Delegation and Efficiency in a Mixed Oligopoly

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ABSTRACT. The present work analyses the effect of delegation on the market outcome when agents have private information about the firms' productivity. Two types of firms are considered: managerial firms (delegation) and entrepreneurial firms (no delegation). Due to the asymmetry of information managerial firms are less efficient (productive efficiency) because they must pay informational rents to their managers. The existence of delegating firms leads the market outcome further away from the competitive one. However the presence of non-delegating firms limits the agents' informational rents.

We show that there is an improvement in terms of *allocative efficiency* in the market when one of the managerial firms is a non-profit maximizer. However this improvement has a negative counterpart in terms of *productive efficiency*: firstly, the welfare maximizing firm leads to an increase of the average cost of each unit of output, due to the large inequality between firms' production and to increasing marginal costs; secondly, the public firm bears a larger informational rent than private firms. The public firm is welfare improving mainly when the number of competing firms is small and/or the proportion of managerial firms is large. In certain cases the entrepreneurial firms substitute the role of the public firm as a regulation mechanism.

Délégation et efficacité dans un oligopole mixte

RÉSUMÉ. – La contribution d'une entreprise publique pour augmenter le bien-être social dans une industrie oligopolistique dépend non seulement de la structure du marché (nombre de firmes et dimension de la demande) mais aussi de l'organisation interne des firmes privées (délégation).

Le rôle de mécanisme de régulation du marché attribué à l'entreprise publique est plus efficace quand la proportion de firmes qui délèguent est grande. Dans certains cas les firmes qui ne délèguent pas peuvent remplacer l'entreprise publique dans son rôle de régulation car elles sont plus efficaces.

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1 Introduction

Delegation, as a contractual arrangement between a principal and an agent, in oligopolistic markets, has received little formal attention. However, we can find many industries where firms in which owners delegate management activities to an agent, compete with firms ruled by the owners themselves. Since delegation is usually connected to asymmetry of information and informational rents, these industries are an interesting case study because firms face different cost structures due to their internal organization.

This problem was treated in some recent works like HART [1983] and SCHARFSTEIN [1988]. They consider a perfectly competitive market where delegating firms (managerial firms) compete with non-delegating firms (entrepreneurial firms) and it is shown how competition can help to solve the agency problems of managerial firms. Some other works (VICKERS [1985], FERSHTMAN and JUDD [1987] and SKLIVAS [1987]) focus on the use of strategic contracts as strategic variables and ignore agency problems. They show how the precommitment value of contracts [1] can improve owners' payoffs.

In the present setting we consider an oligopoly framework where an exogeneous number of managerial firms and entrepreneurial firms compete in quantities. An asymmetric oligopoly emerges because managerial firms must pay informational rents to their managers. The main goal is to analyse the effects of delegation on the market outcome.

Since we do not associate delegation with any advantage to the firm, delegation entails exclusively higher costs, due to agents' informational rents. Hence, managerial firms are less efficient than entrepreneurial firms. Consequently, managerial firms lead to a less competitive market outcome than in the regular Cournot case (when delegation is not considered) and exert a positive externality on entrepreneurial firms which obtain higher payoffs than in the equilibrium without managerial firms [2]. When the market competition increases due to a larger proportion of entrepreneurial firms, two sorts of effects are observed: informational rents decrease (they are positively related to the firm's production) but the *per* firm's profit is reduced. Since entrepreneurial firms make higher profits the industry profitability increases and this corresponds to a gain in terms of market efficiency. In this sense entrepreneurial firms perform a disciplinary role.

In this framework we also take into account the possibility of improving the industry efficiency by substituting a profit maximizing managerial firm by a non-profit maximizing managerial firm (public firm). Recent works like CRÉMER, MARCHAND and THISSE [1989], BEATO and MAS-COLELL [1984] and

1. Under the crucial assumptions of nonrenegotiation and perfect observability of contracts.

2. When agency problems are not considered and contracts are used strategically in the sense of VICKERS [1985], FERSHTMAN and JUDD [1987] and SKLIVAS [1987], the results are inverted: the market outcome is more competitive and if a non-delegating firm competes with a delegating firm, the former obtains lower payoff than the latter.

De FRAJA and DELBONO [1989], [1990] focus on the study of mixed markets, *i. e.* markets where firms have different objective functions. The aim of these works is to analyse whether it is possible to reduce the inefficiencies raised by imperfect competition, through the use of a public firm as an internal regulation mechanism. Since the public firm seeks the maximization of social welfare, there is a gain in terms of allocative efficiency. In the present paper we characterize the market equilibrium when there is a public firm and the conditions for the mixed oligopoly to be welfare improving.

In order to examine the pure effect of a welfare maximizing firm, we assume that property rights do not entail any efficiency differential in terms of both technology and informational structure. Any efficiency differential under private and public ownership is endogenous to the model.

The mixed oligopoly is a quite interesting situation because we have competing firms characterized by different internal organization and different objective functions.

We show that there is an improvement in terms of *allocative efficiency* in the market when one of the managerial firms is a non-profit maximizer. However this improvement has a negative counterpart in terms of *productive efficiency*: firstly, the welfare maximizing firm leads to an increase of the average cost of each unit of output, due to the large inequality between firms' production and to increasing marginal costs [3]; secondly, the public firm bears a larger informational rent than private firms [4].

In some cases the disciplinary role of entrepreneurial firms can substitute the function of the public firm as an internal regulation mechanism due to this trade-off between allocative efficiency and productive efficiency: although these firms are profit maximizers, they are more efficient and have positive effects on competition. Actually, it is mainly in cases where there is low competition in the market (due to either a small number of firms or a large proportion of managerial firms) that the role of the public firm becomes more relevant.

However, if the number of firms is very large, we find cases where the public firm is welfare improving and this is quite interesting since De FRAJA and DELBONO [1989] have shown the opposite result. Their result is due to the fact that the negative effect in terms of productive efficiency (low profits) outweighs the positive effect in terms of allocative efficiency (high consumer surplus). In our setting when the number of firms raises agents' rents are also reduced and, for a number sufficiently large of firms, it might be the case that total informational rents are higher in the private than in the mixed oligopoly, leading to a gain in terms of industry efficiency. Nevertheless DELBONO and De FRAJA's result is retrieved if the proportion of managerial firms is very small and the total number of firms is large: in

3. This is a very known result from the mixed oligopolies literature (see De FRAJA and DELBONO [1989]).

4. Because the public firm produces larger output than private firms and the agent's informational rent is an increasing function of the firm's output.

case where the public firm is the unique managerial type firm it is more efficient to instruct the public firm to maximize profits.

This paper is organized as follows. Section 2 presents the main assumptions of the model. Section 3 studies the equilibrium in contracts when all firms are profit maximizers (private duopoly); section 4 analyses the equilibrium in contracts when one of the managerial firms is a welfare maximizer (mixed duopoly). Section 5 compares the results of the private and mixed oligopoly. Finally, section 6 summarizes the main conclusions.

2 The Model

Assume n firms that produce a homogeneous good and face a linear demand given by

(1) $$\mathrm{P} = a - \mathrm{Q}, \qquad a > \mathrm{Q}$$

where P is the market clearing price, q_{J} is firm's output and $\mathrm{Q} = \sum\limits_{\mathrm{J}=1}^{\mathrm{N}} q_{\mathrm{J}}$. The value of a is large enough for the equilibrium quantities to be always positive.

We assume that there is no production costs and that firms compete in quantities. Each firm is run by a manager (or the owner/manager) whose effort e_{J} is necessary for production according to the production function

(2) $$q_{\mathrm{J}} = e_{\mathrm{J}} - \theta$$

where $\theta \in \Theta = \{\theta_l,\ \theta_h\}$, with $\theta_h < \theta_l$, is a stochastic productivity parameter such that $\Pr[\theta = \theta_l] = p$, $0 < p < 1$ and $e_{\mathrm{J}} \in [\theta_l,\ \infty[$. High productivity is associated with a low value for θ.

At an initial date 0, when the owner of each firm contracts with an agent, θ is already realized and each agent perfectly observes his productivity parameter. Managerial firms' owners cannot observe the realization of θ but the properties of the distribution function are public knowledge.

The productivity parameter of each firm is determined independently of the productivity parameter of the rival firms.

An exogenous number m of firms are managerial firms. The remaining $n - m$ firms are ruled by their owners who exert the effort necessary for production.

Owners do not observe the effort exerted by their agents. Since the productivity parameter is the agent's private information, the owner of a managerial firm cannot infer the value of e_{J} on basis of the information about the quantity produced. Thus, the agent can shirk without being detected. Consequently, the contract cannot be conditioned upon either the agent J's effort or the productivity parameter θ_{J}. It can only be conditioned upon the message delivered by the agent about the firm's productivity parameter. Nevertheless the owner faces the risk of being cheated by the agent. If it is in the interest of the agent to report a false productivity parameter, he will do so. As a result, the contract must stipulate a reward scheme such that the agent has an incentive to report the correct value of θ_{J}. Note that owners do not observe rival agents' messages.

The contract between the agent and the owner stipulates two variables: given the value of θ reported by the agent, the owner commits himself to pay a transfer $w_{\mathrm{J}}(\hat{\theta}_{\mathrm{J}})$ if the agent satisifies a specific production target $q_{\mathrm{J}}(\hat{\theta}_{\mathrm{J}})$.

For simplicity we assume that all players in this game are risk neutral. Each agent has a reservation utility, which is normalized to zero, and bears the effort disutility given by $\varphi(e_{\mathrm{J}}) = e_{\mathrm{J}}^2/2$. The agent is willing to sign the contract if and only if it guarantees at least his reservation utility in both states of nature (the individual rationality constraint).

Entrepreneurial firms are ruled by the owner who is able to observe the realization of θ_{J}. Since the owner/manager is the residual claimant for the firm's profits, he exerts the effort necessary for profit maximization, given the actual realization of θ_{J} and the disutility of the effort.

3 Private Oligopoly

In the section we look for the equilibrium contracts when all firms are profit maximizers.

3.1. The Managerial Sector

We start be deriving the equilibrium for a market where all firms are profit maximizers.

Subscripts M and E designate managerial and entrepreneurial firms, respectively.

At the beginning of the game each owner asks his manager to report the value of θ. By the revelation principle we need only to consider contracts that induce the agent to report θ truthfully and this is a weakly dominant strategy for each firm. Specifically, we restrict our attention to incentive compatible contracts.

In order to select the optimal incentive contract, firm M's owner maximizes his expected payoff (the profits of the firm net of the agent's fixed wage) subject to both the incentive compatibility constraint and the individual rationality constraint. Thus, each owner solves the following program:

$$(3) \qquad \max_{\substack{w_{\mathrm{M}\,l},\, w_{\mathrm{M}\,h} \\ q_{\mathrm{M}\,l},\, q_{\mathrm{M}\,h}}} \mathrm{E}_\theta[\pi_{\mathrm{M}}] = p\left\{\left(a - \sum_{\mathrm{J}\neq\mathrm{M}} \mathrm{E}_\theta[q_{\mathrm{J}}] - q_{\mathrm{M}\,l}\right)q_{\mathrm{M}\,l} - w_{\mathrm{M}\,l}\right\}$$
$$+ (1-p)\left\{\left(a - \sum_{\mathrm{J}\neq\mathrm{M}} \mathrm{E}_\theta[q_{\mathrm{J}}] - q_{\mathrm{M}\,h}\right)q_{\mathrm{M}\,h} - w_{\mathrm{M}\,h}\right\}$$

$$\text{(4)} \qquad \text{s.t.} \quad w_{\mathrm{M}\,h} - \frac{(q_{\mathrm{M}\,h} + \theta_h)^2}{2} \geq w_{\mathrm{M}\,l} - \frac{(q_{\mathrm{M}\,l} + \theta_h)^2}{2}$$

$$\text{(5)} \qquad w_{\mathrm{M}\,l} - \frac{(q_{\mathrm{M}\,l} + \theta_l)^2}{2} \geq w_{\mathrm{M}\,h} - \frac{(q_{\mathrm{M}\,h} + \theta_l)^2}{2}$$

$$\text{(6)} \qquad w_{\mathrm{M}\,l} - \frac{(q_{\mathrm{M}\,l} + \theta_l)^2}{2} \geq 0$$

$$\text{(7)} \qquad w_{\mathrm{M}\,h} - \frac{(q_{\mathrm{M}\,h} + \theta_h)^2}{2} \geq 0$$

$$q_{\mathrm{M}\,l},\ q_{\mathrm{M}\,h},\ w_{\mathrm{M}\,l},\ w_{\mathrm{M}\,h} \geq 0 \qquad \text{and} \quad J = 1, \ldots, M-1,\ M+1, \ldots, N$$

Variables $w_{\mathrm{M}\,l}$, $w_{\mathrm{M}\,h}$, $q_{\mathrm{M}\,l}$, and $q_{\mathrm{M}\,h}$ represent the terms of firm M's contract when the state of nature reported by the manager is l and h, respectively. $\mathrm{E}_\theta\left[q_{\mathrm{J}}\right] = p q_{\mathrm{J}\,l} + (1-p)\, q_{\mathrm{J}\,h}$ is the expected output of firm J. Incentive compatibility constraints (4) and (5) guarantee truthful revelation; constraints (6) and (7) ensure that the agent is willing to accept the contract because the *ex post* utility of the agent is at least equal to his reservation utility.

First-order conditions for this program (see appendix 1) yield the managerial firm's best reply functions for, respectively, state l and h:

$$\text{(8)} \qquad q^*_{\mathrm{M}\,l}(\cdot) = \frac{a - \theta_h - \sum\limits_{\mathrm{J} \neq \mathrm{M}} \mathrm{E}_\theta\left[q_{\mathrm{J}}\right]}{3} - \frac{\theta_l - \theta_h}{3\,p}$$

$$\text{(9)} \qquad q^*_{\mathrm{M}\,h}(\cdot) = \frac{a - \theta_h - \sum\limits_{\mathrm{J} \neq \mathrm{M}} \mathrm{E}_\theta\left[q_{\mathrm{J}}\right]}{3}$$

$$\text{for} \quad J = 1, \ldots, M-1,\ M+1, \ldots, N$$

Notice that for a given expected production of the rival firms, $q_{\mathrm{M}\,h}$ is independent of the terms related to uncertainty. In this state, the informational asymmetry does not affect the firm's decisions. On the contrary, the existence of asymmetry of information decreases the level of $q_{\mathrm{M}\,l}$ and this negative effect is magnified by the gap between θ_l and θ_h.

Given the solution of the maximizing program, constraints (4) and (6) are binding while constraints (5) and (7) are not. This means that, when the less favorable state occurs (state l) the owner succeds in extracting all the agent surplus; conversely, when the more favorable state is realized (state h) the owner must pay a rent in order to induce the agent to report his private information truthfully.

As the individual participation constraint binds in state l, the optimal wage is:

$$\text{(10)} \qquad w^*_{\mathrm{M}\,l} = \frac{(q^*_{\mathrm{M}\,l} + \theta_l)^2}{2}$$

The optimal wage in state h is computed using the binding incentive compatibility constraint for state l and the agent's wage, when θ_h is reported, becomes:

(11) $$w^*_{\mathrm{M}\,h} = w^*_{\mathrm{M}\,l} - \frac{(q^*_{\mathrm{M}\,l} + \theta_h)^2}{2} + \frac{(q^*_{\mathrm{M}\,h} + \theta_h)^2}{2}$$

As it is well known from the agency theory, the state h wage is above the perfect information level, *i.e.* the wage that would be optimal if one could write a state contingent contract. This fact is due to the agent's information monopoly and, consequently, to the existence of informational rents.

3.2. The Entrepreneurial Sector

The owner/agent of an entrepreneurial firm is the residual claimant of the firm's revenues. Since the firm is run by the owner, he observes the realization of his productivity parameter. Thus, each owner selects the level of production that maximizes his payoff taking into account the observed state of nature. His payoff is the firm's profit net of the effort disutility necessary for production. Hence, the entrepreneurial firm solves the following program:

(12) $$\max_{q_{\mathrm{E}\,t}} E_\theta\left[\pi_{\mathrm{E}\,\tau}\right] = \left(a - q_{\mathrm{E}\,t} - \sum_{\mathrm{J}\neq\mathrm{E}} \mathrm{E}_\theta\left[q_{\mathrm{J}}\right]\right) q_{\mathrm{E}\,t} - \frac{\left(q_{\mathrm{E}\,t} + \theta_t\right)^2}{2}$$

for $J = 1, \ldots, E-1, E+1, \ldots, N, \; t = l, h.$

First-order conditions yield the following best reply function

(13) $$q^*_{\mathrm{E}\,t}(\cdot) = \frac{a - \theta_t - \sum\limits_{\mathrm{J}\neq\mathrm{E}} \mathrm{E}_\theta\left[q_{\mathrm{J}}\right]}{3}$$
$$J = 1, \ldots, E-1, E+1, \ldots, N, \; t = l, h$$

Since the payoff function is concave, second-order conditions are satisfied.

3.3. The Market Equilibrium

We start by analysing the equilibrium that emerges in the market when all firms are entrepreneurial, i.e. $m = 0$ (there is no asymmetry of information). Since all firms are equal we can compute the equilibrium output for each firm using the best reply function given by (13) and the definition of $\mathrm{E}_\theta\left[q_{\mathrm{J}}\right]$:

(14) $$\hat{q}^*_{\mathrm{J}\,l} = \frac{a - \theta_l}{n+2} - \frac{2\,(n-1)\,(1-p)\,\overline{\theta}}{6\,(n+2)}$$

(15) $$\hat{q}^*_{\mathrm{J}\,h} = \frac{a - \theta_h}{n+2} + \frac{2\,(n-1)\,p\,\overline{\theta}}{6\,(n+2)} \qquad J = 1, \ldots, N$$

where $\overline{\theta} = \theta_l - \theta_h$.

We call the equilibrium given by (14) and (15) *perfect information equilibrium* since it corresponds to the case where there is no asymmetry of information.

In the case where $m > 0$ we derive the equilibrium contracts for each state of nature and each type of firm. Using the best-reply functions given by (8), (9) and (13) and the definition of a firm's expected output we obtain the equilibrium output stipulated in contracts:

LEMMA 1 : The state contingent optimal quantities in the private oligopoly are the following:

$$q^*_{\mathrm{M}\,l} = \frac{a-\theta_l}{2+n} - \frac{\overline{\theta}\,(1-p)}{6\,(2+n)}\left[\frac{2\,(2+n)}{p} + 3\,(n-m)\right] \tag{16}$$

$$q^*_{\mathrm{E}\,l} = \frac{a-\theta_l}{2+n} + \frac{\overline{\theta}\,(1-p)}{6\,(2+n)}\,[3\,m - 2\,(n-1)] \tag{17}$$

$$q^*_{\mathrm{M}\,h} = \frac{a-\theta_h}{2+n} + \frac{\overline{\theta}}{6\,(2+n)}\,[2\,(n-1)\,p - (1-p)\,(2+n-3\,m)] \tag{18}$$

$$q^*_{\mathrm{E}\,h} = \frac{a-\theta_h}{2+n} + \frac{\overline{\theta}}{6\,(2+n)}\,[2\,(n-1)\,p + 3\,m\,(1-p)\,] \tag{19}$$

The equilibrium wages are obtained by replacing the optimal quantities into (10) and (11).

To guarantee positive quantities we require:

$$(a-\theta_l) > \frac{\overline{\theta}\,(1-p)}{6\,p}\,[3\,(n-m)\,p + 2\,(2+n)] \tag{20}$$

This condition implies that the reduction of the output due to the existence of asymmetry of information cannot be larger than the quantity produced in absence of asymmetry of information.

Note that $q^*_{\mathrm{M}\,l}$, the output of managerial firms in the less favorable state of nature, is lower than the perfect information level, $\hat{q}^*_{\mathrm{J}\,l}$ due to the existence of asymmetry of information. The explanation for this fact is presented in more detail below. In state h, $q^*_{\mathrm{M}\,h}$ is larger than the perfect information level if and only if the number of managerial firms is large enough, more precisely if and only if $m > (n+2)/3$. In case where this condition is verified, managerial firms facing state h benefit from the fact that some of the managerial rivals might face state l, the state where this type of firms is less efficient relatively to entrepreneurial firms. Entrepreneurial firms set larger output than in the perfect information case in both states of nature. Moreover, entrepreneurial firms produce larger output than managerial firms in both states of nature.

It should be noted, too, that when state l occurs less likely, *ceteris paribus*, $q^*_{\mathrm{M}\,l}$ is set further below the perfect information level $\hat{q}^*_{\mathrm{M}\,l}$, *i.e.* the lower is p the smaller is the value of $q^*_{\mathrm{M}\,l}$. By the same reasoning, the lower is

p the larger is the output of entrepreneurial firms relatively to their perfect information level. Further, the output of both type of firms, in both states of nature, is increasing with m.

By using equations (10) and (16) we compute the agents' payoffs for each state of nature. In state l, agents obtain zero payoff but, when the most favorable state occurs, they obtain a positive surplus. In order to induce the manager to tell the truth, the owner must pay him an informational rent in state h otherwise it would be advantageous for the agent to report θ_l, obtaining an extra payoff equal to $\overline{\theta}\,(2\,q^*_{\mathrm{M}\,l} + \theta_l + \theta_h)/2$. Since $\overline{\theta} > 0$, the surplus is strictly positive. For preventing the manager from shirking, $w_{\mathrm{M}\,h}$ must be increased. But, since the benefit from shirking is increasing in $q^*_{\mathrm{M}\,l}$, a reduction in $q^*_{\mathrm{M}\,l}$ decreases the amount by which $w_{\mathrm{M}\,h}$ must be increased to maintain incentive compatibility. Consequently, $q^*_{\mathrm{M}\,l}$ is lower than its perfect information level.

For making these observations more precise, let $\hat{q}^*_{\mathrm{J}\,l}$ and $\hat{q}^*_{\mathrm{J}\,h}$ stand for the perfect information output, as defined above. The equilibrium contracts are such that the principal induces the manager to produce an inefficiently small output in the higher cost state (*i.e.* he sets $q^*_{\mathrm{M}\,l} < \hat{q}^*_{\mathrm{J}\,l}$) to reduce the magnitude of the payment needed to induce truthful revelation in the lower cost state. When designing the contract, the principal weighs the expected benefits of setting $q^*_{\mathrm{M}\,l}$ lower than $\hat{q}_{\mathrm{J}\,l}$ (benefits which accrue when state h is realized) against the cost of inefficiency (costs which are borne if state l is realized). Of course this contract is not first-best and provides strictly less expected profit than does a first-best contract (see SAPPINGTON [1983]).

3.4. **Delegation and Market Efficiency**

Let us define $v \in [0,\, 1]$ as the proportion of managerial firms in the market. We proceed to look for the effect of a variation of v on the owners' payoffs. Computing the managerial firms' payoffs and taking the derivatives with respect to v yields:

$$\frac{\partial \mathrm{E}_\theta\,[\pi_{\mathrm{M}}]}{\partial v} = 3\,\tau\,[2\,(a-\theta_l) - n\,(1-v)\,(1-p)\,\overline{\theta}] \tag{21}$$

where $\mathrm{E}_\theta\,[\pi_{\mathrm{M}}]$ is the firm M's expected payoff and

$$\tau = (1-p)\,\overline{\theta}/[4\,(2+n)^2].$$

Using condition (20) we can show that (21) is positive.

The effect of v on entrepreneurial firms' payoff can be investigated using (22):

$$\frac{\partial \mathrm{E}_\theta\,[\pi_{\mathrm{E}}]}{\partial v} = 3\,\tau\,[2\,(a-\theta_l) + (nv+2)\,(1-p)\,\overline{\theta}] > 0 \tag{22}$$

Clearly a larger proportion of less efficient firms allows entrepreneurial firms to get higher expected payoffs.

The main reason for the direct relationship between v and firms' profits is related to higher expected prices associated with larger proportion of managerial firms. The explanation is quite straightforward: when v goes up, expected total output decreases because managerial firms' expected output is lower than entrepreneurial firms'. Consequently, the expected price rises and all firms have incentive to produce more at the new equilibrium. The impact of increasing v has a positive effect on *per firm* production but total expected output decreases leading to higher clearing market price.

This result is very interesting because it shows that delegation leads the market outcome further away from the competitive one. FERSHTMAN and JUDD [1987] have shown that delegation induces firms to produce larger output and, in consequence, the market outcome becomes more competitive.

Note that the agent's rent is positively related to $q_{\mathrm{M}\,l}$ so, when v increases the agent's rent also increases. Nevertheless, in spite of larger informational rents, managerial firms get larger profits when v increases.

What is the effect of delegation on market efficiency? In order to answer this question we analyse the impact of increasing v on the consumer and producer surplus:

$$\text{(23)} \qquad \frac{\partial \mathrm{E}_\theta\,[\mathrm{S}]}{\partial v} = \frac{4\,n\,\tau}{9\,p}\,\{v\,\overline{\theta}\,(2+n)^2 - 9\,p\,[(a-\theta_l) + \overline{\theta}\,(1-v)] - \overline{\theta}\,(n+5)\,(n-1)\,(1-v)\,p^2\}$$

where S denotes the consumer surplus.

For values of a sufficiently large the expected consumer surplus decreases with v since entrepreneurial firms are substituted by firms that are relatively less efficient. However, in cases where p is low and v is sufficiently large, the expected consumer surplus might increase with v because in such case total production in state h is larger than in the perfect information case.

We have already seen that firms' expected profits are positively related to v but, in order to analyse the effect of increasing v on the producer surplus, we need to know the value of $\mathrm{E}_\theta\,[\pi_{\mathrm{E}}] - \mathrm{E}_\theta\,[\pi_{\mathrm{M}}]$:

$$\text{(24)} \quad \mathrm{E}_\theta\,[\pi_{\mathrm{E}}] - \mathrm{E}_\theta\,[\pi_{\mathrm{M}}] = \frac{\tau\,(2+n)}{6\,p} \times \{3\,p\,\theta_l\,(2+6\,nv+n) - \overline{\theta}\,(10+18\,nv-13\,n)\,p^2 + 36\,ap + 3\,p\,\theta_h\,(2-6\,nv+7\,n) - 4\,\overline{\theta}\,(n+2)\}$$

Since (24) is positive by condition (20) then entrepreneurial firms make larger expected profits than managerial firms. However, if a managerial firm decides to become entrepreneurial v decreases, and consequently, the expected payoffs of all profit maximizing firms diminish. The new entrepreneurial firm gets higher payoff relatively to the previous state but entrepreneurial firms' payoffs are now lower than before, *i.e.* the new entrepreneurial firm exerts a negative externality on the other firms.

Using equations (21), (22) and (24) we can show that the producer surplus decreases when the proportion of managerial firms is larger. In spite of *per firm* larger expected profits, the aggregated expected profit is lower since managerial firms get lower expected profits than entrepreneurial firms.

In consequence, both consumer and producer surplus decrease when the proportion of delegating firms increases and the market demand parameter a is sufficiently large.

$$(25)\quad \frac{\partial \mathrm{E}_\theta\,[\mathrm{W}]}{\partial v} = \frac{\tau}{18\,p}\{4\,(n+2)^2\;(3+2\,nv)\overline{\theta} - 72\,ap\,(3+n) - 9\,p\,\theta_l\,(n+2)^2 - 18\,\overline{\theta}\,p\,(8+3\,n)\,(1-p) - 9\,\theta_h\,p\left(4+20\,n+7\,n^2\right) - \delta\overline{\theta}\,p^2\}$$

where $\delta = -60 - 32\,nv + 116\,n - 32\,n^2\,v + 71\,n^2 - 8\,n^3\,v + 8\,n^3$ is positive.

From (25) it is clear that *the expected welfare is a decreasing function of the managerial firms proportion* except in cases where either the market demand a is very small or the probability p is very low.

PROPOSITION 2 : If the fraction of managerial firms increases then:
(i) the output per firm in each state of nature augments. Consequently, agents' informational rents increase;
(ii) the expected total output diminuishes due to the large fraction of less efficient firms;
(iii) the augmentation of expected prices leads to a rise of net expected profits of both, entrepreneurial and managerial firms;
(iv) the expected social welfare decreases except in cases where either the market demand is very small or the probability p is very low.

The main result of this section extends, in a certain way, HART's [1983] results. In that paper, under perfect competition and perfect correlation of input prices, entrepreneurial firms provide a source of discipline for managerial firms: due to the output expansion (entrepreneurial firms produce larger output), prices fall and agents are forced to reduce slack in order to reach the profit target specified by the contract. In the present case informational rents are reduced when the proportion of entrepreneurial firms raises since these firms are more efficient. In this sense, entrepreneurial firms have a *disciplinary role.*

4 Mixed Oligopoly

In this section we investigate whether it is possible to improve allocative efficiency by introducing a public firm in the market. We maintain the market structure assuming that one of the managerial firms, firm s, is a public firm which maximizes social welfare. It faces the same problem of asymmetry of information as the remaining managerial firms, the only difference is that the owner of firm s has a different payoff function. Social welfare is defined as the sum of consumer and producer surplus.

In order to determine the optimal contract for the agent, owner s solves the following program:

$$\max_{\substack{q_{sl},\, q_{sh} \\ w_{sl},\, w_{sh}}} \; \mathrm{E}_\theta\,[\mathrm{W}] = \frac{\mathrm{E}_\theta \left[\left(\sum\limits_{\mathrm{J}=1}^{\mathrm{N}} q_{\mathrm{J}} \right)^2 \right]}{2} \tag{26}$$

$$+ \, p \sum_{\mathrm{J}=1}^{\mathrm{N}} \pi_{\mathrm{J}\, l} \left(q_{s\, l}, \sum_{\mathrm{J}\neq \mathrm{K}} \mathrm{E}_\theta\,[q_{\mathrm{J}}],\, w_{\mathrm{J}\, l} \right)$$

$$+ \, (1-p) \sum_{\mathrm{J}=1}^{N} \pi_{\mathrm{J}\, h} \left(q_{s\, h}, \sum_{\mathrm{J}\neq \mathrm{S}} \mathrm{E}_\theta\,[q_{\mathrm{J}}],\, w_{\mathrm{J}\, h} \right)$$

$$\text{s.t.} \quad w_{sl} - \frac{(q_{sl}+\theta_l)^2}{2} \geq w_{sh} - \frac{(q_{sh}+\theta_l)^2}{2} \tag{27}$$

$$w_{sh} - \frac{(q_{sh}+\theta_h)^2}{2} \geq w_{sl} - \frac{(q_{sl}+\theta_h)^2}{2} \tag{28}$$

$$w_{sl} - \frac{(q_{sl}+\theta_l)^2}{2} \geq 0 \tag{29}$$

$$w_{sh} - \frac{(q_{sh}+\theta_h)^2}{2} \geq 0 \tag{30}$$

$$q_{sl},\, q_{sh},\, w_{sl},\, w_{sh} \geq 0 \quad J = 1, \ldots, N.$$

From the first-order conditions we obtain the best reply functions of firm s:

$$q^*_{sl}\,(\cdot) = \frac{a - \theta_h - \sum\limits_{\mathrm{J}\neq\mathrm{S}} \mathrm{E}_\theta\,[q_{\mathrm{J}}]}{2} - \frac{\overline{\theta}}{2\,p} \tag{31}$$

$$q^*_{sh}\,(\cdot) = \frac{a - \theta_h - \sum\limits_{\mathrm{J}\neq\mathrm{S}} \mathrm{E}_\theta\,[q_{\mathrm{J}}]}{2} \tag{32}$$

$$\text{for} \quad J = 1, \ldots, S-1,\, S+1, \ldots, N.$$

Due to the existence of asymmetry of information, in state l firm s produces below its perfect information level, for a given expected production of the rival firms. Since constraints (28) and (29) are binding, the optimal state contingent wages for agent s can be computed using (10) and (11) but now with q_{sl} and q_{sh} replacing $q_{\mathrm{M}\, l}$ and $q_{\mathrm{M}\, h}$.

Using (8)-(9), (12) and (31)-(32) we derive the market equilibrium for each state of nature:

LEMMA 3 : The unique Nash equilibrium for the mixed oligopoly leads to the following state contigent quantities:

$$q^*_{sl} = \frac{2\,(a-\theta_l)}{3+n} - \frac{\overline{\theta}\,(1-p)}{2\,(3+n)} \left[2\,(n-m) + \frac{(n+3)}{p} \right] \tag{33}$$

$$q^*_{\mathrm{M}\,l} = \frac{(a-\theta_l)}{3+n} - \frac{\overline{\theta}\,(1-p)}{6\,(3+n)}\left[3\,(n-m) + \frac{2\,(3+n)}{p}\right] \tag{34}$$

$$q^*_{\mathrm{E}\,l} = \frac{(a-\theta_l)}{3+n} + \frac{\overline{\theta}\,(1-p)}{6\,(3+n)}\,[3\,m+1-2\,(n-1)\,] \tag{35}$$

$$q^*_{sh} = \frac{2\,(a-\theta_h)}{3+n} - \frac{\overline{\theta}}{2\,(3+n)}\,[(n-1)\,p-(n+1-2\,m)] \tag{36}$$

$$q^*_{\mathrm{M}\,h} = \frac{(a-\theta_h)}{3+n} + \frac{\overline{\theta}}{6\,(3+n)}\,[3(n-m)\,p-(n-3\,m)] \tag{37}$$

$$q^*_{\mathrm{E}\,h} = \frac{(a-\theta_h)}{3+n} + \frac{\overline{\theta}}{6\,(3+n)}\,[3\,(m+1)\,(1-p)+\,2\,np)] \tag{38}$$

The equilibrium wages are obtained by substituting the optimal quantities into (10) and (11).

where subscripts M and E denote managerial and entrepreneurial firms, as in the previous case. In order to guarantee positive outputs for all firms we must impose the following condition on the parameters:

$$(a-\theta_l) > \frac{\overline{\theta}\,(1-p)}{6\,p}\,[3\,(n-m)\,p+2\,(3+n)] \tag{39}$$

As in the private oligopoly, this condition requires that the reduction of the output, due to the existence of asymmetry of information, cannot be larger than the quantity produced in absence of asymmetry of information. Once again, all managerial firms produce below their perfect information levels, in state l. On the contrary entrepreneurial firms over-produce in both states relatively to the perfect information level. The public firm, in state h, produces below its perfect information level if and only if $m > (n+1)/2$ whereas the remaining managerial firms over-produce in state h relatively to the perfect information level if $m > n/3$. Note that $q^*_{sl} > q^*_{\mathrm{M}\,l}$ and $q^*_{\mathrm{E}\,l} - q^*_{\mathrm{M}\,l} = [(1-p)\,\overline{\theta}\,(2+p)/2\,p] > 0$ but we cannot state that an entrepreneurial firm produces more than firm s in state l; in state h the output set by either firm s or firm E is larger than the output set by firm M. This fact suggests an interesting observation: *while in general the welfare maximizing firm produces higher level of output than the profit maximizer, asymmetry of information leads to situations where this may not be true anymore* (see De Fraja and Delbono [1989], [1990]).

Since the output of firm s is higher than firm M's and the agent's surplus is an increasing function of state l output, then the agent s's informational rent is higher than the agent M's. In particular, the difference between $\mathrm{E}_\theta\,[\mathrm{U}_s]$ and $\mathrm{E}_\theta\,[\mathrm{U}_{\mathrm{M}}]$ is increasing with v, the proportion of managerial firms. Actually, in state l, increasing the number of managerial firms leads to higher production for all types of firms,

$$\partial q^*_{sl}/\partial v = 2\,\partial q^*_{\mathrm{M}\,l}/\partial v = 2\,\partial q^*_{\mathrm{E}\,l}/\partial v = n\,(1-p)\,\overline{\theta}/2\,(3+n).$$

Consequently, $\mathrm{E}_\theta[\mathrm{U}_s]$ and $\mathrm{E}_\theta[\mathrm{U}_\mathrm{M}]$ must raise in order to maintain incentive compatibility. Since the disutility of effort is increasing in effort and $q_{sl} > q_{\mathrm{M}\,l}$ then $\mathrm{E}_\theta[\mathrm{U}_s]$ raises faster than $\mathrm{E}_\theta[\mathrm{U}_\mathrm{M}]$,

$$\partial \mathrm{E}_\theta[\mathrm{U}_s]/\partial v = 2\,(\partial \mathrm{E}_\theta[\mathrm{U}_\mathrm{M}]/\partial v) = n\,(1-p)^2\,\overline{\theta}^2/\,(3+n).$$

Like in the private duopoly entrepreneurial firms have a disciplinary role: by decreasing v managers' informational rents are reduced.

Straightforward computations show that all firms' net profits are strictly increasing with the proportion of managerial firms as we have shown in the previous private oligopoly case. However, the payoff of firm s is the expected social surplus and the derivative with respect to v yields:

$$\begin{aligned}(40)\quad \frac{\partial \mathrm{E}_\theta[\mathrm{W}]}{\partial v} = \frac{-\tau'}{72\,p}\,\{&9\,p\,[8\,a\,(5+n)+\theta_l\left(23+8\,n+n^2\right)\\ &-\theta_h\left(9-32\,n+7\,n^2\right)]+\overline{\theta}\,[18\,nv\,(13+3\,n)\,(1-p)\,p\\ &+\beta\,p^2-4\,(n+3)^2\,(2\,nv+1)]\}\end{aligned}$$

where $\tau' = n\,\overline{\theta}\,(1-p)/(3+n)^2$ and

$$\beta = [-135-72\,nv+216\,n-48\,n^2\,v+87\,n^2+8\,n^3\,(1-v)] > 0.$$

Like in the private oligopoly, if the probability associated with state l is not too small (or a is large enough) the expected social surplus decreases with the proportion of managerial firms. Since the net expected profit is an increasing function of v, the negative impact on the expected total welfare is explained by the effect of increasing v on the consumer expected surplus. Computations yield:

$$\begin{aligned}(41)\quad \frac{\partial \mathrm{E}_\theta[\mathrm{S}]}{\partial v} = \frac{-n\,(1-p)\,\overline{\theta}}{9\,p\,(3+n)^2}\,\{&9\,p\,[(a-\theta_h)\,(n+1)+\overline{\theta}\,n\,(1-v)]\\ &+\overline{\theta}\,[p^2\,n^2\,(6+n)\,(1-v)-(3+n)^2\,(nv-1)]\}\end{aligned}$$

It is clear that the expected consumer surplus decreases with v except in the cases where p is too close to zero. The explanation for this effect is the same as given in the previous section. Consequently, for values of p sufficiently high, the relationship between expected social welfare and v is negative due to the negative effect in both consumer and producer surplus, in spite of the increment of the *per* firm's profit.

5 Private Oligopoly vs. Mixed Oligopoly

In this section we compare the equilibrium outcomes in mixed and private oligopolies, which have been found in the previous sections. To start with, we compare the quantities produced by each type of firm in each market structure:

$$q_{\mathrm{M}\,l}^{\mathrm{MO}} - q_{\mathrm{M}\,l}^{\mathrm{PO}} = q_{\mathrm{E}\,l}^{\mathrm{MO}} - q_{\mathrm{E}\,l}^{\mathrm{PO}} = q_{\mathrm{M}\,h}^{\mathrm{MO}} - q_{\mathrm{M}\,h}^{\mathrm{PO}} = q_{\mathrm{E}\,h}^{\mathrm{MO}} - q_{\mathrm{E}\,h}^{\mathrm{PO}} = -\kappa \tag{42}$$

where $\kappa = [2\,(a-\theta_l) - \overline{\theta}\,n\,(1-v)\,(1-p)]/[2\,(2+n)\,(3+n)]$ is positive by condition (39). Superscripts MO and PO denote mixed oligopoly and private oligopoly, respectively.

Note that each profit maximizing firm reduces its optimal production when the public firm is present in the market. However, the output of firm s is very large and more than offsets the reduction in the profit maximizing firms' production:

$$\mathrm{E}_\theta\,[\mathrm{Q}^{\mathrm{MO}}] - \mathrm{E}_\theta\,[\mathrm{Q}^{\mathrm{PO}}] = 2\,\kappa$$

Since $\kappa > 0$, total output is larger and consumers are better off in the mixed oligopoly. The presence of the public firm leads to an improvement of the allocative efficiency.

We proceed to compare agents' total informational rent in both market structures. Adding-up the informational rents of all managerial firms' agents and computing the difference between those amounts in the mixed and in the private oligopoly, yields:

$$\begin{aligned}\Delta &= \overline{\theta}\,(1-p)\,[q_{sl}^{\mathrm{MO}} + (nv-1)\,q_{\mathrm{M}\,l}^{\mathrm{MO}} - nvq_{\mathrm{M}\,l}^{\mathrm{PO}}] \\ &= \overline{\theta}\,(1-p)\left[\kappa\,(n\,(1-v)+2) - \frac{\overline{\theta}\,(1-p)}{6\,p}\right]\end{aligned}$$

The total informational rent is higher in the mixed oligopoly for $\kappa > \overline{\theta}\,(1-p)/[6\,p\,(n\,(1-v)+2)]$. For very small values of p or for large number of firms the informational rent is larger in the private oligopoly than in the mixed oligopoly. In fact, the derivative of managerial firms' output, in state l, with respect to p is positive but it is larger for the non-profit maximizer than for the profit maximizing firm. In consequence, for very small values of p the level of q_{sl}^* is relatively smaller so the difference between $q_{sl}^{\mathrm{MO}} - q_{\mathrm{M}\,l}^{\mathrm{PO}}$ is not enough to outweigh the negative term $nv\,(q_{\mathrm{M}\,l}^{\mathrm{MO}} - q_{\mathrm{M}\,l}^{\mathrm{PO}})$. Remark that managerial firms' agents obtain a lower rent in the mixed oligopoly than in the private oligopoly.

However, we are particularly interested in the impact of the non-profit maximizing firm in terms of expected social surplus. We want to determine the set of conditions under which the mixed oligopoly generates higher expected social welfare than the private oligopoly. Computing the difference between total surplus at the equilibrium in each market structure yields:

$$\begin{aligned}\Delta_{\mathrm{W}} = \frac{1}{72\,\gamma}\,\{&-36\,p\,(a-\theta_l)^2\left(n^2-n-8\right) \\ &+ 9\,\overline{\theta}\,(1-p)\,[\gamma\overline{\theta}\,(8\,nv-10) - 8\,np\,(a-\theta_l)\,(1-v)\,(3\,n+7) \\ &+ \overline{\theta}\,pn^2\left(n^2+11\,n+20\right)(1-v)^2\,(1-p)]\}\end{aligned}$$

where $\Delta_{\mathrm{W}} = \mathrm{E}_\theta\,[\mathrm{W}^{\mathrm{MO}}] - \mathrm{E}_\theta\,[\mathrm{W}^{\mathrm{PO}}]$ and $\gamma = (2+n)^2\,(3+n)^2/9\,p$.

For the range of parameters that satisfy $\Delta W \geq 0$ the industry configuration is mixed because the public firm is welfare improving, otherwise it is private.

In figure 1 we represent the locus of $\Delta_{\mathrm{W}} = 0$ in the space $(\hat{a}, n)$, where $\hat{a} = a - \theta_l$, for different industry configurations: $v = 1/n$, $v = 1/2$ and $v = 1$, assuming $p = 1/2$ and $\overline{\theta} = 1$.

Consider first the number of firms $n < 4$ as given and take into account increases in market demand $\hat{a}$. In a point like B, Δ_{W} is positive for $v = 1$ but it is negative when v is at least lower than 1/2. When the market demand is small (low informational rents) and the market is less competitive, the public firm is welfare improving if and only if the proportion of managerial firms is large enough. In situations where v is not very high entrepreneurial firms perform better the disciplinary role and there is no room for a public firm.

A further increase of $\hat{a}$ to a point like C implies that for $v = 1/2$, Δ_{W} is positive but it is still negative for small values of v; however at point D the public firm is welfare improving irrespective of the proportion of managerial firms. Intuitively one would think that a rise in $\hat{a}$ would make equilibria with a public firm less likely because informational rents increase and the public firm bears higher informational rents. However, because the public firm produces larger quantities, the allocative efficiency gain more than offsets the loss in terms of productive efficiency. *In the case where n is low, the regulating role of the public firm becomes more relevant when the proportion of managerial firms is high because these firms reduce competition in the market.*

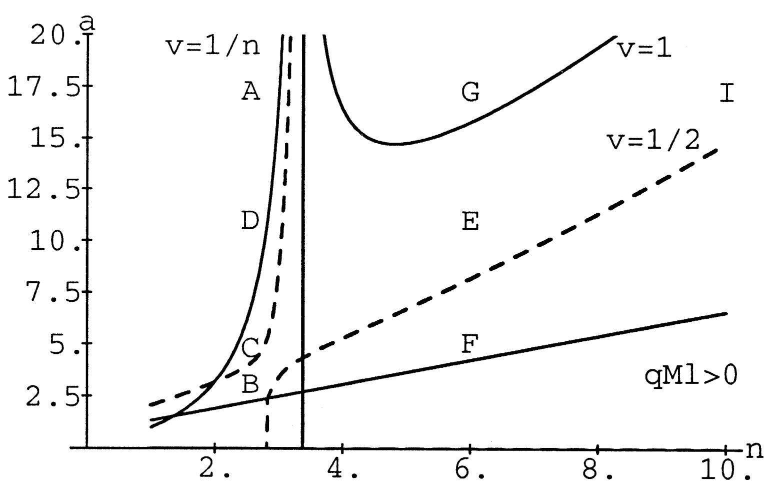

FIGURE 1

$p = 1/2$ ***and*** $\theta = 1$***. The curves represent the locus of*** $\Delta_{\mathrm{W}} = 0$ ***when*** $v = 1$***,*** $v = 1/n$ ***(solid lines) and*** $v = 1/2$ ***(dashed line)***

For cases where n is larger, like in point F, conclusions are turned upside down: at F there is a mixed industry configuration for $v = 1$ and $v = 1/2$ but for $v = 1/n$ the industry configuration is private. Here $\Delta_{\mathrm{W}} > 0$ for the points below the curves and it is negative for the points above the curves. A further increase in the demand size to point E shifts the industry configuration from mixed to private oligopoly when v is at least equal to 1/2: since there is a direct relationship between informational rents and demand size, entrepreneurial firms substitute the public firm as a regulation mechanism. Note that at points D and E the level of $\hat{a}$ is the same but, for $v = 1/2$, the public firm is welfare improving only when competition in the market is low. Finally, if demand size is so large, like point G, the industry configuration is always private: no matter the proportion of managerial firms the informational rent of the public agent is too large.

Take now $\hat{a}$ as fixed and consider successive increases of n: at a point like A the industry configuration is a mixed oligopoly. A further increase of n to point G changes the configuration from mixed to private oligopoly. At point I, Δ_{W} is positive except when v is very small. Once again the large proportion of entrepreneurial firms perform better the regulatory role. It is interesting to point out that De FRAJA and DELBONO [1987] conclude that when there are many firms in the market the negative effect in terms of productive efficiency, due to the presence of the public firm, outweighs the positive effect in terms of allocative efficiency. The main reason is that they consider firms that are equally efficient and the increase in consumer surplus is not sufficient to offset the loss in private firms' profits due to the presence of the public firm. In our framework when n is large and $\hat{a}$ is not too large there are two kind of effects that are not present in De FRAJA and DELBONO [1989]; first, when the number of firms raises agents' rents are also reduced and for a number sufficiently large of firms, total informational rents might be higher in the private than in the mixed oligopoly [5]; second, this situation is more likely to occur when the proportion of less efficient firms is large.

Summing up, the relationship between industry configuration and number of firms, for a fixed demand size, is not monotonic.

We have considered the role played by variables $\hat{a}$ and n for given values of the probability p and the measure of uncertainty $\overline{\theta}$. We do not provide a full analysis here of the way these variables affect equilibrium outcomes; however, it is easy to show that the mixed oligopoly is more likely for small values of p (low informational rents).

Summing-up the main results of this example we conclude that:

(i) if all profit maximizing firms are managerial firms and there is low competition in the market then the mixed oligopoly is welfare improving:

(ii) when the market demand is small (low informational rents) and the market is less competitive (low number of firms), the public firm is welfare improving if and only if the proportion of managerial firms (less efficient

5. This situation corresponds to the case where a profit maximizing managerial firm, in a mixed oligopoly, produces larger output in the less favorable state of nature than a public firm, implying larger informational rents for the profit maximizer.

firms) is large enough, otherwise entrepreneurial firms perform better the disciplinary role and there is no room for a public firm;

(iii) in cases where the market demand is large and the number of firms is not very high, the market configuration is clearly private oligopoly because, under such conditions, informational rents are very high;

(iv) if the market is very competitive then the public firm might be welfare improving because in such case the total informational rents are larger in the private oligopoly than in the mixed oligopoly. This situation is more likely to occur the larger the number of managerial firms.

Finally, let us determine which is the influence of changing the proportion of entrepreneurial firms on the value of Δ_{W}. The derivative of Δ_{W} with respect to v yields:

$$\begin{aligned}(43)\quad \frac{\partial \Delta_{\mathrm{W}}}{\partial v} = \frac{-n}{36\,\gamma\,p}\,&\{\overline{\theta}\,(1-p)\,[-36\,(a-\theta_l)\,(3\,n+7)\,p \\ &+ 9\,\overline{\theta}\,n\,(n^2+11\,n+20)\,(1-v)\,p\,(1-p) - 36\,\overline{\theta}\gamma]\}\end{aligned}$$

The sign of (43) is always positive which means that for $\Delta_{\mathrm{W}} > 0$, the augmentation of v reinforces the positive effect of the public firm in the market. As we expected, for $\Delta_{\mathrm{W}} > 0$ the advantage of the mixed oligopoly, in expected social welfare terms, decreases with the proportion of entrepreneurial firms. In this case *the disciplinary role of the entrepreneurial firms substitutes the role of the public firm as an internal regulation mechanism.*

6 Conclusion

The present paper examines the effect of competition on the design of contracts between firms' owners and their managers. We assume that a proportion of firms does not face the usual monitoring problems associated with the asymmetry of information. By increasing the proportion of these firms the informational rents in the market are reduced and consumers are better off. However, changing the proportion of firms in this way is welfare improving if and only if the probability of occuring the less favorable state is not too low.

Considering that one of the managerial firms is a non-profit maximizer – a public firm – we obtain some interesting results. First of all, there is an improvement in terms of allocative efficiency in spite of the fact that the public agent obtains higher rent than the profit maximizer's agent, corresponding to a loss in terms of productive efficiency. Is this trade-off between the allocative efficiency gain and productive efficiency loss that determines the net effect of the presence of the public firm. We conclude that:

(i) if all the profit maximizing firms are managerial firms and there is low competition in the market then the mixed oligopoly is welfare improving;

(ii) when the market demand is small (low informational rents) and the market is less competitive (low number of firms), the public firm is welfare improving if and only if the proportion of managerial firms (less efficient firms) is large enough, otherwise entrepreneurial firms perform better the disciplinary role and there is no room for a public firm;

(iii) in cases where the market demand is large and the number of firms is not very high the market configuration is clearly private oligopoly because, under such conditions, informational rents are very high;

(iv) if the market is very competitive then the public firm might be welfare improving, because in such case the total informational rents are larger in the private oligopoly than in the mixed oligopoly. This situation is more likely to occur the larger the number of managerial firms.

Note first that we can simplify the nonlinear program by dropping (7) because it is implied by (4) and (6):

$$w_{\mathrm{M}\,h} - \frac{(q_{\mathrm{M}\,h} + \theta_h)^2}{2} \geq w_{\mathrm{M}\,l} - \frac{(q_{\mathrm{M}\,l} + \theta_h)^2}{2} \geq w_{\mathrm{M}\,l} - \frac{(q_{\mathrm{M}\,l} + \theta_l)^2}{2}$$

Let $x = \{q_{\mathrm{M}\,l},\, q_{M\,h},\, w_{\mathrm{M}\,l},\, w_{\mathrm{M}\,h}\}$ and $\lambda = \{\lambda_1,\, \lambda_2,\, \lambda_3\}$ and define the Lagrangian function as follows:

$$\begin{aligned}
\mathrm{L}\,(x,\,\lambda) = p &\left[\left(a - \sum_{h \neq \mathrm{M}} q_{h1} - q_{\mathrm{M}\,l}\right) q_{\mathrm{M}\,l} - w_{\mathrm{M}\,l}\right] \\
&+ (1-p)\left[\left(a - \sum_{h \neq \mathrm{M}} q_{h2} - q_{\mathrm{M}\,h}\right) q_{\mathrm{M}\,h} - w_{\mathrm{M}\,h}\right] \\
&+ \lambda_1 \left[w_{\mathrm{M}\,h} - \frac{(q_{\mathrm{M}\,h} + \theta_h)^2}{2} - w_{\mathrm{M}\,l} + \frac{(q_{\mathrm{M}\,l} + \theta_h)^2}{2}\right] \\
&+ \lambda_2 \left[w_{\mathrm{M}\,l} - \frac{(q_{\mathrm{M}\,l} + \theta_l)^2}{2} - w_{\mathrm{M}\,h} + \frac{(q_{\mathrm{M}\,h} + \theta_l)^2}{2}\right] \\
&+ \lambda_3 \left[w_{\mathrm{M}\,l} - \frac{(q_{\mathrm{M}\,l} + \theta_l)^2}{2}\right]
\end{aligned}$$

The problem is that the constraints set is not convex. However, if it verifies: $\mathrm{L}\,(x,\,\lambda^*) \leq \mathrm{L}\,(x^*,\,\lambda^*) \leq \mathrm{L}\,(x^*,\,\lambda)\ \forall\, x \geq 0,\ \lambda \geq 0$ then $(x^*,\,\lambda^*)$ solves the nonlinear program by the Kühn-Tucker theorem. This means that the vector $(x^*,\,\lambda^*)$ must be a saddle point of the Lagrangian function, maximizing it relative to all the nonnegative elements of x and minimizing it relative to all nonnegative Lagrange multipliers.

The Kühn-Tucker conditions are the first-order conditions for a saddle point of $\mathrm{L}\,(x,\,\lambda)$ and they are given by equations (A.1)-(A.4):

$$\begin{aligned}
\text{(A.1)}\quad \frac{\partial \mathrm{L}}{\partial q_{\mathrm{M}\,l}} &= p\left(a - 2\,q_{\mathrm{M}\,l} - \sum_{h \neq \mathrm{M}} \mathrm{E}_\theta\,[q_h]\right) - (\lambda_2 + \lambda_3)\,(\theta_l + q_{\mathrm{M}\,l}) \\
&\quad + \lambda_1\,(\theta_h + q_{\mathrm{M}\,l}) \leq 0, \qquad \frac{\partial \mathrm{L}}{\partial q_{\mathrm{M}\,l}}\, q_{\mathrm{M}\,l} = 0, \quad q_{\mathrm{M}\,l} \geq 0
\end{aligned}$$

$$\begin{aligned}
\text{(A.2)}\quad \frac{\partial \mathrm{L}}{\partial q_{\mathrm{M}\,h}} &= (1-p)\left(a - 2\,q_{\mathrm{M}\,h} - \sum_{h \neq \mathrm{M}} \mathrm{E}_\theta\,[q_h]\right) \\
&\quad - (\lambda_1 + \lambda_4)\,(\theta_h + q_{\mathrm{M}\,h}) \\
&\quad + \lambda_2\,(\theta_l + q_{\mathrm{M}\,h}) \leq 0, \qquad \frac{\partial \mathrm{L}}{\partial q_{\mathrm{M}\,h}}\, q_{\mathrm{M}\,h} = 0, \quad q_{\mathrm{M}\,h} \geq 0
\end{aligned}$$

(A.3) $\dfrac{\partial \mathrm{L}}{\partial w_{\mathrm{M}\,l}} = \lambda_3 - \lambda_1 + \lambda_2 - p \leq 0, \qquad \dfrac{\partial \mathrm{L}}{\partial w_{\mathrm{M}\,l}}\, w_{\mathrm{M}\,l} = 0, \quad w_{\mathrm{M}\,l} \geq 0$

(A.4) $$\frac{\partial \mathrm{L}}{\partial w_{\mathrm{M}\,h}} = \lambda_4 + \lambda_1 - \lambda_2 - (1-p) \leq 0, \qquad \frac{\partial \mathrm{L}}{\partial w_{\mathrm{M}\,h}}\, w_{\mathrm{M}\,h} = 0, \qquad w_{\mathrm{M}\,h} \geq 0$$

$$\lambda_t \geq 0 \; \frac{\partial \mathrm{L}}{\partial \lambda_t} \geq 0 \qquad \frac{\partial \mathrm{L}}{\partial \lambda_t}\, \lambda_t = 0 \quad t = 1,\, 2,\, 3,\, 4$$

It cannot be optimal to choose $q_{\mathrm{M}\,l} = q_{\mathrm{M}\,h} = 0$, for a given output of the other firms. It yields a negative payoff since the firm pays always a minimum effort level to the agent. Suppose (A.3) holds as a strict inequality. The complementary slackness condition implies that $w_{\mathrm{M}\,l} = 0$ and condition (6) is violated. Then, (A.3) must hold as equality and $\lambda_1 = \lambda_2 + \lambda_3 - p$.

Now we have to distinguish two cases:

(i) (A.4) holds as an equality then

(A.5) $$\lambda_1 = \lambda_2 + 1 - p > 0 \qquad \text{and} \qquad \lambda_3 = 1$$

In this case both (4) and (6) are binding and yield:

(A.6) $$w_{\mathrm{M}\,l} = \frac{(q_{\mathrm{M}\,l} + \theta_l)^2}{2}$$

(A.7) $$w_{\mathrm{M}\,h} = \frac{(q_{\mathrm{M}\,l} + \theta_l)^2}{2} + \frac{(q_{\mathrm{M}\,h} + \theta_h)^2}{2} - \frac{(q_{\mathrm{M}\,l} + \theta_h)^2}{2}$$

Substituting (A.6) and (A.7) into (5) yields:

(A.8) $$-(\theta_l - \theta_h)\,(q_{\mathrm{M}\,h} - q_{\mathrm{M}\,l}) \leq 0$$

In order to (A.8) to be verified it must verify either $q_{\mathrm{M}\,l} = q_{\mathrm{M}\,h}$ or $q_{\mathrm{M}\,h} > q_{\mathrm{M}\,l}$. Let check the first case. Substituting (A.5) into (A.1) and (A.2) yields $(\theta_l - \theta_h)\,[\lambda_2 + (1-p)] = 0$. This can be true only for $\theta_l = \theta_h$ which is a contradiction. Then $q_{\mathrm{M}\,h} > q_{\mathrm{M}\,l}$ in order to satisfy (A.8). Since (A.8) holds as a strict inequality then (5) is not binding and so $\lambda_2 = 0$. Consequently, $\lambda_1 = 1 - p$. From (A.1) and (A.2) we obtain the output levels given by (8) and (9).

(ii) (A.4) holds as a strict inequality. Complementary slackness implies $w_{\mathrm{M}\,h} = 0$. Using (6) we can write:

(A.9) $$-w_{\mathrm{M}\,l} + \frac{(q_{\mathrm{M}\,l} + \theta_l)^2}{2} > -w_{\mathrm{M}\,l} + \frac{(q_{\mathrm{M}\,l} + \theta_h)^2}{2} > \frac{(q_{\mathrm{M}\,h} + \theta_h)^2}{2}$$

which violates condition (6).

It is straightforward to check that the Lagrange parameters defined in (i) minimize $\mathrm{L}\,(x^*,\, \lambda)$ given x^*. To prove that x^* defined in (i) indeed maximizes $\mathrm{L}\,(x,\, \lambda^*)$ we have to show that $\mathrm{L}\,(x,\, \lambda^*)$ is everywhere defined negative semidefinite. To see this note that the Hessian matrix becomes:

$$H = \begin{bmatrix} \mathrm{L}_{q_{\mathrm{M}\,l}\,q_{\mathrm{M}\,l}} & 0 \\ 0 & \mathrm{L}_{q_{\mathrm{M}\,h}\,q_{\mathrm{M}\,h}} \end{bmatrix}$$

while other second derivatives of L vanishe. Since $\mathrm{H}_1 = -3\,p < 0$ and $\mathrm{H}_2 = 9\,p\,(1-p) > 0$ we have solved the optimization problem.

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