexample showing that risk aversion invalidates the general possibility theorem. The bargaining rule is assumed to satisfy the following properties:

Optimality up to Bargaining Costs:	$b_1(t, s_1, s_2) + b_2(t, s_1, s_2) = t$
Monotonicity:	b_1 and b_2 non decreasing in s_1, s_2
Domination of the status quo:	$b_1(t, s_1, s_2) \geq s_1$
	$b_2(t, s_1, s_2) \geq s_2$.
Continuity:	$b_1(.)$ and $b_2(.)$ continuous.

3.1. Delegation

There are two sets of classes of economic environments where the first best is inducible without renegotiation by means of contracts that are particularly simple. These are either when one of the player's utility function is separable in (or independent of) θ, or when the distribution of θ is concentrated on two points. A delegated contract will be effective in these cases. A contract is *delegated* to one player if the other has a trivial (one-point) message space. The use of delegated contracts avoids issues of Nash equilibrium and selection among them, in favor of something strategically simpler.

PROPOSITION 1: Il player 1's utility function is either independent of θ or additively separable in θ, then a contract exists that induces the first-best without renegotiation for all b satisfying the assumptions of this section. This contract is delegated to player 2.

Proof: Suppose that $v(x, \theta) \equiv \psi(x)$ and let $\psi(x_0) = \bar{V}$. Then we can delegate the selection of (x, y) to player 2 and obtain the first-best as follows: player 2 is allowed to choose $(x, y) \in \{(x, y) | y - \psi(x) = \bar{V}\}$. For any choice (x, y) player 1's utility will be constant. Player 2 knowns that the bargaining rule is monotonic. Therefore, his best achievable utility level will be obtained when (x, y) maximizes $u(x, \theta) - y$ for (x, y) in this set. But this just means that the choice is Pareto efficient, and hence is invariant to the renegotiation rule.

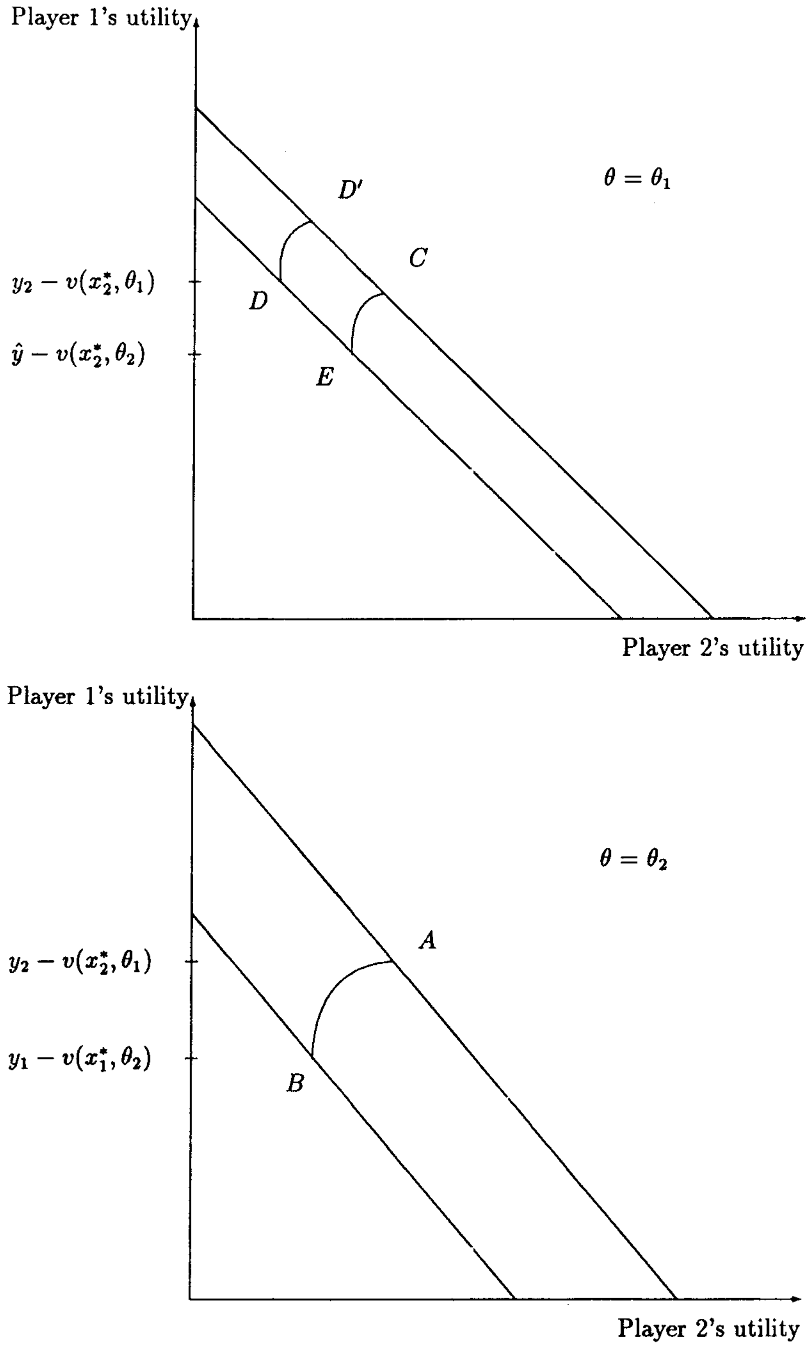

FIGURE 3.1

Observe also that if $v(x, \theta) \equiv \psi(x) - \phi(\theta)$ the same argument works, where $\bar{V}$ is replaced by $\bar{V} + E\phi(\theta)$. Player 1 then receives the utility

$y - \psi(x) - \phi(\theta)$ which is $\bar{V} + \phi(\theta) + E\phi(\theta)$ for all choices player 2 might make. Thus the monotonicity argument still applies. □

PROPOSITION 2: Let the distribution of θ be concentrated on two points θ_1 and θ_2. There exists a contract that induces the first best without renegotiation. The contract will depend upon b. It will be a delegated contract, but the identity of the individual to whom it is delegated cannot be determined independently of utility functions, the bargaining rule b, and the distribution of θ. This contract will induce the first best without renegotiation for all renegotiation cost functions ℓ.

Proof: In the first step of the proof, we proceed as if there was no bargaining cost, ℓ. Bargaining costs are reintroduced at the end of the proof.

Let x_2^* be optimal given θ_2. Fix y_2 arbitrarily. This produces the utility pair denoted A. The bargaining rule determines the locus of points which would be renegotiated to A. Let x_1^* be optimal given θ_1 and consider the utilities achievable by varying y given $x = x_1^*$ and $\theta = \theta_2$. Select y_1 such that the utility pair associated with (x_1^*, y_1) would be renegotiated to the same utilities associated with (x_2^*, y_2) when $\theta = \theta_2$. Thus both players would regard these two status quo agreements as equivalent at θ_2.

Now consider $\theta = \theta_1$, which, without loss of generality, we take to be below θ_2. Since $x_i^*,\ y_i,\ i = 1, 2$ are already fixed, the location of the utility pairs induced by these possible choices are fixed at C, for $i = 1$, and D, for $i = 2$ respectively. The bargaining solution fixes the locus of pairs that are renegotiated to C. Consider the point E which is obtained at θ_1 by x_2^* and some transfer $\hat{y}$. If $\hat{y} < y_2$, (as shown) player 2 would prefer (x_1^*, y_1) to the point D' that would be the result of the renegotiation of the inefficient choice (x_2^*, y_2). Thus, if player 2 were delegated the choice between (x_1^*, y_1) and (x_2^*, y_2) he would choose the efficient pair in each instance, and no renegotiation would ensue.

Conversely, if $\hat{y} > y_2$ player 1 can be delegated the choice. Note now that the same choices are made by the players if bargaining costs exist, since they make even less attractive the alternatives (x_i^*, y_i) in state $\theta_j,\ j \neq i$.

Finally, note that players's ex ante individual rationality constraints can be satisfied by making a shift in y_1 and recomputing y_2 as described above (by continuity an appropriate y_1 exists if the relationship is valuable). □

Quite clearly, the power of delegation is limited and the two state case is very special. As soon as there are three or more states, as we will see in the next subsection, we cannot expect that a first best be achieved by delegation to one of the players. A necessary condition for a delegation game to induce a first best allocation without renegotiation is that this allocation be incentive compatible for the player to which the decision is delegated. So in any environment where no first best allocation is incentive compatible for either player, delegation cannot work.

There remains, of course, the possibility that a non-delegated contract will work. And indeed it will, as we show in section 3.2.

3.2. A General Possibility Theorem

This section contains the main positive result -a constructive proof of the fact that the first-best can be induced without renegotiation in any game. In our general possibility result we construct a game form using message spaces which are copies of Θ. The main idea is that all the off-diagonal entries are the same, and the diagonal constitutes a selection from the Nash equilibria. Thus, in equilibrium, both players have a choice between the fixed off-diagonal alternative and the equilibrium on the diagonal. We arrange the transfers $y(\theta)$ so that this is always a matter of indifference, excluding bargaining costs. Therefore, including bargaining costs the diagonal, which is efficient, is strictly preferred over the off-diagonal which would have to be renegotiated. (There will be one state where the diagonal and off-diagonal are the same, and in that state they are efficient).

THEOREM 3: There exists a contract C that induces the first-best without renegotiation.

Proof: Fix θ_0 arbitrarily and set $y^*(\theta_0) = 0$. Select $y^*(\theta)$ to satisfy:

(1) $$b_1\Big(t(\theta), s_1, s_2\Big) = b_1\Big(t(\theta), s_1^0, s_2^0\Big)$$

where

$$\begin{aligned} s_1 &= y^*(\theta) - v\Big(x^*(\theta), \theta\Big) \\ s_2 &= u\Big(x^*(\theta), \theta\Big) - y^*(\theta) \\ s_1^0 &= y^*(\theta_0) - v\Big(x^*(\theta_0), \theta\Big) \\ s_2^0 &= u\Big(x^*(\theta_0), \theta\Big) - y^*(\theta_0). \end{aligned}$$

Note that, by the optimality of $x^*(\theta)$, the left hand side of (1) is just s_1.

Now construct the contract as follows: For message spaces, (A_1, A_2), take copies of the parameter space, Θ. For the outcome function.

$$\begin{aligned} f(\theta_1, \theta_2) &= \Big(x^*(\theta), y^*(\theta)\Big) \quad \text{if } \theta_1 = \theta_2 = \theta \\ &= \Big(x^*(\theta_0), y^*(\theta_0)\Big) \quad \text{if } \theta_1 \neq \theta_2. \end{aligned}$$

To show that $\Big(x^*(\theta), y^*(\theta)\Big)$ is a Nash equilibrium outcome at θ, observe simply that by construction, at any $\theta \neq \theta_0$, both players prefer $\Big(x^*(\theta), y^*(\theta)\Big)$ to $\Big(x^*(\theta_0), y^*(\theta_0)\Big)$ because of the renegotiation costs. For θ_0 they are indifferent to their message but whatever message they choose $\Big(x^*(\theta_0), y^*(\theta_0)\Big)$ is achieved without renegotiation.

Adjust $y^*(\theta_0)$ if necessary to obtain ex ante individual rationality of both agents.

3.3. Extension of the Inducibility Result when the Bargaining Rule is Unknown

The previous section has demonstrated that a contract can be designed so that the first-best is implemented without renegotiation. The design of the contract depends upon the bargaining rule b. One might be interested in situations in which b is not known at the time the contract is written. In this section we show that a straightforward extension of the method of section 3.2 can be used to provide a positive result in this case as well[2].

The timing we envision is one in which the bargaining rule, which may reflect the realization of outside opportunities that influence threat points as well as the players' "bargaining abilities", is not known when the contract is written. However, at the time the game form defined by the contract must be played, after θ has been determined, the bargaining rule b will have become common knowledge as well.

COROLLARY 4: Given a family of possible bargaining rules B, there exists a contract under which the first-best is induced, without renegotiation, for all $b \in B$ and all $\theta \in \Theta$.

Proof: The method of the theorem applies. The strategy spaces are copies of $\Theta \times B$. The equilibrium outcomes are defined by fixing $(\bar{\theta}, \bar{b})$ and $y^*(\bar{\theta}, \bar{b})$ and then setting $y^*(\theta, b)$ to solve

$$y^*(\theta, b) - v\Big(x^*(\theta), \theta\Big) = b_1(t, s_1, s_2)$$

where $t = t(\theta)$

$$s_1 = y^*(\bar{\theta}, \bar{b}) - v\Big(x^*(\bar{\theta}), \theta\Big)$$

$$s_2 = u\Big(x^*(\bar{\theta}), \theta\Big) - y^*(\bar{\theta}, \bar{b}).$$

Then the outcome function f is defined by

$$\begin{aligned} f\Big((\theta_1, b_1), (\theta_2, b_2)\Big) &= \Big(x^*(\theta), y^*(\theta, b)\Big) \text{ if } \theta_1 = \theta_2 = \theta \text{ and } b_1 = b_2 = b. \\ &= \Big(x^*(\bar{\theta}), y^*(\bar{\theta}, \bar{b})\Big) \text{ if } \theta_1 \neq \theta_2 \text{ or } b_1 \neq b_2. \quad \square \end{aligned}$$

This shows that, because of risk-neutrality, the uncertainty about b can be treated as just another aspect of the unknown state θ. However, as we shall see in the next section, these results break down under risk aversion.

2. We are grateful to E. Maskin for suggesting this extension. It can be shown that if we keep the same strategy spaces as in Theorem 3, it is not always possible to induce the first best without renegotiation independently of the bargaining solution.

3.4. Impossibility Result with Risk Aversion

In this section we give a simple example showing that the first best cannot always be induced without renegotiation. In particular, when the renegotiation cost is small it may not be possible to avoid it.

Suppose that $\Theta = \{\theta_1, \theta_2\}$ and that player 1 has the utility function:

$$w(y) - v(x, \theta), \text{ where } w' > 0,\ w'' < 0,\ v_x > 0,\ v_{xx} > 0,\ v_{x\theta} > 0$$

and player 2 has the same utility function as in section 3.3:

$$u(x, \theta) - y,$$

with $u_x > 0,\ u_{xx} < 0,\ u_{x\theta} > 0$.

If π_1 and π_2 are the probabilities of θ_1 and θ_2, the first best $(x_1^*, x_2^*, \bar{y})$ is characterized by the unique solution to:

$$\frac{v_x(x_1, \theta_1)}{u_x(x_1, \theta_1)} = \frac{v_x(x_2, \theta_2)}{u_x(x_2, \theta_2)} = w'(y)$$

$$w(y) - \pi_1 v(x_1, \theta_1) - \pi_2 v(x_2, \theta_2) = \bar{V}.$$

Assuming that $\theta_1 < \theta_2$ and $w' u_{x\theta} - v_{x\theta} > 0$, we have $x_1^* < x_2^*$. We assume, as above, that only these two values of x are possible (remember we are simply constructing a counter-example).

In the diagonal we have no choice; the allocation must be $(x_1^*, \bar{y})$ if θ_1 and $(x_2^*, \bar{y})$ if θ_2. Then we have two possibilities from which to choose the value of x in the upper right cell.

Case 1: $x = x_1^*$

Player 2 Player 1	θ_1	θ_2
θ_1	$(x_1^*, \bar{y})$	(x_1^*, y_3)
θ_2		$(x_2^*, \bar{y})$

FIGURE 3.2

No deviation by player 2 if θ_1 requires $y_3 \geq \bar{y}$. But this induces a deviation by player 1 if θ_2 because player 1 gets a strictly better x and a transfer which is at least as good. For small enough renegotiation cost he gains from the deviation.

Case 2: $x = x_2^*$

Player 2 Player 1	θ_1	θ_2
θ_1	$(x_1^*, \bar{y})$	(x_2^*, y_3)
θ_2		$(x_2^*, \bar{y})$

FIGURE 3.3

No deviation by player 2 if θ_1 requires $y_3 \geq \bar{y}$. But this induces a deviation by player 1 if θ_2 for a small enough renegotiation cost. □

It is clear from this example that the first best cannot be induced in general without renegotiation. A characterization of the limits to risk sharing at inducible allocations required as a result of the non-verifiability of information, is thus a most important open question. However, as explained in the introduction, any step in this direction requires a precise model of the enforcement agency and of the bargaining rule. Results will be highly dependent on the costs of these institutions.

The argument used in this example does point out a basic relationship between inducibility without renegotiation and implementation in Nash equilibrium. High renegotiation costs are beneficial because of the control they place on equilibrium (diagonal) strategies. Implementation in Nash equilibrium is the limiting case where $\ell \equiv b - s$. As renegotiation becomes less costly it is harder to avoid defections. Our theorem shows that this can be done by setting $y^*(\theta)$ appropriately. Controlling y is socially costless under risk neutrality but cannot be used in general without inefficiently imposing risks of fluctuation in y on the players.

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