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Network Compatibility: Joint Adoption Versus Individual Decisions

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ABSTRACT. – In this paper we consider the problem of compatibility between networks when the consumers of a network service value the capacity of the network itself rather than the number of users. Compatibility decisions are modelled as the outcome of a three stages game where strategies include compatibility, capacity levels and prices. Two main questions are studied. First, will the result of this game differ from the outcome of a joint adoption procedure, where compatibility is adopted if and only if all the firms in the industry agree? Second, what are the effects of compatibility adoption on the network sizes? These questions are examined in a duopolistic framework where competition occurs between vertically differentiated products whose perceived qualities depend on their respective producers and on the competitor's network capacity when compatibility prevails.

Compatibilité de réseaux : décisions concertées ou décentralisation des choix ?

RÉSUMÉ. – Les externalités de réseau sont souvent modélisées en posant que la qualité d'un bien, perçue par l'utilisateur, est fonction du nombre de ses usagers. Dans de nombreux cas cependant c'est moins le nombre des usagers qui importe au client, que la taille du réseau. On examine dans cet article le problème du choix de la compatibilité entre réseaux par deux firmes qui mettent sur le marché des substituts différenciés verticalement, et qui se font concurrence en prix. On montre que le choix de la compatibilité est toujours une stratégie dominante pour chaque firme. On étudie aussi les conséquences du passage de réseaux incompatibles à des réseaux compatibles sur les tailles mêmes de ceux-ci. Ces effets dépendent de façon critique de paramètres de qualité et de coûts propres à chaque firme.

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1 Introduction

Compatibility appears as an important phenomenon in a lot of industries supplying goods as well as services. In fact the concept of compatibility has many facets and it is not sure that the numerous situations in which the problem of compatibility occurs, may receive a unified treatment. Let us give some examples. First, some goods are in fact complex systems which involve complementary components (*see* MATUTES and REGIBEAU [1987, 1988 and 1989]). The producers of these components may decide to make them compatible or to let them independent. Even if compatibility enlarges the variety of supplied goods, it may also strengthen the competition between the firms and as such it may not be adopted through independant actions of the market participants. Second, in other situations compatibility results in the adoption of some standard to which all the users must conform. The process of achievement of this standard is in general intricate and may involve either excess monentum or excess inertia (FARREL and SALONER [1985, 1986, 1987]). Third, many compatibility problems occur in industries where network externalities are present (KATZ and SHAPIRO [1985, 1986], FARREL and SHAPIRO [1988], JEANNERET [1990], PERROT [1991]. In this paper we investigate a problem related to this last category. Externalities we consider are associated to situations where a consumer of some network values directly the size of the network instead of the total number of users. For example, some airlines take advantage from a network connecting a large number of city pairs (LEVINE [1987]). Thus the quality of a service supplied appears as an increasing function of the network size. Other examples, such as credit card dispensers, automobile dealer networks, computerized reservation systems, ... illustrate the direct effect ot the capacity of a network, on the perceived quality attached to its products. In all such situations, the different networks supply substitute products which are generally mutually exclusive from the consumer point of view. When such network capacity externalities are present, the decision to achieve compatibility may result either from explicit coordination between producers or from independent choices. In this paper we investigate this last procedure, modelling compatibility decisions as the outcome of a three stages game where strategies include compatibility, capacity levels and prices. In a companion paper (ENCAOUA, MICHEL and MOREAUX [1991], EMM [1991] thereafter) we have analyzed the joint adoption procedure. Two main questions are examined. First, to what extent will the two procedures lead to different compatibility decisions? Second, may the outcome of one of the procedures be dominated by the outcome of the other? These questions are examined in a duopolistic framework where competition occurs between vertically differentiated products whose perceived qualities depend on their respective producers and on the competitor's network capacity when compatibility prevails.

In section 2 we present the framework. Section 3 is devoted to the analysis of price competition and the choices of capacity. The outcome of the game, concerning the compatibility decisions, is examined in section 4. Section 5 gives some insights on the effect of compatibility on the network sizes and section 6 concludes.

2 The Framework

In the partial duopoly model we consider, there are two brands, vertically differentiated according to their perceived qualities, which are sold to maximising surplus consumers. Each consumer buys at most one brand and purchases either one or zero unit of the good. The choice results from the maximization of the net surplus.

The market is made of a continuum of consumers uniformly distributed on the segment [0, 1] with density 1, differing by their willingness-to-pay for the perceived quality. For a consumer $\theta \in [0, 1]$, $u(\theta, h)$ is the gross surplus in terms of money generated by a good of perceived quality h. For analytical tractability we take the following specification, often used in the vertical differentiation litterature (JASKOLD-GABSCEWICZ and THISSE [1979], SHAKED and SUTTON [1982], CHAMPSAUR and ROCHET [1987]).

$$u(\theta, h) = \theta h$$

Hence the net surplus $S(\theta, h, p)$ obtained by a consumer of type θ, buying one unit of the good of perceived quality h at price p, is:

$$S(\theta, h, p) = \theta h - p \quad (1)$$

The vertically differentiated brands are supplied by two networks. The goods marketed by the two networks differ first by their "basic" qualities b_i, $i = 1, 2$, which are exogenously given. Basic quality of good 1 is taken as the highest one ($b_1 > b_2$). Besides this basic differentiation parameter the firms have the possibility to alter the perceived quality of their brands by increasing the size of their network. This is the essence of the externality capacity effect. Moreover by deciding to achieve compatibility, perceived quality can be related to the joint network size. As in ESSER and LERUTH [1988] we capture the externality and compatibility effects by the following expression linking the perceived quality of good i, denoted h_i, to the basic quality parameter b_i:

$$h_i = b_i [1 + r (k_i + \mu_i k_j)] \quad (2)$$

where:

r = externality rate parameter, $r \geq 0$

k_i = capacity or size of network i, $i = 1, 2$

μ_i = compatibility rate of the network i with the network j, $j \neq i$, $\mu_i \in \{0, 1\}$

The two compatibility rates μ_i and μ_j are chosen either independently by the two firms or according to a joint adoption procedure which corresponds to a standard accepted by all the participants in the industry.

In the first case (the decentralized procedure) four situations may arise depending on the non-coordinated decisions μ_i and μ_j:

$\mu_i = \mu_j = 1$ defines fully compatible networks

$\mu_i = 1$, $\mu_j = 0$, $j \neq i$ defines semi-compatible networks

$\mu_i = \mu_j = 0$ defines incompatible networks

Semi-compatible networks occur in situations where only one of the two firms chooses to increase its perceived quality by allowing its customers to benefit from the size of the joint network

In the second case (the joint adoption procedure) two situations only arise: $\mu_i = \mu_j = \mu \in \{0, 1\}$. In this case, $\mu = 1$ if and only if the two firms agree on compatibility. On the contrary when only one firm has a private incentive to compatibility, the two networks remain incompatible. So the joint adoption procedure corresponds to the peculiar specification of (2):

(2′) $$h_i = b_i[1 + r(k_i + \mu k_j)]$$

The good 1 with the highest basic quality index is supposed to have also the highest perceived quality:

(3) $$h_1 > h_2$$

One must be aware of the restrictive nature of this assumption. Indeed, the ranking of the two brands according to their perceived and basic qualities may differ. A lower basic quality brand could compensate its initial handicap by having a larger network size. However, note that this can happen only for a sufficiently high value of the externality rate r.

Three types of costs have to be considered. First, there are production costs. We assume that the variable unit costs are constant and the same for the two firms. So whithout loss of generality, we take the variable cost of production to be equal to zero. Seconds costs of capacity expansion differ between the two firms. We assume constant unit costs of capacity and we will denote by c_i the unit capacity cost of firm i, $i = 1, 2$. Last, there are costs of achieving compatibility. We assume that variable costs of compatibility are negligible. This assumption is appropriate inasmuch as compatibility involves only negotiation costs between the two firms or any other fixed cost and no other scale dependant compatibility cost.

In order to avoid no trade situations, the basic quality index is supposed to satisfy:

(4) $$c_i < b_i, \qquad i = 1, 2$$

The decentralized choice procedure where the perceived quality is given by (2), is modelled as a three stage game. At each stage both firms move simultaneously and then they observe the decision of the other one before the beginning of the following stage. At stage one the firms take their compatibility decisions μ_1 and μ_2, at stage two they choose the capacity of their networks k_1 and k_2 and at stage three they compete in prices p_1 and p_2. We are looking for subgame perfect equilibria, so we proceed by backward induction.

The joint adoption procedure involves only two stages corresponding to the choices of capacities (stage one) and prices (stage two). At the subgame perfect equilibrium of the game, the profit functions depend on the compatibility parameter μ. The compatibility results from the rate of variation of the profit functions according to μ (*see* EMM [1991]).

3 Price Competition and Choices of Capacity

Suppose that the decisions on the capacities and the compatibility parameters have been taken. Then, the perceived qualities h_i, $i=1, 2$, are determined. In the situation where the two brands have positive market shares, the demand functions D_i, $i=1, 2$, are given by:

(5.1) $$D_1(p_1, p_2)=1-\frac{p_1-p_2}{h_1-h_2}$$

(5.2) $$D_2(p_1, p_2)=\frac{p_1-p_2}{h_1-h_2}-\frac{p_2}{h_2}$$

Thus the sales, x_i, $i=1, 2$, amount to:

(6) $$x_i(p_1, p_2)=\min\{D_i(p_1, p_2), k_i\}, \qquad i=1, 2$$

and profits are equal to:

(7) $$\pi_i(p_1, p_2)=p_i x_i(p_1, p_2)-c_i k_i, \qquad i=1, 2$$

We denote by $p_i^*(h_1, h_2)$, $i=1, 2$, the equilibrium prices.

Clearly in this model it is impossible to have excess demand at equilibrium. Indeed, if $D_i(p_1^*, p_2^*)>k_i$, firm i could increase its price and continue to sell k_i obtaining higher profits. So for any pair (h_1, h_2) the equilibrium prices are the equilibrium strategies of a game the payoff functions of which are:

$$\bar{\pi}_i(p_1, p_2)=p_i D_i(p_1, p_2)$$

Hence the equilibrium prices are:

(8.1) $$p_1^*(h_1, h_2)=\frac{2h_1(h_1-h_2)}{4h_1-h_2}$$

(8.2) $$p_2^*(h_1, h_2) = \frac{h_2(h_1 - h_2)}{4h_1 - h_2}$$

leading to the corresponding demands:

(9.1) $$D_1^*(h_1, h_2) = \frac{2h_1}{4h_1 - h_2}$$

(9.2) $$D_2^*(h_1, h_2) = \frac{h_1}{4h_1 - h_2}$$

For any perceived qualities h_1 and h_2, the demand for good 1 is twice the demand for good 2.

We now state the first result showing that for r sufficiently small the equilibrium prices involve no excess capacity.

LEMMA 1: There exists $\bar{r} > 0$ such that for all $r < \bar{r}$ and all μ_i's, equilibrium prices satisfy:

$$D_i(p_1^*, p_2^*) = k_i, \qquad i = 1, 2$$

Proof: Since we have shown that no excess demand equilibrium can occur, it remains to demonstrate that the case $D_i(p_1^*, p_2^*) < k_i$ is impossible. For $r = 0$ the profit functions π_i are independent of the capacity levels:

$$p_1^* D_1(p_1^*, p_2^*) = \frac{4b_1^2(b_1 - b_2)}{(4b_1 - b_2)^2}$$

$$p_2^* D_2(p_1^*, p_2^*) = \frac{b_1 b_2(b_1 - b_2)}{(4b_1 - b_2)^2}$$

The derivatives of these profit functions according to the corresponding capacity levels are thus null.
For $(k_1, k_2) \in (0, 1)^2$ and $(\mu_1, \mu_2) \in \{0, 1\}^2$ there exists $\bar{r} > 0$ such that for $r \in [0, \bar{r})$:

$$\frac{\partial(p_i^* D_i(p_1^*, p_2^*))}{\partial k_i} < c_i, \qquad i = 1, 2$$

Hence:

$$r \in [0, \bar{r}) \Rightarrow \frac{\partial(p_i^* D_i(p_1^*, p_2^*) - c_i k_i)}{\partial k_i} < 0$$

which implies that for any strictly positive capacity, each demand (at equilibrium prices) exactly balances the corresponding capacity when r is sufficiently low. □

The intuition of the result is clear: when r is small, the benefit obtained by increasing capacity and improving the perceived quality is smaller than the cost of capacity expansion.

In the remaining part of the paper we will confine ouselves to situations where no excess capacity prevails.

By equating demand and capacities one can get the equilibrium prices as functions of the capacities:

$$(10.1) \qquad p_1^*(k_1, k_2) = h_1(1-k_1) - h_2 k_2$$

$$(10.2) \qquad p_2^*(k_1, k_2) = h_2(1-k_1-k_2)$$

where h_1 et h_2 are given by (2). The corresponding profits are equal to:

$$(11) \qquad \pi_i(k_1, k_2) = [p_i^*(k_1, k_2) - c_i]\, k_i, \qquad i=1, 2$$

It is easy to show that for low values of the externality rate r, $\pi_i(k_1, k_2)$ is a concave function of k_i (*see* EMM (1991)). However, note that these profit functions are polynomials of degree three in k_1 and k_2, so that the first order conditions for the capacity equilibrium are of order two in k_1 and k_2, which implies analytically untractable solutions. By restricting attention to low values of the externality rate, one can make a Taylor expansion of the profit functions around $r=0$ and obtain the equilibrium capacities of the linearized profits. To validate this Taylor expansion one has to show the existence and unicity of the equilibrium capacities for $r=0$.

Let us introduce the following parameters:

$$(12) \qquad s_i = 1 - \frac{c_i}{b_i}, \qquad i=1, 2. \qquad t = \frac{b_2}{b_1}$$

Parameter s_i is an index, with range (0, 1), of the quality/cost ratio for the good i, where quality refers to the basic one. Parameter t is the basic qualities ratio: $t \in (0, 1)$.

LEMMA 2: For $r=0$ the equilibrium of the second stage of the game exists and is unique. The two equilibrium capacities k_1^0 and k_2^0 are positive iff:

$$s \equiv \frac{s_2}{s_1} \in \left(\frac{1}{2}, \frac{2}{t}\right)$$

Proof: The proof is straightforward. The equilibrium capacities are unique and given by:

$$(13.1) \qquad k_1^0 = \begin{cases} \dfrac{s_1}{2} & \text{if} \quad s \leqslant \dfrac{1}{2} \\ \dfrac{2s_1 - ts_2}{4-t} & \text{if} \quad s \in \left(\dfrac{1}{2}, \dfrac{2}{t}\right) \\ 0 & \text{if} \quad s \geqslant \dfrac{2}{t} \end{cases}$$

$$(13.2)\qquad k_2^0=\begin{cases} 0 & \text{if} \quad s\leqslant \frac{1}{2} \\ \frac{2s_2-s_1}{4-t} & \text{if} \quad s\in\left(\frac{1}{2},\frac{2}{t}\right) \\ \frac{s_2}{2} & \text{if} \quad s\geqslant \frac{2}{t} \end{cases}$$

When $s\in\left(\frac{1}{2},\frac{2}{t}\right)$ the equilibrium capacities k_i^0, $i=1, 2$ are thus unique and strictly positive. □

The corresponding profits have the following expressions:

$$(14)\qquad \pi_i^0=b_i(k_i^0)^2, \qquad i=1, 2$$

Consider the first order Taylor expansion of the equilibrium capacities and profits around $r=0$:

$$(15)\qquad \tilde{k}_i(r, \mu_1, \mu_2)\simeq k_i^0+rv_i(\mu_1, \mu_2)$$

$$(16)\qquad \tilde{\pi}_i(r, \mu_1, \mu_2)\simeq \pi_i^0+r\,\sigma_i(\mu_1, \mu_2)$$

After some tedious computations one can obtain the first order derivatives which appear in (15) and (16):

$$(17.1)\quad v_1(\mu_1, \mu_2)=\frac{1}{4-t}[4k_1^0-2tk_2^0-6(k_1^0)^2+t(k_2^0)^2+2tk_1^0k_2^0 + \mu_1[2k_2^0(1-2k_1^0)]+\mu_2[tk_1^0(k_1^0-2k_2^0-1)]]$$

$$(17.2)\quad v_2(\mu_1, \mu_2)=\frac{1}{4-t}[4k_2^0-2k_1^0+(t-6)(k_1^0)^2-4k_1^0k_2^0 - \mu_1[k_2^0(1-2k_1^0)]+\mu_2[2k_1^0(1-k_1^0-k_2^0(2-t))]]$$

$$(18.1)\quad \sigma_1(\mu_1, \mu_2)=b_1k_1^0[-tv_2(\mu_1, \mu_2)+k_1^0(1-k_2^0)-t(k_2^0)^2 + \mu_1k_2^0(1-k_1^0)-\mu_2tk_1^0k_2^0]$$

$$(18.2)\quad \sigma_2(\mu_1, \mu_2)=b_2k_2^0[-v_1(\mu_1, \mu_2)+k_2^0-(k_2^0)^2-k_1^0k_2^0 + \mu_2k_1^0(1-k_1^0-k_2^0)]$$

The expressions (15) and (16), where $v_i(\mu_1, \mu_2)$ and $\sigma_i(\mu_i, \mu_j)$ have been replaced by their values given by (17) and (18) are thus the linear approximations of the equilibrium capacities and profits.

Note that $v_i(\mu_1, \mu_2)$ and $\sigma_i(\mu_1, \mu_2)$ are linear in μ_1 and μ_2. The effect of the compatibility decision μ_i on $\sigma_i(\mu_1, \mu_2)$ appears as the sum of:

– a direct effect of the compatibility decision on $\sigma_i(\mu_i, \mu_j)$ for given capacities k_i and k_j. This direct effect measures the marginal profit $\frac{\partial\pi_i}{\partial\mu_i}$

which has the same sign as $\frac{\partial \sigma_i}{\partial \mu_i}$

$$(19.1) \qquad \frac{\partial \sigma_1}{\partial \mu_1} = b_1 k_1^0 k_2^0 (1 - k_1^0)$$

$$(19.2) \qquad \frac{\partial \sigma_2}{\partial \mu_2} = b_2 k_1^0 k_2^0 (1 - k_1^0 - k_2^0)$$

– a strategic effect of the compatibility decision on $\sigma_i(\mu_i, \mu_j)$ via the influence of μ_i on the capacity k_j. This strategic effect measures the marginal profit $\frac{\partial \pi_i}{\partial k_j} \frac{\partial k_j}{\partial \mu_i}$ which has the same sign as $\frac{\partial \pi_i}{\partial k_j} \frac{\partial v_j}{\partial \mu_i}$. According to (17.1) and (17.2) the expressions of $\frac{\partial v_j}{\partial \mu_i}$ are:

$$(20.1) \qquad \frac{\partial v_2}{\partial \mu_1} = \frac{1}{4-t} k_2^0 [2 k_1^0 - 1]$$

$$(20.2) \qquad \frac{\partial v_1}{\partial \mu_2} = \frac{1}{4-t} t k_1^0 [k_1^0 - 2 k_1^0 - 1]$$

These identifications of the direct and strategic effects allow us to analyze the first stage equilibrium corresponding to the compatibility decisions.

4 Compatibility Decisions at Equilibrium

The linearized profits around $r=0$ given by (16) are the payoffs of a non-cooperative game in μ_1 and μ_2. The two following results state that $\mu_1 = 1$ and $\mu_2 = 1$ are dominant strategies.

| LEMMA 3: Choosing $\mu_1 = 1$ is a dominant strategy for firm 1.

Proof: First, the direct effect of firm 1's compatibility on its own profit is always positive. This results directly from inspection of (19.1). The reason is clear. For given capacities, by choosing $\mu_1 = 1$ instead of $\mu_1 = 0$, firm 1 increases the perceived quality of its brand. Since it is the highest quality producer, its profit increases with its quality (*see* SHAKED and SUTTON [1982]). Second, the strategic effect of firm 1's compatibility decision, $\mu_1 = 1$, has the same sign as the product $\frac{\partial \pi_1}{\partial k_2} \frac{\partial v_2}{\partial \mu_1}$. The first term $\frac{\partial \pi_1}{\partial k_2}$ is negative since the two products are substitutes. Indeed, by identification, one obtains:

$$\frac{\partial \pi_1}{\partial k_2} = -b_1 t k_1^0 < 0$$

The second term $\frac{\partial v_2}{\partial \mu_1}$ measures the effect of firm 1's compatibility decision on its rival capacity. According to (20.1), we have the following characterization:

$$k_1^0 < \frac{1}{2} \Leftrightarrow \frac{\partial v_2}{\partial \mu_1} < 0$$

Hence, the sign of the strategic effect is as follows: if $k_1^0 < \frac{1}{2}$ the strategic effect is positive and if $k_1^0 > \frac{1}{2}$ the strategic effect is negative. So, when k_1^0 is lower than half of the potential capacity of the market, the sum of the direct and strategic effects is positive. In this case $\mu_1 = 0$ is dominated by $\mu_1 = 1$. When $k_1^0 > \frac{1}{2}$ a trade-off occurs between the positive direct effect and the negative strategic effect. However, we show that the direct effect outweighs the strategic effect, giving rise to a positive total effect. The total effect has the same sign as:

$$(1 - k_1^0) + \frac{t(1 - 2k_1^0)}{4 - t}$$

This sum is positive iff:

$$k_1^0 < \frac{4}{4 + t}$$

Substituting for k_1^0 by its value $\frac{2s_1 - ts_2}{4 - t}$ (*see* (13.1)) we obtain:

$$k_1^0 < \frac{4}{4 + t} \Leftrightarrow s_1 < \frac{2(4 - t)}{4 + t} + \frac{ts_2}{2}$$

Let us denote $\frac{2(4 - t)}{4 + t} + \frac{ts_2}{2}$ by $\varphi(t, s_2)$. We have:

$$\varphi(0, s_2) = 2, \qquad \varphi(1, s_2) = \frac{6}{5} + \frac{s_2}{2} > 1$$

and $\frac{\partial \varphi(t, s_2)}{\partial t} < 0$.

So when t varies from 0 to 1, $\varphi(t, s_2)$ decreases from 2 to a value which is higher than 1. As $s_1 \in (0, 1)$, then $\varphi(t, s_2) > s_1$. □

A similar result holds for firm 2, the lowest quality brand producer.

Lemma 4: Choosing $\mu_2 = 1$ is a dominant strategy for firm 2.

Proof: The same proof as for lemma 3 applies. Note, however, that for firm 2 the direct effect of its compatibility decision on its own profits and the strategic effect have always the same signs which are positive. The direct effect has the sign of $1 - k_1^0 - k_2^0$. The strategic effect has the sign of $2k_2^0 + 1 - s_2$ which is positive since $s_2 < 1$. □

It is important to note the fundamental asymetry between the two firms concerning the strategic effects of compatibility. The lowest quality producer always benefits from compatibility with the highest quality network while the highest quality producer takes a strategic advantage by deciding compatibility only if its basic equilibrium capacity is not too high (k_1^0 below half of the market).

Theorem 5: $\mu_1 = \mu_2 = 1$ is the only Nash equilibrium of the game corresponding to the decentralized procedure.

Proof: It is a direct consequence of the two preceeding lemmas. □

It is interesting to compare this result with the configuration of compatibility decisions resulting from the joint adoption procedure (*see* EMM [1991]). Recall that under this procedure, compatibility is adopted if and only if it is profitable for both the firms. Some insights concerning the comparison between the equilibria obtained in the two procedures may be gained by the examination of the cross-effects.

Lemma 6: Choice of $\mu_2 = 1$ by the lowest quality firm 2 always decreases the profit of the highest quality firm 1:

$$\frac{\partial \sigma_1}{\partial \mu_2} < 0, \qquad \forall s \in \left(\frac{1}{2}, \frac{2}{t}\right), \qquad \forall t \in (0, 1)$$

The effect of $\mu_1 = 1$ on firm 2's profit depends on the quality cost index of firm 2:

$$\frac{\partial \sigma_2}{\partial \mu_1} < 0 \Leftrightarrow k_1^0 < \frac{1}{2}$$

Proof: From (18.1) one gets:

$$\text{sign}\,\frac{\partial \sigma_1}{\partial \mu_2} = \text{sign}\left(-t\frac{\partial v_2}{\partial \mu_2} - tk_1^0 k_2^0\right)$$

From (17.2) we have:

$$\text{sign}\,\frac{\partial v_2}{\partial \mu_2} = \frac{2k_1^0(1 - s_2 + tk_2^0)}{4 - t} > 0$$

since $s_2 < 1$. So, $\frac{\partial \sigma_1}{\partial \mu_2} < 0$.

Now, from (18.2) we have:

$$\text{sign}\,\frac{\partial \sigma_2}{\partial \mu_1} = \text{sign} - \frac{\partial v_1}{\partial \mu_1}$$

and from (17.1):

$$\frac{\partial v_1}{\partial \mu_1}=\frac{2k_2^0(1-2k_1^0)}{4-t} \quad \square$$

Let us denote by L the lengh of the interval $\left(\frac{1}{2},\frac{2}{t}\right)$: $L=\frac{4-t}{2t}$. The condition $k_1^0<\frac{1}{2}$ is equivalent to either $s_2>\frac{2s_1}{t}-L$ or $s_1<\frac{s_2+L}{2/t}$

Lemma 6 asserts that the semi-compatibility case resulting from compatibility adoption by only the lowest quality firm 2 never benefits to firm 1. However, the reverse case is not always true: firm 2 may benefit from the compatibility adoption by firm 1 if the basic capacity of firm 1, k_1^0, is greater than one half of the total potential capacity of the market. This last condition amounts to say that the quality/cost index of firm 2, s_2, is greater than some threshold which depends on the quality cost index of firm 1, s_1, according to the expression given above.

All these results may be summarized in the following payoff matrix giving the pairs of profits corresponding to the different values of μ_1 and μ_2.

	$\mu_2=0$	$\mu_2=1$
$\mu_1=0$	a_1, b_1	a_2, b_2
$\mu_1=1$	a_3, b_3	a_4, b_4

In this bimatrix the profits of firm 1 are given by:

$$a_1=\pi_1(0,0)=\pi_1^0+r\sigma_1(0,0) \qquad a_3=\pi_1(1,0)=\pi_1^0+r\sigma_1(1,0)$$
$$a_2=\pi_1(0,1)=\pi_1^0+r\sigma_1(0,1) \qquad a_4=\pi_1(1,1)=\pi_1^0+r\sigma_1(1,1)$$

The payoffs of firm 2 are defined analogously. Now:

by lemma 3 we have: $a_3>a_1$ and $a_4>a_2$

by lemma 4 we have: $b_2>b_1$ and $b_4>b_3$

by lemma 6 we have: $a_2<a_1$ and $a_4>a_3$

$b_3<b_1$ and $b_4<b_2$ if $k_1^0<\frac{1}{2}$

$b_3>b_1$ and $b_4>b_2$ if $k_1^0>\frac{1}{2}$

Before proceeding to the comparison of the compatibility decisions which occur at the equilibrium of the decentralized procedure game, with the different configurations obtained in the joint adoption procedure, let us make an important remark. In the companion paper (EMM [1991]) where emphasis is put on the last procedure, we have shown that the two firms never both agree on incompatibility. Indeed, either they are both in favor of compatibility or they have conflicting incentives concerning compatibility. This implies that the above matrix game is not a prisoner's dilemma: full compatibility is not Pareto dominated by total incompatibility. So, the decentralized procedure for which adoption of compatibility is a dominant strategy for each firm, leads to a pair of profits which are never both worse than those obtained under the joint adoption procedure. In this case, EMM [1991] have shown that compatibility adoption depends ont the relative performance index measured by the ratio s of the quality/cost indexes of the two networks $\left(s \equiv \frac{s_2}{s_1} \in \left(\frac{1}{2}, \frac{2}{t}\right)\right)$.

If the ratio s is in the neighbourhood of 1, which means that the quality/cost indexes of the two networks are not too remote, both firms are in favor of compatibility. Then the two procedures lead to the same result (full compatibility). Moreover, the equilibrium of the decentralized compatibility game gives rise to Pareto dominant payoffs. According to lemma 6, we have in this case:

for firm 1: $a_2 < a_1 < a_4 < a_3$

and

for firm 2:

$$b_3 < b_1 < b_4 < b_2 \quad \text{if} \quad k_1^0 < \frac{1}{2}$$

$$b_1 < b_2 < b_3 < b_4 \quad \text{if} \quad k_1^0 > \frac{1}{2}$$

(*see* figures 1 and 2).

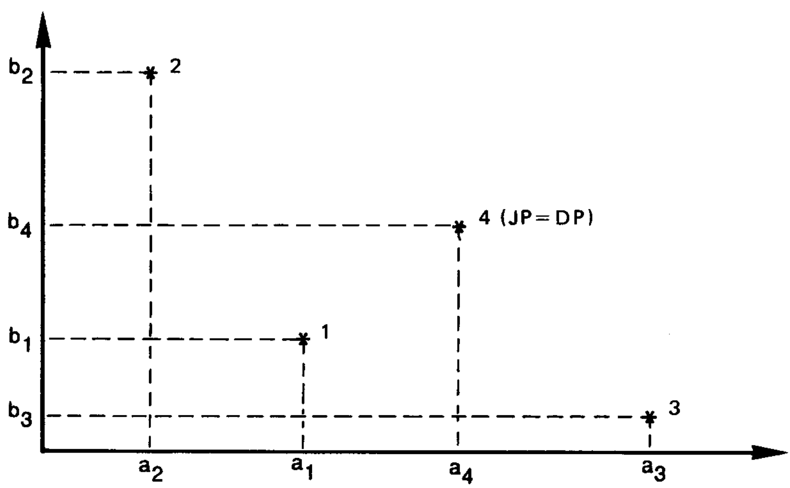

FIGURE 1

Case: $a_2 < a_1 < a_4 < a_3$, $b_3 < b_1 < b_4 < b_2$. $\left(k_1^0 < \frac{1}{2}\right)$

If the quality/cost ratio s is sufficiently far from 1, the two firms have conflicting interests in the joint adoption procedure, while the only Nash

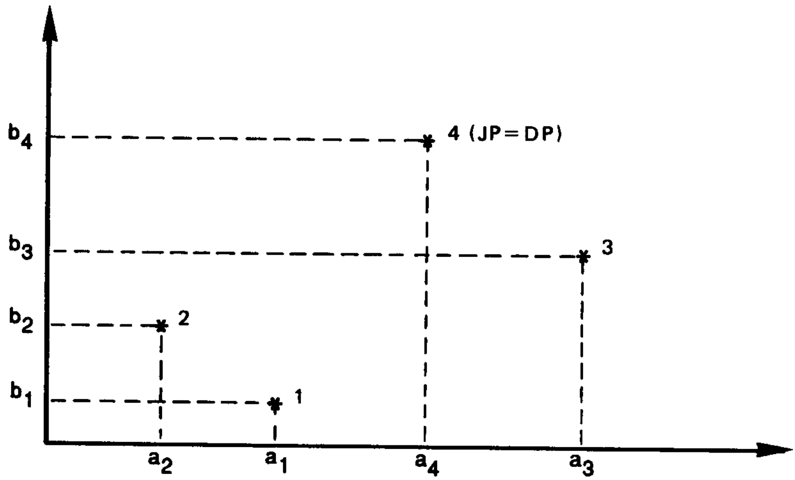

FIGURE 2

Case: $a_2 < a_1 < a_4 < a_3, b_1 < b_2 < b_3 < b_4.$ $\left(k_1^0 > \frac{1}{2}\right)$

equilibrium of the decentralized game leads them to adopt compatibility. One extreme case is when s is near to the left bound of the interval $\left(\frac{1}{2}, \frac{2}{t}\right)$, which means that firm 1 has a largely higher quality/cost ratio than firm 2 $(s_2 > s_1)$. In this case firm 2 is in favor of compatibility while firm 1 is against it: we have $a_4 < a_1$ and $b_4 > b_1$. The networks remain incompatible in the joint adoption procedure. According to lemma 6:

for firm 1: $a_2 < a_4 < a_1 < a_3$

for firm 2: $b_3 < b_1 < b_2 < b_4$

(*see* figure 3).

In the other extreme case where s is near to the right bound of the interval $\left(\frac{1}{2}, \frac{2}{t}\right)$, which means that it is now the lowest basic quality producer, firm 2, which has an advantage over firm 1 in terms of quality/cost ratio $(s_2 > s_1)$, firm 1 favors compatibility while firm 2 is against it. In this case, we have $a_4 > a_1$ and $b_4 < b_1$. The two networks remain incompatible in the joint adoption procedure. According to lemma 6 in this case:

for firm 1: $a_2 < a_1 < a_4 < a_3$

for firm 2: $b_3 < b_4 < b_1 < b_2$

(*see* figure 4).

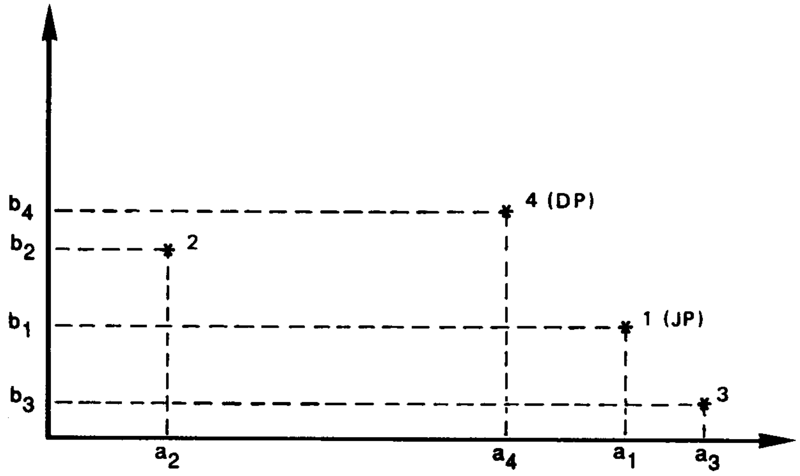

FIGURE 3

Case: $a_2 < a_4 < a_1 < a_3$, $b_3 < b_1 < b_2 < b_4$.

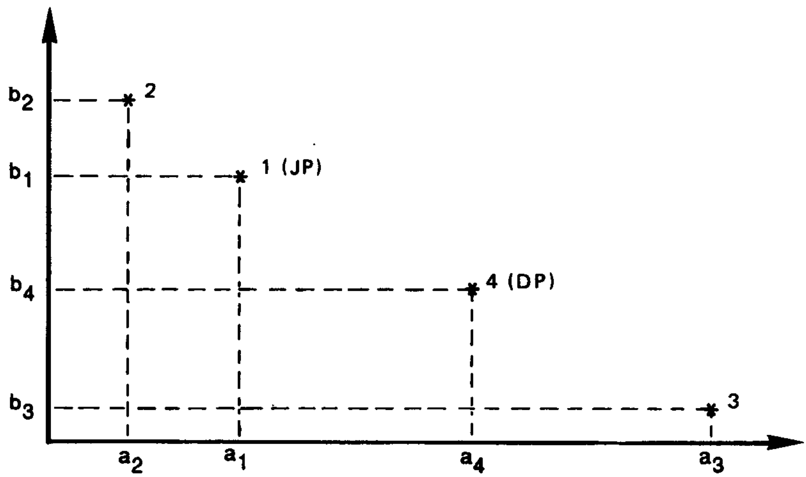

FIGURE 4

Case: $a_2 < a_1 < a_4 < a_3$, $b_3 < b_4 < b_1 < b_2$.

Note that when the two firms agree on the compatibility in the joint adoption procedure (figure 1 and 2) the outcome (a_1, b_1) is Pareto dominated. Furthermore as we see on figures 3 and 4, when the two firms have conflicting interests in the joint adoption procedure, payoffs (a_1, b_1) and (a_4, b_4) belong to the Pareto frontier of the game. But the decentralized procedure selects (a_4, b_4) (full compatibility) while the joint procedure leads to (a_1, b_1) (incompatibility). In order to compare the two outcomes (a_4, b_4)

and (a_1, b_1), one has to analyze how the sizes of the networks vary with the compatibility choices.

5 Network Sizes and Compatibility

Under the joint adoption procedure ($\mu_1 = \mu_2 = \mu$) the linearization around $r=0$, leads to:

$$k_i = k_i^0 + r\tilde{v}_i(\mu) \tag{21}$$

$$\pi_i = \pi_i^0 + r\tilde{\sigma}_i(\mu) \tag{22}$$

$\mu=1$ will be adopted in this procedure iff $\tilde{\sigma}_1(\mu)$ and $\tilde{\sigma}_2(\mu)$ are increasing functions of μ, while $\mu=0$ will be the outcome if either $\tilde{\sigma}_1(\mu)$ or $\tilde{\sigma}_2(\mu)$ decreases with μ. In EMM [1991] we have shown that:

$$\frac{d\tilde{v}_1}{d\mu} = \frac{1}{4-t}[2k_2^0(1-2k_1^0) - tk_1^0(1-k_1^0+2k_2^0)] \tag{23.1}$$

$$\frac{d\tilde{v}_2}{d\mu} = \frac{1}{4-t}[2k_1^0 - k_2^0 - 2(k_1^0)^2 - 2(1-t)k_1^0 k_2^0] \tag{23.2}$$

By using (17.1) and (17.2) one can easily verify that:

$$\frac{d\tilde{v}_1}{d\mu} < \frac{\partial v_1}{\partial \mu_1}$$

and

$$\frac{d\tilde{v}_2}{d\mu} < \frac{\partial v_2}{\partial \mu_2} \Leftrightarrow k_1^0 < \frac{1}{2}$$

Hence the partial effects of compatibility on capacity differ in the two procedures. Nevertheless one has to take the total effects of compatibility on capacity to obtain valid comparisons. The total effect of compatibility on firm i's capacity in the decentralized procedure is given by:

$$v_i(1, 1) - v_i(0, 0) = \frac{\partial v_i}{\partial \mu_i} + \frac{\partial v_i}{\partial \mu_j} \equiv \Delta v_i$$

By contruction: $\Delta v_i = \dfrac{d\tilde{v}_i}{d\mu}$.

If $k_1^0 > \frac{1}{2}\left(\Leftrightarrow s_2 < \frac{2s_1}{t} - \mathrm{L}\right)$ the signs of Δv_1 and Δv_2 are easily obtained:

$$\Delta v_1 < 0 \qquad \text{and} \qquad \Delta v_2 > 0$$

So, if the quality/cost ratio s_2 of firm 2 is lower than the above threshold, compatibility leads firm 1 to decrease its capacity relatively to the situation without compatibility and firm 2 to increase its capacity. The same result applies for firm 1 with the joint adoption procedure.

If $k_1^0 < \frac{1}{2}$, computations are more tedious. However, for specific values of the ratio $s = \frac{s_2}{s_1}$ one can obtain some results.

• **Case 1**: $s = 1$, the two networks have the same quality/cost ratio.

In this case we know that the two procedures lead the two network producers to adopt compatibility. Moreover we have shown (EMM [1991]) that there exists a function $\bar{s}_1(t)$ given by:

$$\bar{s}_1(t) = \frac{-t^3 + 6t^2 - 10t + 8}{(2-t)(t^2+4)}$$

$$\lim_{t \downarrow 0} \bar{s}_1(t) = 1 \qquad \lim_{t \uparrow 1} \bar{s}_1(t) = \frac{3}{5}$$

such that:

$$\Delta v_1 = \frac{d\tilde{v}_1}{d\mu} > 0 \quad \Leftrightarrow \quad s_1 < \bar{s}_1(t)$$

$$\Delta v_2 = \frac{d\tilde{v}_2}{d\mu} > 0 \quad \text{for all } s \in \left(\frac{1}{2}, \frac{2}{t}\right) \text{ and } t \in (0, 1)$$

It is clear that when the two procedures lead to the same compatibility decisions, as in the present case, the network sizes are the same.

• **Case 2**: $s = 2$, the lowest basic quality firm has an advantage in terms of quality/cost index.

In this case we have:

$$\Delta v_1 = \frac{d\tilde{v}_1}{d\mu} > 0 \quad \text{for all } s \in \left(\frac{1}{2}, \frac{2}{t}\right) \text{ and } t \in (0, 1)$$

It exists a function $\bar{s}_2(t)$:

$$\bar{s}_2(t) = \frac{4t^2 - 17t + 4}{10(t-1)^2}.$$

$$\lim_{t \downarrow 0} \bar{s}_2(t) = \frac{2}{5} \qquad \lim_{t \uparrow 1/4} \bar{s}_2(t) = 0$$

such that:

$$\Delta v_2 = \frac{d\tilde{v}_2}{d\mu} > 0 \quad \Leftrightarrow \quad s_2 < \bar{s}_2(t)$$

Note that in this case the joint adoption procedure would had lead to different results. If s_2 is higher than $\hat{s}_2(t)=\frac{2(4t^2-17t+4)}{17t^2-25t+8}$, the joint adoption procedure leads to incompatibility $\left(\frac{d\tilde{v}_2}{d\mu}<0\right)$ while the decentralized procedure leads to full compatibility.

6 Conclusion

We have shown in this paper that when network capacity externalities are present, non cooperative behavior leads the two producers to achieve compatibility of their networks. This occurs even in cases where an explicit coordination mechanism between the two producers prevents such a compatibility, due to conflicting interests between the high and the low quality/cost producers. Moreover, in our model where the perceived quality of each product is related to the sizes of the two networks capacities, the equilibrium of the decentralized procedure is obtained as the outcome of dominant strategies, such that the compatibility game is solvable.

This result has been obtained by restricting the analysis to the case where the rate of externality is sufficiently low to prevent excess capacity. When the rate of externality is high and the costs of capacity expansion are low, excess capacity situations may become profitable, since they soften price competition. However, an exact resolution of these cases is analytically untractable and one has to use numerical simulation to obtain some results. This must be contrasted with our approach where low values of the rate of externality allow first order approximations leading to a complete analytical resolution of the problem.

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