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Earnings-Related Borrowing Restrictions: Empirical Evidence from a Pseudo Panel for the U.K.

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ABSTRACT. – The life-cycle model with liquidity constraints produces an Euler equation with unobservable Kuhn-Tucker multipliers. If borrowing restrictions depend on earnings and leisure is a choice variable one can derive an Euler equation involving only observable variables.

This paper presents estimates of the Euler equation on a pseudo (or "synthetic") panel of UK households. Most parameters are well determined and in agreement with the model's predictions. They can therefore be used to evaluate each cohort's Kuhn-Tucker multiplier over the sample period.

Endettement contraint par les revenus du travail : estimations sur un pseudo-panel britannique

RÉSUMÉ. – Le modèle de cycle de vie avec contraintes de liquidité conduit à une équation d'Euler avec des multiplicateurs de Kuhn et Tucker non observables. Si l'endettement est contraint par les revenus du travail et que le loisir est une variable de choix, on obtient une équation d'Euler ne faisant intervenir que des variables observables.

Dans cet article, on estime une équation d'Euler sur un pseudo-panel de ménages britanniques. La plupart des paramètres sont estimés avec précision et les estimations sont en accord avec les prédictions du modèle. Elles peuvent donc être utilisées pour évaluer le multiplicateur de Kuhn et Tucker de chaque cohorte en la période d'échantillonnage.

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1 Introduction

Recent studies of intertemporal behaviour under uncertainty have adopted the Euler equation approach introduced by HALL [1978]. Assuming perfect capital markets and rational expectations, Hall showed that the marginal utility of consumption is a martingale process. If preferences are additively separable in goods and leisure, utility is quadratic and the real interest rate equals the time preference parameter, consumption itself is a random walk.

However, several empirical papers have rejected the restrictions implied by Hall's version of the Life-cycle/Permanent Income Model (FLAVIN, 1981, HALL and MISHKIN, 1982, HAYASHI, 1985). Likely explanations of such rejection is the presence of capital markets imperfections affecting a non negligible proportion of the population and/or preference interactions between goods and leisure.

As MUELLBAUER [1983], ZELDES [1989], BROWNING and ROBB [1985] and others point out, the Euler equation for consumption with borrowing restrictions contains an additional (unobservable) explanatory variable, the Kuhn-Tucker multiplier associated with the net wealth condition. In this paper, I argue that a simple analytical framework for incorporating both earnings-dependent liquidity constraints and non-separable preference is readily available, and show that under weak conditions it produces an Euler equation involving only observable variables.

The method used in this paper to evaluate the importance of liquidity constraints relies on a very simple intuition: if the borrowing limit depends on (current) earnings, the price of leisure is higher for those households who are liquidity constrained, as they forego consumption both directly (lower earnings) and indirectly (lower borrowing ability). This is exactly equivalent to a change in the marginal tax-rate on earnings, and one can use the induced distortion in the leisure-goods choice to identify liquidity constrained consumers.

The main attraction of this method is that it does not require to split the sample according to some arbitrary criterion (as in HAYASHI, 1985, and ZELDES, 1989). Its major drawback is its reliance on the ability of consumers to vary their hours of work – if this condition is not satisfied one is likely to underestimate the importance of liquidity constrained behaviour.

This paper is organized as follows: section 2 (which draws on ALESSIE, MELENBERG and WEBER, 1988) derives the individual Euler equation for consumption when leisure is a choice variable and the borrowing limit depends on earnings. Section 3 presents the equations corresponding to a Cobb-Douglas utility function coupled with a multiplicative household technology and discusses estimation issues. Section 4 describes the data used, while section 5 presents empirical evidence. Section 6 draws some conclusions and indicates promising areas for future research.

2 Earnings-Dependent Borrowing Restrictions

To establish notation, let me first of all consider the standard case where the borrowing limit is fixed and goods and leisure are choice variables. Consumers solve the following optimization problem:

(1) $$\max_{\{c_i, l_i\}} E_t \sum_{i=t}^{L} (1+\tau)^{t-i} u(c_i, l_i)$$

s. t.

(2) $$A_i = (1+r_{i-1}) A_{i-1} + m_i + w_i (T - l_i) - p_i c_i, \qquad i = t \ldots L$$

(3) $$A_i \geqslant M_i, \qquad i = t \ldots L$$

(4) $$l_i \leqslant T, \qquad i = t \ldots L$$

$$A_{i-1} \text{ given}, \qquad A_L = 0,$$

where:

$u_i(.,.)$ = intratemporal utility function [1]

τ and r_{i-1} = time preference and interest rates

c_i, l_i = goods and leisure in period i

A_i = non human wealth at the end of period i

m_i = non-labour income in period i

p_i, w_i = goods price and wage rate in period i

L, T = length of life and time endowment

M_i = fixed borrowing limit.

First order conditions for period t are:

(5) $$\partial u(c_t, l_t)/\partial c_t = \lambda_t p_t$$

(6) $$\partial u(c_t, l_t)/\partial l_t = \lambda_t w_t + \nu_t$$

(7) $$\lambda_t - \mu_t = E_t [\lambda_{t+1} (1+r_t)/(1+\tau)]$$

(8) $$\mu_t (A_t - M_t) = 0; \qquad \nu_t (T - l_t) = 0$$

(9) $$\mu_t \geqslant 0; \qquad \nu_t \geqslant 0.$$

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1. I assume enough regularity conditions on $u(.,.)$ to ensure a strictly positive solution for both leisure and goods.

The variables λ_t, λ_{t+1} denote the Lagrange multipliers associated to (2), whereas μ_t and ν_t are the Kuhn-Tucker multipliers corresponding to the borrowing and time constraints, (3) and (4), respectively.

By using (5) one can rewrite equation (7) as:

$$\text{(10)} \qquad \frac{1+r_t}{1+\tau}\,\frac{p_{t+1}}{p_t}\,\frac{\partial u(c_{t+1}, l_{t+1})}{\partial c_{t+1}} = \frac{\partial u(c_t, l_t)}{\partial c_t} - p_t\,\mu_t + \varepsilon_{t+1}$$

where the error has zero conditional mean.

For estimation purposes, equation (10) is unsatisfactory in that it contains the unobservable, endogenous variable μ_t. Only in special cases can a closed form solution be found. Moreover, in general no information can be gleaned from equation (10) as to whether liquidity constraints are operating, *i. e.* whether μ_t is positive or zero.

The only case where equation (10) can be used to assess the importance of liquidity constraints is where prior information is available: if one knows that for some observations μ_t is zero, then one can estimate (10) on this subsample and thus compute predicted μ_t for the remaining observations. One can then check whether μ_t has a positive mean for the liquidity constrained as the model predicts [ZELDES, 1989]. The trouble with the method is its reliance on usually unavailable sample separation information.

Now, assume instead that the credit limit is linked to earnings: the higher current earnings are, the larger the credit the consumer is granted. This rule is widely employed in practice in several developed countries, probably because current earnings are positively associated with long term earnings ability, hence with life time resources.

A simple way to write the liquidity constraint is:

$$\text{(11)} \qquad A_i \geqslant \Phi_0 + \Phi_1\, w_i (T - l_i), \qquad i = 1 \ldots L$$

where Φ_0 and Φ_1 are parameters. In view of the considerations sketched above, one expects the lower limit to liquid asset holdings to be inversely related to current earnings, *i. e.* $\Phi_1 < 0$. Its absolute value is instead likely to be the smaller, the higher the proportion of earnings volatility that can be induced by consumers via changes in hours of work. It is worth noting that Φ_0 could be a general function of individual characteristics (profession, education, family composition etc.) and business cycle indicators (monetary restrictions, e. g.): the formal analysis would be unaffected. [2]

In this setting the first order condition (6) becomes:

$$\text{(12)} \qquad \partial u(c_t, l_t)/\partial l_t = \lambda_t\, w_t - \Phi_1\, \mu_1\, w_t + \nu_t$$

2. The borrowing limit may well depend on the value of collateral provided. In this case, Φ_0 could be made a function of the value of the durable stock owned by the household. This in turn would affect the durable-nondurable and durable-leisure margins of consumer choice, as shown by ALESSIE, DEVEREUX and WEBER [1992].

i.e. labour market variables provide information on the operation of the liquidity constraint. Combining (5) and (12) one obtains an expression for μ_t:

$$\mu_t = \frac{1}{\Phi_1}\left[\frac{1}{p_t}\frac{\partial u(c_t, l_t)}{\partial c_t} - \frac{1}{w_t}\left(\frac{\partial u(c_t, l_t)}{\partial l_r} - \nu_t\right)\right]. \tag{13}$$

Finally, substitution of equation (13) into (10) gives:

$$\left[\frac{1+r_t}{1+\tau}\frac{p_{t+1}}{p_t}\frac{\partial u(c_{t+1}, l_{t+1})}{\partial c_{t+1}}\right] = \frac{\Phi_1 - 1}{\Phi_1}\frac{\partial u(c_t, l_t)}{\partial c_t} + \frac{p_t}{\Phi_1 w_t}\left(\frac{\partial u(c_t, l_t)}{\partial l_t} - \nu_t\right) + \varepsilon_{t+1} \tag{14}$$

where $E_t[\varepsilon_{t+1}] = 0$.

In this Euler equation μ_t does not appear. In its place, there is now the Kuhn-Tucker multiplier on leisure, ν_t, which is going to be positive when a corner solution obtains in the labour market. Once again, a closed form solution for this endogenous variable is unlikely to exist: however, contrary to the case of capital markets, in the case of the labour market sample separation information is readily available. [3]

If panel data on individual households are available, the parameters of equation (14) can be estimated by restricting the sample to the employed in period t and correcting for the resulting selection bias (the error term is not orthogonal to the selection rule if there are fixed effects and individual specific shocks). With aggregate or average cohort data sample selection is difficult to correct for (impossible if individual employment histories are unknown), and a better estimation strategy is to choose a definition of leisure for which corner solutions are infrequent.

In either case, once equation (14) has been estimated, one can formally test for the complete absence of liquidity constraints (*i.e.* for $1/\Phi_1 = 0$) by standard statistical methods.

Some further remarks on equation (14) are in order:

1) equation (14) holds whether a consumer is liquidity constrained or not and whether he is rationed in his labour supply choice in period $t+1$;

2) equation (14) cannot be derived when Φ_1 is zero. However, a similar equation (where both sides are multiplied by Φ_1) can be derived by substituting (10) into (12). This highlights that within-period allocation between leisure and goods is distorted whenever Φ_1 differs from zero. If $\Phi_1 = 0$, we are back to the case where standard rules for within period allocation of full expenditure into goods and leisure are unaffected by the presence of

3. This more general setting does not preclude estimation of the traditional Euler equation for those who are neither liquidity constrained nor at a corner solution in their leisure choice. I can in fact rewrite (11) as $l_i \leq T'$ where $T' = T - (A_i - \Phi_0)/\Phi_1 w_i$. Then $T' < T$ whenever $A_i < \Phi_0$, *i.e.* if either (4) or (11) is binding, and $\mu_i = 0$ if $A_i > \Phi_0$. One could then estimate (10) on the subsample for which $A_i > \Phi_0$, after correcting for selectivity bias.

a binding constraint in the capital market (the one considered by BLUNDELL and WALKER, 1986, and ALTONJI, 1986);

3) one can use equations (10) or (13) and the parameter estimates from (14) to compute μ_t for each individual household.

The relationship between the formal test for $(1/\Phi_1)=0$ and the informal computation of μ_t is worth exploring. Inspection of equation (14) and consideration of the underlying model suggest that rejection of the null in the formal test does not imply that the estimated μ's should be non zero: it is in fact possible for consumers not to be bound by the liquidity constraint in period t even though the earnings-related constraint exists in a non-trivial form (*i.e.* $\Phi_1 > -\infty$).

3 Estimation Issues

The model in section 2 produces an estimable equation which does not require information on the current or future ability to borrow of consumers. Such an equation could in principle be estimated on aggregate data, if one was prepared to neglect the problem of corners in leisure choice. However, credit market imperfections are likely to affect consumers to different extents, depending on their desire to borrow. In so far as such differences are traceable to observable factors (like age, presence of children in the household or the type of occupation), more disaggregated data are likely to afford important efficiency gains.

This argument could lead one to use panel data (as in ZELDES, 1989). A problem with panels, however, is that consumption data is normally partial and of low quality: the often used US Panel Study of Income Dynamics (PSID) contains information on food expenditure only, and this is based on interview questions, not a detailed diary. Another problem is that their time dimension is normally inadequate (identification of rational expectations models requires in general many observations in time, rather than across individuals). [4] A more down-to-earth problem is that no UK household panel is as yet available.

An alternative strategy relies on constructing pseudo-panels on the basis of high quality cross-sectional information spanning long periods of time (as suggested by BROWNING, DEATON and IRISH, 1985). In my application, I follow this approach.

4. This could explain why ALESSIE, KAPTEYN and MELENBERG [1989] encounter difficulties in identifying parameters relating to intertemporal decisions.

For the purpose of estimation, I treat the household as the decision unit, and take (per-capita) total expenditure on goods other than durables and housing as the relevant choice variables. As for labour supply decisions, I consider two alternative procedures: in a first attempt, total household leisure is treated as a homogeneous commodity; later, male and female leisure are kept separate, with the former treated as the relevant choice variable and the latter as a conditioning variable. Both definitions satisfy the requirement that the occurrence of a corner solution on the labour market choice be relatively infrequent, so that one can neglect the maximum hours constraint while deriving estimable equations. As for the consumption variable one should observe that data on services from durables are not available, thus forcing the assumption either of additive separability of durables services from goods, other services and leisure, or that total consumption is proportional to the sum of its non-durable components. Finally, housing services are also not included in consumption, because of serious measurement problems.

Given the fairly aggregate nature of the data, I follow BEAN [1986] and choose a Cobb-Douglas utility function, but allow some flexibility by introducing seasonal and demographic effects in the household production technology. The main advantage of the chosen functional form is that the estimable equation is linear in known functions of the observable variables, so exact aggregates can be constructed from the micro data. The disadvantage in terms of implied unit elasticities is mitigated by the introduction of the household production technology – whereby unitary elasticity of substitution is not imposed between non-durables expenditure and leisure, but on general functions of these and of demographic variables.

In the sequel I present the derivations for the general case of male and female leisure. The case where household leisure is the appropriate choice variable is similar.

Let individual households solve the following maximization problem:

$$\max_{(C_i,\, L_i^m\, L_i^f)} E_t \left\{ U_t \equiv \sum_{i=1}^{L} (1+\tau)^{t-1} \frac{u_i^\gamma}{\gamma} \right\}, \qquad \gamma \neq 0, \gamma < 1 \tag{15}$$

where

$$u_i = c_i^\alpha \, l_i^{m\beta} \, l_i^{f\,(1-\alpha-\beta)}, \qquad 0 < \alpha, \beta < 1$$

c denotes consumption services, C expenditure, l^m and l^f leisure services (male and female), L^m and L^f their observable counterparts.

I specify the following household technology (which generalizes Miron's [1986] treatment of seasonality):

$$c_i = C_i \, e^{\sum_s X_i^s \delta_c^s + \sum_k Z_i^k \psi_c^k} \tag{16}$$

$$l_i^m = L_i^m \, e^{\sum_s X_i^s \delta_m^s + \sum_k Z_i^k \psi_m^k} \tag{17}$$

$$l_i^f = L_i^f \, e^{\sum_s X_i^s \delta_f^s + \sum_k Z_i^k \psi_f^k} \tag{18}$$

In the above, the X's are seasonal variables, while the Z's are demographic indicators (such as presence of children in the household, or educational status); the δ's and ψ's are parameters of the household technology function, *i.e.* they account for the effect seasons and demographic characteristics have on the way the observable quantities translate into different utility levels.

The following constraints also apply:

$$(19)\quad A_i = (1+r_{i-1})A_{i-1} + m_i + w_i^m(T - L_i^m) + w_i^f(T - L_i^f) - p_i C_i, \qquad i = t \ldots L$$

$$(20)\quad A_i \geqslant \Phi_0 + \Phi_1 w_i^m(T - L_i^m) + \Phi_2 w_i^f(T - L_i^f), \qquad i = t \ldots L$$

$$A_{i-1} \text{ given}, \qquad A_L = 0,$$

where

A_i = liquid assets at the end of period i

m_i = non labour income in period i

p_i, w_i^m, w_i^f = goods price and wage rates in period i

L, T = length of life and time endowment.

The Euler equation [5] is:

$$(21)\quad E_t\left\{\frac{1+r_t p_t}{1+\tau p_{t+1}}\left(\frac{C_{t+1}}{C_t}\right)^{\gamma\alpha-1}\left(\frac{L_{t+1}^m}{L_t^m}\right)^{\gamma\beta}\left(\frac{L_{t+1}^f}{L_t^f}\right)^{\gamma(1-\alpha-\beta)} e^{\sum_s \Delta X_{t+1}^s[\alpha\delta_c^s+\beta\delta_m^s+(1-\alpha-\beta)\delta_f^s]\gamma}\, e^{\sum_k \Delta Z_{t+1}^k[\alpha\psi_c^k+\beta\psi_m^k+(1-\alpha-\beta)\psi_f^k]\gamma}\right\}$$

$$\frac{\Phi_1-1}{\Phi_1}\left[1+\frac{\beta}{\alpha(\Phi_1-1)}\frac{p_t C_t}{w_t^m L_t^m}\right]$$

Given the large number of parameters in (21) and the limited number of observations available for estimation it may be preferable to assume joint log-normality of the variables on the left hand side, and rely on the standard approximation $\log(1+x)\cong x$ for the term on the right hand side (this procedure is valid if Φ_1 is large in absolute value, *i.e.* if one expresses all growth rates in monthly or weekly terms). This gives the following estimable equation:

$$(22)\quad \Delta\ln C_{t+1} = a_0 + \sum_k \psi_k^* \Delta Z_{t+1}^k + \sum_s \delta_s \Delta X_{t+1}^s + \frac{1}{1-\alpha\gamma} r_{t+1}^* + \frac{\gamma\beta}{1-\alpha\gamma}\Delta\ln L_{t+1}^m + \frac{\gamma(1-\alpha-\beta)}{1-\alpha\gamma}\Delta\ln L_{t+1}^f + \frac{\beta}{\alpha(\Phi_1-1)(\gamma\alpha-1)}\frac{p_t C_t}{w_t^m L_t^m} + \varepsilon_{t+1}$$

where Δ denotes the first difference operator, a^0 is a constant term, r_{t+1}^* is the (ex-post) real interest rate, ε_{t+1} is the expectational error (a white noise

5. This corresponds to equation (14), when $v_t = 0$.

process) and:

$$\psi_k^* = \frac{\gamma}{(1-\alpha\gamma)}[\alpha\psi_c^k + \beta\,\psi_m^k + (1-\alpha-\beta)\,\psi_f^k] \tag{23}$$

$$\delta_s^* = \frac{\gamma}{(1-\alpha\gamma)}[\alpha\delta_c^s + \beta\delta_m^s + (1-\alpha-\beta)\,\delta_f^s]. \tag{24}$$

The corresponding expression for the Kuhn Tuker multiplier μ_t is:

$$\mu_t = \frac{u_t^\gamma}{\Phi_1 p_t c_t}\left[\alpha - \beta\frac{p_t C_t}{w_t^m L_t^m}\right] \tag{25}$$

and μ_t is positive if $\alpha w_t^m L_t^m < \beta p_t C_t$, i.e. if within period allocation is "tilted" away from (male) leisure towards goods. By construction μ_t reflects the nominal character of the constraint (20), and will therefore decrease as prices grow, *ceteris paribus*: to obtain a more useful measure of the severity of liquidity constraints over time I will look at the real variable $\mu_t^* = \mu_t p_t$.

Equation (22) can be estimated by a method of moments estimator, such as Instrumental Variables. However, some care must be taken in the choice of the instruments, as the error on the estimable equation may exhibit a Moving Average structure of order 1. This will arise if the decision period is shorter than the observation period (Hall, 1988), but can also be induced by taking averages over relatively small sub-groups of the population (Deaton, 1985). The former argument would suggest a positive root of the MA structure, the latter a negative one. In view of the expectational character of (at least part of) the error term standard backward filtering techniques lead to inconsistent estimates (Hayashi and Sims, 1983). A simple estimation strategy which I take here requires avoiding first lags as instruments, and computing MA (1) corrected standard errors throughout.

4 Data Description

The data used in this paper are drawn from the UK Family Expenditure Surveys 1970-84. Four year-of-birth cohorts were constructed: the first cohort is made up of all households headed by someone born in the 7 year interval 1941-1947, the second covers the previous seven-year band, and so does the third. The fourth cohort includes households whose head was born in the 1948-54 interval and enters the sample in 1977. In this way, nobody aged less than 23 or more than 57 is included, thus reducing the risks of attrition bias. Also, the relatively large number of households included (about 350 per semi-annual observation) should make measurement error corrections unnecessary. A full description of the cohorts is in the Appendix.

Consumption is defined as the half-yearly geometric mean of the sum of expenditure on food, alcohol, tobacco, fuel, clothing, transport, services and "other" by all household members. Leisure is instead constructed on the assumption that the weekly time endowment for each adult member is 90. [6] Two definitions of leisure are employed: household leisure, which is the sum of unworked hours by all adult members; male and female leisure, which cover the first two adults in each household. The nominal interest rate adopted is the Building Society Deposits rate, on the grounds that most consumers have positive holdings of this asset, and that it hardly differs from the after-tax rate on mortgages. A household specific price index is computed to express expenditure and the interest rate in real terms.

Household consumption is expressed in per capita terms by defining household size as the sum of the number of adults and half the number of children. My empirical results suggest that different definitions have little effect. Other demographic variables used in the equation are the proportion of households whose head is in white-collar occupation, the proportion of multiple and single adult households, the proportion of households with young children and of owner occupiers. In practice only the first two of these variables turn out to have significant explanatory power.

Also, the annual cohort average of the ratio of expenditure on goods to expenditure on leisure is calculated for each leisure definition. Where no wage rate is available (households with no workers), the observation is not used to compute this particular variable. In an attempt to check if biases arise from this procedure, the cohort proportion of households of this type was allowed in as an extra regressor but generally found to be insignificant (a typical *t*-ratio would be .5). A formal selectivity bias correction is however impossible on these data, because employment history is not known.

5 Empirical Results

Tables 1 and 2 present estimation results. The first column gives the Instrumental Variables estimates, columns 2 and 3 the MA (1) corrected

6. Key parameter estimates are unaffected when T takes values in the 80-120 range.

standard error and t-statistic. At the bottom of each table corresponding OLS results are also presented.

TABLE 1

Dependent Variable is: $\Delta \ln C$ ***(Consumption per Equivalent Adult) Household leisure***

$\sigma = 0.0014$
number of instruments = 33
number of observations = 90
Sargan test (26) = 25.75

Variable	coef.	s. e.	t-ratio
r^*	0.2791	0.1595	1.7501
$\Delta \ln l$	−0.7528	0.3153	−2.3874
pcwl	0.0001	0.0004	0.3093
Δs 1	0.0018	0.0002	7.8461
Δma	0.2789	0.2298	1.2141
Δwhc	0.3218	0.1857	1.7322
cnst	0.0003	0.0006	0.4501

First order serial correlation is −0.2545
Structural parameters, s. e.'s and t values

α	0.4892	0.1235	3.9594
γ	−5.2806	3.6438	−1.4492
Φ_1	−2,204.8339	6,280.3647	−0.3511

Results for OLS
$\sigma = 0.0014$
mean of dep. v.ble = 0.0002476322

	Estimate	se	t-ratio	Mean of regressor
r^*	0.3549	0.1355	2.6186	−0.0008
Dln L	−0.5659	0.1403	−4.0347	0.0002
pcwl	0.0000	0.0003	0.1649	1.5839
Δs 1	0.0017	0.0002	9.6775	0.0000
Δma	0.3344	0.1467	2.2795	0.0003
Δwhc	0.2939	0.1047	2.8075	0.0000
cnst	0.0004	0.0004	1.0372	1.0000

Analysis of liquidity constraints

obs.	mean	st.dev.	max.	min.
90	0.026	0.026	0.134	−0.003

Note: Instruments are differences and levels lagged two and three times. The following levels are used: r, *pcwl*, cohort dummies, proportions of owner occupiers, of net lenders, of net borrowers, of households with young children, of households without earnings. First differences of the following variables are used: $\ln C$, $\ln p$, ln (rateable value), ln (net normal income), ln L, ln RPI (last month), proportions of wite collar and multiple adults households.

Table 1 presents results for a fairly parsimonious version of equation (22) specified in terms of household leisure. The first explanatory variable is the real interest rate, followed by the variation in leisure and by the ratio of goods to leisure (*pcwl*). The first difference of a seasonal dummy (Δs 1) comes next, followed by the demographic indicators Δma (proportion of multiple adult households) and Δwhc (proportion of white collar households). This specification is not rejected by the Sargan test for overidentifying restrictions.

The structural (preference) parameters are also computed, and their standard errors are evaluated by using Cramer's General transformation theorem. If we denote by V the (MA (1) heteroskedasticty corrected) variance- covariance matrix of the IV estimates of the equation (household leisure definition, say)

$$\Delta \ln C_{t+1} = a_0 + a_1 r^*_{t+1} + a_2 \Delta \ln L_{t+1} + a_3 pcwl + b \Delta s1 + c \Delta Z_{t+1} + \varepsilon_{t+1} \tag{26}$$

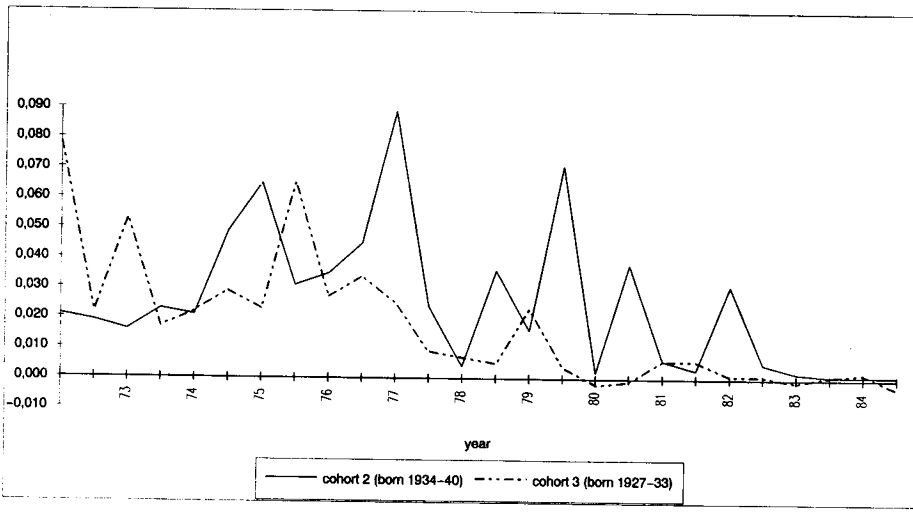

FIGURE 1

Kuhn-Tucker Multipliers — Older Cohorts

where Z is a vector of demographic characteristics and by J the jacobian of the transformation of the a's into the deep parameters (α, γ and Φ_1), then the standard errors are the square roots of the diagonal elements of $(J'VJ)$.

Of the preference parameters, α and γ are well determined: the elasticity of intertemporal substitution corresponding to $\gamma = -5.3$ is .16, which broadly agrees with previous empirical findings (see Alogoskoufis, 1987, who reports an elasticity of .231). The estimate of Φ_1 is correctly signed but poorly determined: its magnitude is also far from expectations (it implies that the borrowing limit is 42 times annual earnings) and may suggest that the liquidity constraint is unlikely to be binding.

Another way of determining the importance of borrowing restrictions is to look at the (real) Kuhn-Tucker multipliers associated to the net-wealth condition: these are mostly positive, but vary considerably across cohorts and in time (see Fig. 1-2). The younger cohorts (1 and 4), for instance, appear to have been affected most in the early 80's, when their earnings were still on a steeply ascending path. In that period inflation peaked, while housing prices remained stable: home-owners with large outstanding mortgages might have tried to resort to personal credit unsuccessfully (financial deregulation started in the second half of 1982). Cohorts 2 (aged 30 to 36 in 1970) and 3 (aged 38 to 44 in 1970) were also liquidity constrained, but mainly in the mid-70's, when there was a sudden rise in their μ. For all cohorts, Kuhn-Tucker multipliers fall to zero from the second half of 1982, *i.e.* from the time when financial liberalisation was introduced in Britain (see Davies and Weber, 1991, for the institutional detail).

TABLE 2

Dependent Variable is Dln C (Consumption per Equivalent Adult) Male leisure

$\sigma = 0.0018$
number of instruments = 37
number of observations = 90
Sargan test = 35.52

Variable	coef.	s. e.	*t*-ratio
r^*	0.4475	0.2061	2.1713
$\Delta \ln L^m$	−0.0733	0.2405	−0.3049
$\Delta \ln L^f$	−0.0009	0.0628	−0.0143
pcwlm	0.0005	0.0004	1.2262
Δs 1	0.0017	0.0003	6.6755
Δww	0.2619	0.1673	1.5648
Δwhc	0.4636	0.2057	2.2536
cnst	−0.0006	0.000,8	−0.6738

First order serial correlation is −0.3075
Structural parameters, s.e.'s and *t* values

α	0.8816	0.3551	2.4825
β	0.1170	0.3330	0.3514
γ	−1.4004	1.3368	−1.0476
Φ_1	−116.8762	14.6266853	−0.0080

Results for OLS
$\sigma = 0.0015$
mean of dep. v.ble = 0.000,247,897,4

	Estimate	se	*t*-ratio	Mean of regressor
r^*	0.5174	0.1643	3.1492	−0.0008
Δlm	−0.1554	0.1518	−1.0236	0.0002
Δlf	0.0164	0.0382	0.4294	−0.0005
pcwlm	0.0005	0.0004	1.5154	2.2900
Δs 1	0.0017	0.0002	8.1494	0.0000
Δww	0.1883	0.1063	1.770,4	0.0000
Δwhc	0.2861	0.1122	2.5496	0.0000
cnst	−0.0005	0.0008	−0.7198	1.0000

Analysis of liquidity constraints

obs.	mean	st.dev.	max.	min.
90.000,000	−0.041	0.008	−0.028	−0.063

Notes: see Table 1 for instruments list. Male and female leisure replace household leisure throughout.

Table 2 presents a similar set of results for the case where male and female leisure are two distinct commodities (the only difference being in the demographics: the multiple adult variable is replaced by the proportion of

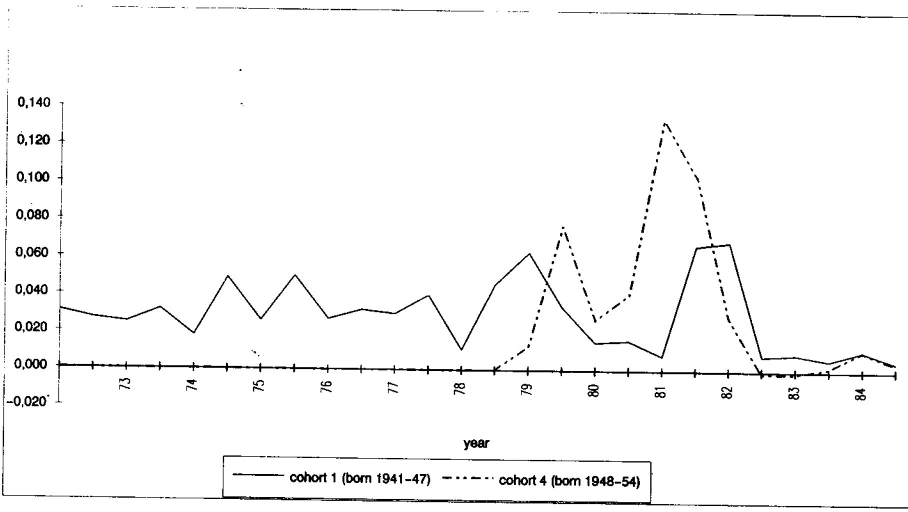

FIGURE 2

Kuhn-Tucker Multipliers – Younger Cohorts

household where the wife is working: Δww). The striking features of this set of estimates are: little effect of leisure on consumption growth, better determined estimates of the coefficients on r^* and *pcwlm*. The elasticity of intertemporal substitution is now .42. The point estimate of Φ_1 is much smaller in absolute terms, but the Kuhn Tucker multipliers are all negative, and are not reported. Finally, the Sargan test is close to rejecting at the 90% significance level.

A possible explanation for the difference in the estimated Kuhn Tucker multipliers across the two Tables lies in the often quoted institutional feature of the British labour market, whereby primary workers are less able to choose their hours than secondary workers. If this was indeed true, the specification behind Table 1 would be better suited at capturing distortions in the hours-goods choice than that behind Table 2, which uses first order conditions for male leisure alone.

Other sets of estimates obtained for different definitions of equivalent adults and of demographic variables are not reported because empirical results are largely unaffected.

6 Conclusions

In this paper I have used the well known fact that the life-cycle model with liquidity constraints produces an Euler equation with a Kuhn-Tucker multiplier to show that if borrowing restrictions depend on earnings this equation involves only observable variables.

I have then estimated the Euler equation on a pseudo panel of UK households, and found that most parameters are well determined. The estimated coefficients are used to evaluate each cohort's Kuhn-Tucker multiplier over the sample period. While the estimates are not always conclusive, they broadly confirm the importance of financial constraints for young households up to the second half of 1982. After that (and until the end of the sample period, in 1984), I find instead that consumer expenditure does not reflect binding credit restrictions. This could well be due to financial deregulation, or to high real interest rates, or instead reflect a change in expectations at the time of a deep recession.

The estimates presented here provide some *prima facie* evidence on the empirical importance of credit market imperfections and illustrate the potential usefulness of the method advocated in this paper. However, the differences across the models where household leisure and male leisure are treated as the relevant choice variable, respectively, point to the desirability of directly exploiting the large variability of female hours of work.

Future research could concentrate on the effects of credit market imperfections on female labour supply. Female leisure is more likely to be a choice variable than its male counterpart, but is obviously affected by a large number of corner solutions. Sample selectivity corrections would then be needed. In data where employment histories are known, this could be achieved by estimating current and past employment probability equations as functions of deterministic, individual-specific regressors and predetermined choice variables (at the cohort level, if individual observations do not exist). Dynamic Euler equations explaining consumption growth between period $(t+1)$ and t could then be estimated on the subsample of households where the wife worked in period t. With data sets where past employment is not reported, but financial wealth is accurately recorded, one could instead estimate within-period conditions for female hours on repeated cross sections of (married) working women, by selecting only those households who report high liquid asset holdings. This approach would require making some arbitrary assumptions on the nature of the error term, but would allow evaluating the shadow price of the borrowing restriction for all working wives.

APPENDIX

The data used in this study are drawn from (UK) Family Expenditure Surveys run over the 15 years period 1970-84. The characteristics of the survey suggested the following selection rule: households were excluded if the head is selfemployed (no data on hours of work is recorded) or their residence is Northern Ireland (response in the Province is patchy).

Four year-of-birth cohorts are constructed, grouping together households whose head was born in the same seven year band. For three cohorts observations cover the whole sample period (cohorts 1-3, born in the intervals 1941-47, 1934-40, 1927-33). The remaining cohort (born 1948-54) appears in the second half of the sample (1977 onwards). The chosen data frequency is semi-annual: thefore, the total number of observations is 102, of which 90 are available for estimation purposes.

The following variables are retrieved for each cohort: logarithm of real expenditure (excl. durables and housing), logarithm of leisure (household, male and female), logarithm of nominal expenditure, ratio of expenditure on goods to expenditure on leisure (*pcwl* and *pcwlm* respectively), proportion of households where no adult reports hours of work, proportion of households who report positive interest income, proportion of net-borrowers, proportion of owner occupier households, logarithm of rateable value of the house, proportions of single and multiple adult (>2) households, proportion of households with young children (aged 0-5), logarithm of net normal income, inclusive of imputed rent from owner occupation, number of adults, number of equivalent adults, proportion of households where the wife is working, logarithm of $1+$nominal interest rate (beginning of period).

In estimation, consumption growth is defined in per capita terms, either per-adult or per-equivalent adult (where children count as half an adult). All growth and interest rate are expressed in weekly terms.

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