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Rational Escalation

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ABSTRACT. – In many conflicts, protagonists commit resources that will not be returned. These situations, which often lead to apparently wasteful escalation, are well captured by the following "all-pay" auction. Two bidders bid repeatedly for a prize until one drops out. As usual the prize goes to the highest bidder but both bidders, the winner and the looser, pay their bids. Not only a process of escalation may be rational but it may be the only reasonable rational issue. We indeed prove that, if there is some uncertainty about the strength of the players, the only stable equilibrium may entail escalation. This result corroborates the idea that escalation is primarily a struggle to determine which player is the strongest one.

Rationalité des surenchères

RÉSUMÉ. – Dans de nombreux conflits – course aux armements, investissement en recherche et développement – les protagonistes se trouvent engagés dans un engrenage de dépenses de plus en plus grandes. De tels comportements, apparemment irrationnels, sont observés dans l'enchère suivante. Deux joueurs enchérissent successivement jusqu'à l'abandon de l'un d'entre eux. Le prix est attribué au plus fort enchérisseur mais aussi bien le perdant que le gagnant doivent payer leur enchère. Nous montrons qu'un comportement de surenchère pouvant conduire chaque joueur à dépenser plus que la valeur du prix est la seule issue « raisonnable » (dans le sens de Kohlberg-Mertens). Pour cela, il suffit que les joueurs soient incertains quant à la force de leur adversaire.

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1 Introduction

Escalation occurs when two or more agents become locked into a decision process resulting in spending more resources than what the outcome is worth whatsoever. Observation of everyday events provides many examples of such behaviour: International conflicts, arms races, investment processes, competitions for government contracts,... The emergence of escalation processes in real-life poses something of a puzzle, since it looks inconsistent with individual rationality. Some of the critical questions are: Which situations favor escalation? Can escalation be rational?

Game theory is of course an appropriate tool to get some insight on these difficult questions. They should be first adressed in models that capture the essential ingredients necessary for escalation but are simple enough to be analyzed. The dollar auction is one of them. It can be played in a classroom, usually to the great advantage of the auctioneer (Shubik [1971]). Players alternately bid for a prize, say ten dollars, given that all bids are lost and that the prize goes to the highest bidder. Often two students engage into an escalation process which can be understood as follows. A bidder always bids up over his opponent's last bid by less than ten dollars. So the difference between the current bids is less than ten dollars. Suppose it is five. The second highest bidder is tempted to bid up by six, hoping to get the prize. But then, his opponent is tempted to become again the highest bidder, by bidding up by two, and so on. Such an escalation process is hard to stop and each player, winner or not, ends up paying more than ten dollars.

The mechanism at work here is typical of escalation: A sequence of actions that look locally rational results in a globally irrational behaviour. If, from the very beginning, the players realized that, escalation should not occur. The game-theoretic analysis proposed by O'Neill [1986], and Leininger [1988] seems to show this point. In all equilibria in pure strategies, only one player bids and always wins the prize. Actually, escalation can be justified in this game because there are equilibria with escalation but in mixed strategies. However it is not clear that escalation should occur: Equilibria are numerous and no argument allows one to select a particular equilibrium.

The aim of this paper is to test the idea that escalation is primarily a struggle to determine which of the participants is the stronger. In the real world, strength comes from superior skill, or greater resources, and so on... In our sketchy model, strength is represented by a card, which may be high or low. More precisely we study a variant of the dollar auction where each of the two players receives privately a card, high or low, at the beginning of the auction. If all resources are exhausted, cards are shown and a player gets the prize only if his card is high and his opponent's card is low. The value of the prize is less than their resources so that if cards are shown, even the winner of the prize looses money.

We show that not only escalation may be rational in this auction but even sometimes it may be the only rational issue (in a sense to be precised later). Quite interesting are the two basic ingredients necessary for this result. First, the bids a player is allowed to make at each step is smaller than the value of the prize. It agrees with the intuition that a cycle of escalating commitment is more likely to occur when, at each step, a player hopes to cut his losses by a further commitment. Second, players are uncertain at the beginning of the game about their opponent's strength. So that bidding is the only way for players to signal their strength.

To understand how these two ingredients generate escalation, it is important to see that **rational** escalation requires some uncertainty. If a player's behaviour could be predicted for sure at some point, for example if it was known that he would never bid more than a given amount, then a backward argument shows that he should not bid at all. Therefore, when a player has bid once he is expected to bid again. By induction, when both players have bid at least once they have engaged themselves into an irreversible escalation process in which they exhaust their resources with positive probability.

This leaves only two possible kinds of equilibria: the **winning equilibria**, in which only one of the players bids and always wins the prize, or the **escalation equilibria**, in which escalation occurs with some probability. These equilibria are very different. Winning equilibria are based on threats: the auction stops at once because the looser does not bid, by the fear that the winner will overbid. On the contrary, in an escalation equilibrium, the auction may continue until exhaustion of the resources; furthermore information about the strength of the participants may be partially and gradually revealed.

Often both kinds of equilibria exist, and the question is to know which one is the more sensible. This is where the initial uncertainty about opponent's strength plays a role. Bidding allows a player to signal his strength and it can destabilize the winning equilibria. Indeed the looser is not expected to bid at all. By bidding he may try to signal that he is strong. To be convincing his signal should not be ambiguous: The winner, after observing an unexpected bid, should arrive at the conclusion that the looser is indeed strong. This would induce him to drop out from the auction and destabilize the equilibrium. Such an argument is based on the winner's belief off the equilibrium path. Since this belief is not given by Bayes' rule, we need a theory of how it is formed or at least of which beliefs can be surely discarded as unplausible. Such a theory is provided by the concepts of forward induction and stability.

The paper is organized as follows. Section 2 illustrates, by means of examples, some properties of escalation. It is rational only if each opponent's behaviour is uncertain. If there is no exogenous uncertainty as in O'Neill's model, escalation can be understood only through the use of mixed strategies. But if there is exogenous uncertainty bids may convey information and, for that reason, escalation may be the only stable rational issue. It is shown by using the logic of forward induction as in Cho and Kreps [1987]. Section 3 sets up the model. Section 4 proves some properties of rational escalation, in particular the irreversibility of escalation once both players have started to bid. Section 5 describes the equilibria of the auction and studies their stability. In particular it is shown that, for some values of the initial assessments, an equilibrium with escalation exists and is the unique stable equilibrium. A direct forward induction argument is difficult to carry out when the game has many steps. This is why we use the less intuitive concept of stable equilibrium since, as it is well known now, it captures some features of forward induction (KOHLBERG and MERTENS [1986], VAN DAMME [1989]). It turns out to be very performant here and to select the more plausible issue. Finally we discuss some related work.

2 Examples

2.1. Example 1: The Dollar Auction

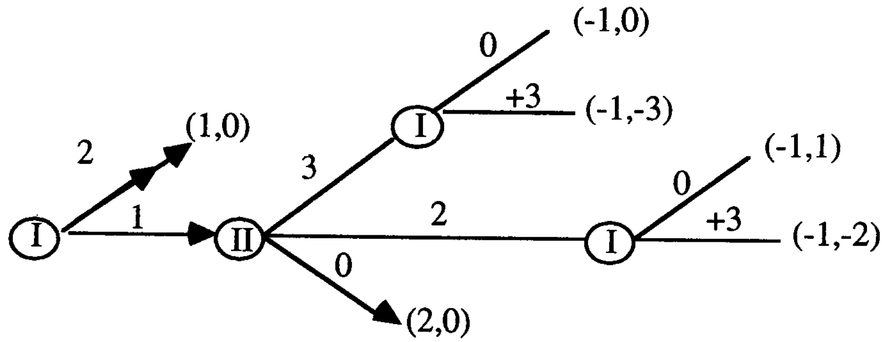
We consider the dollar auction game as modelled by O'Neill. The auction is as follows: Two players bid alternately for S dollars. Each one has a budget of B dollars, $S < B$, and is free to choose the number of dollars he bids. Bids are lost and a player should bid up by at least one dollar over the other's last bid.

It is a game with perfect information and the usual backward procedure can be performed. It consists of eliminating the strictly dominated moves, starting from the terminal nodes and going backward through the tree. However here it does not single out a unique outcome because players are indifferent at some nodes between several actions. For instance, in the case of $S=3$ and $B=4$ the elimination of the strictly dominated moves leads to the extensive form depicted in figure 1.

At this point, depending on the way indifferences are broken, two perfect equilibrium outcomes in pure strategies are obtained. They are represented respectively by arrows and double arrows. Player I bids by one or two dollars and player II always drops out.

More generally, for any values of S and B, there are several perfect equilibria in pure strategies, but in all of them player I wins the prize and player II never bids. On this basis, O'Neill argues that escalation is not rational in this game. But his argument holds only if pure strategies are considered. Escalation cannot occur in an equilibrium in pure strategies: The winner of the prize is known for sure, and the looser should not bid at all. However, equilibria with escalation exist in mixed strategies. They are even numerous. To see this, remark that any randomization is rational at the final nodes because of the indifference of the players. Appropriate randomization values then make the player who plays the step before indifferent between his choices. And so on, so that indifference propagates through the game tree.

FIGURE 1



For example consider in figure 1 the situation where player I has bid 1 \$. If I drops out with probability 2/3 when II bids 2 \$, player II is indifferent between bidding 2 \$ or dropping out. Now, going backward, II may bid 2 \$ with probability less than 1/3 and dropping out with the complementary probability, thereby inciting I to initially bid 1 \$ and mixed equilibria are obtained. But II may also bid 2 \$ with probability exactly equal to 1/3, so as to make player I indifferent between initially bidding either 1 \$ or 2 \$ and other mixed equilibria are obtained.

Therefore there are lots of perfect equilibria and no refinement theory proposed so far allows one to select one of them.

2.2. Example 2

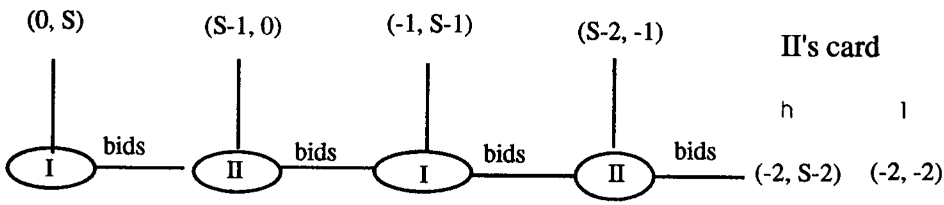
Our second example will be generalized in the rest of the paper. It illustrates the signalling role that bids may play in a game with imperfect information. This signalling role is used to pick up a particular equilibrium, namely the equilibrium with escalation. We consider the auction depicted in figure 2: Two players bid for S dollars, $1<S<2$. They both have the same budget of 2 dollars. At the beginning of the game player II receives privately a card, high (*h*) or low (*l*). Then they alternately either bid by one dollar or drop out. Player I starts and all bids are lost. If a player drops out, the auction is over and the prize is awarded to the opponent. Otherwise, both players bid twice, and II's card is shown. Player II gets the prize if his card is high. Otherwise the prize is lost.

We shall see that there are three kinds of Nash equilibria in this game:

– the **I-winning equilibria** where player I bids and player II drops out wathever his card is;

– the **II-winning equilibria** where player I drops out;

FIGURE 2



— an **equilibrium with escalation** where both players spend their two dollars with positive probability (recall that the value of the prize S is smaller than 2). Contrary to the other equilibria, this one conveys some information: Player II is more likely to bid with a high card rather than with a low one. An important parameter of this equilibrium is the value $1-1/S$. Suppose that a player expects his opponent to drop out with probability $1/S$ and overbid with probability $1-1/S$. Then he expects to recover immediately exactly his bid. In the sequel we set $\pi=1-1/S$. The equilibrium exists if the initial probability of II having a high card is small enough, more precisely if $q<\pi^2$. It can be described as follows.

First bids:

Player I bids for sure;

Player II overbids for sure if his card is high. If his card is low, he randomizes: he overbids with the probability β^l defined by $q/(q+(1-q)\beta^l)=\pi$. Therefore, after observing II's bid, I's assessment about II having a high card is updated exactly to π.

Second bids:

Player I bids his second unit with probability π;

Player II overbids for sure if his card is high and drops out for sure if his card is low.

These strategies are indeed in equilibrium. At his first bid II builds his "reputation" of having a high card. This requires him to drop out sufficiently often when his card is low. As a consequence, player I expects an immediate strictly positive gain by entering into the auction because he expects II to overbid with probability $(q+(1-q)\beta^l)=q/\pi$, which by assumption is smaller than π. Therefore it is optimal for him to bid first. At his second bid he expects II to overbid him with probability π, which makes him indifferent between bidding and dropping out. II's strategy is also optimal: with a high card he expects a strictly positive payoff by bidding at both steps; with a low card he expects a null payoff so that he is indifferent between bidding and dropping out.

We claim that the equilibrium with escalation is the only reasonable one if $q<\pi^2$. For example, we show how to eliminate the I-winning equilibria. They are supported by threats of overbidding from Player I in case of an unexpected bid of II: I's probability of overbidding II's bid should be high enough to refrain II from bidding. More precisely, suppose that I threatens to overbid with probability λ. Then player II expects by bidding his first unit:

$$(1-\lambda)(S-1)-\lambda \text{ if his card is low}$$

(because he optimally drops out at last step)

$$(1-\lambda)(S-1)+\lambda(S-2) \text{ if his card is high}$$

(because he optimally bids at last step).

II is refrained from bidding if his expectations are negative. This requires that the threat λ is at least equal to $S-1$. And for such threats, II surely strictly looses with a low card but may be tempted to bid with a high one (when λ is just equal to $S-1$). Thus, if I ever observes a bid, he should conclude that II's card is high and should not overbid at all. Player II knows that and the equilibria are destabilized.

The argument here is usually called a **forward induction** argument. If II ever bids, player I tries to find a rational explanation to this bid, instead of thinking it is a mistake. He looks forward to seeing who surely looses by bidding, and he concludes that II's card must be high. Such an introspection leads to nothing in example 1 because information is complete. It is the very fact that bids may convey information which allows here to select one equilibrium by forward induction.

It is interesting to understand why the concepts of perfection, and properness do not capture this signalling role. In these approaches unexpected moves are perceived as transmission errors and players' strategies are required to be robust against such errors. Therefore if I ever observes a bid from II, he gives no meaning to that bid: He has no reason to believe that II is more likely to "tremble" with a high card rather with a low one. He may even think the contrary, in which case it is rational for him to overbid.

In section 5, the argument is generalized to the case where uncertainty bears on both players and where they can bid more than twice. The above forward induction argument is difficult to carry out when the game has many steps. Therefore, we use the concept of stable equilibrium. Roughly speaking, I-winning equilibria are unstable if by slightly perturbing players' strategies in some direction no equilibrium is close to the I-winning equilibria. As was pointed out by Van Damme, perturbing the strategies and not only the actions allows to correlate the moves off the equilibrium path. This explains why an unexpected bid may be interpreted and becomes informative.

3 The Model

3.1. The Rules of the Auction

Two players bid for S dollars; $S>1$. They both have the same budget equal to B. At the beginning of the game each player receives privately a card, high (h) or low (l). Let p (resp. q) the probability of player I (resp. II) drawing a high card. Then they alternately either bid by one dollar or drop out. Player I starts to play. All bids are lost. Whenever a player drops out, the auction is over and the prize is awarded to his opponent. Otherwise, both players bid **B** times until exhausting their budget. In that case, they show their cards and receive payoffs according to table 1 (bids are excluded).

TABLE 1

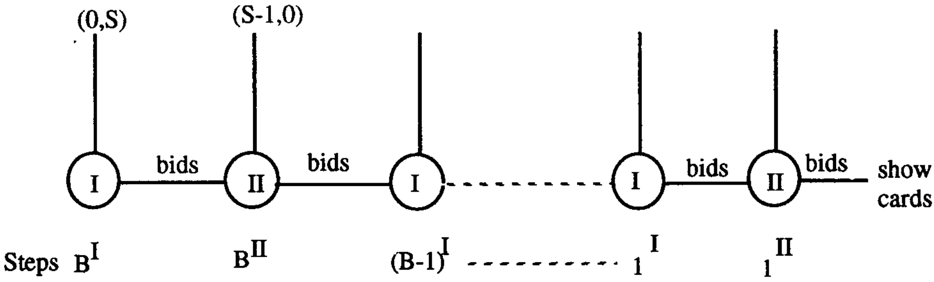
I \ *II*	h	l
h	$\varepsilon S, \varepsilon S$	$S, 0$
l	$0, S$	$0, 0$

We shall assume $0<\varepsilon S<1$. The strict positivity of ε is taken for technical convenience and does not alter the spirit of the results. The assumption $\varepsilon S<1$ is more important: It captures extremely competitive situations where there can be at most one winner. Even if a player's card is high, it is not clear that he should bid even one dollar. The drawings of the cards are independent. The information is *perfect* if cards are known by both players at the start, namely if each assessment p, q is equal to 0 or 1. Otherwise the information is *imperfect*. The auction is depicted in figure 3: Player I starts and acts at steps B^{I}, $(B-1)^{I}, \ldots, 1^{I}$, player II acts at steps B^{II}, $(B-1)^{II}, \ldots, 1^{II}$.

3.2. Notation and Definitions

A **behavioral strategy** for a player specifies his probability of bidding, depending on his card, at any step he may be called to play. We denote by $\alpha=(\alpha_b^c)_{b=1,\ldots,B;\, c=h,l}$ a player I's strategy where α_b^c represents his probability of bidding at step b^{I} if his card is c. Similarly $\beta=(\beta_b^c)$ represents a II's strategy.

FIGURE 3



Suppose that players follow the strategies α and β. As the auction continues, their **probability assessments** about each other card evolve according to Bayes' rule. We denote by q_b the probability assessment by I that II's card is high when II's resources are exactly equal to b; and similarly p_b for II's assessment about I's card.

If there is a positive probability to reach any step of the auction, all these assessments are well defined. For the same reason, the payoff that a player expects if the auction starts at a given step and players move according to the strategies is also well defined: Let v_b^c the payoff that I expects at step b^{I} if his card is c (bids that have already been already made are excluded). At step b^{I}, he expects II to overbid with probability μ_b equal to:

$$\mu_b = (1 - q_b)\,\beta_b^l + q_b\,\beta_b^h.$$

Therefore, if he bids, he expects an **immediate net gain** equal to $(1-\mu_b)\mathrm{S}-1$, which can also be written as $(\pi-\mu_b)\mathrm{S}$ where $\pi = 1 - 1/\mathrm{S}$ as in example 2. (Remark that π is between 0 and 1). Since I expects to reach step $(b-1)^{\mathrm{I}}$ with probability μ_b his payoffs satisfy the recursive formula:

$$(1)\qquad \begin{cases} v_b^c = \alpha_b^c\,((\pi-\mu_b)\,\mathrm{S} + \mu_b\,v_{b-1}^c) & b>1 \\ v_0^l = 0,\ v_0^h = (1 - q_0 - q_0\,\varepsilon)\,\mathrm{S} \end{cases}$$

Similarly, II's expectation at step b^{II} about I's overbidding is equal to:

$$\lambda_{b-1} = (1 - p_{b-1})\,\alpha_{b-1}^l + p_{b-1}\,\alpha_{b-1}^h.$$

and II's expected payoff w_b^c satisfy:

$$(2)\qquad \begin{cases} w_b^c = \beta_b^c\,((\pi-\lambda_{b-1})\,\mathrm{S} + \lambda_{b-1}\,w_{b-1}^c), & b>2 \\ w_1^l = -\beta_1^l,\ w_1^h = \beta_1^h\,(\pi - p_0\,(1-\varepsilon))\,\mathrm{S} \end{cases}$$

Remark that w_{B}^c is II's expected payoff at step B^{II}. At the beginning of the game, he expects $(1-\lambda_{\mathrm{B}})\,\mathrm{S} + \lambda_{\mathrm{B}}\,w_{\mathrm{B}}^c$.

Before starting any formal analysis remark that "winning" equilibria always exist. In such an equilibrium the winner always gets the prize at once because his opponent never enters into the auction. The winner threatens his opponent to overbid with a high enough probability if ever he bids. Since the value of the prize is smaller than the budget, it is worthless for the opponent to ever bid. However these equilibria do not always seem reasonable. For example if player II is *a priori* in a strong position, *i.e.* if p is small and q is large, player I should not always win. This explains why we now study the other equilibria and carry out a stability analysis. We first start by some properties on rational escalation. Apart from their onw interest, they will be used to build equilibria with escalation

4 Some Properties of Rational Escalation

In this section, we assume that players act rationally in the usual Nash sense. That is, we consider Bayesian Nash equilibria. Roughly speaking, at each step where a player is called to bid if the strategies are followed, he uses an action that is optimal against the anticipated future moves given by the strategies. We prove here two important properties of "rational" escalation. They imply that escalation necessarily entails some uncertainty in each opponent's behaviour. As the proofs show, they are valid in more general auctions than the one we consider here: It suffices that the bid a player is allowed to make at each step is strictly smaller than the value of the prize.

4.1. At the Start of the Escalation Process

PROPOSITION 1: In an equilibrium, whenever a player bids, his opponent expects him to bid again with positive probability if his budget is not exhausted.

Proof: Suppose that some step b^i is reached ($b > 1$) and that player i bids. If his opponent expects him to drop out for sure at step $(b-1)^i$, he anticipates to get the prize at a cost of one unit, whatever his card is. Thus the rationally bids up. But then, by bidding at step b^i, player i is sure to loose his bid against nothing: He should not bid at all. □

Suppose that a player bids. Proposition 1 does not say that he will bid at next step with positive probability, but that his opponent expects him to bid with positive probability. When information is imperfect it makes a difference: a player may bid with a low card knowing that he will not bid again but his opponent does not know it if he does not know his opponent's card.

By induction, whenever both players have bid once, a process of escalation has started. This allows one to classify the equilibria: Either one of the players never bids or they both exhaust all their budget with positive probability. The result does not depend on the specific rules used at the end of the auction. This proves the following corollary.

COROLLARY 2: There are only three kinds of Nash equilibrium:

– the I-winning equilibria where I bids and II drops out, whatever their cards are;

– the II-winning equilibria where I never bids, whatever his card is;

– the equilibria with escalation where both players are expected *ex ante* to exhaust all their budget with positive probability.

Apart from classifying the equilibria, corollary 2 is very useful to study those with escalation: Since all steps are reached, the probability that a player assesses about his opponent's card, and the payoff he expects if he bids are defined at all steps.

4.2. Along the Escalation Process

We show here that once the escalation process has started there is not much freedom in player's expected behaviour: Each one drops out in such a way that this opponent's immediate expected gain is zero when he bids. It is a usual feature when players randomize and can be understood as follows. In order that a player continues to bid with some probability until the end of the auction, it must be that his opponent drops out with some probability. Therefore bidding once more should not give either player a positive gain (Formal proofs are given in the appendix).

PROPOSITION 3: In an equilibrium with escalation, no player is expected to overbid for sure, *i.e.*:

$$\lambda_b < 1 \text{ for } b = B-1, \ldots, 1 \qquad \text{and} \qquad \mu_b < 1 \text{ for } b = B, \ldots, 1.$$

Consider a player, say I, who bids at step b^I knowing that his opponent surely bids up. He expects an immediate loss of 1. So he must expect to recover at least his bid from the auction starting at step $(b-1)^I$. Thus at step $(b-1)^I$ he expects a strictly positive payoff and he bids for sure. But now, the same argument works for player II since at step b^{II}, he expects player I to overbid for sure. Repeating the argument, all players are expected to bid for sure until exhausting their resources. We get the contradiction: Player II bids his last unit only if his card is high. Knowing that, player I expects $\varepsilon S - 1$ at step 1^I by bidding. Since we assumed $\varepsilon S < 1$, he should not bid at all contradicting $\lambda_1 = 1$. □

COROLLARY 4: In an equilibrium with escalation, a player who bids expects to recover exactly his bid at the next step, if any, except possibly at the first step:

$$\lambda_b = \mu_b = \pi, \qquad b = B-1, \ldots, 1.$$
$$0 < \mu_B \leqslant \pi \quad \text{and} \quad \lambda_B = 1 \quad \text{if} \quad \mu_B < \pi$$

Consider a player who is still in the auction at step b. If he expects to recover immediately strictly more than his bid, he rationally bids for sure. It contradicts proposition 3 except if it is player I at the first step of the auction. On the contrary, suppose he expects an immediate loss by bidding. When he bids, he must expect a strictly positive payoff from the auction starting at step $b-1$. So he bids for sure at step $b-1$ which contradicts again proposition 3. □

Therefore players can expect a positive gain only at the very beginning or at the end of the auction. These results are again fairly general: Proposition 3 and the statement of corollary 4 (but not the specific values for λ_b and μ_b) hold whenever it is not optimal for both players to simultaneously bid for sure their last unit.

Corollary 4 has an important consequence about the evolution of the "reputation" of the players: Their reputation of having a high card cannot increase too much along the auction after the first bids: By Bayes' rule $p_{b-1} = \alpha_b^h p_b / \lambda_b$, so that $p_{b-1} \leqslant p_b / \lambda_b$ with an equality if player I bids for sure with a high card. By a repeating argument assessments always satisfy the following constraints: $p_{b-1} \lambda_b \lambda_{b+1} \ldots \lambda_B \leqslant p_B = p$. Corollary 5 is then easily obtained from corollary 4:

COROLLARY 5: In an equilibrium with escalation, reputations satisfy:

$$p_0 \pi^{B-1} \lambda_B \leqslant p \quad \text{and}$$

$p_0 \pi^{B-1} \lambda_B = p$ if player I bids for sure with a high card,

$$q_1 \pi^{B-2} \mu_B \leqslant q \quad \text{and}$$

$q_1 \pi^{B-2} \mu_B = q$ if player II for sure with a high card.

5 Equilibria with Escalation

An equilibrium with escalation may not exist. Consider for instance the auction where it is known at the start that II's card is low. When the bidders have only one unit left, player II drops out for sure. So by corollary 2, escalation equilibria do not exist. Theorem 6 states the conditions on the initial uncertainty under which escalation is rational.

5.1. Existence

THEOREM 6: An equilibrium with escalation exists

$$\text{either if } p < \pi^B/(1-\varepsilon) \quad \text{and} \quad 0 < q < \pi^B \text{ (case 1)}$$
$$\text{or if } p \leqslant \pi^B/(1-\varepsilon) \quad \text{and} \quad q \geqslant \pi^B \text{ (case 2).}$$

In case 1, the equilibrium is unique.

To find an equilibrium with escalation, a backward induction argument can be used since, by corollary 2, all steps are reached. The construction is laborious and is given in the appendix. It is technically similar in spirit to KREPS and WILSON [1982] or CHATTERJEE and SAMUELSON [1987]. The basic point to understand is that rational escalation is primarily a battle on reputations. But on this respect, player II has the disadvantage of being the last player to spend his last unit. Indeed II bids rationally his last unit only under two conditions: his card is high and his assessment about I's card being high is low enough. But in an escalation equilibrium I expects him to bid his last unit with probability equal to π. Therefore **it must be that at the last step I's reputation is not too high and II's reputation is not too bad.**

However all along the auction the evolutions of the players' reputation are constrained. If I's initial reputation is too good, his final one will be too high and an equilibrium with escalation cannot exist: This explains the condition on p. As for II, two cases occur depending on his initial reputation.

Case 1: II's initial reputation is not good, more precisely $0 < q < \pi^B$.

As in example 2, II needs to increase his reputation at the very first bid, by dropping out with a high enough probability when his card is low. But, since there are good chances that his card is indeed low, player I expects him to drop out with a high probability so that he initially bids for sure. The equilibrium is unique and can be described as follows:

(i) both players always bid when their cards are high;

(ii) with a low card, player I bids for sure at step B^I; thereafter he randomizes so that he is expected to bid with a total probability equal to π;

(iii) with a low card, player II always randomizes: At step B^{II}, so that I's assessment q_{B-1} after observing his bid is exactly equal to π^{B-1}; thereafter, so that he is expected to bid with a total probability equal to π.

At the very beginning of the game, I's reputation does not increase and II's reputation makes a jump: $p_{B-1} = p$ and $q_{B-1} = \pi^{B-1}$. Then, as long as the auction continues, the assessments increase linearly: $p_{b-1} = p_b/\pi$ and $q_{b-1} = q_b/\pi$ for $b = 1, \ldots, B-1$. Player I gets his payoff at the beginning of the auction. On the contrary, player II, if his card is high, has to wait until the end to take advantage of bad I's reputation.

Case 2: II's initial reputation is not bad, more precisely $q > \pi^B$.

This time, II's reputation is high enough and player I builds his reputation at the very first bid by dropping out with some probability when his card is low. Thereafter an equilibrium is very similar to case 1. Details are given in the appendix.

5.2. Stable Equilibria

We know from above that the three kinds of equilibria, with either I winning or II winning or with escalation all exist if I's initial reputation is not too good; otherwise only both winning equilibria exist. The next theorem shows that the stable equilibria are the more plausible one; If I is not in a too good position an escalation process determines the winner; otherwise I wins.

THEOREM 7: The stable equilibria are:

the equilibria with escalation if $(1-\varepsilon)p<\pi^{B}$,

the I-winning equilibria if $(1-\varepsilon)p>\pi^{B}$.

The proof is given in the appendix. To destabilize for example the I-winning equilibria when I's initial reputation is not good, we have to find small perturbations of the strategies which incite players to deviate largely from them. Perturb the II's strategies such that, with a small probability, II plays until the end and far more often with a high card rather than with a low one. Then player I cannot deter II from bidding. Because, if he can, II's bids come from the perturbed strategy only. Player I should therefore stop after observing a bid but then II should always bid. So there is an internal inconsistency in assuming that player II never bids. This implies that II bids with some probability. Then one can show that an equilibrium of the perturbed game is an equilibrium with escalation which is far from any I-winning equilibrium if I's initial reputation is small enough.

Remark that the I-winning equilibria are sequential and perfect. To be sequential, it must be that, if II ever bids, he is "rationally' punished. Namely, he gets less than 1 dollar in a sequential equilibrium starting at step $(B-1)^{I}$, whatever his card is. Since II is not expected to bid, player I may assess any probability that II has a high card if II bids. In particular he may believe that II's card is surely low and it is rational for him to bid. The concept of stable equilibria precisely prevents such unplausible beliefs.

5.3. Discussion with Related Work

Our game can be interpreted as a war of attrition, for example as a model of exit in a duopoly market. Firms compete for a natural monopoly market. Whenever both are active they loose money. Information bears on private costs. It is interesting to compare our result with FUDENBERG and TIROLE'S [1986]. Their model is technically different: Bids are placed in a continuous way, horizon is infinite, and players' types take an infinite number of values. They prove that there exists a unique Bayesian equilibrium, which is similar to our escalation equilibrium. Escalation allows firms to demonstrate their power: The lower the cost of a firm, the longer it intends to stay in the market. But their uniqueness result is driven by a different argument than ours. They assume that for some low enough costs, a firm makes money even as a duopoly. For these costs, bidding at all steps is a strictly dominant strategy so that in all equilibria, surely each firm bids with a positive probability, and by the escalation principle no winning equilibria exist. In contrast, our uniqueness result is driven by the signalling effect of a bid.

Our model is also technically close to the bargaining model of Chatterjee and Samuelson where players alternate offers until one of the player accepts it. Accepting the opponent's offer corresponds to dropping out in our auction. But bargaining models are different since reaching an agreement is always profitable for both parties and more profitable than disagreeing forever. The main problem is therefore the date of the agreement.

Proof of Proposition 3

Suppose for example that player I has card c and is still in the auction at step b^{I}. In an equilibrium he chooses α_b^c so as to maximize his expected payoff, which is given by (1): $v_b^c = \alpha_b^c\{(\pi - \mu_b)S + \mu_b v_{b-1}^c\}$. Therefore:

$$\alpha_b^c > 0 \Rightarrow v_b^c = (\pi - \mu_b)S + \mu_b v_{b-1}^c$$

(3) and

$$v_b^c > 0 \Rightarrow \alpha_b^c = 1$$

So now suppose that player I bids at step b^{I} knowing that his opponent surely bids up so that $\mu_b = 1$. The following implications hold

$$\alpha_b^c > 0 \Rightarrow v_b^c = (\pi - 1)S + v_{b-1}^c \geqslant 0 \Rightarrow v_{b-1}^c > 0 \Rightarrow \alpha_{b-1}^c = 1.$$

This means that $\alpha_b^c > 0$ implies $\alpha_{b-1}^c = 1$ so that λ_{b-1} is equal to 1. But now, the same argument works for player II since at step b^{II}, he expects player I to overbid for sure. Thus μ_{b-1} is equal to 1. Repeating the argument, all players are expected to bid for sure until exhausting their resources. Since player II optimally bids his last unit only if his card is high player I expects $\varepsilon S - 1$ at step 1^{I} by bidding. Since we assumed $\varepsilon S < 1$, he should not bid at all contradicting $\lambda_1 = 1$. □

Proof of Corollary 4

Suppose for example that player I has card c and that step b^{I} is reached. From (3):

– If $\mu_b < \pi$, whatever his card is, he bids for sure (since $v_{b-1}^c \geqslant 0$). This is impossible from proposition 2 except if it is the first step of the auction.

– If $\mu_b > \pi$, if he bids, it must be that $v_{b-1}^c > 0$. As in the proof of proposition 2, it implies that he surely bids at step $(b-1)^{\mathrm{I}}$, which again gives a contradiction. □

Proof of Theorem 6

In an equilibrium with escalation, all steps are reached so that a backward induction argument can be used. We first prove in lemma 8 that final assessments p_0 and q_1 are constrained. Since the behaviour along the auction is also constrained, the behaviour at the first bids is fixed so as to satisfy the final requirements on the assessments.

LEMMA 8: In an equilibrium with escalation,

$$\text{either } p_0 < \pi/(1-\varepsilon) \quad \text{and} \quad q_1 = \pi \text{ (case 1)}$$
$$\text{or } p_0 = \pi/(1-\varepsilon) \quad \text{and} \quad q_1 \geqslant \pi \text{ (case 2).}$$

Proof: We know from corollary 4, that μ_1 is equal to π. But, from (2), II surely drops out at step $1''$ if his card is low. Thus $\mu_1 = \beta_1^h q_1 = \pi$. This implies $q_1 \geqslant \pi$ and $\beta_1^h > 0$. Using that β_1^h maximizes $\beta_1^h(\pi - p_0(1-\varepsilon))S$ over [0, 1] we get that $p_0 \leqslant \pi/(1-\varepsilon)$. Moreover, if $p_0 < \pi/(1-\varepsilon)$ then $\beta_1^h = 1$ so that $\mu_1 = q_1 = \pi$. □

Intuitively one would expect in an equilibrium with escalation that players always bid with a high card. To check that, remark that player I expects after the first bid

$$\gamma\varepsilon S \text{ with a high card and 0 with a low one}$$

where γ is the probability that cards are shown. Similarly player II expects:

$$\gamma(\pi - p_0(1-\varepsilon))S \text{ with a high card and 0 with a low one.}$$

Therefore player I expects all along the auction a strictly positive gain if his card is high so that he surely bids at all steps. As for player II it is only true in case 1. It implies at once that an equilibrium with escalation may exist only if $p \leqslant \pi^B/(1-\varepsilon)$. Indeed corollary 5 gives: $p = \lambda_B \pi^{B-1} p_0$ and using that $p_0 \leqslant \pi/(1-\varepsilon)$ implies $p \leqslant \pi^B/(1-\varepsilon)$. Now the nature of the equilibrium depends on q.

Case 1: It occurs if and only if $p < \pi^B/(1-\varepsilon)$ and $0 < q \leqslant \pi^B$.

– *Necessary conditions:* We just saw that in case 1 both players surely bid at all steps with a high card. Using also $q_1 = \pi$, corollary 5 gives:

$$p = \lambda_B \pi^{B-1} p_0 \quad \text{and} \quad q = \mu_B \pi^{B-1}$$

Now $p_0 < \pi/(1-\varepsilon)$ implies $p < \pi^B/(1-\varepsilon)$. As for q, recall that $0 < \mu_B \leqslant \pi$ (corollary 4). Since $\mu_B = q/\pi^{B-1}$ we get $0 < q \leqslant \pi^B$.

– *Sufficient conditions:* Assume that the necessary conditions hold we show that an equilibrium exists and that it is moreover unique if $0 < q < \pi^B$. It should satisfy:

(i) both players surely bid at all steps when their cards are high:

(4) $$\alpha_b^h = 1, b = B, \ldots, 1$$

and

(5) $$\beta_b^h = 1, b = B, \ldots, 1$$

(ii) except at first bids, their probability of bidding with a low card are determined by corollary 4:

(6) $$\lambda_b = (1-p_b)\alpha_b^l + p_b = \pi, b = 1, \ldots, B-1$$

and

(7) $$\mu_b=(1-q_b)\,\beta_b^l+q_b=\pi, b=1, \ldots, B-1$$

where assessments are given by: $p_b=p_0\,\pi^b$, $q_b=\pi^b$, $b=1, \ldots, B-1$.

(iii) at first steps we know that II bids so as to build his reputation:

(8) $$\mu_B=(1-q)\,\beta_B^l+q=q/\pi^{B-1}$$

Finally I always bids whatever his card if $\mu_B<\pi$ so that

(9) $$\lambda_B=1 \quad \text{if} \quad q<\pi^B$$

If $p<\pi^B/(1-\varepsilon)$ and $q\leqslant\pi^B$ strategies (α, β) which satisfy (4) to (9) exist. One easily verifies that they constitute an equilibrium. If moreover $q<\pi^B$ they are unique.

Case 2: It occurs if and only if either $p=\pi^B/(1-\varepsilon)$ or $p<\pi^B/(1-\varepsilon)$ and $\pi^B\leqslant q$.

– *Necessary conditions:* The argument is similar except that, as we above saw, player II expects now a null payoff after the first bid whatever his card. Thus he is indifferent between bidding and dropping out so that corollary 5 gives now:

(10) $$p=\lambda_B\,\pi^{B-1}\,p_0=\lambda_B\,\pi^B/(1-\varepsilon) \qquad \text{and} \qquad q\geqslant\mu_B\,\pi^{B-1}$$

By the first equation $\lambda_B=p(1-\varepsilon)/\pi^B$ so that $p\leqslant\pi^B/(1-\varepsilon)$. Moreover if $p<\pi^B/(1-\varepsilon)$ then $\lambda_B<1$. By corollary 4 this requires that $\mu_B=\pi$. Therefore by the second inequality of (10) $q\geqslant\mu_B\,\pi^{B-1}=\pi^B$.

– *Sufficient conditions:* By a limiting argument it suffices to prove that an equilibrium exists in the non degenerate case where $p<\pi^B/(1-\varepsilon)$ and $\pi^B\leqslant q$. Equilibria are no more unique since there is some freedom in II's behaviour as long as the final assessment q_1 is higher than π. The conditions on his behaviour (5) and (7) are replaced by

(11) $$(1-q_b)\,\beta_b^l+q_b\,\beta_b^h=\pi, \qquad q_b\geqslant\pi^b, \qquad b=1, \ldots, B$$

As for player I his strategy should still satisfy (4) and (6) along the auction and his first bid is now determined by (10). Now, one easily checks that strategies that satisfy (4), (6), (10), and (11) exist under the conditions on p and q and constitute an equilibrium. Equilibria can be described as follows:

(i) player I always bids when his card is high;

(ii) with a low card, player I always randomizes: At step B′ so that II's assessment p_{B-1} after observing his bid is exactly equal to $\pi^B/(1-\varepsilon)$; thereafter, so that he is expected to bid with a probability equal to π;

(iii) player II is expected to bid at all steps with a probability equal to π. He may drop out with a high card as long as the assessments still satisfy $q_b\geqslant\pi^b$.

Payoffs are:

for player I, 0 if his card is low and $\pi^B \varepsilon S$ if his card is high;

for player II, $(1-p(1-\varepsilon)\pi^{-B})S$ whatever his card is.

Stable Equilibria

Stability is studied by slightly perturbing the strategies of the players and looking at the equilibria of the perturbed game. More precisely one chooses a completely mixed strategy for each player, $\hat{\alpha}$ for player I and $\hat{\beta}$ for player II, and perturb the initial game as follows: If player I chooses a strategy α, it is replaced by $\alpha^* = \delta\hat{\alpha} + (1-\delta)\alpha$, meaning that α is played only with probability $(1-\delta)$ and $\hat{\alpha}$ is played with probability δ. And similarly a player II's strategy β is perturbed into β^*. We call the strategies choosen by the players the "real" strategies.

Real strategies (α, β) form an equilibrium of this perturbed game if α is a best response to β^* and β is a best response to α^*. The I-winning equilibria are unstable if, for some strategies, $\hat{\alpha}$, $\hat{\beta}$ all equilibria of the perturbed game are far from them whenever δ is small enough. And similarly for the other types of equilibria.

We choose a completely mixed strategy $\hat{\alpha}$ which generates posterior beliefs and bidding probabilities such that for any $b < B$:

$$\hat{p}_b > \pi^b \qquad \text{and} \qquad \hat{\lambda}_b > \pi \tag{12}$$

when the initial assessment is p_B. Such a strategy exists because (12) is not required at step B so that player I can build his reputation at that step. For example one can choose $\hat{\alpha}_b^l = \gamma$ and $\hat{\alpha}_b^h = 1-\gamma$ for γ positive and small enough. Roughly speaking $\hat{\alpha}$ is close to a revealing strategy because I bids far more often with a high card than with a low one. Moreover it is a "tough" strategy since with a high card I bids with a high probability until the end. Therefore if II knows that $\hat{\alpha}$ is played and if he observes a bid he should stop. Consider a similar strategy $\hat{\beta}$ for player II. We denote by p^* and q^* the beliefs generated by the perturbed strategies.

The proof of theorem 7 can be sketched as follows. We first show that players start to bid (lemma 9). Then, since each player chooses to bid once with a positive probability, we expect an escalation process to start. The proof is a little more complicated than for the original game. Crucial variables are the probabilities assessed by the players that their opponent's strategy is real. Indeed whenever this assessment is too small the player should stop. But we prove that, for δ small enough, this assessment remains high for at least one of the player as long as the real players continue to bid with some probability. It implies that any equilibrium of the perturbed game is "nearly" an escalation equilibrium, meaning that both players bid with positive probability at all steps except possibly at the very last one (lemma 11). Finally we prove that if II stops at the last step the equilibrium is close to a I-winning equilibrium. If II does not stop, the equilibrium is an escalation one. Each case appears depending on p being greater or smaller than $\pi^B/(1-\varepsilon)$.

Formally, let s^i be the "stopping step", that is the first step, if any, where the auction surely stops if the real strategies are played (*i.e.* for example if $i=\mathrm{I}$ player I surely stops when his remaining budget is $s:\lambda_s=0$, but no player surely stops before: $\lambda_b>0$ and $\mu_b>0$ for any $b>s$).

LEMMA 9: In an equilibrium of the perturbed game, no player surely drops out: $\lambda_B>0$ and $\mu_B>0$.

Proof: Suppose, by contradiction, $\lambda_B=0$. After observing I's first bid, player II knows that strategy $\hat{\alpha}$ will be played if the auction continues and his assessment about I's card is equal to $\hat{p}_{B-1}$. He optimally stops so that $\mu_B=0$. But player I expects II to overbid with probability μ_B^*, which is equal to $(1-\delta)\mu_B+\delta\hat{\mu}_B$. Since $\mu_B=0$, this probability is smaller than δ so that I should bid: this contradicts $\lambda_B=0$. An analogous argument shows that $\mu_B=0$ is also impossible. □

LEMMA 10: In an equilibrium of the perturbed game, as long as a player bids, he expects a null immediate gain, except maybe player I at first step. So that:

$$\text{(13)} \qquad \mu_B^* \leqslant \pi \quad \text{and} \quad \lambda_B=1 \quad \text{if } \mu_B^*<\pi$$

$$\text{(14)} \qquad \begin{cases} \lambda_b^*=\pi \quad \text{and} \quad \mu_b^*=\pi \quad \text{for } b=B-1, \ldots s, \\ \lambda_{s-1}^*=\pi \quad \text{if } i=\mathrm{II} \end{cases}$$

Proof: It suffices to adapt proposition 3 and corollary 4 to the perturbed game. □

We denote by P_b (resp. Q_b) the probability assessed by II (resp. I) that the real strategy α (resp. β) is being played when step b^{I} (resp. b^{II}) of the auction is reached. Of course $P_B=Q_B=1-\delta$.

LEMMA 11: In an equilibrium of the perturbed game, assessments about the opponent's real strategy satisfy:

$$(1-P_b)\leqslant\delta/(\pi^{B-b-1}\lambda_B^*) \quad \text{and} \quad (1-Q_b)\leqslant\delta/(\pi^{B-b-1}\mu_B^*)$$

until the stopping step.

Proof: The assessment about the strategy $\hat{\alpha}$ being played evolves according to $1-P_b=(1-P_{b+1})\hat{\lambda}_{b+1}/\lambda_{b+1}^*$ so that $(1-P_b)\leqslant(1-P_{b+1})/\lambda_{b+1}^*$.

From (14), a repeating argument yields: $(1-P_b)\leqslant\delta/(\pi^{B-b-1}\lambda_B^*)$ until step s^i. And similarly for Q_b. □

LEMMA 12: Consider an equilibrium of the perturbed game for δ small enough. Either the stopping step does not exist or it is the last step of the game, *i.e.* $s^i=1^{\mathrm{II}}$.

Proof: Suppose that the stopping step exists. From (13) at least one of the player is expected to bid with a probability higher than π at first step: $\lambda_B^*\geqslant\pi$ or $\mu_B^*\geqslant\pi$. Suppose w.l.o.g. it is player I. Then his opponent assesses a high probability that he is playing his real strategy until the stopping step. Indeed if δ is small enough surely from lemma 11.

$$\text{(15)} \quad P_b>\pi \quad \text{for all} \quad b\geqslant s \quad \text{if} \quad i=\mathrm{I} \quad \text{and for all} \quad b\geqslant s-1 \quad \text{if} \quad i=\mathrm{II}$$

We claim that I is not the first to stop (*i.e.* he is not player i). Suppose by contradiction he is. At step s^i one has $\lambda_s^* = (1-P_s)\hat{\lambda}_s + P_s\lambda_s$. Using (15) and $\lambda_s = 0$ gives $\lambda_s^* < \pi$, which contradicts lemma 10.

So it must be player II who stops. Suppose by contradiction that s^{II} is not the end of the auction (*i.e.* $s > 1$). Player I knows after observing a bid from player II at step s^{II} that he now faces the strategy $\hat{\beta}$ and he optimally stops. So that $\lambda_{s-1} = 0$. But by (15) this implies $\lambda_{s-1}^* < \pi$ and again we get a contradiction: player II should not have stopped at step s^{II} since he could get an immediate strictly positive gain. This proves that the stopping step, if any, is the last step of the game. □

Proof of Theorem 7

Consider an equilibrium of the perturbed game for δ small enough. From lemma 12 two cases may occur: either player II stops for sure at the last step or does not.

- **Case a:** player II stops for sure at the last step. Then surely $p \geqslant \pi^B/(1-\varepsilon)$ and the equilibrium is close to a I-winning equilibrium.

We first show that player II gives up often at his first step. By definition: $\mu_1^* = (1-Q_1)\hat{\mu}_1 + Q_1\mu_1$. But $\mu_1^* = \pi$ and $\mu_1 = 0$ so that $\pi = (1-Q_1)\hat{\mu}_1$. Therefore by lemma 11:

$$\pi \leqslant (1-Q_1) \leqslant \delta/(\pi^{B-2}\mu_B^*)$$

Hence μ_B^* tends to zero with δ. This implies that I always bids at first step: $\lambda_B = 1$ and that II gives up with a high probability at his first step since μ_B also tends to zero with δ. The equilibrium is indeed close to a I-winning equilibrium.

By the usual argument we have: $p = p_B^* \geqslant \lambda_B^* \pi^{B-1} p_0^* \geqslant (1-\delta)\pi^{B-1} p_0^*$. But it must be that $p_0^* \geqslant \pi/(1-\varepsilon)$ otherwise II would not stop at last step. Therefore $p \geqslant (1-\delta)\,\pi^B/(1-\varepsilon)$.

- **Case b:** player II never stops for sure.

In that case both players are expected to play until the end. By adapting the argument used in the non perturbed game, it must be that:

$$\text{either } p_0^* < \pi/(1-\varepsilon) \quad \text{and} \quad q_1^* = \pi \text{ (case } b-1)$$
$$\text{or } p_0^* = \pi/(1-\varepsilon) \quad \text{and} \quad q_1^* \geqslant \pi \text{ (case } b-2).$$

By an argument very close to the proof of theorem 6 we show that this case occurs only if $p \leqslant \pi^B/(1-\varepsilon)$ and that the nature of the equilibrium depends on the position q with respect to π^B.

Case b occurs only if $p \leqslant \pi^B/(1-\varepsilon)$ whenever γ is small enough.

In an equilibrium player I always bids with a high card if he uses his real strategy and with probability greater than $1-\gamma$ if he uses $\hat{\alpha}$. Hence $\alpha_b^{*h} > 1-\gamma$ which implies $p(1-\gamma)^B \leqslant \lambda_B^* \pi^{B-1} p_0^* \leqslant \pi^B/(1-\varepsilon)$.

• **Case b − 1:** In that case $p_0^*(1-\varepsilon)<\pi$ so that II expects a strictly positive payoff if cards are shown and his card is high. Since, as usual, he expects a nonnegative immediate gain all along the auction he surely bids when his card is high. In particular $\beta_B^h=1$. This proves that the equilibrium is bounded away from the I-winning equilibria. Moreover the assessments satisfy: $q(1-\gamma)^B \leqslant \pi B-1\, \mu_B^* q_1^* \leqslant q$. But $\mu_B^* \leqslant \pi$ always holds and in case 1 $q_1^*=\pi$. This implies

$$q(1-\gamma)^B \leqslant \pi^B \quad \text{and if} \quad q<\pi^B \quad \text{then} \quad \mu_B^*<\pi.$$

So if $q<\pi^B$ player I bids at the beginning: $\lambda_B=1$. This proves that the equilibrium is bounded away from the II-winning equilibria.

• **Case b − 2:** In that case player I should reach the assessment $p_0^*=\pi/(1-\varepsilon)$. So surely $p>0$. Suppose moreover that $p \leqslant \pi^B/(1-\varepsilon)$ holds strictly. Since

$$p(1-\gamma)^B \leqslant \lambda_B^* \pi^{B-1} p_0^* \leqslant p$$

it implies

$$p(1-\gamma)^B (1-\varepsilon)/\pi^B \leqslant \lambda_B^* \leqslant p(1-\varepsilon)/\pi^B.$$

Thus, by the assumption on p, λ_B^* is bounded away from 1 and 0 and γ small enough. But λ_B^* and λ_B differ from at most δ so λ_B is also bounded away from 1 and 0 for δ and γ small enough. Therefore the equilibrium is bounded away from the I and II-winning equilibria.

Moreover it must be that $\mu_B^*=\pi$ (by (13)). As usual, $q \geqslant \pi B-1\, \mu_B^* q_1^*$ so that $q_1^* \geqslant \pi$ and $\mu_B^*=\pi$ imply $q \geqslant \pi^B$.

We can now end of the proof. If $p>\pi^B/(1-\varepsilon)$ only case a can occur so that the I-winning equilibria are the only stable equilibria. If $p<\pi^B/(1-\varepsilon)$ and $q<\pi^B$ only case $b-1$ can occur. If $p<\pi^B/(1-\varepsilon)$ and $q>\pi^B$ only case $b-2$ can occur for γ small enough. In both cases we proved that the equilibrium is bounded away from the I and II-winning equilibria: the escalation equilibria are the only stable equilibria. □

• References

Chatterjee, K., and Samuelson, W. F. (1987). – "Bargaining with Two-sided Incomplete Information", *Review of Economic Studies*, 54, pp. 175-179.

Cho, I. K., and Kreps, D. M. (1987). – "Signalling Games and Stable Equilibria", *Quarterly Journal of Economics*, CII, pp. 179-221.

Fudenberg, D., and Tirole, J. (1986). – "A Theory of Exit in a Duopoly", *Econometrica*, 54, pp. 943-960.

Kohlberg, E., and Mertens, J. F. (1986). – "On the Strategic Stability of Equilibria", *Econometrica*, 54, pp. 1003-1037.

Kreps, D., and Wilson, R. (1982). – "Reputation and Imperfect Information", *J. of Econ. Theory*, 27, pp. 253-279.

Leininger, W. (1989). – "Escalation and Cooperation in Conflict Situations – the Dollar Auction Revisited", *J. of Conflict Resolution*, 33, 2, pp. 231-254.

O'Neill, B. (1986). – "International Escalation and the Dollar Auction", *J. of Conflict Resolution*, 30, 1, pp. 33-50.

Selten, R. (1975). – "Re-Examination of the Perfectness Concept for Equilibrium Points in Extensive Games", *International Journal of Game Theory*, 4, pp. 25-55.

Shubik, M. (1971). – "The Dollar Auction Game: A Paradox in Non Cooperative Behavior and Escalation", *J. of Conflict Resolution*, 15, pp. 545-547.

Van Damme, E. (1989). – "Stable Equilibria and Forward Induction", *J. of Econ. Theory*, 48, pp. 476-496.