ANNALES D'ÉCONOMIE ET DE STATISTIQUE. – N° 29 — 1993

Independent of a Base Equivalence Scales Estimation Using United States Micro-Level Data

Julie A. NELSON *

ABSTRACT. – Problems of identification of equivalence scales, on both philosophical and mathematical grounds, are discussed. Scales are estimated using an Almost Ideal Demand System and data from the 1980's United States Consumer Expenditure Survey. Resulting parameter estimates are reasonable for income and demographic effects, but the estimated price elasticities and the scales themselves are less appealing and less robust, especially since the validity of the scale estimates depends on strong assumptions about functional form.

Estimation d'échelles d'équivalence indépendantes d'une base utilisant des données microéconomiques américaines

RÉSUMÉ. – Les problèmes d'identification d'échelles d'équivalence sont discutés, au niveau philosophique et au niveau de la formalisation mathématique. Les échelles sont estimées en utilisant un Almost Ideal Demand System et les données d'une enquête de 1980 sur les dépenses des Ménages aux États-Unis. Les paramètres estimés sont raisonnables pour les effets revenus et démographiques, mais les estimations d'élasticié en prix et les échelles elles-mêmes sont moins satisfaisantes et moins robustes, en particulier parce que la validité des estimations des échelles dépend d'hypothèses fortes sur les formes fonctionnelles.

---

* J. A. NELSON : University of California, Davis, California, USA. This work was undertaken for a conference on "Recent Advances in Consumer Theory" sponsored by the Département et Laboratoire d'Économie Théorique et Appliquée, in Paris, France, October 3-5 1991. The results presented in this article differ somewhat from those presented at the conference due to the correction of a programming error in the data preparation and testing of sensitivity of the results to the level of the "α_0" parameter. This material is based upon work supported by the National Science Foundation under Award No. SES-8921634.

1 Introduction

Household equivalence scales adjust measures of household income for differences in household composition. This paper discusses the argument that derivation of "true" equivalence scales from demand data is impossible, on the conceptual ground that demand patterns reveal only "conditional" preferences. The empirical section describes some of the practical problems, as well as insights that can be gained, from attempting to estimate household equivalence scales of the "Independent of a Base" form using U.S. micro-level data on household demands.

2 Conditional or Unconditional Scales?

POLLAK and WALES [1979] and FISHER [1987] argue against the estimation of equivalence scales through demand analysis on the following basis. Demand behavior may be thought of as being generated by a utility function

$$U(\mathbf{x}, \boldsymbol{\alpha}) \tag{1}$$

where $\mathbf{x}$ is a vector of commodities and $\boldsymbol{\alpha}$ is a vector describing household composition. However, household welfare, it is argued, should be thought of as depending on household composition directly, as well as through the effect of household composition on commodity demands. If household welfare is determined by a function like

$$U^*[U(\mathbf{x}, \boldsymbol{\alpha}),\boldsymbol{\alpha}] \tag{2}$$

then information about $U(\mathbf{x}, \boldsymbol{\alpha})$ derived from analysis of demands does not provide a sufficient basis for welfare comparisons. "The expenditure level required to make a three-child family as well off as it would be with two children and \$ 12,000," write POLLAK and WALES, "depends on how the family feels about children." An approach which ignores the direct effects of composition on household welfare is said to the merely "conditional", while true scales must depend on the more inclusive, "unconditional" welfare function U^*. Such an approach to equivalence scales has been adopted by LEWBEL [1989], BLUNDELL and LEWBEL [1991], BOURGUIGNON [1989], and BLACKORBY and DONALDSON [1991 *b*], as well as further elaborated by POLLAK [1990].

One point at issue here, on a conceptual level, is the definition of "welfare." The "unconditional" approach equates welfare with *happiness*. In contrast, most of the historical literature on equivalence scales, as well as the policy situations to which they are applied, are concerned with welfare in the sense of the *standard of living* (NELSON [1991]). A cogent philosophical discussion of the distinction between "happiness" and the "standard of living" can be found in SEN [1987]. The pleasure one (for example, a parent) may get from supporting someone else (for example, her or his child) may be part of a broad conception of welfare, or what SEN calls "well-being," but it is justly excluded from judgment of one's "standard of living" or well-being related to one's *own* life (SEN, p. 28-29, examples added). A particular household member may certainly be made happier by the presence of other members, but (with limited resources) the additional demands on household resources also certainly reduce the member's ability (or "capability" in Sen's terms) to be, himself or herself, well-clothed, well-fed, well-rested, etc. Most distributional or policy studies center around questions of the distribution of economic means relative to needs in this latter sense (for example, the adjustment of poverty line income standards for family size). As questions of the distribution of pure subjective happiness are rarely raised in practical application, equivalence scales in the older, more materialistic, and more objective sense remain of great practical interest in their own right. [1]

A second point at issue, this time at a practical level, is the question of the identification of household equivalence scales. While it is only one of many possible ways of approaching the estimation of equivalence scales, many economists have tried to infer them from observing variations in household consumption patterns, as household composition changes (and usually with the goods defined at a very aggregated level). [2] As one popular example, the "food share" (or "Engel") approach takes the share of income spent on food as a measure of the household standard of living, and then compares the levels of income at which different types of households attain the same proportion of income spent on

1. This is not a claim that (for example) a parent's happiness from having children is irrelevant to public policy. It is only a claim that such considerations are irrelevant to standard of living, and hence equivalence scale, comparisons. Policy-makers may very well want to consider, in addition to the standard of living consequences, the possible incentive effects on fertility of programs such as family allowances. Attempts to force the two issues into one, however, tend to lead to confusion between the concern with welfare (and perhaps especially children's welfare) and the concern about choice (on the part of adult members).
2. Some of the most popular methods which use observed consumption are the "Barten," "Rothbarth," and "Engel" methods. These are compared in DEATON and MUELLBAUER [1986] and in NELSON [1991, 1992*a*]. Others include the "Prais-Houthakker" method, which however, suffers from an underidentification problem [MUELLBAUER, 1986], and such methods as the "Iso-prop" [WATTS, 1967] which (along with the "Engel" or "food share" method) lack a clear theoretical base. For a review of these models see NELSON [1986]. Inference from observed consumption expenditure patterns is, however, only one of several possible approaches. BUHMANN *et al.* [1988] compare the values of scales derived from surveys of household expenditures, with those derived from surveys of household subjective income needs and from expert opinion.

food. Obviously, a first requirement for such inference from demand patterns to work is that there must be systematic relationships among demands, composition, and income. While special cases such as homothetic preferences (which would vitiate such inference) may be of theoretical interest, systematic relationships among demand shares, income, and composition, are, fortunately, nearly always found in cross-section data. But even with these patterns, equivalence scales cannot be recovered purely from demand data alone. As with any empirical project, the examination of the data cannot be accomplished entirely free of assumptions, from the theoretical and over-arching to the mundane and practical.

The type of identifying assumption made is important. Instead of using welfare proxies (such as using the food share as an indicator of welfare-an example of an a priori identifying assumption), many researchers have sought to more explicitly use neoclassical consumer theory. Defining the cost function conditional on household composition as

$$C(u, \mathbf{p}, \boldsymbol{\alpha}) = \min_{\mathbf{x}} \{ \mathbf{p} \cdot \mathbf{x} \mid U(\mathbf{x}, \boldsymbol{\alpha}) \geqslant u \} \tag{3}$$

an equivalence scale can then be defined in terms of the ratio of the cost functions of comparison ($\boldsymbol{\alpha}$) and a reference ($\boldsymbol{\alpha}^r$) households as

$$\frac{C(u, \mathbf{p}, \boldsymbol{\alpha})}{C(u, \mathbf{p}, \boldsymbol{\alpha}^r)} \tag{4}$$

Since cost functions can be recovered from estimated demands by integration, this would seem to allow identification of the equivalence scale. However, if one has no theory about how goods demanded are converted into household welfare, one may judge one utility or cost function to be as good as the next, as long as they yield the same observed demands. Specifically, the functions $U^*[U(\mathbf{x}, \boldsymbol{\alpha}), \boldsymbol{\alpha}]$ and $U(\mathbf{x}, \boldsymbol{\alpha})$ both yield the same demands, and thus are equally acceptable, in this view. (Once a particular functional form is specified it may, in fact, be that a function of the "conditional" form may be made to look like one of the "unconditional" form, or vice versa, simply through rearrangement.) As argued in much of the "unconditional scale" literature (especially Pollak and Wales [1979]; Blundell and Lewbel, [1991]), equivalence scales estimated from demands can in this case be made to equal any arbitrary value. Suppose, as an example, we let

$$U^*[U(\mathbf{x}, \boldsymbol{\alpha}), \boldsymbol{\alpha}] = g(\boldsymbol{\alpha})\, U(\mathbf{x}, \boldsymbol{\alpha}).$$

To see the effect on equivalence scales, note that the following ratio of the cost functions between the comparison and reference households,

$$\frac{C(g(\boldsymbol{\alpha})\, u, \mathbf{p}, \boldsymbol{\alpha})}{C(g(\boldsymbol{\alpha}^r)\, u, \mathbf{p}, \boldsymbol{\alpha}^r)} \tag{5}$$

which is consistent with the same observed demands which generated equation (3), can be made to take any arbitrary value (regardless of the data or functional form with which the cost function is estimated) by a suitable choice of $g(\alpha)$.[3] In the absence of a theory about how goods demanded are converted into household welfare, equivalence scales can be "identified" by choosing a particular functional representation for utility – but there is no justification for making one choice over another.

What has been ignored in the "unconditional scale" literature, however, is that, if one has a *theory* of intra-household allocation and how those allocations translate into demands, one may be able to justify a preference for one specific utility or cost function. Suppose, for example, that the world were such that all agents were identical and treated symmetrically within households, and that there were no economies of scale in household formation. Then, if $C(u, \mathbf{p})$ is the cost function for a single individual, $J \cdot C(u, \mathbf{p})$ is the cost function for a J person household. The equivalence scale for a five person household is five. One cannot arbitrarily change u to $g(\alpha)u$, since now there is a theory which says that each individual gains a standard of living u from their per capita share of goods x. (If one made such a change, it would open up the nonsensical possibility that it may cost 9,999 times as much to bring two of these identical agents to a given utility level, as compared to just one of them.) While this simple example, which might be called a "per capita theory" of allocation, is crude and unrealistic, it illustrates how a theory of allocation and welfare may put structure on the choice of the utility or cost function.[4] Such a use of the theory of allocation and welfare in the identification of equivalence scales is no less justified than, for example, the use of assumptions about preferences in deciding what goods to exclude and how to aggregate over goods, when one is pursuing the more standard exercise of seeking to identify price and income elasticities. Further research is need to develop more adequate theories of household behavior, and to investigate the combinations of theories and functional forms which may lead to non-trivial identification.[5]

3 Independent of Base Equivalence Scales

An addition to the equivalence scale literature which is relatively recent, and hence relatively untried in empirical application,[6] is the suggestion that

3. I thank Arthur LEWBEL and David DONALDSON for correspondence which proved helpful in setting out his issue.
4. The popular "Barten" model also includes a theory of intra-household resource allocation and welfare determination, which may be a candidate for an identifying theory. Unfortunately, except in the case where agents are identical, the model implies rather perverse intra-household behavior (NELSON [1991]).
5. This is not the only question which remains. On a conceptual level, the most troubling is the question of whether a standard of living exists on a *household* level at all, since individual members may enjoy different standards (SEN [1983]; HADDAD and KANBUR, [1990]).
6. Previous attempts at empirical application include PHIPPS [1991] and BLUNDELL and LEWBEL [1991].

equivalence scales be estimated under the assumption that the relative income needs of households of various sizes are invariant to the level of welfare at which they are compared. The theoretical possibility of deriving equivalence scales under this assumption has been suggested independently by LEWBEL [1989] who calls the assumptions "Independent of a Base" (IB), and BLACKORBY and DONALDSON [1991 *a*] who call it "Equivalence Scale Exactness." Since the theory is a "black box" as far as intra-household allocation and welfare determination are concerned, it suffers from the identification problem discussed above. The plausibility of the assumption is also debatable. It may be that beyond some level of consumption on the part of minor children, parents may be likely to limit their children's current consumption (for fear of "spoiling" them) in favor of increasing their welfare through such means as later-in-life investments in higher education, or transfers of assets. If such is the case, so that (loosely speaking) the children's "share" of household consumption drops as income rises, IB is violated (for welfare defined over current consumption). While these points must be kept in mind in evaluating the scales which result from the following empirical specification, the use of IB scales is still interesting as an experiment with a way of incorporating demographic effects into a demand system.

Following BLACKORBY and DONALDSON [1991 *a*, *b*], it is assumed that all household members have equal welfare levels within the household. Assume, also, that all households have same utility function (where "utility" is understood in the "standard of living" sense) over goods, conditional on their household composition, as given in equation (1). (Since, in theory, the vector describing household composition, $\boldsymbol{\alpha}$, could include great detail, even down to the name of each member, this is not yet very restrictive). In general, the household equivalence scale, that is, the factor by which a household's income must be multiplied for it to achieve the same welfare level as a "reference" household of a particular composition, can (under the preceding assumptions) be expressed as [in equation (4)] the ratio of the cost function for that household to the cost function for the reference household, at the same prices and utility level. The assumption of "independent of a base," however, makes the value of the scale independent of the utility level chosen. If the cost function is restricted to be of the form

$$C(u, \mathbf{p}, \boldsymbol{\alpha}) = \bar{C}(u, \mathbf{p})\, \hat{C}(\mathbf{p}, \boldsymbol{\alpha}) \tag{6}$$

then the equivalence scale (where the r subscript indicates the reference household) and be defined as:

$$\Delta(\mathbf{p}, \boldsymbol{\alpha}) = \frac{\hat{C}(\mathbf{p}, \boldsymbol{\alpha})}{\hat{C}(\mathbf{p}, \boldsymbol{\alpha}^r)} \tag{7}$$

which is independent of u. The homogeneity of degree one of the cost function implies that the equivalence scale $\Delta(\mathbf{p}, \boldsymbol{\alpha})$ must be homogeneous of degree zero in $\mathbf{p}$. For all price vectors, $\Delta(\mathbf{p}, \boldsymbol{\alpha}^r)$ must be equal to one. Rearrangement yields

$$C(u, \mathbf{p}, \boldsymbol{\alpha}) = C(u. \mathbf{p}, \boldsymbol{\alpha}^r)\, \Delta(\mathbf{p}, \boldsymbol{\alpha}) \tag{8}$$

As pointed out by BLACKORBY and DONALDSON [(1991 *a*, *b*], this implies strong interrelationships among the cost functions for different household types. Once $C(u, \mathbf{p}, \boldsymbol{\alpha}^r)$ is chosen, as, say $C^r(u, \mathbf{p})$, the preferences of all other household types are determined by this and the equivalence scale.

4 Model Specification

For simplicity, only a scalar indicator of household characteristics, the number of children (N), is used to derive household equivalence in this paper. The reference household is chosen to be that of a childless couple. Elaborating on a suggestion made in BLACKORBY and DONALDSON [1991 *b*], the functional form chosen for the equivalence scale is

$$(9) \qquad \Delta(\mathbf{p}, \boldsymbol{\alpha}) = (N+1)^{\theta} \prod_i p_i^{\gamma_i N}$$

where $\sum_i \gamma_i = 0$. It can be checked that this is homogeneous of degree zero in prices and equal to one for the reference (N = 0) household for all price vectors.

Although BLACKORBY and DONALDSON do not discuss plausible parameter values for this specification, reasonable values for the parameters θ and γ can be ascertained by considering the shape of the function. The price-independent term, $(N+1)^{\theta}$, will be increasing in N if θ is greater than zero. It will be concave (reflecting possible economies of scale) for all N if θ is also less than one. While the first and second derivatives of the price-dependent term, $\prod_i p_i^{\gamma_i N}$, are not easily signed, some insight can be gained by writing γ_i as

$$(10) \qquad \gamma_i = \frac{\partial \ln \left(\prod_i p_i^{\gamma_i}\right)^{N}}{\partial \ln p_i} \times \frac{1}{N}$$

The first term on the right-hand-side is the elasticity of the (price-dependent portion of the) equivalence scale with respect to the price of good *i*. As this is positive or negative, so is the corresponding γ_i positive or negative. A positive γ_i, then, might be expected on goods "needed" relatively more by the larger households: a rise in their price increases the difference (*i.e.* the equivalence ratio) between the reference and larger household. A negative γ_i, on the other hand, implies that an increase in the price of good *i* reduces the expenditure needs of the larger household relative to those of the reference household. One may expect that among the goods with negative γ_i's would be goods with large household economies of scale.

or goods for which children (the "marginal" members in this specification) have relatively low needs.

The form of the cost function chosen for the reference household is that corresponding to the Almost Ideal Demand System (DEATON and MUELLBAUER, [1980]). Putting equation (8) into logarithmic form and imposing the chosen specifications for the cost function and equivalence scale yields:

$$(11)\quad \ln C(u, \mathbf{p}, \boldsymbol{\alpha}) = \alpha_0 + \sum_i \alpha_i \ln p_i + \frac{1}{2} \sum_i \sum_j \eta^*_{ij} \ln p_i \ln p_j + u\, \beta_0 \prod_i p_i^{\beta_i} + \theta \ln (N+1) + N \sum_i \gamma_i \ln p_i$$

The share-form demand equations implied by this cost function are found by taking the derivative of (12) with respect to $\ln p_j$, and then substituting out the utility term using the fact that $C(u, \mathbf{p}, \boldsymbol{\alpha}) = Y$, total expenditure. With suitable rearrangement,

$$(12)\quad w_i = \alpha_i - \beta_i \alpha_0 + \gamma_i N - \beta_i \theta \ln (N+1) + \sum_j (\eta_{ij} - \beta_i \gamma_j N) \ln p_j + \beta_i \left\{ \ln Y - \sum_j \alpha_j \ln p_j - \frac{1}{2} \sum_j \sum_k \eta_{jk} \ln p_j \ln p_k \right\}$$

where

$$(13)\quad \eta_{ij} = \frac{1}{2} (\eta^*_{ji} + \eta^*_{ij})$$

The following restrictions, in addition to those imposed on the γ_i's, assure that the demand system satisfies Slutsky symmetry, homogeneity, and adding up for all household sizes:

$$\sum_i \alpha_i = 1; \qquad \sum_i \eta_{ij} = 0; \qquad \sum_i \beta_i = 0; \qquad \eta_{ij} = \eta_{ji} \quad (13)$$

These restrictions are not tested in the present paper.

The stochastic specification is derived by assuming that the share equations include additive error terms which have mean zero, are i.i.d. across households, and have a singular covariance matrix (due to adding up). The interdependence of the share equations in any household gives the model a "seemingly unrelated regressions" form, while the singularity of the covariance matrix requires that one equation be dropped before the system is estimated. In practice, the α_0 parameter is rarely identified by the data and is set at a particular value representing the natural logarithm of "the outlay required for a minimal standard of living when prices are unity" (DEATON and MUELLBAUER [1980]). The endogeneity of the total expenditure variable on the right hand side should be dealt with using instrumental variable techniques; however, this was not attempted in the present paper. With the additional assumption of multivariate normality of the error terms, maximum likelihood estimates can be derived by using an algorithm the combines the Gauss-Newton method of minimizing the sum of squared residuals weighted by the residual covariance matrix, with iterative (Zellner) reestimation of this covariance matrix until convergence of the estimates of both the parameters and the residual covariance matrix is achieved. The ITSUR estimation method in the SAS (Version 6) MODEL procedure has been used to accomplish this task.

5 Data

The expenditure and household composition data used in this paper are from the Interview portion of the Consumer Expenditure Survey (CEX), conducted by the U.S. Department of Labor, Bureau of Labor Statistics (BLS). Beginning in 1980, this survey has been conducted on an ongoing basis, with households introduced into the sample, interviewed for up to five quarters, and then dropped from the sample on a rotating basis. Roughly 7,000 households begin the survey during a calendar year; of these, somewhat more than half complete all the interviews. The Interview portion collects detailed data on major items of expenditure that can be expected to be recalled over a three-month period, and global estimates of expenditures on small iterms such as food purchases. In addition to information on expenditures, the survey gathers information on the composition of the "Consumer Unit" (roughly equivalent to, and henceforth referred to, as a "household"), employment, wages and salaries, other forms of income such as public assistance and child support, outlays for gifts and contributions, the characteristics of the housing unit, changes in assets and liabilities, and consumer durables already owned. The primary purpose of the CEX is to provide weights for construction of the Consumer Price Index. The version of it that is released to the public for research purposes has selected information suppressed and selected expenditures topcoded, to ensure confidentiality, and has information on expenditures from only the second through fifth interviews. Conversion of many of the 1980-1989 Public Use datafiles to a form congenial to micro-level economic research (e. g., with the data on households matched across calendar quarters, and aggregates defined to correspond to price index series) has been done by Nelson [1992 *b*].

For this paper, a household was included in the analysis only if it

– completed the full set of five interviews,

– did not change in composition during the survey year,

– lived in an urban area (since the price data used are for urban areas only),

TABLE I

Descriptive Statistics ([a])

Sample Size	Count 5,480
Number of observations with number of children	
0.	1,365
1.	1,306
2.	1,814
3.	733
4.	202
5.	49
6 or more.	25

	Mean	Standard Deviation
Total Expenditure.	$17,991	7,325
Shares:		
Food.	.24	.07
Rent.	.31	.09
Fuel.	.12	.05
Apparel.	.08	.05
Trans.	.16	.07
HFO.	.09	.06
Prices:		
Food.	1.06	.09
Rent.	1.10	.14
Fuel.	1.01	.06
Apparel.	1.05	.07
Trans.	1.01	.04
HFO.	1.03	.05

([a]) Food: food at home and food away from home (excluding alcohol).

Rent: rent of dwelling (renters) or owner's equivalent rent (owners).

Fuel: household fuels (fuel oil, bottled gas, etc.) and utilities (electricity, sewer, etc.).

Apparel: apparel commodities (including footwear) and services (e. g. dry cleaning).

Trans.: transportation nondurables (gasoline, motor oil, etc.) and services (e. g. auto repair, auto insurance, airline fares).

HFO: household furnishings (furniture, textiles, appliances, etc.) and operations (baby-sitting, lawn care, appliance repair, etc.).

– consisted of a husband age 60 or younger, and wife age 50 or younger, and their own children under the age of 18, if any,

– was considered a "complete income reporter" by BLS standards (since, while income data is not directly used in the present study, "complete income reporter" status tends to be a good overall indicator of cooperation with the survey),

– reported positive values for expenditures on food consumed at home,

– had no topcoded expenditures in the categories analyzed.

Information on the sample sizes is contained in Table 1. Imposition of these criteria has implications for the representativeness of the final sample. Households who participate for the full course of interviews are more likely to be homeowners and have older members, than households for whom data is incomplete (NELSON [1992*b*]). Topcoding was most severe in the category of "rent", where the cap was put at $ 1,000 per month and over three hundred observations were lost. While imputation of rental values for these households could be attempted in the future, the present exclusion of topcodes from the analysis implies that high-expenditure households are somewhat underrepresented. [7]

In choosing the categories of goods to include in the demand system, the three main considerations were a focus on flows of consumption from services and nondurable goods, the existence of suitable corresponding price indices, and computational feasibility. This paper analyzes expenditure on six categories of consumption expenditures: food, rent or rental equivalence for the dwelling (abbreviated as "rent"), household fuels and utilities ("fuel"), apparel, transportation nondurables and services ("trans."), and household furnishings and operations ("HFO"). Expenditures are measured as the total over the twelve months of the household's participation in the survey. Descriptive statistics on these categories, and of the total expenditure on the included categories, are included in Table 1. The percentage of households reporting zero expenditures was under .3% for all categories except HFO, for which 1.5% of the households recorded no expenditure.

The availability of a measure of rental equivalence for homeowners, starting in 1982, is theoretically preferable to alternative measures of owner's housing costs used in the past (such as mortgage interest plus taxes). However, not all respondents were able to estimate what their house would earn on the rental market, and as a result a substantial proportion of the rental equivalence values were created by BLS imputations. In keeping with a focus on consumption services, vehicle purchases were excluded from the transportation category. Imputation of service flows was not attempted here. The category of household furnishings and operations also includes many consumer durables or semi-durables (e. g. washing machines, VCR's), but these were admitted to the analysis on the assumption that the flow of these expenditures (which are likely smaller and more frequent than expenditures for vehicles) may more nearly approximate their service flows.

A number of service and nondurable expenditures are not included in the analysis, largely because proliferation of categories makes estimation

---

7. High-expenditure households are not *excluded* from the analysis due to a peculiarity of the data: the value reported by a respondent as the likely market rent for a "similar" house in the same neighborhood was not topcoded, and was substituted (if available) for the estimate of rent for the respondent's own house when that was topcoded or otherwise missing. It could be argued, however, that the "Independent of Base" equivalence scale assumption would actually be more plausible if the data were limited to households in the middle or lower-to-middle income ranges.

more difficult; these included medical care, entertainment, alcohol, tobacco, lodging away from home, and personal and educational expenses. Inclusion of these in a single "other" category would be possible, although it would require care in the construction of a suitable price index. While the CEX also gathers data on other outlays such as cash contributions and capital improvements to an owned dwelling, these are usually not considered to be part of consumption. Expenditure on the six categories included in the analysis amounts to, on average, 74% of household expenditures on all consumer goods, services and durables. Approximately 17 percentage points of the difference is attributable to excluded nondurables and services, and 9 percentage points to the excluded vehicle purchases. While the sum of expenditures on the six included categories will henceforth be referred to as "total expenditure," or, for short, "income," it should be kept in mind that these names are not entirely accurate. The exclusion of some expenditures may not be innocuous where equivalence scale estimation is concerned: analysis of sample means indicates that the six included goods are relatively more important in larger households (rising from accounting for 73% of household expenditures for households with no children, to 80% for households with five children).

The α_0 parameter in the AIDS specification was set at 8.5, which is approximately the natural logarithm of 4800. The number 4800 was chosen, as it is approximately the natural logarithm of 4800. The number 4800 was chosen, as it is approximately equal to 74% of the 1983 poverty threshold for a household of two persons, as determined by the U.S. Department of Health and Human Services.

The price data used in this paper are U.S. City Average, U.S. Consumer Price Indices, supplied in raw from on magnetic tape by the U.S. Bureau of Labor Statistics, for 1980-1989. The base period is 1982-1984. As households may enter the survey in any month of the year, the price index for a particular good and household was constructed as the average of monthly indices for that good over the twelve months of the household's expenditure reporting. In the case of transportation nondurables and services, a composite index had to be constructed by constructing a weighted average of the separate nondurable and services indices (where the weights were the mean shares in the sample, over the period 1982-1984). No interarea price variation was introduced since geographic information is largely suppressed in the public use CEX data. Mean prices and their standard deviations are reported in Table 1.

6 Results

Results of the estimation of the Independent-of-Base, AIDS model are given in Tables 2 to 7. Only five equations were estimated at a time, but the procedure was followed three times with a different good excluded, as a computer-intensive way of estimating a complete set of parameters and their standard errors. Even though the convergence criteria were tightly specified, this led to some variation in parameter estimates due entirely to computational differences. Differences occurred at the fourth or fifth significant digit for most parameters, but at the second or third for some of the demographic parameters. Reported results are taken largely from the estimation run which excluded household furnishings and operations.

Table 2 reports on the estimated parameter values and their standard errors. All α and β estimates, except for the β estimates for transportation and rent, are statistically significant at the 1% level. The η_{ij} estimates show a more spotty pattern of statistical significance. Of the demographic variables, only the γ_i for food, rent, fuel, and transportation seem to be at all precisely estimated. The estimate of θ is statistically significantly different from zero at the 1% level and its value is within the [0, 1] range considered most plausible in theory.

The signs of the γ_i terms seem to accord well with what one might expect according to economies of scale. Rent and transportation have negative γ_i's, indicating that rises in their prices make it relatively cheaper to live in a larger household (relative, that is, to the smaller reference household). Since the use of houses and cars can accommodate a good deal of sharing, this makes sense. Fewer economies of scale would be expected with food, and indeed it's positive γ_i indicates that arise in its price makes it relatively more expensive to live in a larger household. Further research (not reported in detail here) revealed that the signs and magnitudes of the γ_i terms are not sensitive to the choice of α_0 (which, in the AIDS system, must be set by the researcher), and the signs tend to be insensitive to the choice of households and goods.

Table 3 presents the predicted shares of expenditure going to each of the six goods, for a selection of household sizes, when prices and total expenditure are set at their sample means. Correspondent with patterns in the raw data, the shares of food and household fuels and utilities increase and the rent, transportation nondurables and services, and household furnishings and operations shares fall as the number of children increases. The income (*i. e.* total expenditure) elasticities presented in Table 4 follow an intuitively sensible pattern of luxury vs. necessity classification. For all goods but rent and transportation, the small sizes of the estimated asymptotic standard errors imply that a hypothesis of unitary elasticity can be rejected at a confidence level of 95%. The variation across household sizes in income elasticities is due entirely to variation in the shares predicted for each household size at mean prices and income. Formulas for all income and price elasticity calculations are included in the Appendix. The income elasticities appear to be robust to changes in the choice of α_0, to the choice of households (e. g. excluding homeowners) and, in a relative sense, to the choice of goods to be included in the demand system.

A comparison of Tables 3 and 4 also yields some insights into the general question of the use of demand data to estimate equivalence scales. A very old and very popular approach to testimating household equivalence (called

TABLE 2

Demand System Estimation

Parameter	Good	Estimate	Approximate Standard Error
α_i.	Food	.27**	.003
	Rent	.31**	.004
	Fuel	.16**	.002
	Apparel	.04**	.002
	Trans.	.19**	.003
	HFO	.03**	.003
β_i.	Food	−.052**	.002
	Rent	.006*	.003
	Fuel	−.046**	.002
	Apparel	.038**	.002
	Trans.	−.001	.002
	HFO	.054**	.002
η_{ij}.	Food-Food	.097	.13
	Food-Rent	−.087	.07
	Food-Fuel	−.069	.04
	Food-Apparel	.217**	.08
	Food-Trans.	.052	.05
	Food-HFO	−.210*	.08
	Rent-Rent	.139**	.04
	Rent-Fuel	.048*	.03
	Rent-Apparel	−.054	.05
	Rent-Trans.	−.086*	.04
	Rent-HFO	.040	.05
	Fuel-Fuel	.082**	.02
	Fuel-Apparel	−.009	.04
	Fuel-Trans.	−.034	.02
	Fuel-HFO	−.018	.04
	Apparel-Apparel	−.219*	.11
	Apparel-Trans.	.014	.04
	Apparel-HFO	−.018	.04
	Trans.-Trans.	.064*	.04
	Trans.-HFO	.051	.07
	HFO-HFO	.149*	.08
γ_i.	Food	.018**	.001
	Rent	−.008**	.001
	Fuel	.002*	.001
	Apparel	.001	.001
	Trans.	−.012**	.001
	HFO	−.002	.001
θ.		.144**	.06

Adjusted R^2. Food: .20; Rent: .02; Fuel: .16, Apparel: .10; Trans.: .06; HFO: .11.

**: Statistically significant at 1% level.

*· Statistically significant at 10% level.

the "Engel method" by DEATON and MUELLBAUER [1986]) is to take the share of income spent on food as a measure of the household standard of living, and then compare the levels of income at which different types of

TABLE 3

Predicted Shares (a)

Good	Number of Children			
	0	1	2	4
Food.	.21	.23	.26	.30
Rent.	.32	.31	.31	.29
Fuel.	.11	.11	.12	.13
Apparel.	.08	.08	.08	.08
Trans.	.18	.17	.16	.14
HFO.	.10	.09	.08	.08

[a]: evaluated at sample mean prices and total expenditure.

TABLE 4

Estimated Income Elasticities (a)

Good	Number of Children			
	0	1	2	4
Food.	.75 (.01)[b]	.78	.80	.82
Rent.	1.02 (.01)	1.02	1.02	1.02
Fuel.	.57 (.02)	.59	.61	.63
Apparel.	1.46 (.02)	1.47	1.48	1.48
Trans.	1.00 (.01)	1.00	1.00	1.00
HFO.	1.57 (.02)	1.62	1.65	1.71

[a]: evaluated at sample mean prices and total expenditure.

[b]: numbers in parentheses are approximate standard errors.

household attain the same food share. Since food is inelastically demanded with respect to income, higher incomes imply lower shares, and hence higher imputed welfare. In Table 3 one sees that smaller households have smaller food shares than larger households, and would hence (as would be hoped) be judged as being able to achieve a higher standard of living for a given income. Two problems with the food share approach are well-illustrated in the comparison of the tables. One problem is the question of why a measure based on food should be preferred over a measure based on some other good. A comparison of the income elasticities in Table 4 with the patterns of shares according to household size in Table 3 reveals that fuel and household furnishings and operations would also be contenders with food for the designation as a welfare proxy in this approach: shares respond to increases in household size in each case in a manner that is inverse to the response to increases in income. The second problem with the good-share approach is the fact that food (or other) shares might vary over household composition for reasons other than differences in the standard of living. For example, economies of scale may come into play (Nelson [1988]), or children might be relatively larger consumers of some goods than others, compared to adults (Deaton and Muellbauer, [1986]). This can also be seen in a comparison of Tables 3 and 4, for the cases of rent and clothing. Rent has an income elasticity statistically indistinguishable from one, but the share of income devoted to rent falls with household size. If rent were considered a necessity, the share-proxy approach would have to conclude that the larger households, needing a lower share of this necessity, are therefore better off. A more plausible explanation would be that larger households can reduce rental expenditures by exploiting economies of scale not available to the smaller households. The share of apparel, rather than falling with household size as might be expected by its status as a luxury good, stays constant. A likely explanation for this is that growing children are more "clothing-purchase intensive" than adults who are more able to consume from previously acquired clothing stocks (especially at low to moderate income levels; see Nelson [1989]).

Tables 5 and 6 present estimated uncompensated and compensated price elasticities for a childless household, evaluated at mean prices and total expenditure. The estimated asymptotic standard deviations for the uncompensated own-price elasticities indicate that the hypothesis of unitary price elasticity can be rejected at a 5% significance level for rent, fuel, household furnishings and operation (in the direction of inelasticity) and apparel (in the direction of high elasticity). The positive own-price elasticity for household furnishings and operations, and the seemingly very large negative price elasticity for apparel must be considered in the light of (as for food) low precision in estimation. Further warnings, however, are in order. The estimates of price elasticities seems to be sensitive to variations in α_0, to the households selected for inclusion, and (in a relative as well as absolute sense) to changes in the selection of goods included in the system. For example, replacing the α_0 of 8.5 by one of 4800 causes three of the estimated elasticities to change little, but the uncompensated own-price elasticities for fuel, transportation, and household furnishing to change from (−.16, −.65, +.55) to (+.22, −.04, −1.6). The reported price elasticities should hence be treated with skepticism.

Table 7 presents the estimated scales, $\Delta(\mathbf{p}, \boldsymbol{\alpha})$. The household of a couple and one child is estimated to need less than 11% higher expenditures to reach the same welfare as a childless couple (at mean prices, and *any* – because of the IB assumption – total expenditure level). Subsequent children add even less. While the scales rise with increasing numbers of children, and at a decreasing rate (as would be implied by economies of scale), the scales overall seem low. Given the problems of identification of the model, these scales must be taken with a grain of salt. The estimated scales were also very sensitive to the choices of households and goods used in the analysis, though not to the choice of α_0.

TABLE 5

Estimated Uncompensated Price Elasticities ([a])

	Number of Children = 0					
	Food	Rent	Fuel	Apparel	Trans.	HFO
Food.	−.47 (.65)[b]	−.33	−.29	1.03	.29	−.99
Rent.	−.28	−.57 (.14)	15	−.17	0.27	.12
Fuel.	−.53	.59	−.16 (.23)	−.07	−.24	−.16
Apparel.	2.49	−.80	0.18	−3.66 (1.27)	09	.60
Trans.	.28	−.46	−.19	.08	−.65 (.20)	−.06
HFO.	−2.38	.24	−.29	.52	−.22	.55 (.76)

[a]: evaluated at sample mean prices and total expenditure.

[b]: numbers in parentheses are approximate standard errors

TABLE 6

Estimated Compensated Price Elasticities ([a])

	Number of Children = 0					
	Food	Rent	Fuel	Apparel	Trans.	HFO
Food.	−.31	−.09	−.21	1.10	.43	−.92
Rent.	−.06	−.25	.26	.09	−.08	.22
Fuel.	−.41	.77	−.10	−.02	−.14	−.10
Apparel.	2.80	−.33	−.02	−3.54	.36	.74
Trans.	.49	−.14	−.08	.16	−.47	.04
HFO.	−2.04	.74	−.12	.65	.07	.70

[a]: evaluated at sample mean prices and total expenditure.

TABLE 7

Estimated Scales ([a])

Number of Children	Scale	Change
0.	1	–
1.	1.106	.106
2.	1.173	.067
3.	1.225	.052
4.	1.268	.043

[a]: evaluated at sample mean prices.

7 Summary and Directions for Future Research

While it was possible to estimate household equivalence scales using U.S. micro-level data and an Almost Ideal Demand System model, the scales suffer from a problem of identification (resting on an essentially arbitrary functional representation), the value of these scales seems low in the favored specification, and the scales are not robust to changes in the empirical specification. The validity of the estimated price elasticities may also be questionable. Only the estimated income elasticities, and (to a lesser extent) the goods-specific demographic parameters seem to be robust to changes in specification.

A number of extensions to this sort of empirical analysis would be possible with the available data. For example, instrumental variables could be applied to the total expenditure variable; the equivalence scale could be made to depend on a vector of composition characteristics rather than only on the number of children; leisure/home-production could be included as a consumption category; an attempt could be made to compute flows of services from durables (such as vehicle purchases) so that these could be included in the analysis; or a finer breakdown of goods could be used. However, while such extensions would likely lead to interesting insights into household behavior, the identification problem and the sensitivity of estimated equivalence scales to model assumptions and empirical specification suggests that the use of IB scales within an AIDS model is not a satisfactory method of determining scales. Further research into combinations of theories and functional forms which could overcome these problems is needed.

APPENDIX

Elasticity formulas

In general, the income (*i. e.* total expenditure) elasticity, ψ_i, can be expressed as:

$$\phi_i = 1 + (\partial \ln w_i / \partial \ln Y) = 1 + (\partial w_i / \partial \ln Y)(1/w_i)$$

Applying this to equation (12) yields:

$$\phi_i = 1 + (\beta_i / w_i)$$

Note that the income elasticity does not *directly* depend on the number of children (N).

In general, the uncompensated price elasticity for good i with respect to price j, ε_{ij}, can be expressed as

$$\varepsilon_{ij} = -\delta_{ij} + (1/w_i) \times (\partial w_i / \partial \ln p_j)$$

where $\delta_{ij}=$ if $i=j$, $=0$ otherwise. Applied to equation (10) this yields:

$$\varepsilon_{ij} = -\delta_{ij} + (1/w_i) \times [\eta_{ij} - \beta_i \gamma_j N - \beta_i (\alpha_j + \sum_k \eta_{kj} \ln p_k)]$$

Using the Slutsky equation, the formula for the compensated elasticity (ε^*_{ij}) can be expressed in terms of the uncompensated price elasticity, income elasticity, and good share as:

$$\varepsilon^*_{ij} = \varepsilon_{ij} + \phi_i w_j$$

Elements of the Slutsky matrix can be expressed in terms of the compensated elasticity:

$$\frac{\partial^2 C(u, \mathbf{p}, \boldsymbol{\alpha})}{\partial p_i \partial p_j} = \varepsilon^*_{ij} \times (x_i / p_j)$$

Approximate asymptotic standard errors for income and own uncompensated price elasticities were computed using Taylor series expansions (Greene, [1990], pp. 228).

• References

Blackorby, Ch. and Donaldson, D. (1991 *a*). – "Adult Equivalence-Scales and Interpersonal Comparisons of Well-Being, and Applied Welfare Economics," *in Interpersonal Comparisons and Distributive Justice*, Elster J. and Roemer J., Eds. Cambridge University Press.

BLACKORBY, Ch. and DONALDSON, D. (1991 *b*). – "Adult Equivalence-Scales and the Economic Implementation of Interpersonal Comparisons of Well-Being," Univ. of British Columbia, Department of Economics Discussion Paper No. 91-08.

BLUNDELL, R. and LEWBEL, A. (1991). – "The Information Content of Equivalence Scales," *Journal of Econometrics*, 50.

BOURGUIGNON, F. (1989). – "Family Size and Social Utility: Income Distribution Dominance Criteria," *Journal of Econometrics*, 42, pp. 67-80.

BUHMANN, B., RAINWATER, L., SCHMAUS, G. and SMEEDING, T. M. (1988). – "Equivalence Scales, Well-Being, Inequality, and Poverty: Sensitivity Estimates Across Ten Countries Using the Luxembourg Income Study (LIS) Database," *Review of Income and Wealth*, pp. 115-142.

DEATON, A. and MUELLBAUER, J. (1980). – "An Almost Ideal Demand System," *The American Economic Review*, 70, pp. 312-326.

DEATON, A. and MUELLBAUER, J. (1986). – "On Measuring Child Costs: With Applications to Poor Countries," *Journal of Political Economy*, 94, pp. 720-744.

FISHER, F. M. (1987). – "Household Equivalence Scales and Interpersonal Comparisons," *Review of Economic Studies*, 54, pp. 519-524.

GREENE, W. H. (1990). – *Econometric Analysis*, New York, Macmillan.

HADDAD, L. and KANBUR, R. (1990). – "How Serious is the Neglect of Intrahousehold Inequality?", *Economic Journal*, 100, pp. 866-881.

LEWBEL, A. (1989). – "Household Equivalence Scales and Welfare Comparisons," *Journal of Public Economics*, 39, 1989, pp. 377-391.

MUELLBAUER. John (1980). – "The Estimation of the Prais-Houthakker Model of Equivalence Scales," *Econometrica*, 48, pp. 153-176.

NELSON, J. A., (1986). – "A Model of Household Joint Utility Maximization with Public Goods," *Ph. D. dissertation*, University of Wisconsin, Madison.

NELSON, J. A. (1988). – "Household Economies of Scale in Consumption: Theory and Evidence," *Econometrica*, 56, pp. 1301-1314.

NELSON, J. A. (1989). – "Individual Consumption Within the Household: A Study of Expenditures on Clothing," *Journal of Consumer Affairs*, 23, pp. 21-44.

NELSON, J. A. (1991). – "Household Equivalence Scales: Theory Versus Policy?" forthcoming, *Journal of Labor Economics*.

NELSON, J. A. (1992 *a*). – "Methods of Estimating Household Equivalence Scales: An Empirical Investigation," *Review of Income and Wealth*, 38, pp. 295-310.

NELSON, J. A. (1992 *b*). – "Consumer Expenditure Survey, Interview Survey, for Household-Level Analysis, (1980-1989)"

PHIPPS, S. A. (1991). – "Price-Sensitive Adult-Equivalence Scales for Canada," *mimeo*, Department of Economics, Dalhousie University.

POLLAK, R. A. and WALES, T. J. (1979). – "Welfare Comparisons and Equivalence Scales," *American Economic Review*, 69, pp. 216-221.

POLLAK, R. A., (1990). – "Welfare Comparisons and Situations Comparisons," University of Washington, Institute for Economic Research Discussion Paper #90-11.

SEN, A. (1983). – "Economics and the Family," *Asian Development Review* 1, Reprinted in SEN, 1984, *Resources, Values and Development*, Cambridge, Harvard University Press, pp. 369-385.

SEN, A. (1987). – *The Standard of Living*, Cambridge, Cambridge University Press.

WATTS, H. W. (1967). – "The Iso-Prop Index," *Journal of Human Resources*, 2, pp. 3-18.