

Empirical Tests of a Simple Pricing Model for Sugar Futures

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ABSTRACT. – In this paper we test the empirical implications of a simple pricing model for commodity futures for the marginal process of prices of sugar futures. According to the pricing model, the futures price bias depends linearly on the conditional variance. We find significant coefficients, from monthly as well as daily data, if the conditional variance is modelled using the GARCH-M model. These estimates imply contango in the futures market and a net hedging demand on the long side of it.

Tests empiriques d'un modèle de prix pour des contrats à terme de sucre

RÉSUMÉ. – Dans cet article, nous testons un modèle de détermination des prix pour des contrats à terme concernant le sucre. Selon ce modèle, la déviation des prix prévisionnels dépend linéairement de la variance conditionnelle. En utilisant le modèle de GARCH-M pour la variance conditionnelle, nous trouvons des coefficients significatifs, à partir des données aussi bien mensuelles que quotidiennes. Ces estimations impliquent un « contango » sur le marché et une demande nette de diminution des risques du côté des acheteurs.

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1 Introduction

Many models consider relations between prices of futures contracts and corresponding spot prices (see e. g. ANDERSON and DANTHINE [1983], HIRSCHLEIFER [1989, 1990]). In this paper, we test whether a simple special case of these models, which is discussed at length e. g. in the recent textbook by DUFFIE [1989], is relevant for the futures market for sugar. Because of data limitations the tests are directed towards the implications for the marginal process of the futures prices only. According to the model under consideration, the futures price bias depends linearly on the conditional variance. A straightforward generalization of the ARCH-M model proposed in ENGLE, LILIEN and ROBINS [1987] is used to model the conditional variance. Results on monthly data from January 1972 to June 1989 as well as from daily data since 1979 are reported. The GARCH-M models yield significant coefficients for the conditional variance, which imply contango in the sugar futures market and a net hedging demand on the long side of the market.

The plan of this paper is as follows. In section 2 we present the pricing model under consideration. The stylized facts in the data are described in section 3. The significance of the estimates of the monthly conditional variance is tested in section 4. Daily data are analyzed in section 5, while section 6 concludes.

2 A Simple Pricing Model

Consider the demand for commodity futures contracts of agents with mean variance utility functions, who possibly have positions on next periods spot market for the commodity under consideration. Assume that the agents can trade the corresponding futures contract as well as a stock market portfolio. Let $q_{t+1}^{(i)}$ denote agent i 's next period's spot market position, while s_{t+1} denotes next periods spot price. We make the simplifying assumption that $q_{t+1}^{(i)}$ is known in period t , which can be motivated by the observation that the uncertainty in $q_{t+1}^{(i)}$ is probably small compared to the price-uncertainty. If furthermore $f_t^{[k]}$ denotes the price of the corresponding futures contract k periods before expiration, $B_t^{(i)}$ is the amount invested in the stock market portfolio and r_{t+1} is the return on the stock market portfolio, the wealth of agent i in the subsequent period, $W_{t+1}^{(i)}$, can be expressed as

$$(1) \quad W_{t+1}^{(i)} = B_t^{(i)}(1 + r_{t+1}) + q_{t+1}^{(i)}s_{t+1} + y_t^{(i)}(f_{t+1}^{[k-1]} - f_t^{[k]}),$$

where $y_t^{(i)}$ denotes the agent's futures position in period t . Note that it is assumed that there are no margin requirements in cash. Moreover we assume for simplicity that the returns on the stock market portfolio are uncorrelated with the spot and futures prices, which is not in conflict with the empirical evidence for commodity futures (DUSAK [1973]). Every agent is assumed to choose his position in the futures market by maximizing a utility function defined over the expected value and the variance of end-of-period wealth W_{t+1} , *i. e.*

$$(2) \quad y_t^{(i)*} = \operatorname{argmax}_{y_t^{(i)}} U^{(i)} \{ E_t [W_{t+1}^{(i)}], \operatorname{Var}_t [W_{t+1}^{(i)}] \},$$

where $U^{(i)}$ is the utility function of agent i and $E_t[\cdot]$ and $\operatorname{Var}_t[\cdot]$ denote the conditional expectation and conditional variance respectively. One way to motivate (2) is to assume utility functions with constant absolute risk aversion for all agents as well as lognormality of wealth (see e. g. DEATON and MUELLBAUER [1980, p. 403]). Note that the agents are allowed to have different degrees of risk aversion, but that homogeneity of expectations is assumed throughout. Differentiation with respect to $y_t^{(i)}$ in (2) after substitution of (1) yields the first order condition

$$(3) \quad y_t^{(i)*} = 0.5 \{ E_t [f_{t+1}^{[k-1]}] - f_t^{[k]} \} / \{ \operatorname{Var}_t [f_{t+1}^{[k-1]}] * \rho^{(i)} \} \\ - q_{t+1}^{(i)} \operatorname{Cov}_t [s_{t+1}, f_{t+1}^{[k-1]}] / \operatorname{Var}_t [f_{t+1}^{[k-1]}],$$

where $\rho^{(i)} = -U_2^{(i)}/U_1^{(i)}$ is the risk aversion coefficient. The first and second term in (3) are known as the pure speculative demand and the pure hedge demand respectively, as the second term vanishes for speculators ($q_t^{(i)} = 0$) while the first term vanishes if $\rho^{(i)} \rightarrow \infty$. Equilibrium on the futures market requires

$$(4) \quad \sum_{i=1}^N y_t^{(i)*} = 0$$

where N is the number of agents in the economy. From (3) and (4) one can easily derive the following expression for the futures price bias $f_t^{[k]} - E_t [f_{t+1}^{[k-1]}]$,

$$(5) \quad E_t [f_{t+1}^{[k-1]}] - f_t^{[k]} = 2 \rho q_{t+1} \operatorname{Cov}_t [s_{t+1}, f_{t+1}^{[k-1]}],$$

where

$$\rho = \left(\sum_{i=1}^N (\rho^{(i)})^{-1} / N \right)^{-1}$$

is the markets risk aversion and $q_{t+1} = \sum_{i=1}^N q_{t+1}^{(i)} / N$ is the net hedging pressure on the market. Two additional assumptions will be imposed to obtain a univariate model which can be estimated from data on futures prices only. Admittedly, these two assumptions are crude approximations at the best and they can be avoided in a multivariate model. The first additional assumption is that in every period a futures contract expires with exactly

the specifications (place of delivery, quality etc.) which the hedgers require. Using an arbitrage argument this assumption of the absence of basis risk implies $\text{Cov}_t[s_{t+1}, f_{t+1}^{[k-1]}] = \text{Var}_t[f_{t+1}^{[k-1]}]$. The second additional assumption is that net hedging pressure does not vary over time: $q_{t+1} = q$. If these assumptions are made equation (5) yields testable implications on the marginal process for $\Delta f_{t+1} = f_{t+1}^{[k-1]} - f_t^{[k]}$,

$$(6) \quad \Delta f_{t+1} = \delta \text{Var}_t[\Delta f_{t+1}] + \varepsilon_{t+1},$$

where $\delta = 2\rho q$ and where ε_{t+1} is independent of past realizations of Δf_{t+1} . CARTER *et al.* [1983] have considered a model similar to (5) and avoided the assumption of a constant net hedging pressure by making use of data on the percentage of speculators who were net along. CARTER *et al.* (1983) however assumed conditional homoskedasticity.

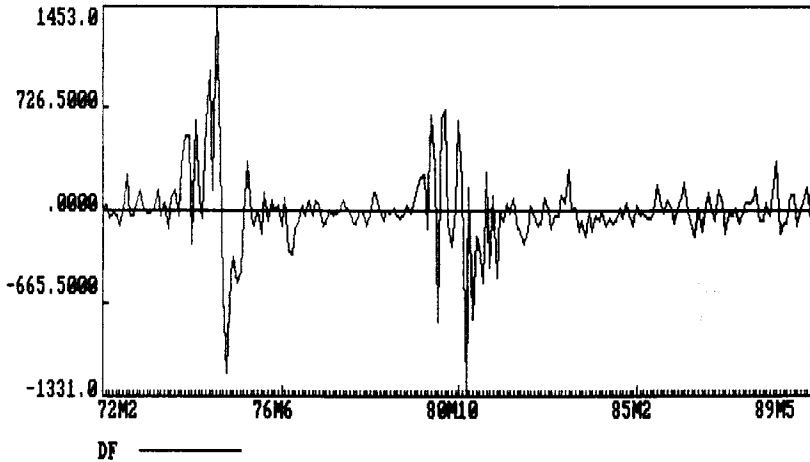
The basic model that is outlined above has been refined by many authors (see e.g. ANDERSON and DANTHINE [1983] and HIRSCHLEIFER [1989, 1990]) taking into account e.g. production uncertainty and covariance of commodity prices with the market portfolio. In this paper we will restrict ourselves to testing the relevance of the basic model for sugar futures.

3 Some Stylized Facts on Sugar Futures

Sugar-11 contracts which expire in January, March, May, July, September and October are traded on the New York Coffee, Sugar and Cocoa Exchange (NYCSCE). A data-series of daily observations on price changes in the contract which is closest to expiration (excluding last month observations to avoid potential delivery obligations) will be analyzed in sections 4 and 5 in order to test the pricing model that was presented in the previous section. The series of monthly changes which is constructed from this data-set is presented in figure 1. Throughout the paper all prices are in dollars per 10,000 lbs. Two periods of bad weather (1973/1974 and 1979/1980) caused high international sugar prices (see FAO [1985]), which are evidently reflected in the futures prices. Figure 1 also clearly shows that volatility of the sugar futures prices is time varying and that the marginal distribution of the price changes is fat tailed. These are well known stylized facts, which hold true for many futures (compare e.g. TAYLOR [1986]). The kurtosis of the monthly series is estimated as 11.9, while the standard Lagrange Multiplier test statistic for sixth order ARCH yields the very significant value of 63.6. The LM test for sixth order autocorrelation yields a value of 17.6 which is close to the 1% critical value of a χ^2 -distribution with six degrees of freedom. Note however that standard tests for serial correlation in the mean will be biased upward in the presence of conditional heteroskedasticity (see e.g. DIEBOLD [1987]) and

FIGURE 1

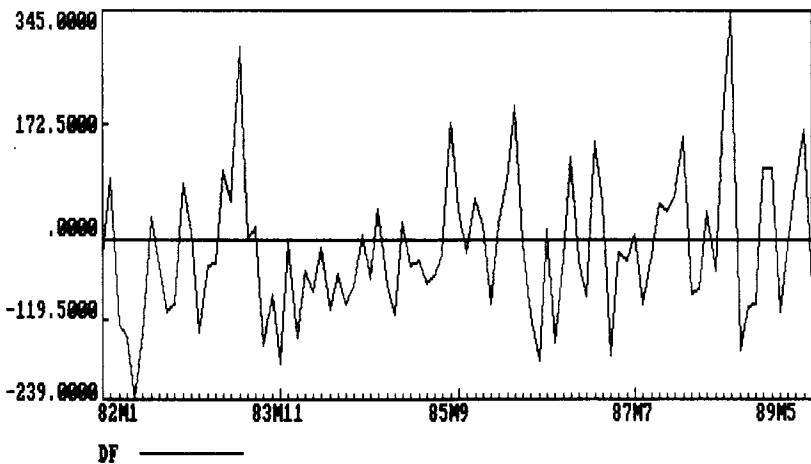
Changes in Monthly Sugar Futures Prices



thus lead to over-rejection. In figure 2 the monthly price changes are plotted for the sub-sample from January 1982 to June 1989 during which period the sugar prices were much more stable. For this sub-period the estimated kurtosis is 3.8 and the LM tests for ARCH and autocorrelation yield the insignificant values of 2.8 and 5.7.

FIGURE 2

Changes in Monthly Sugar Futures Prices



The two daily series, which will be analyzed in section 5, are plotted in figures 3 and 4. The conditional heteroskedasticity is obviously even more pronounced for the full sample than it is for the less volatile sub-sample. Note that, according to the results on temporal aggregation of

FIGURE 3

Daily Price Differences for Sugar Futures 72-89

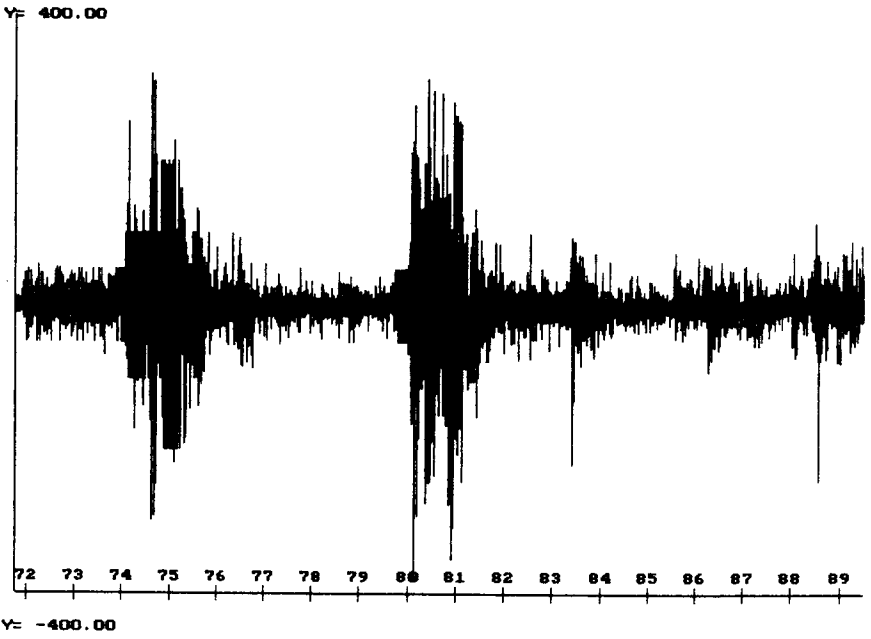
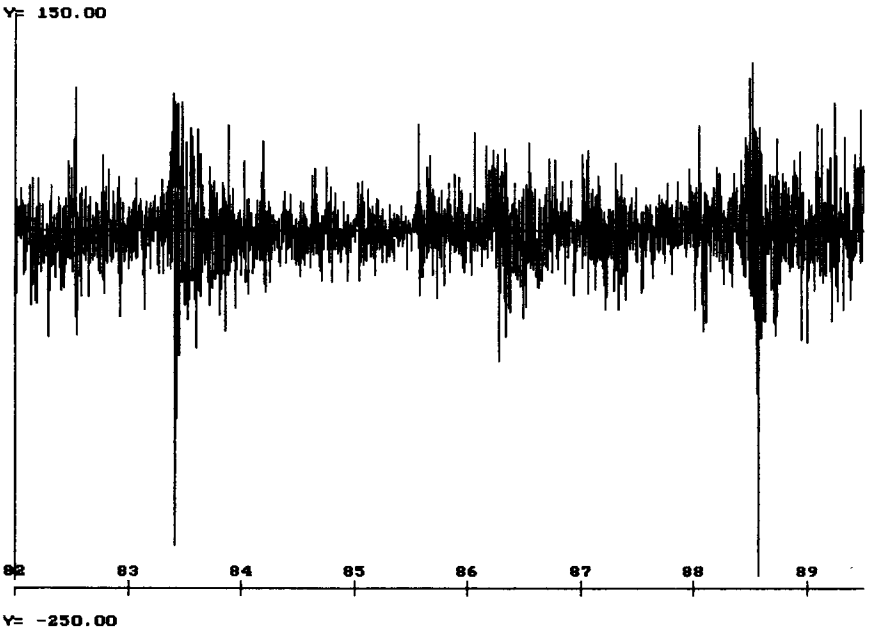


FIGURE 4

Daily Price Differences for Sugar Futures 82-89



GARCH processes in DROST and NIJMAN [1990], the conditional heteroskedasticity in the daily data in the subsample is not conflicting with the apparent homoskedasticity of the monthly data.

4 Tests Based on Monthly Data

According to the pricing model discussed in section 2, the futures price bias depends linearly on the conditional variance. As the conditional variance is unobservable, estimation of the parameter δ in (6) requires estimates of this conditional variance. At least two approaches to derive estimates of the monthly conditional variance have been proposed in the literature. A first suggestion (see e.g. POTERBA and SUMMERS [1986] or FRENCH, SCHWERT and STAMBAUGH [1987]) is the use of a moving average of squared daily returns. We have computed estimates of the monthly conditional variance in this way and subsequently estimated (6) using ordinary and generalized least squares for various subsamples. The estimates of δ , which are reported in NIJMAN and BEETSMA [1990], tend to be negative but insignificant. We do not report the estimates here because of space limitations, but also because this approach is valid only if the daily expected return can be assumed to be negligible. Another drawback of this approach is that it can not be used to generate daily variance forecasts from daily data.

A second approach to the estimation of the conditional variance which has been taken in the literature is the use of the GARCH model proposed by BOLLERSLEV [1986]. The simplest GARCH model, the GARCH(1,1) model, assumes that the conditional variance depends with exponentially declining weights on squared unexpected changes in the futures price. If in addition to the GARCH(1,1) assumption normality of the conditional distribution is imposed the complete model reads as

$$(7) \quad \Delta f_t = \delta h_t + \xi_t h_t$$

$$(8) \quad \xi_t | \xi_{t-1}, \xi_{t-2}, \dots \sim N(0, 1)$$

$$(9) \quad h_t = \psi + \beta h_{t-1} + \alpha \xi_{t-1}^2 h_{t-1}.$$

The model in (7), (8) and (9) is a straightforward generalization of the ARCH-M (ARCH in mean) model proposed by ENGLE, LILJEN and ROBINS [1987]. The parameters in the GARCH-M model can be estimated by numerical maximization of the likelihood. The consistency of the maximum likelihood estimator in this model, which has long been a mere conjecture, has been recently proved by BOLLERSLEV and WOOLDRIDGE [1990].

Estimates of (7)-(9) from monthly data are presented in table 1. The numbers in parentheses are *t*-statistics derived from estimates of the large

sample variance which are routinely computed from the Hessian of the log-likelihood. The numbers in square brackets are the t -statistics proposed by WEISS [1986], which are robust to departures from normality in the rescaled innovations ξ_t in GARCH models. We conjecture that these t -statistics are also more reliable in GARCH-M models if conditional normality does not hold. The divergence from normality can be judged from the estimate of the kurtosis of the rescaled disturbances in the last column of the table. Finally the sixth column of the table presents the Lagrange multiplier test, LM_A , for up to sixth order autocorrelation on the rescaled residuals.

TABLE 1

Monthly GARCH-M models

Sample	ψ	α	β	δ	μ	LM_A	κ
72-1 89-6	2.324 (3.21) [3.37]	0.43 (5.73) [7.17]	0.52 (7.44) [10.05]	-0.00055 (-3.62) [-3.63]		16.8	4.5
72-1 81-12	2.358 (2.38) [2.12]	0.44 (5.88) [8.55]	0.51 (7.24) [10.41]	-0.00054 (-3.13) [-2.85]		18.9	6.0
82-1 89-6	2.789 (0.64) [0.37]	0.12 (0.55) [0.25]	0.60 (1.01) [0.53]	-0.00064 (-0.51) [-0.48]		5.0	3.6
72-1 89-6	2.320 (3.22) [3.26]	0.43 (5.74) [6.99]	0.52 (7.45) [9.90]	-0.00053 (-3.04) [-2.87]	-0.0043 (-0.50) [-0.45]	17.0	4.5

The estimates of δ in table 1 are negative, which suggests a net hedging demand on the long side of the market: The hedging demand from buyers on the spot market exceeds the hedging demand of sellers. The order of magnitude of the estimates of δ in table 1 does not differ from the results which are obtained if estimates of the monthly conditional variance are derived from squared daily excess returns, but the important point to note here is that for both the full sample (January 1972-June 1989) and the high volatility subsample (January 1972-December 1981) the impact of the conditional variance on the futures price bias appears to be significant. Note however that the Lagrange Multiplier test autocorrelation, LM_A , indicates some autocorrelation in the disturbances, which might result in biased t -ratio's. Under the null hypothesis of no autocorrelation this test statistic is asymptotically centrally χ^2 distributed with 6 degrees of freedom. The critical values of this distribution at the 5 and 1% level are 12.6 and 16.8 respectively.

Attempts to estimate a GARCH-M model for January 1982-June 1989 failed as the assumption of no persistence in variance ($\alpha + \beta < 1$) which is imposed by the software was violated, which probably has to do with erratic observations in the beginning of the sample and a small sample size. If the sample is chosen as January 1984-June 1989 convergence is achieved. As in this case the estimated variance parameters are insignificant, it is not surprising to find an insignificant estimate of the mean parameter as

well. In order to show that the evidence on the time varying risk premium is not caused by the presence of a constant risk premium and absence of a constant in (7), we have also estimated the model including the possibility of a time-invariant risk premium, denoted by μ . As shown in the final row of table 1 this parameter is insignificant and does not affect the significance of the remaining parameters.

5 Tests Based on Daily Data

In the previous section the GARCH-M model, which was motivated by the pricing model presented in section 2, was estimated for monthly data. It is not clear from the pricing model, however, how long the appropriate time periods are. If the agents solve a simple single period optimization problem, as they do in section 2, the use of monthly data already appears to imply a short planning horizon. On the other hand, however, the agents obviously have access to high frequency information on the underlying variables and have incentives to use this information. This argument suggests the use of high frequency, e. g. daily, data. Moreover, the behavior of agents who in fact maximize a multiperiod additively separable mean variance criterion function, can probably be closely mimicked by the behaviour of myopic agents if subsequent changes in asset prices are uncorrelated, which is roughly the case for the sugar futures (compare also INGERSOLL [1987], pp. 255-258). This implies that the GARCH-M model might well be a valid specification at the daily frequency as well.

Estimates of the daily GARCH-M model are presented in table 2. Because of limitations of our software, daily observations have only been analyzed

TABLE 2

Daily GARCH-M models

Sample	ψ	α	β	δ	μ	LM_A	κ
780101	6.14	0.084	0.906	-0.0008		8.6	4.5
-	(4.22)	(9.75)	(96.9)	(-2.10)			
890601	[3.02]	[6.60]	[65.1]	[-2.02]			
820101	10.41	0.081	0.900	-0.0025		3.6	4.7
-	(3.36)	(7.27)	(63.5)	(-2.70)			
890601	[2.21]	[4.60]	[41.0]	[-2.53]			
780101	6.07	0.085	0.906	-0.0005	-0.72	8.5	4.5
-	(4.20)	(9.75)	(96.8)	(-1.01)	(-1.74)		
890601	[3.00]	[6.63]	[65.3]	[-0.91]	[-1.70]		
820101	10.44	0.081	0.900	-0.0026	0.059	3.6	4.7
-	(3.30)	(7.25)	(62.8)	(-2.12)	(0.14)		
890601	[2.23]	[4.53]	[39.8]	[-2.08]	[0.21]		

from January 1, 1978 onwards. The estimates for the full sample imply (once more) a significant negative impact of the conditional variance on the futures price bias. Moreover the estimate of δ that is obtained is close to the estimates from monthly data. As suggested by the results on temporal aggregation of GARCH processes in DROST and NIJMAN [1990], the daily model is close to integration in variance and the estimate of β from daily data exceeds the estimates from monthly data, while the opposite is true for the estimates of α . For the subsample from 820101-890601 similar results are obtained, although the estimate of δ is somewhat small. For the full sample the problem of multicollinearity between a time-variant and a time-invariant component of the risk premium reappears. For the subsample, however, the significance of the conditional variance term is not affected by the presence of a constant in (7).

6 Concluding Remarks

In this paper we have tested the empirical implications for the marginal process of prices of Sugar futures of a simple pricing model. A significant impact of the conditional variance on the change in futures prices was obtained in monthly as well as daily GARCH-M models. These estimates imply contango in the futures markets and a net hedging demand on the long side of the futures market. Moreover our results suggest that the simple pricing model points at at least one important aspect of the pricing of Sugar futures: the risk premia depend on the time varying volatility. Tests of the more detailed pricing models which have been proposed in the literature are left for future research.

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