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Investment Dynamics and Capacity Utilization under Monopolistic Competition

Omar LICANDRO *

ABSTRACT. – In the tradition of Tobin's q models, the influence on investment of demand uncertainty and capacity constraints is analyzed in a monopolistically competitive economy. Under these conditions, the degree of capacity utilization has a positive effect on the markup rate and explains the difference between average q and marginal q. In the aggregate economy, when the representative firm faces only specific demand uncertainty, it is shown that the degree of capacity utilization is strictly smaller than one at steady state. Expected changes in aggregate demand or in demand price elasticity are shown to accelerate the accumulation process.

RÉSUMÉ. – Dans le cadre d'une économie en concurrence monopolistique et dans la tradition des modèles du q de Tobin, ce papier analyse les effets que l'incertitude sur la demande et les contraintes de capacités ont sur l'investissement. Au niveau de la firme, il est montré que le degré d'utilisation des capacités a un effet positive sur le taux de markup et qu'il explique les différences entre le q moyen et le q marginal. Au niveau agrégée, il est montré que le degré d'utilisation des capacités est strictement inférieur à un en état stationnaire et que des changements attendus de la demande agrégée, ou de l'élasticité prix de la demande, accélèrent le processus d'investissement.

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* O. LICANDRO: Universidad Carlos III de Madrid, Espagne.

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1 Introduction

As shown by empirical economists, the degree of capacity utilization plays an important role in investment decisions.[1] The question is then, what is the simplest theoretical way of stressing this empirical finding? As shown by MALINVAUD [1987] [1989], sufficient conditions in the case of a monopolistic firm are demand uncertainty and short-run factor complementarity. When factors are complementary, technical coefficients are rigid and the capital stock imposes an upper bond on production, giving sense to the idea of "capacities". Moreover, since marginal cost can equal marginal revenue for a production level smaller than total capacities, even with price flexibility it can be optimal for the monopolistic firm to use its capacities only partially. In this framework, the underemployment of capacities is a desired outcome and involuntary excess capacity does not exist.

In a previous paper, LICANDRO [1990], we added to the Malinvaud model investment adjustement costs and nominal rigidities in an intertemporal setup. As in d'AUTUME [1988], the introduction of adjustement costs produces a smooth path for the capital stock. The addition of nominal rigidities to Malinvaud's assumptions allows for *ex post* involuntary excess capacity. The firms choose prices and capacities before demand is known and thus fix an optimal *ex ante* expected use of capacities. But the observed degree of capacity utilization depends on the realization of the demand shocks, leading to undesired excess or shortage of capacities. Moreover, we showed that even if HAYASHI's [1982] conditions hold,[2] demand uncertainty implies that average q exceeds marginal q, the difference depending on the degree of capacity utilization. Only if the firm expects to use fully its capacities does the equality between marginal and average q holds. This result is very close to BLANCHARD and SACHS [1982].

Nominal rigidities under monopolistic competition are now a current assumption in macroeconomics and are based on menu costs or information costs.[3] The price behavior of monopolistically competitive firms under demand uncertainty and factor complementarity has been analyzed by SNEESSENS [1987], showing that the markup rate depends on demand pressures in addition to the demand elasticity.

For the analysis of macroeconomic aggregates, another interesting contribution was made by LAMBERT [1988], who assumes an economy with price

1. See in particular DRÈZE and BEAN [1990] and CHAN-LEE and TORRES [1987].
2. HAYASHI [1982] imposes the following three conditions for the equality between marginal and average q: (1) perfect competition in the goods market, (2) constant returns to scale in production, and (3) linear homogeneity of the adjustement cost function. We will discuss these in more detail below.
3. For a reference *see* BLANCHARD and FISCHER [1989].

rigidity and firms operating on different micromarkets, e. g., because they produce differentiated goods.[4] Under plausible assumptions aggregate production can be approximated by a CES function of aggregate demand and potential output. This theoretical approach was shown to be of empirical relevance, in particular considering the recent results of the "European Unemployment Program".[5]

This paper is concerned with the steady state equilibrium and the dynamic effects of changes in demand in an economy where the firms behave as in LICANDRO [1990] and the aggregates are defined as in LAMBERT [1988]. We try to answer two important questions. First, we ask whether capacities are fully employed in the long run or whether firm's optimal behavior pushes the economy to underemploy capacities. In real economies aggregate capacities are always underutilized, even in "booms". Demand uncertainty, and in particular firm's specific demand uncertainty, play an important role in this respect. Specific demand uncertainty is related to the distribution of aggregate demand among firms. We will show that even in the extreme case where aggregate demand is known and only firm-specific demand uncertainty exists, aggregate capacity always exceeds aggregate production.

Second, we study the effect of changes in both aggregate demand and demand elasticity. We show that just as in the accelerator model demand has a Keynesian effect on the accumulation process. In particular, all increases in aggregate demand push up long-term capacities and production in the same proportion.

Section 2 is devoted to the definition of demand uncertainty and capacity constraints. Section 3 is concerned with the representative firm behavior and optimal conditions. The relation between average and marginal q is analyzed. In Section 4 we define the aggregate economy, solve the steady state equilibrium, and analyze the dynamic evolution of the aggregates. Finally, Section 5 is devoted to the analysis of changes in demand.

2 Demand Uncertainty and Capacity Constraints

The essential elements of the model are contained in the three following assumptions: (*a*) the goods market is in monopolistic competition; (*b*) the

4. Lambert analyzes the problem in the context of aggregation over micromarkets in disequilibrium. See QUANDT [1988] and GOURIEROUX *et al.* [1984] for a survey.
5. See DRÈZE and BEAN [1990] in this respect. The first references in this matter are SNEESSENS and DRÈZE [1986] and LAMBERT [1988].

firm faces demand uncertainty; and (*c*) prices and capacities are set at the beginning of each period.

Assumptions (*a*) and (*b*) are reflected in the following demand functions,[6]

(1) $$\mathrm{YD}_t = \left(\frac{p_t}{\mathrm{P}_t}\right)^{-\varepsilon} \overline{\mathrm{YD}}_t \, u_t;$$

where YD is demand, p is the firm's own price, P is the aggregate price level, $\overline{\mathrm{YD}}$ is expected aggregate demand, and u follows a purely stochastic process, which is assumed lognormally distributed with mean 1 and variance σ^2. All firms are identical, which implies that the own and the aggregate price are the same in equilibrium. The demand elasticity ε is assumed constant over time.

Moreover, it is assumed that only demand is random and all other variables are known by the firm. This assumption allows us to concentrate on the consequences of demand uncertainty without the complexities of a more realistic specification.

The production function is Leontief. This assumption inhibits the firm to modify capacities during the current period. Capacities can then be written as

(2) $$\mathrm{YP}_t = \mathrm{B}_t \mathrm{K}_t;$$

where YP represents potential output, B is the technical coefficient for capital and K is the capital stock.

Excess capacity and excess demand are possible since capacities and prices are determined before the stochastic demand is known. This implies that production is defined by

(3) $$\mathrm{Y}_t = \min\{\mathrm{YD}_t, \mathrm{YP}_t\},$$

where Y is production.

Since the stochastic term in YD is lognormally distributed, LAMBERT's [1988] theorem applies and expected production can be approximated by

(4) $$\mathrm{E}_t(\mathrm{Y}_s) = [\mathrm{E}_t(\mathrm{YD}_s)^{-\rho} + \mathrm{YP}_s^{-\rho}]^{-1/\rho} \, \forall s > t,$$

where $\mathrm{E}_t(.)$ represents expectations conditional on the information set at time t. The parameter ρ depends inversely on σ^2. In particular, when ρ goes to infinity there is no uncertainty at all and expected production approaches the minimum of expected demand and potential output.[7]

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6. See DIXIT and STIGLITZ [1977] for a general reference on monopolistic competition and SNEESSENS [1987] for an application to price determination under demand uncertainty and capacity constraints.
7. In this case the model is related to the investment models under fixed prices. See MALGRANGE and VILA [1984], MICHEL [1986] and PRECIOUS [1987].

An interesting characteristic of equation (4), as shown by LAMBERT [1988], is that the elasticity of expected production with respect to potential output, denoted Φ_p, is

(5) $$\Phi_{p,s} = (E_t(DUC_s))^{\rho},$$

where

$$DUC_s = \frac{Y_s}{YP_s}.$$

The variable $\Phi_{p,s}$ is a function of expected values conditional on the information set at time t. The index t is omitted for simplicity. In the same way Φ_d defines the elasticity of expected production to expected demand. LAMBERT [1988] also shows that the elasticities Φ^p and Φ^d are equal to the weighted probability of capacity constraint and demand constaint, respectively. From equation (4) we can easily show that

(6) $$\Phi_{p,s} + \Phi_{d,s} = 1.$$

The expectations about the degree of capacity utilization, denoted $E_t(DUC)$ and the elasticity Φ_p play an important role in the model. This will become clear below when we compute the optimality condition for investment and analyze the relation between marginal and average q.

Under these assumptions, expectations at time t about the time-s profits π_s can be written as

(7) $$E_t(\pi_s) = (p_s - MPC_s)\, E_t(Y_s) - P_s \Omega\left(\frac{I_s}{K_s}\right) K_s.$$

Marginal production cost, MPC, is assumed given and known to the firm. The price of investment goods is assumed equal to the aggregate price level.[8] Let α denote the ratio of investment to capital stock, I/K. The adjustement cost function $\Omega(\alpha)$ satisfies the following conditions: $\Omega(\delta) = \delta$, $\Omega'(\alpha) > 0$, $\Omega'(\delta) = 1$, $\Omega''(\alpha) > 0$ and $\Omega''(\delta) = 1$, where δ is the depreciation rate.

Because the depreciation rate determines the steady state of the model, the function $\Omega(\alpha)$ is normalized at $\alpha = \delta$. When the firm invests to replace the depreciated capital, it is assumed that it does not face adjustement costs, *i.e.*, investment spending is equal to $P\delta K$.

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8. Since this paper analyses investment in a partial equilibrium set-up, it could be assumed that the investment price is exogenous. So are the marginal production cost and the nominal interest rate. The study of the dynamics of the model (Appendix 3) would be easier with an exogenous real interest rate. However, because we have in mind to build a general equilibrium model it seems to us better to assume that investment prices are endogenous and to be sure that the more complex dynamic could be solved.

3 Representative Firm's Behavior

Under the previous assumptions the firm's optimization problem can be written as

(8) $$V_t = \underset{\{I_s,\, p_s\}_{s=t}^{\infty}}{\text{Max}} \int_t^{\infty} \left(a_s E_t(Y_s) - P_s \Omega\left(\frac{I_s}{K_s}\right) K_s \right) e^{-r(s-t)} ds,$$

where

$$a_s = p_s - MPC_s;$$

(4) $$E_t(Y_s) = [E_t(YD_s)^{-\rho} + YP_s^{-\rho}]^{-1/\rho};$$

(1′) $$E_t(YD_s) = \left(\frac{p_s}{P_s}\right)^{-\varepsilon} \overline{YD}_s;$$

(2) $$YP_s = B_s K_s;$$

(9) $$\dot{K}_s = I_s - \delta K_s;$$

and the initial value for K is given. As indicated above, the firm takes as given the path for the marginal production cost MPC, the aggregate price level P, the technical coefficient for capital B and the aggregate demand $\overline{YD}$. The discount rate r, the depreciation rate δ, the parameter ρ and the demand elasticity ε are assumed constant. The control variables are the firm's own price p and investment I.

The Hamiltonian for this problem is

$$H_s = \left\{ a_s E_t(Y_s) - P_s \Omega\left(\frac{I_s}{K_s}\right) K_s + \mu_s (I_s - \delta K_s) \right\} e^{-r(s-t)}$$

Note that μ is the costate variable associated with the capital stock. The first-order conditions for price, investment and capital are

(10) p: $$\frac{\partial H_s}{\partial p_s} = a_s \frac{\partial E_t(Y_s)}{\partial p_s} + E_t(Y_s) = 0;$$

(11) I: $$\frac{\partial H_s}{\partial I_s} = -P_s \Omega'_s + \mu_s = 0;$$

(12) K: $$-\frac{\partial H_s}{\partial K_s} = \frac{d}{ds}(\mu_s e^{-r(s-t)}).$$

The optimality condition (10) implies a price equation where the firm sets a markup on marginal production cost. This price equation takes the form:

(13) $$p_s = (1 - (\varepsilon \Phi_{d,s})^{-1})^{-1} MPC_s.$$

This price equation was first derived by SNEESSENS [1987]. It stresses the influence of demand pressures on the markup rate through the elasticity of expected production to expected demand, Φ_d. Moreover, since Φ_d depends negatively on expected DUC, the optimal price for a monopolistically competitive firm is an increasing function of the expected utilization of capacities. Finally, as in all monopolistic models the elasticity of output with respect to prices must be greater than one, *i.e.*, $\varepsilon \Phi_d > 1$, to avoid infinite prices.

Condition (11) allows a first characterization of the optimality condition for investment:

$$\Omega'\left(\frac{I_s}{K_s}\right) = q_s. \tag{14}$$

Just as in standard q theory, optimal investment is a function of marginal q, which is defined as the ratio of the marginal value of capital μ to the replacement cost P.

For the optimality condition (12) we must take into account the fact that

$$\frac{\partial H_s}{\partial K_s} = \left[a_s \frac{\partial E_t(Y_s)}{\partial K_s} - P_s\left(\Omega_s - \Omega'_s \frac{I_s}{K_s}\right) - \delta \mu_s\right] e^{-r(s-t)};$$

and that

$$\frac{d}{ds}(\mu_s e^{-r(s-t)}) = (\dot{\mu}_s - r\mu_s) e^{-r(s-t)}.$$

In addition, if we use the definition of the potential output elasticity to deduce the marginal effect of capital on expected production,

$$\frac{\partial E_t(Y_s)}{\partial K_s} = \Phi_{p,s} \frac{E_t(Y_s)}{K_s} = B_s \Phi_{p,s} E_t(DUC_s),$$

we see that the marginal productivity of capital, which is equal to the technical coefficient B, is weighted by the derivative of expected production with respect to capacities. By equation (5) we know that this derivative is smaller than or equal to one depending on DUC, for plausible values of ρ.

Using also the first-order condition for investment, it follows that

$$\dot{\mu}_s = (r + \delta - \alpha_s)\mu_s - a_s B_s \Phi_{p,s} E_t(DUC_s) + P_s \Omega_s. \tag{15}$$

The only difference with respect to standard q theory lies in the marginal productivity of capital. Only if the firm expects to use its capacities fully is the marginal productivity of capital exactly equal to the marginal effect of capital on expected production. Only in this case is equation (15) identical to the standard result.

Taking into account the condition for marginal productivity implicit in equation (15), that is

$$\frac{\partial E_t(Y_s)}{\partial K_s} = \frac{c_s}{a_s}, \tag{15'}$$

we can deduce the user cost of capital c, which is

$$c_s = (r + \delta - \alpha_s)\,\mu_s + P_s \Omega_s - \dot{\mu}_s.$$

The user cost of capital is defined exactly as in standard q models, implying that equation (15′) differs from the standard marginal productivity condition because capacities are not necessary fully employed.

As stated above, the introduction of stochastic demand shocks in the investment decision problem of a monopolistic competitive firm produces two important changes in optimal behavior. The first has already been stressed by SNEESSENS [1987]: the markup rate depends on demand pressures. The second difference is in equation (15): the actual marginal effect of capital on expected production can be smaller than marginal productivity; equality holds only when capacities are fully employed.

3.1. Average Q and Marginal Q

HAYASHI [1982] shows that linear homogeneity of the installation function in addition to constant returns in production and perfect competition in the goods market are sufficient conditions for the equality between marginal q and average q. In the presence of both demand uncertainty and capacity constraints, this proposition no longer holds, because *expected* production is no longer linearly homogeneous in all factors, even if the production function itself is.

If the adjustement cost function is linearly homogeneous in I and K, the relation between marginal and average q can be written as:

$$q_t = Q_t - \frac{1}{P_t K_t} \int_t^{\infty} \Phi_{d,s}\, a_s\, E_t(Y_s)\, e^{-r(s-t)}\, ds; \tag{16}$$

where q_t and Q_t are marginal and average q, respectively, at time t. A formal proof is given in Appendix 1.

Marginal q is smaller than average q, the difference being explained by all future profits weighted by the probability of being demand constrained, Φ_d. This result is similar to PRECIOUS' [1987], who analyzes investment under nonstochastic demand constraints. In Precious' solution time-s expected production is weighted by the marginal value of the demand constraint at time s. In the same way, $(\Phi_d a)$ can be interpreted as the marginal value of an increase in expected demand.

4 Steady State and Dynamics

4.1. Aggregation

Let us assume that the aggregate demand is known by the firm and rewrite equation (1) in terms of the firm market share, *i.e.*,

$$\frac{YD_t}{\overline{YD}_t} = \left(\frac{p_t}{P_t}\right)^{-\varepsilon} u_t.$$

Since firms are assumed ex-ante identical and prices are set before the stochastic shocks take place, firm prices are equal and market shares depend only on u_t. The market shares add-up to one and u_t can be interpreted as the distribution of aggregate demand among firms. When firms are not ex-ante identical or whan there is no price rigidity, prices could be different and the distribution of aggregate demand among firms could depend on both the distribution of prices and the distribution of the term u.

Under the previous assumptions, this economy can be interpreted as follows: the representative firm knows the level of aggregate demand and faces demand uncertainty simply because it does not know what its position will be in the distribution of aggregate demand among firms. In this sense, the stochastic demand term faced by the representative firm in equation (1) represents the distribution of aggregate demand among firms (if the relative price is equal to one for all firms). Demand uncertainty is firm-specific.

Under this assumption, aggregate demand is equal to the expected demand perceived by the representative firm in equilibrium (when $p = P$). For the same reason, aggregate production and aggregate DUC are equal to representative firm's expected production and expected DUC in equilibrium. The aggregate level of production, denoted by $\overline{Y}$, is given by equation (4) and can be written as

(4′)
$$\overline{Y}_s = [\overline{YD}_s^{-\rho} + YP_s^{-\rho}]^{-1/\rho}.$$

Aggregate DUC is denoted $\overline{DUC}$ and Φ^d and Φ^p represent also the elasticities of aggregate production to aggregate demand and aggregate capacities, respectively. Moreover, all the optimality conditions for the representative firm can be reinterpreted as the aggregate equilibrium of the economy.

4.2. Steady State

In steady state the capital stock and its marginal value remain constant: $\dot{\mu} = \dot{K} = 0$. The $\dot{K} = 0$ condition imposes that, at steady state, investment only replaces the depreciated capital, *i.e.*, the steady state investment rate α^* is equal to δ. The optimality condition for investment, equation (14), implies that steady state marginal q is given by

$$q^* = \Omega'(\delta) = 1. \tag{17}$$

This is a standard result in q models. In particular, the normalization assumptions lead us to TOBIN's [1969] proposition that capital does not grow when marginal q is equal to one.

As it is shown in Appendix 2, the steady state value of aggregate DUC is defined by the following relation:

$$\frac{\overline{\mathrm{DUC}}^{*\rho+1}}{1-\overline{\mathrm{DUC}}^{*\rho}} = (r\,\Omega'(\delta) + \Omega(\delta))\frac{\varepsilon}{\mathrm{B}}. \tag{18}$$

Under our previous normalization assumptions on $\Omega(\alpha)$, it becomes

$$\frac{\overline{\mathrm{DUC}}^{*\rho+1}}{1-\overline{\mathrm{DUC}}^{*\rho}} = (r+\delta)\frac{\varepsilon}{\mathrm{B}}. \tag{18'}$$

Since B and $(r+\delta)\varepsilon$ are positive and ρ is greater than one, equation (18′) has a unique positive real solution, which is strictly positive and smaller than one. It becomes equal to one in the extreme case when ε or ρ go to infinity. This result implies that the steady state level of capacities, determined by firms in an optimal way, exceeds aggregate production. Only if the firms operate in a competitive market ($\varepsilon \rightarrow \infty$) or if there is no demand uncertainty ($\rho \rightarrow \infty$) do they expect to fully use their capacities at steady state. [9]

Equation (18′) can be interpreted in terms of the standard relation between marginal productivity and the user cost of capital. Equation (15′) evaluated at steady state implies

$$\mathrm{B}\,\Phi_p^*\,\overline{\mathrm{DUC}}^* = \frac{(r+\delta)\,\mathrm{P}}{a} = (r+\delta)\,\varepsilon\,\Phi_d^*,$$

where $\mathrm{P}(r+\delta)$ is the steady state value for the user cost of capital, as shown in Appendix 2. At steady state the firm chooses capacities in such a way that the marginal effect of capital on expected production is equal to the real value of the user cost of capital. It is obvious that there is a positive relation between r, δ and ε on one hand and the steady state value of aggregate DUC on the other.

This steady state condition is equivalent to the "modified golden ruel" in growth theory and it is compatible with ABEL and BLANCHARD [1983] if we impose the same normalization conditions on the adjustement cost

9. In this context, to assume that there is no demand uncertainty is equivalent to assuming that there is only one firm. It is the opposite case of a competitive market.

function.[10] Moreover, MALINVAUD'S [1987] solution with putty-clay technology is identical to the steady state result for our model.

4.3. Dynamics

The dynamic system is defined by two differential equations: equations (9) for the capital stock and equation (15) for the marginal value of capital. Nevertheless, as in the analysis of the steady state, we are concerned with the evolution of the aggregate degree of capacity utilization and of marginal q. For this reason, the differential equation system in K and μ is transformed into a differential equation system in $\overline{\text{DUC}}$ and q. The mathematical derivations for this section are in Appendix 3.

As said above, the dynamics of the system is defined by the following two differential equations

$$\dot{\overline{\text{DUC}}}=(\delta-\alpha)\,\Phi_d\,\overline{\text{DUC}}, \tag{19}$$

$$\dot{q}=\left[r+(\delta-\alpha)\left(1-\frac{\rho\,\Phi_p}{\varepsilon\,\Phi_d-1}\right)\right]q-\frac{\text{B}\,\Phi_p}{\varepsilon\,\Phi_d}\overline{\text{DUC}}+\Omega(\alpha), \tag{20}$$

and the initial condition $\overline{\text{DUC}_0}=\left(1+\left(\frac{\overline{\text{YD}_0}}{\text{YP}_0}\right)^{-\rho}\right)^{-1/\rho}$. with given initial values for $\overline{\text{YD}_0}$ and YP_0.

The elasticities Φ_p and Φ_d depend on $\overline{\text{DUC}}$ as stated in equations (5) and (6) and the rate of investment α is a function of q, given by equation (14). The time index was suppressed for simplicity. The linearization of the system around the steady state implies

$$\begin{pmatrix}\dot{q}\\ \dot{\overline{\text{DUC}}}\end{pmatrix}=\begin{pmatrix}\gamma_2 & -\gamma_3\\ -\gamma_1 & 0\end{pmatrix}\begin{pmatrix}(q-q^*)\\ (\overline{\text{DUC}}-\overline{\text{DUC}}^*)\end{pmatrix},$$

where

$$\gamma_1=\frac{\Phi_d^*\,\overline{\text{DUC}}^*}{\Omega''(\delta)}>0;$$

$$\gamma_2=r+\frac{\Omega'(\delta)}{\Omega''(\delta)}\,\frac{\rho\,\Phi_p^*}{\varepsilon\,\Phi_d^*-1}>0;$$

$$\gamma_3=\frac{\text{B}\,\Phi_p^*}{\varepsilon\,\Phi_d^*}\left(\frac{\rho}{\Phi_d^*}+1\right)>0.$$

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10. In ABEL and BLANCHARD [1983] the steady state value of the marginal productivity is greater than the "modified golden rule" because they impose adjustement costs when α is equal to δ. If the firm has no adjustment costs when it invests just to replace the depreciated capital, then the "modified golden rule" holds.

The characteristic roots are $\lambda=[\gamma_2 \pm \sqrt{(\gamma_2)^2+4\gamma_1\gamma_3}]/2$, yielding a saddle point solution with one positive and one negative root. The phase diagram for this system is represented in Figure 1. The initial level of capital and aggregate demand define the initial value for aggregate DUC. The initial value for q lies on the convergent path. If marginal q exceeds q^* firms invest and aggregate DUC decreases, moving the economy to the steady state equilibrium.

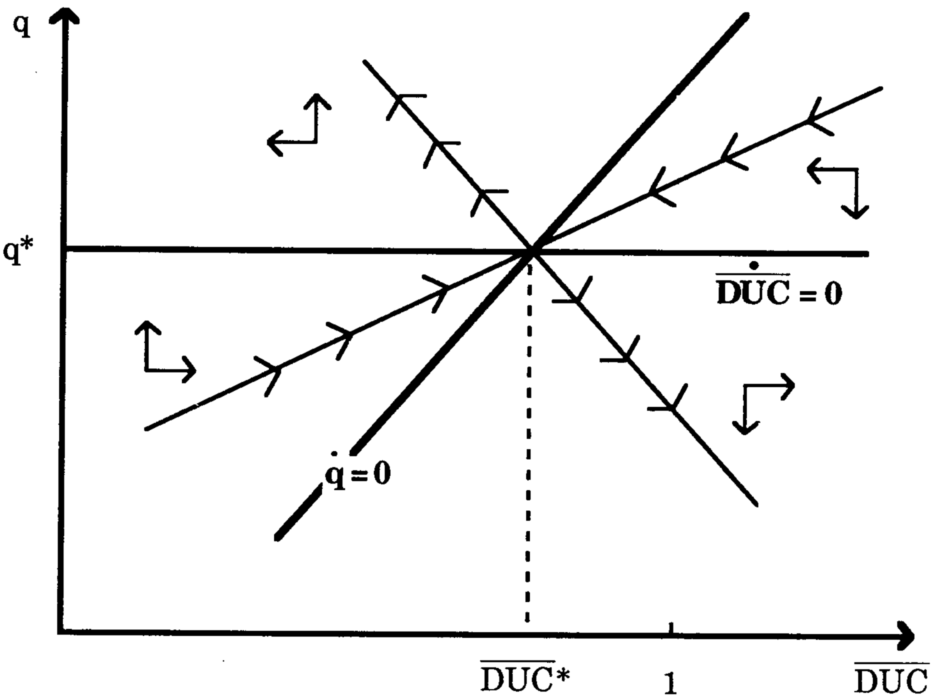

FIGURE 1

The Dynamics of DUC and q.

5 Dynamic Effects of Demand Shocks

In this last section we analyze the effects of changes in demand on the accumulation process. A word of warning is necessary. The model developed in this paper is a partial equilibrium model, which concentrates on the firm's investment behavior. The labor supply is assumed infinitely elastic, allowing the firm to hire any amount of workers at the given wage rate. Moreover, the firm is assumed to be the representative one with its own price equal to the aggregate price level. Nevertheless, the behavior of consumer-shareholder agents is supposed exogeneous. It eliminates all feed-backs through demand, which is consequently taken as an exogenous variable. [11]

5.1. Permanent Shocks in Aggregate Demand

A permanent demand shock is represented by an increase in the aggregate demand $\overline{YD}$, in equation (1). Let us assume that the economy is initially at steady state. The steady state values for q and $\overline{DUC}$ do not depend on $\overline{YD}$, which implies that the steady state of the model is not modified. However, because initial production capacities are given, an increase in aggregate demand raises the contemporaneous degree of capacity utilization. Furthermore, marginal q rises as a consequence of the effect of increased demand on future profits.

Figure 2 shows the dynamic effects of a permanent change in aggregate demand. At time 0 demand increases, raising $\overline{DUC}$ from the steady state value to $\overline{DUC}_0$. At the same time marginal q jumps as a consequence of increased profitability from q^* to q_0. They later come back to the steady state equilibrium A along the convergent path. During the whole process, the investment rate is greater than the depreciation rate, driving the capital stock to a higher steady state level.

Capacities increase in response to an increase in aggregate demand, but they rise slowly as a consequence of the adjustement costs. In the long run capacities increase in the same proportion as demand, moving aggregate DUC back to its initial level.

As stated in Section 4.2, the short-run elasticity of aggregate production to aggregate demand is smaller than one, and it is negatively related to DUC. This implies that the initial effect of an increase in aggregate demand has a partial effect on aggregate output. But as a result of the accumulation process, capacities increase in the same porportion as demand, yielding a long-term demand elasticity of one: increased aggregate demand is totally reflected in aggregate production. The model behaves like an accelerator model in steady state, and in this sense it is purely Keynesian in the long run.

11. ABEL and BLANCHARD [1983] analyze the behavior of competitive markets under adjustement costs in a general equilibrium model.

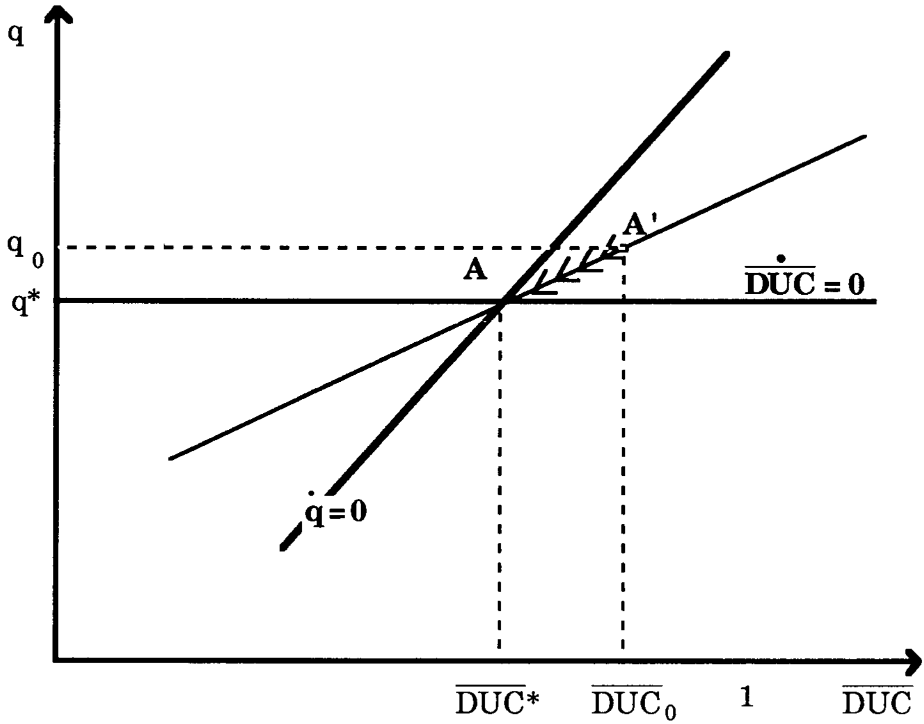

FIGURE 2

Aggregate Demand Shock.

5.2. Permanent Shocks to the Demand Price Elasticity

Another type of demand shock comes from the parameters of the demand function, *i.e.*, the price elasticity parameter ε. It is a pure change in preferences. Contrary to aggregate demand, the price elasticity influences the steady state value of DUC and has no incidence on contemporaneous DUC, since the relative price is always one. (Remember that all firms are identical.)

When ε decreases firm monopoly power increases, allowing the firm to increase prices and profitability. Because all firms are identical, aggregate price also increases without any negative effect on firm demand. Under these improved conditions the firms try to avoid, more than in the past, excess demand. For this reason they are willing, in an optimal sense, to accumulate more capital – and thus to run a higher risk of *ex post* capacity underutilization – in order to reduce the *ex ante* probability of an excess demand.

Assume that in the initial period the economy is at steady state and a positive demand elasticity shock, *i.e.*, a permanent decrease in ε, occurs. The steady state equilibrium shifts from A to B in Figure 3. Because initial capacities are given and the demand shock does not modify contemporaneous demand, aggregate DUC stays unchanged. On the other hand, because the firm increases prices and profitability, the good news increases the marginal value of the firm. During the adjustement process the firms invest, raising the capital stock and reducing the degree of capacity utilization to the new steady state level.

Figure 3 represents the dynamic effects of a permanent drop in the demand elasticity ε. At the initial point capacities are utilized at a rate of DUC* and marginal q is at the steady state level, inhibiting the capital stock to grow. Since the firms know that ε was increased, their marginal value jumps from point A to A′. The capital stock starts growing until marginal q comes back to q^*. The new steady state is B, with an aggregate value for the degree of capacity utilization of DUC**. At the end of the process, for a given level of aggregate demand a permanent decrease in ε has produced an increase in the long-run values of both capital stock and production.

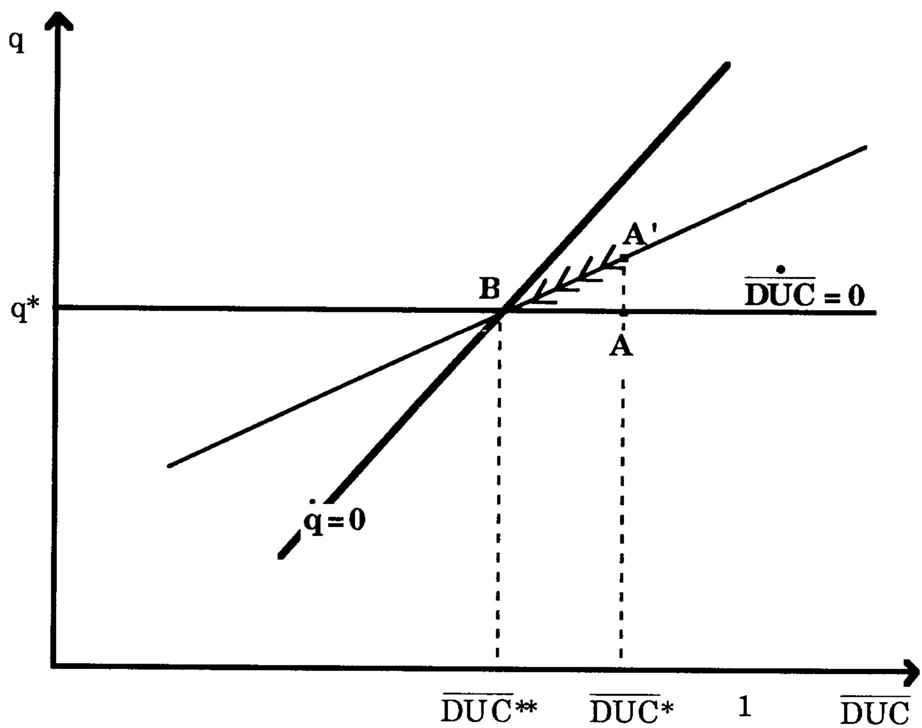

FIGURE 3

Demand Price Elasticity Shock.

Both types of permanent demand shocks have positive effects on the accumulation process for two different reasons. Aggregate demand calls for additional capacities in order to avoid the undesired increase in the expected utilization of capacities. A reduction in the price elasticity increases monopoly power, the markup rate and pure profits, making it more profitable to decrease expected DUC at steady state in order to reduce the probabilitiy of an excess demand.

6 Conclusion

This paper stresses the role of demand uncertainty in a world where firms face capacity constraints. Implicit in our assumptions is the notion that capacities are not necessary fully employed *ex post*, *i.e.*, after the realization of the stochastic component of demand. More importantly, the optimal behavior of the firm is associated with aggregate excess capacity.

Optimal behavior implies an investment ratio depending on marginal q, as in standard q models. Nevertheless, there are two differences. First, as shown by SNEESSENS [1987], in the determination of the markup rate the demand elasticity is weighted by an indicator of demand pressures, which depends inversely on the degree of capacity utilization. This implies that **prices are an increasing function of the degree of capacity utilization.** Second, the marginal effect of capital on expected production differs from the marginal productivity of capital, because demand uncertainty implies that any new unit of capital has a positive probability of nonutilization. For this reason **Marginal q is generally smaller than average q**.

When demand uncertainty is firm-specific and there is no uncertainty on aggregate demand, the firm's behavior leads the economy to produce less than total capacities. At steady state, the "modified golden rule" holds and it determines the steady-state degree of capacity utilization, which is necessarily below unity. It can be unitary only in two opposite cases, when the market is perflectly competitive or when it is monopolistic (since only specific demand uncertainty is allowed and aggregate demand is known, the monopolist faces no demand uncertainty at all).

As a direct consequence of demand uncertainty, permanent shocks on expected aggregate demand have a short-run elasticity to production which is smaller than one. This is a standard result of "aggregation over micro-markets" models, as shown by LAMBERT [1988]. Moreover, we show in this paper that **the long-run elasticity is equal to one.** This purely Keynesian long-run effect is related to two implicit assumptions: a constant return to scale technology and an infinitely elastic labor supply. Both assumptions imply that all increases in demand can be satisfied by an increase in the capital stock. This is because labor is assumed available in infinite quantities at the given wage rate, and because both factors can be increased in the same proportion without modifying marginal productivities in the long run. Another specification of the labor market could weaken this result even if the constant returns assumption is kept. To combine investment under demand constraints with wage bargaining can be an interesting research topic. *See* in particular ARNSPERGER and DE LA CROIX [1990 *a*] and [1990 *b*], where monopolistic competition is combined with wage bargaining.

Permanent shocks to the price elasticity of demand also have long-run effects on the accumulation process, but these effects follow a totally different propagation mechanism. A fall in the demand elasticity increases the markup rate, leading to an increase in capacities even if aggregate demand remains unchanged.

Finally, we think that the construction of a general equilibrium model of the ABEL and BLANCHARD [1983] type, adding demand uncertainty and factor complementarity, can illuminate the influence of permanent shocks on expected demand on the accumulation process. The paper by LAFFARGUE, MALGRANGE and PUJOL [1990] could be useful in this direction, since the authors solve a large general equilibrium model with monopolistic competition and adjustement costs.

APPENDIX 1

Average q and Marginal q

The proof for equation (16) is very similar to the one in HAYASHI [1982]. Let us first take the time derivative of the discounted marginal value for the capital stock,

$$\frac{d}{ds}(\mu_s K_s e^{-r(s-t)}) = (\dot{\mu}_s K_s + \mu_s \dot{K}_s - r \mu_s K_s) e^{-r(s-t)}.$$

Equations (9) and (15) define the laws of motion for K and μ, which implies

$$\frac{d}{ds}(\mu_s K_s e^{-r(s-t)}) e^{r(s-t)}$$
$$= (\{(r+\delta-\alpha_s)\mu_s - a_s B_s \Phi_{p,s} E_t(DUC_s) + P_s \Omega_s\} K_s + \mu_s \{I_s - \delta K_s\} - r \mu_s K_s).$$

After some obvious simplifications, we obtain

$$\frac{d}{ds}(\mu_s K_s e^{-r(s-t)}) e^{r(s-t)} = -a_s \Phi_{p,s} E_t(Y_s) + P_s \Omega_s K_s.$$

By integration between t and ∞, and taking into account the transversality condition, we get

$$\mu_t K_t = \int_t^{\infty} (a_s \Phi_{p,s} E_t(Y_s) - P_s \Omega_s K_s) e^{-r(s-t)} ds.$$

Finally, dividing both sides by PK and using the definition of marginal and average q, we obtain

$$(16) \qquad q_t = Q_t - \frac{1}{P_t K_t} \int_t^{\infty} \Phi_{d,s} a_s E_t(Y_s) e^{-r(s-t)} ds;$$

where

$$q_t = \frac{\mu_t}{P_t} \quad \text{and} \quad Q_t = \frac{V_t}{P_t K_t}. \quad \square$$

APPENDIX 2

The Steady State Conditions

The steady state conditions for the firm are given by $\dot{\mu}=\dot{K}=0$. The time derivatives of K and μ are

(15) $$\dot{\mu}=(r+\delta-\alpha)\mu-a\,B\,\Phi_p\,\overline{DUC}+P\,\Omega,$$

(9) $$\dot{K}=I-\delta K.$$

The time index has been suppressed to simplify the notation. As in the main text, $\overline{DUC}$ represents aggregate DUC and Φ_p the elasticity of aggregate production to agregate capacities. At steady state the aggregate price level P and the firm's own price p are assumed equal. Moreover, the optimality condition for prices, equation (13), can be written as

$$a=\frac{P}{\varepsilon\,\Phi_d}.$$

Dividing both sides of equation (15) by P, and introducing the optimal price condition and $\dot{\mu}=0$. we obtain

(15′) $$(r+\delta-\alpha)\,q+\Omega=\frac{B}{\varepsilon\,\Phi_d}\Phi_p\,\overline{DUC}.$$

Finally, given the optimality condition for investment, equation (11), and given that the steady state rate of investment is equal to the depreciation rate, equation (15′) becomes

(15″) $$r\,\Omega'(\delta)+\Omega(\delta)=\frac{B}{\varepsilon}\,\frac{\Phi_p\,\overline{DUC}}{\Phi_d}.$$

Taking into account the fact that $\Phi_p+\Phi_d=1$, and using the relation between Φ_p and E(DUC) given by equation (5), the steady state value for aggregate DUC is given by

(18) $$\frac{\overline{DUC}^{*\rho+1}}{1-\overline{DUC}^{*\rho}}=(r\,\Omega'(\delta)+\Omega(\delta))\frac{\varepsilon}{B}. \quad \square$$

The User Cost of Capital

For equation (15), and recalling that the marginal effect of capital on aggregate production is given by

$$\frac{\partial\overline{Y}}{\partial K}=B\,\Phi_p\,\overline{DUC},$$

we can deduce the user cost of capital, denoted by c, as

$$(r+\delta-\alpha)\,\mu+P\,\Omega-\dot{\mu}=c.$$

Equation (15) can then be written as

$$B\,\Phi_p\,\overline{DUC}=\frac{c}{a}.$$

The optimality condition for the firm states that the marginal effect of capital on production must be equal to the user cost of capital divided by the net price. At steady state the user cost of capital is $c^*=(r+\delta)\,P$.

APPENDIX 3

Dynamic Analysis

The dynamic conditions for the system are given by

(15) $$\dot{\mu} = (r + \delta - \alpha)\,\mu - a\,B\,\Phi_p\,\overline{DUC} + P\,\Omega,$$

(9) $$\dot{K} = I - \delta K.$$

The time index has been suppressed to simplify the notation. We can transform the dynamic conditions for K and μ into the dynamic conditions for q and aggregate DUC by using the definitions

$$\overline{DUC} = \frac{\overline{Y}}{BK} \qquad \text{and} \qquad q = \frac{\mu}{P}.$$

$\overline{Y}$ is aggregate production. Because it is equal to expected production, we can use equation (4) to write

(4′) $$\overline{Y} = [\overline{YD}^{-\rho} + YP^{-\rho}]^{-1/\rho}.$$

The derivatives with respect to time are

$$\dot{\overline{DUC}} = -\Phi_d \frac{\dot{K}}{K} \overline{DUC} \qquad \text{and} \qquad \frac{\dot{q}}{q} = \frac{\dot{\mu}}{\mu} - \frac{\dot{P}}{P}.$$

This result in

$$\dot{\overline{DUC}} = (\delta - \alpha)\,\Phi_d\,\overline{DUC}$$

and

$$\dot{q} = (r + \delta - \alpha - \hat{P})\,q - \frac{a\,B}{P}\Phi_p\,\overline{DUC} + \Omega.$$

Furthermore, it is assumed that the firm's price is equal to the aggregate price level, a usual condition in monopolistically competitive markets. Moreover, the optimality condition for prices, equation (13), can be rewritten as

$$a = \frac{P}{\varepsilon\,\Phi_d}.$$

By derivation of equation (13) with respect to time, the growth rate of prices is

$$\hat{P} = \frac{\rho\,\Phi_p\,\dot{\overline{DUC}}}{(\varepsilon\,\Phi_d - 1)\,\Phi_d\,\overline{DUC}}.$$

Under the optimality conditions given by equations (13) and (14), the dynamic system becomes

$$\dot{\overline{\text{DUC}}} = (\delta - \alpha)\,\Phi_d\,\overline{\text{DUC}} \tag{19}$$

and

$$\dot{q} = \left[r + (\delta - \alpha)\left(1 - \frac{\rho\,\Phi_p}{\varepsilon\,\Phi_d - 1}\right)\right] q - \frac{B\,\Phi_p}{\varepsilon\,\Phi_d}\overline{\text{DUC}} + \Omega(\alpha). \quad \square \tag{20}$$

The Phase Diagram

The slope of the $\dot{q}=0$ locus in ($\overline{\text{DUC}}$, q) space is obtained by differentiating the right-hand side of equation (20), under the optimal investment condition

$$q = \Omega'(\alpha).$$

Valuated at the steady state equilibrium, the slope of $\dot{q}=0$ takes the form

$$\left[r + \frac{\Omega'(\delta)}{\Omega''(\delta)}\,\frac{\rho\,\Phi_p}{\varepsilon\,\Phi_d - 1}\right] dq = \frac{B\,\Phi_p}{\varepsilon\,\Phi_d}\left(\frac{\rho}{\Phi_d} + 1\right) d\overline{\text{DUC}},$$

which is positive since $\varepsilon\,\Phi_d > 1$. The slope of the $\dot{\overline{\text{DUC}}}=0$ locus is given by the relation $\alpha=\delta$, which determines a constant value for $q=\Omega'(\delta)$. The cases $\overline{\text{DUC}}=0$ and $\overline{\text{DUC}}=1$ are axcluded. The first would imply infinite capacities. The second yields an indetermination in the price condition, equations (10) and (13), which implies that expected production must be zero.

Above the $\dot{\overline{\text{DUC}}}=0$ locus, from the properties of the $\Omega(\alpha)$ function, aggregate DUC is decreasing. Below, $\overline{\text{DUC}}$ is increasing. Moreover, in the neighborhood of the steady state, to the right of the $\dot{q}=0$ locus q is decreasing and increasing to the left. These results are confirmed by the analysis of the stability conditions of the linearized differential equations around the steady state.

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