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Testing Linearity of Economic Time Series against Cyclical Asymmetry

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ABSTRACT. – In this paper we study the possible asymmetry of business cycles using time series techniques. The hypothesis of linearity is tested against various forms of nonlinearity, some of which are capable of representing processes generating nonsymmetric cycles. Quarterly unemployment and industrial production series are used as business cycle indicators, and the data consists of 13 OECD countries. The test results indicate that many macroeconomic time series are nonlinear and may be asymmetric.

Tests de la linéarité des séries temporelles économiques contre des hypothèses d'asymétrie cyclique

RÉSUMÉ. – Le sujet de cet article est l'asymétrie possible des cycles économiques, que nous recherchons par des méthodes d'analyse des séries temporelles. La linéarité des séries est testée contre des non linéarités diverses, quelques-unes capables de représenter des processus produisant des cycles asymétriques. Les séries trimestrielles du chômage et de la production industrielle des treize pays de l'OCDE sont utilisées comme des indicateurs du cycle. Les résultats indiquent qu'il y a plusieurs séries macroéconomiques non linéaires qui présentent des asymétries.

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1 Introduction

The idea of asymmetric business cycles is not new. MITCHELL [1927, pp. 330-334 and pp. 407-412] has discussed it and presented statistical evidence both in favour of and against it. KEYNES [1936, p. 314] has argued that the contractions in an economy are more violent but also more short-lived than the expansions. Yet, modern theoretical work does not seem to be much influenced by such a possibility. For instance, there is no mention of cyclical asymmetry in the entry on business cycles (DOTSEY and KING [1987]) in the New Palgrave Dictionary of Economics describing recent work. On the other hand, STOCK [1987] termed a paper on nonlinearities in aggregate output supply and demand equations by CHETTY and HECKMAN [1986] an important foundation for his investigation of business cycle asymmetry. Thus the question of linearity or symmetry of business cycles is relevant from the economic theory point of view.

Recently, some investigators armed with modern time series tools have returned to discussing the asymmetry or nonlinearity of business cycles. We shall describe these efforts briefly in section 2. The testing approach we shall adopt in this paper is presented in section 3. In section 4 we reanalyze the U.S. unemployment series considered by NEFTÇI [1984] and others. They turn out to be nonlinear: all our tests reject linearity. The strongest evidence for cyclical asymmetry seems to come from the observations before 1960. The analysis of unemployment series continues in section 5 where we consider international time series and find the evidence mixed. Section 6 reveals that these results are sensitive to the logistic transformation. In section 7 we study international industrial output series, and section 8 concludes.

2 Assessing Nonlinearities in Business Cycles

Cyclical asymmetry is a descriptive rather than a theoretical, uniquely defined concept. Asymmetry is said to occur if the two main phases of the business cycle, contraction and expansion, are not equally long. There is no unique method of defining contractions and expansions and many types of data may be used to describe and represent a business cycle. NEFTÇI [1984], who pioneered in this field, suggested the use of finite state Markov processes for studying the problem in the U.S. economy. The procedure can be described as follows. Consider the first differences of a quarterly, seasonally adjusted and stationary time series representing the business cycle. Dichotomize the differenced series according to the signs of the differences. Study the length of runs of positive and negative signs indicating the length of expansions and contractions. If they differ, the two conditional probabilities of remaining in the same state as before may not be equal, and that is tantamount to cyclical asymmetry. Thus the empirical conclusions may be based on estimated transition probabilities.

NEFTÇI [1984] used three different unemployment series as business cycle indicators. After estimating the relevant transition probabilities using a second-order Markov process as his model and constructing appropriate confidence ellipsoids, he concluded that his evidence favoured the asymmetry assumption. However, SICHEL [1989] recently pointed out computational errors in the confidence ellipsoids of NEFTÇI [1984] and recomputed them. The recomputed ellipsoids did not indicate asymmetry. [1] FALK [1986] applied the same Markov process framework for testing the symmetry of the business cycle but used other U.S. series than unemployment to serve as indicators of the cycle. His evidence did not support the asymmetry assumption. Falk also investigated the issue using quarterly industrial production series from four European countries and Canada. The results again supported the cyclical symmetry rather than the asymmetry hypothesis. Using another definition, DELONG and SUMMERS [1986] studied the asymmetry by calculating the skewness of the distribution of growth rates for a variety of production series in several countries. They found little evidence favouring asymmetry: the only notable exception seemed to be the U.S. unemployment rate 1950-1979.

STOCK [1987] based his approach on the idea of time deformation. Assume that latent economic variables evolve according to a linear time-invariant process in economic time. Economic time is non-trivially different from calendar time if the transformation from economic to calendar time is nonlinear. If this is the case, then the observable processes representing the latent variables may behave asymmetrically in calendar time. STOCK [1987] was mainly interested in finding out whether there is a common cyclical indicator indicating a cyclical time scale for all variables of interest. Although the null hypothesis of a linear time scale was rejected for some of the systems he considered, there was not evidence in favour of a systemwide cyclical time scale. FRANK *et al.* [1988] tested the linearity of some international GNP series against nonlinearity in general by applying the linearity test of BROCK *et al.* [1987]. The test is based on the correlation dimension as defined in the analysis of nonlinear dynamic systems. Our

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1. The considerations of NEFTÇI [1984] were based on the assumption that the sign process was a second-order Markov process. In a footnote, SICHEL [1989] remarks that in unpublished work Neftçi also applied first-order Markov processes to the same U.S. unemployment series and the results favoured cyclical asymmetry. ROTHMAN [1989] recently also did this extending the analysis to sectoral time series, and his conclusions were the same as Neftçi's.

analysis is similar to that of Frank *et al.* in that it is also based on a pure time series approach, but we shall be interested in a fully specified (parametric) type of nonlinearity as the alternative to linearity.

3 The Testing Approach to Cyclical Asymmetry

Assume a white noise process $\{u_t\}$ and a general time series $\{X_t\}$ observed at $t=0, \pm 1, \pm 2, \ldots$ Suppose there is a relationship between $\ldots, X_{t-2}, X_{t-1}, X_t, X_{t+1}, \ldots$ such that

$$h(\ldots, X_{t-2}, X_{t-1}, X_t, X_{t+1}, \ldots)=u_t \tag{1}$$

where h is a given function. If

$$h=\sum_{j=-\infty}^{\infty} h_j X_{t-j} \tag{2}$$

the model describing the relationship is linear; see PRIESTLEY [1988, pp. 13-16] for discussion. In practice, $h_j=0$ for all $j<0$, so that

$$\sum_{j=0}^{\infty} h_j X_{t-j}=u_t. \tag{3}$$

If the process is stationary and ergodic, the linear autoregressive model (3) may be approximated by a finite lag version

$$\sum_{j=0}^{p} h_j X_{t-j}=u_t, \qquad p<\infty \tag{4}$$

where the roots of $\sum_{j=0}^{p} h_j z^j=0$ lie outside the unit circle.

Stationary, cyclically symmetric differences of a time series may often be adequately represented by (4). However, cyclically asymmetric series are a problem. If we want to retain a linear structure, then a non-symmetric error distribution is necessary to account for asymmetry. If we prefer a symmetric, possibly normal, error distribution, then we have to give up (4). Taking the latter route, we want to test (4) against a choice of h in (1) such that the model generates asymmetric cycles. As (1) indicates, the set of alternatives may be large, and to construct an applicable test we have to focus on a sufficiently small subset of non-linear models. TONG and LIM [1980], TONG [1983] and TSAY [1989], among others, discussed a class of non-linear time series models called self-exciting threshold autoregressive (SETAR) models. Assume a stationary business cycle indicator y_t. It is said to follow a SETAR $(2; p, p)$ model if

$$y_t + \pi_0 + \sum_{j=1}^{p} \pi_j y_{t-j} + \left(\theta_0 + \sum_{j=1}^{p} \theta_j y_{t-j}\right) F(z_t) = \varepsilon_t \tag{5}$$

where $\varepsilon_t \sim \text{nid}(0, \sigma^2)$, and $F(.)$ is a Heaviside function:

$$F(z_t) = \begin{cases} 1, & z_t > 0 \\ 0, & z_t \leqslant 0. \end{cases} \tag{6}$$

Furthermore $z_t = y_{t-d} - c$, where c and d are fixed but usually unknown parameters. If $\theta_j = 0$, $j = 0, 1, \ldots, p$, (5) collapses into an ordinary AR(p) model.

It is intuitively obvious that (5) can generate cyclically asymmetric time series. When $y_{t-d} > c$, the process follows an AR(p) regime and it switches from this to another AR(p) regime the next period if $y_{t-d+1} \leqslant c$. Because the two regimes have different dynamic structures, the conditional probability of remaining in one regime given the past is not necessarily the same as the corresponding probability of remaining in the other. That, according to NEFTÇI's definition, is equivalent to cyclical asymmetry. For further discussion, see TONG and LIM [1980]. NEFTÇI [1984] was indeed dealing with similar conditional probabilities, implicitly assuming $c = 0$ in (5), estimating them and testing their equality using a Markov chain approach. On the other hand, what we have just said implies that the validity of the symmetry assumption could also be investigated by testing the linearity of $\{y_t\}$ against the nonlinear alternative (5). However, even if we assume p and d known in (5), deriving a feasible test is difficult if c is unknown. This is because the likelihood function of (5) is then irregular. K. S. CHAN and TONG [1986] suggested a numerical evaluation of the likelihood function and a likelihood ratio test based on that numerical approximation. For two simple special cases, theoretical results allow tabulating the asymptotic null distribution of the likelihood ratio statistic; see MOEANADDIN and TONG [1988] for discussion.

Recently, LUUKKONEN *et al.* [1988 *a*] explored another avenue which opens up through a generalization of (5) and leads to a simple testing procedure. Assume instead of (6) that in (5)

(i) $F: \mathbb{R} \to \mathbb{R}$ is an odd, monotonically increasing function possessing a non-zero derivative of order $(2s+1)$ in an open interval $(-a, a)$, $a > 0$, $s \geqslant 0$.

(ii) $d^k F(z_t)/dz_t^k |_{z_t=0} \neq 0$ for k odd and $1 \leqslant k \leqslant 2s+1$.

(iii) $z_t = \gamma(y_{t-d} - c)$, $\gamma > 0$.

Assumptions (i)-(iii) generalize the SETAR model by making the transition from one regime to the other smooth. Parameter γ in (iii) controls the swiftness of the transition. In fact, the system is described by a mixture of the two regimes. Model (1) where (i)-(iii) define $F(.)$ is called a smooth transition autoregressive or a STAR $(2; p, p)$ model. Cumulative distribution functions of continuous random variables with a density symmetric around the mean are suitable candidates for $F(.)$. W. S. CHAN and TONG [1986] suggested the cdf of the standard normal distribution whereas

LUUKKONEN *et al.* [1988 *a*] recommended that of the logistic distribution. This distribution approximates the normal one quite closely and has clear computational advantages over the normal alternative.

The STAR model can create asymmetric fluctuations just like the SETAR model. Thus it may be used as a parameterized nonlinear alternative in our investigation. LUUKKONEN *et al.* [1988 *a*], starting from assumptions (i)-(iii) recently derived Lagrange multiplier type tests for testing linearity against STAR. They also assumed that d, the delay parameter, is unknown. The testing problem is nonstandard in the sense that (5) is identified only under the alternative and not under the null hypothesis

$$\text{(7)} \qquad H_0: \ \theta_j = 0, \qquad j = 0, 1, \ldots, p.$$

DAVIES [1977] contains a general discussion of this situation. His solution was to derive a test first holding the unidentified parameters fixed. He then suggested using the supremum of the statistic over the unidentified parameters as the final test statistic. A major problem with this approach is that the asymptotic null distribution of the final statistic is usually unknown and has to be approximated. On the other hand, LUUKKONEN *et al.* [1988 *a*] replaced F in (i) by its first-order Taylor expansion about the origin. This and the application of the Lagrange Multiplier principle lead to an auxiliary regression with $p(p+1)/2$ quadratic terms. The coefficients in this regression are functions of the parameters in (5). The residual sum of squares is minimized over all the parameters and the test statistic thus maximized over the set of unidentified parameters. The solution therefore corresponds to the spirit of DAVIES [1977], and it has the useful feature that the asymptotic distribution of the test statistic under the null of linearity is known. The test is in fact equivalent to the well-known linearity test of TSAY [1986]. Assuming we have an observed time series $y_{-p+1}, \ldots, y_0, y_1, \ldots, y_T$, the test of (7) may be carried out as follows:

(i) Regress y_t on $1, y_{t-j}$; $j = 1, \ldots, p$, using ordinary least squares, form the residuals $\hat{\varepsilon}_t$, $t = 1, \ldots, T$, and the residual sum of squares $SSE_0 = \sum_{t=1}^{T} \hat{\varepsilon}_t^2$.

(ii) Regress $\hat{\varepsilon}_t$ on $1, y_{t-i}, y_{t-i}y_{t-j}$; $i = 1, \ldots, p$; $j = i, \ldots, p$, form the residuals $\hat{\eta}_t$, $t = 1, \ldots, T$, and $SSE_1 = \sum_{t=1}^{T} \hat{\eta}_t^2$.

(iii) Compute the test statistic

$$S_1 = T(SSE_0 - SSE_1)/SSE_0.$$

Under H_0, S_1 has an asymptotic χ^2 distribution with $p(p+1)/2$ degrees of freedom. As a LM type test the test has generally good power properties. However, if the main source of non-linearity in (5) is θ_0 so that $\theta_1, \ldots, \theta_p$ are zero or small in absolute value, the test lacks power. Realizing this, LUUKKONEN *et al.* [1988 *a*] derived two other LM type tests which use more degrees of freedom than S_1 but have power against the situation we have just described. They are based on using the third-order Taylor expansion of F rather than the first-order one. We shall present the more parsimonious test procedure of the two. It consists of the following steps:

(i) Same step as before.

(ii) Regress $\hat{\varepsilon}_t$ on 1, y_{t-i}, $y_{t-i}y_{t-j}$, y_{t-i}^3; $i=1, ..., p$; $j=i, ..., p$, form the residuals $\hat{v}_t$, $t=1, ..., T$, and the residual sum of squares $SSE_3 = \sum_{t=1}^{T} \hat{v}_t^2$.

(iii) Compute the test statistic $S_3 = T(SSE_0 - SSE_3)/SSE_0$.

Under H_0, S_3 has an asymptotic χ^2 distribution with $p(p+1)/2+p$ degrees of freedom. This test is not quite as powerful as S_1 if $\theta_0 = 0$ but it is generally more useful of the two if that assumption cannot be made *a priori*. In the applications that follow we shall make use of both S_1 and S_3. As the simulation results in LUUKKONEN *et al.* [1988 *a*] and PETRUCCELLI [1990] demonstrate, these tests are powerful in small samples also when the true alternative is SETAR, *i.e.*, when $\gamma \to \infty$ in assumption (iii).

Rejecting linearity against a well-specified STAR model using a Lagrange multiplier type test does not entitle us to accept cyclical asymmetry. The reason is that the tests against STAR may have power against some of these other non-linearities as well; for discussion see e.g. LUUKKONEN *et al.* [1988 *b*]. It is therefore necessary to learn more about the situation before drawing any definite conclusions. One way of obtaining more information is to test linearity against more than one type of nonlinearity.

A relevant form of nonlinearity which our symmetry tests may well respond to is autoregressive conditional heteroskedasticity (ARCH). It is quite conceivable that the conditional variance of the error process varies according to the phase of the business cycle. Carrying out an ARCH test for the time series we consider, could thus be useful. The test we shall apply is the one MCLEOD and LI [1983] based on the squared residuals of a linear model. The authors derived it as a general linearity test. However, as LUUKKONEN *et al.* [1988 *b*] pointed out, it is asymptotically equivalent to the ARCH test of ENGLE [1982]. The simulation results reported in LUUKKONEN *et al.* [1988 *b*] indicate that in small samples this test, here called $Q(n)$ where n is the lag length, often has little power against many other types of nonlinearity. [2] Thus, if the ARCH test does not reject linearity but the STAR tests do, we may at least exclude conditional heteroskedasticity from our set of alternatives. On the other hand, the same simulation experiments showed that a few linearity tests have power against ARCH at sample sizes appearing in this paper, although they were designed for some other nonlinear alternative. Thus, if all tests reject linearity, we cannot exclude the possibility that the STAR tests actually respond to conditional heteroskedasticity. This is so in particular if the probability value of the ARCH test is considerably smaller than that of the STAR tests. Naturally, both STAR and ARCH may appear simultaneously as well, although models accommodating this possibility have not yet been extensively applied in time series analysis.

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2. LUUKKONEN *et al.* [1988 b] also show that, asymptotically, the local power of $Q(n)$ against bilinearity or exponential autoregression is not higher than the size of the test.

Another nonlinear time series model which is of interest here is the bilinear model. It is defined as

$$(8) \qquad y_t+a_1 y_{t-1}+...+a_t y_{t-p}=\mu+\varepsilon_t+\sum_{i=1}^{m}\sum_{j=1}^{k} c_{ij}\varepsilon_{t-i}y_{t-j}$$

where $\varepsilon_t \sim \text{nid}(0, \sigma^2)$. We call (8) the BL$(p;\ m, k)$ model. This model may generate stationary realizations with amplitude changes, *i.e.*, sharp peaks and troughs, which may also be present in business cycle data. For discussion see SUBBA RAO and GABR [1984, pp. 150-151]. Our STAR tests may well respond to amplitude changes or conspicuous outliers in time series. With that in mind it seems useful to run linearity tests against bilinearity as an additional check. WEISS [1986] and SAIKKONEN and LUUKKONEN [1988] constructed tests against bilinearity: we shall apply the test as defined by the latter authors. The null hypothesis is

$$H_0':\ c_{ij}=0, \qquad i=1, ..., m; \qquad j=1, \ldots, k,$$

in (8). The test may be carried out in three steps as follows.

(i) Same step as before.

(ii) Regress $\hat{\varepsilon}_t$ on $1, y_{t-i}, i=1, ..., p;\ \hat{\varepsilon}_{t-i}\, y_{t-j}, i=1, ..., m; j=1, ..., k$, form the residuals $\hat{\gamma}_t$, $t=1, ..., T$, and the residual sum of squares $SSE_4=\sum_{t=1}^{T}\hat{\gamma}_t^2$.

(iii) Compute the test statistic $BLT=T\,(SSE_0\text{-}SSE_4)/SSE_0$.

Under H_0', BLT has an asymptotic χ^2 distribution with mk degrees of freedom. For further details, see WEISS [1986] and SAIKKONEN and LUUKKONEN [1988]. Two bilinear models are used as possible alternatives. The first one is the BL$(p;\ 1, 1)$ model so that $m=k=1$. The second one, BL$(p;\ 2, 2)$, contains two diagonal bilinear terms and a third term, $y_{t-1}\varepsilon_{t-2}$. The corresponding LM test statistics are called BLT$(p;\ 1, 1)$ and BLT$(p;\ 2, 2)$, respectively.

4 U.S. Unemployment Series

NEFTÇI [1984] used U.S. unemployment series in considering the possible asymmetry of business cycle fluctuations and gave two main reasons for that. First, series like unemployment related to production give a better indication of the business cycle than, say, consumption. Second, there is no need to estimate and eliminate a trend component because the unemployment variables do not contain a trend. Thus he used the first differences of unemployment series, u_t, say, in his considerations.

We use the same quarterly, seasonally adjusted time series from Business Conditions Digest as NEFTÇI [1984] does but first extend the period from

1948(i) to 1985(iv). The series are differenced once before applying the tests. In setting up any of these tests there is the problem of determining p, the unknown order of autoregression. It is solved by starting with a relatively high order, in this case 7, and fitting AR models for all orders not higher than that to the data. The model with the lowest value of an order selection criterion is selected. We have used both AIC (Akaike [1974]) and SBIC (Schwarz [1978], Rissanen [1978]) as our order selection criteria. The general form of these criteria is

$$\text{SC}(g) = \ln \hat{\sigma}_p^2 + g(\text{T})\, p/\text{T} \tag{9}$$

where $\hat{\sigma}_p^2$ is the residual variance of the linear AR(p) model. If $g(\text{T})=2$, (9) is AIC, whereas setting $g(\text{T})=\ln \text{T}$ yields to SBIC. AIC is the less parsimonious criterion of the two, because even asymptotically there is a positive probability of selecting too large a model if the true model is a finite-parameter linear autoregressive model. For SBIC this probability equals zero, and as neither of the criteria asymptotically underestimates the dimension of the model, SBIC is dimension consistent. For further discussion, see e.g. Teräsvirta and Mellin [1986].

Table 1

Observed Significances or p-Values of Linearity Tests Against STAR and Other Nonlinearities Using Quarterly, Seasonally Adjusted U.S. Unemployment Data 1948 (i)-1985 (iv) (1948 (i)-1981 (iv))

BCD Series	AR order p	Test statistics				
		S_1	S_3	BLT ($\hat{p}$; 1, 1)	BLT ($\hat{p}$; 2, 2)	Q(4)
37 ÷ 441	2	0.003	0.008	0.133	0.008	0.007
	(2)	(0.002)	(0.002)	(0.037)	(0.001)	(0.013)
44	3	0.001	0.004	0.082	0.009	0.11
	(2)	(<0.0001)	(<0.0001)	(0.003)	(0.0003)	(0.010)
45	2	0.019	0.031	0.028	0.026	<0.0001
	(2)	(0.028)	(0.050)	(0.030)	(0.038)	(0.0009)

• The series are from Business Conditions Digest and cover the years 1948-1985. Series 37 (in the coding system of BCD) is "number of persons unemployed", 441 is "total civilian labour force", 44 is "unemployment 15 weeks and over" (unemployment rate), and 45 is "average weekly insured employment".

• The figures in parenthesis are based on data from 1948 (i)-1981 (iv), the observation period used by Neftçi [1984].

The test results are in this first case rather close to each other, and the ones reported in Table 1 are based on SBIC. In this and subsequent tables we report the observed significances or p-values of the test statistics rather than the values of the test statistics themselves. The results indicate that the differenced series are nonlinear. The McLeod and Li (ARCH) test with $n=4$ strongly rejects the null in two cases out of three, which points towards ARCH. The linearity is rejected in favour of STAR at the 0.05 level of significance in one case and at the 0.01 level in the remaining two. Even the test against bilinearity rejects linearity, if the bilinear model contains enough parameters ($m=k=2$). Because Q(n) is often not very powerful against nonlinearities other than ARCH, we may not reject the notion that heteroskedasticity is present except for series 44. However, the linearity against STAR is strongly rejected even in that case, which lends some support to cyclical asymmetry or at least nonlinearity in the mean.

To facilitate comparison with Neftçi's paper, we performed the same tests using his observation period, 1948 (i) to 1981 (iv). The results did not change much. Linearity is generally rejected at the 0.05 level of significance. For series 44 the rejection against STAR is very strong, and there is now also a rejection against ARCH at the 0.05 level. Bilinearity is generally rejected slightly less strongly than STAR but remains a possible cause of nonlinearity. The rejections against STAR do not agree with the corrected results in SICHEL [1989] but are in accord with those obtained with the sign process using the first-order Markov process as in ROTHMAN [1989].

TABLE 2

Observed Significances or p-Values of Linearity Tests Against STAR and Other Nonlinearities Using Quarterly, Seasonally Adjusted U.S. Unemployment Data 1960 (i)-1985 (iv)

BCD Series	AR order $\hat{p}$	Test statistics S_1	S_3	BLT ($\hat{p}$; 1, 1)	BLT ($\hat{p}$; 2, 2)	Q(4)
37 ÷ 441	1	0.329	0.598	0.639	0.008	0.002
44	1	0.776	0.957	0.134	0.020	0.016
	(3)	(0.021)	(0.018)	(0.509)	(0.191)	(0.007)
45	1	0.795	0.768	0.272	0.002	0.0004
	(3)	(0.020)	(0.030)	(0.037)	(0.011)	(0.005)

- The BCD series are the same as in Table 1.
- The figures in parenthesis are related to tests in which the AR model has been selected by using AIC. If there are no values in parenthesis, SBIC and AIC yield the same AR model.

In anticipation of the following section, we next drop the early observations and base our tests on data from 1960 (i) to 1985 (iv). The rather ambiguous results are in Table 2. Using SBIC we end up having a parsimonious AR (1) model as our linear alternative. Our conclusion then is that there is no trace of asymmetry in the unemployment series we are considering. However, if we let AIC guide us, the outcome is in two cases a less parsimonious AR model, and the linearity hypothesis is rejected in favour of STAR at the 0.05 level of significance. This result seems surprising. We may except a linearity test sometimes reject the null hypothesis when the lag length is too short simply because the test may also have power against misspecifying the lag length; see TERÄSVIRTA [1990] for discussion. A false acceptance of the linearity is more likely if the lag length is too long because that causes a loss of power. In the present case the linearity is rejected when the lag length is increased. This may indicate that for describing the nonlinearity, the longer lags are crucial, and the short ones do not give enough power to the test. Nevertheless, this is essentially a small-sample argument if the model with a shorter lag is indeed misspecified. At any rate, two conclusions emerge. First, the test results are sensitive to the AR specification. Second, the evidence for nonlinearity of STAR type is weaker if the observations before the sixties are excluded than it is if they are retained. The case for heteroskedasticity remains strong even if the early observations are excluded, and linearity also continues to be rejected against the less parsimonious of the two bilinear alternatives.

5 International Unemployment Series

Next we shall study cyclical asymmetry by looking at unemployment rates elsewhere. This is done by analyzing unemployment data from 13 OECD countries. The time series are quarterly, seasonally unadjusted unemployment rates 1960(i)-1986(iv) published in OECD Main Economic Indicators. The seasonally unadjusted U.S. series is included in the sample for comparison. We shall make the series approximately stationary by seasonal differencing, *i.e.*, the time series to be analyzed are of the type $\nabla_4 u_t = u_t - u_{t-4}$. No other seasonal adjustment procedure is applied.

The results of the linearity tests appear in Table 3. Taking the U.S. first, it can be seen that the results are in accord with those in Table 2. Selecting the AR model by SBIC and testing linearity with S_1 and S_3 leads to accepting linearity as before. On the other hand, the less parsimonious (AIC) AR model using more parameters to characterize seasonality allows us to reject linearity at the 0.05 level in favour of STAR, when the test statistic is S_1. Note that linearity is not rejected against ARCH or bilinear alternatives. Conditional heteroskedasticity thus seems more like an artifact due to a particular seasonal adjustment procedure than a phenomenon inescapably present in cyclical quarterly U.S. economic data.

As to the other countries, Japan and the three Scandinavian countries, Finland, Norway and Sweden, distinguish themselves from the others in that linearity is accepted. In Norway, Sweden and Japan the unemployment has been very low during the whole period of observation, so that we may not expect strong cycles there, symmetric or asymmetric. Yet, the Norwegian data resemble American in that using more parameters to model seasonality does lead to rejecting the linearity in favour of STAR.

In many European countries, the unemployment has risen dramatically at the end of the seventies or early eighties and started to fluctuate around a higher country-specific level. It is conceivable that this remarkable increase in unemployment affects test results: at least the appearance of ARCH may be expected. Indeed, the ARCH tests for Belgium, West Germany, France, Italy and U.K. reject linearity in favour of ARCH. But then, for France and the Netherlands, the rejection of linearity against STAR is

TABLE 3

Observed Significances or p-Values of Linearity Tests Against STAR and Other Nonlinearities Using Quarterly, Seasonally Adjusted Unemployment Rates of 13 OECD Countries, 1960 (i)-1986 (iv)

Country	AR order $\hat{p}$	Test statistics S_1	S_3	BLT ($\hat{p}$; 1, 1)	BLT ($\hat{p}$; 2, 2)	Q(4)
Austria	1	0.078	0.006	0.102	0.382	0.030
Belgium	2	0.938	0.024	0.529	0.077	0.0027
	(6)	(0.176)	(0.172)	(0.290)	(0.011)	(0.100)
Canada	2	0.011	0.027	0.156	0.342	0.107
	(5)	(0.001)	(0.002)	(0.019)	(0.092)	(0.306)
FR Germany	5	0.118	0.120	0.415	0.024	0.027
Finland	4	0.908	0.287	0.053	0.440	0.265
France*	5	<0.0001	<0.0001	0.477	0.0001	0.009
Italy	5	0.256	0.056	0.452	0.390	0.0004
	(6)	(0.082)	(0.068)	(0.287)	(0.566)	(0.086)
Japan	1	0.206	0.393	0.508	0.208	0.919
	(4)	(0.612)	(0.413)	(0.613)	(0.184)	(0.400)
The Netherlands*	6	0.0002	<0.0001	0.860	0.019	0.099
Norway	1	0.692	0.748	0.216	0.150	0.064
	(5)	(0.042)	(0.044)	(0.078)	(0.274)	(0.027)
Sweden †	1	0.709	0.181	0.892	0.885	0.439
	(4)	(0.552)	(0.487)	(0.697)	(0.823)	(0.020)
United Kingdom	6	0.007	0.024	0.975	0.568	0.035
United States	2	0.381	0.390	0.777	0.690	0.131
	(7)	(0.032)	(0.124)	(0.591)	(0.962)	(0.358)

• The figures in parenthesis are related to tests in which the AR model has been selected by using AIC. If there are no values in parenthesis, SBIC and AIC yield the same AR model.

* Results are based on the logarithm of the number of unemployed, as the unemployment rate has not been available for the whole period of observation.

† Results are based on data from 1962 (i) to 1986 (iv).

really overwhelming, and the possibility of cyclical asymmetry can hardly be ignored. Note, however, that these are the two countries for which the logarithm of unemployed has replaced the quarterly unemployment rate, because the latter has not been available for the whole observation period.

The Belgian test results are apparently contradictory as S_1 strongly favours the null, whereas S_3 clearly rejects it. However, as LUUKKONEN *et al.* [1988 *a*] pointed out, S_1 has little power if the dominating nonlinearity parameter in the STAR model is θ_0. The results indicate this to be the case if we model the Belgian unemployment rate by a STAR (2; 2, 2) model. If a less parsimonious AR model is used, S_3 loses power and does not any longer reject linearity. The most clear-cut evidence in favour of possible cyclical asymmetry comes from Canada. When the AR (2) model forms the base for inference, linearity is rejected only against STAR. When the AR (5) model selected by AIC is used, linearity is also rejected against the BL (5; 1, 1) model, but the rejection against STAR is stronger and also much stronger than the corresponding rejection in the AR (2) case. For many European countries we cannot avoid the suspicion that the increase in the natural rate of unemployment occurring during the observation period may have influenced the results. It is therefore difficult to argue in favour of cyclical asymmetry of business cycles in those countries solely on the basis of the evidence in Table 3.

6 Transformed Unemployment Series

Since the increase in the natural unemployment rate may pose a problem, it seems reasonable to transform the data in such a way that the relative importance of the increase diminishes. The unemployment rate (in per cent) u_t is a variable bounded between 0 and 100%. For time series bounded like this, WALLIS [1987] considered the logistic transformation $v_t = \log(u_t/(100-u_t))$. It is a useful transformation in modelling u_t, because it ensures consistency with the definition of the bounds. We shall apply it here mainly because it enhances small absolute but large relative changes in the unemployment rate. Thus it puts a larger weight than previously on cyclical fluctuations before mid-seventies or early eighties in countries where the natural rate of unemployment has soared. The seasonal differences $\nabla_4 v_t$ to be analyzed have the same sign as $\nabla_4 u_t$.

The test results appear in Table 4. It is seen that in the light of our tests, the U.S. data now show no signs of nonlinearity. If anything, the logistic transformation seems an excellent way of linearizing this series. The same is true for the Austrian and Canadian unemployment rates. For the countries with a low unemployment rate in our sample, Japan, Sweden and Norway, the linearity hypothesis not unexpectedly continues to receive support against STAR. However, the transformed Swedish series seems conditionally heteroskedastic, as linearity is very strongly rejected against ARCH. As to Finland where the seasonally unadjusted unemployment rate has not exceeded 7.8%, the less parsimonious (AIC) model of the two hints at STAR, but linearity is still accepted at the 0.05 level of significance.

We half expected the logistic transformation make the unemployment series more linear in the case of European countries with increased natural rate of unemployment. For Belgium and to some extent U.K. this seems to have been the case. However, there are remarkable examples of the series evidently becoming more nonlinear after the transformation. Emphasizing fluctuations at low rates of unemployment more reveals important nonlinearity for West Germany and Italy: linearity is rejected against both STAR and ARCH. Besides, the German data still show indications for bilinearity as in Table 3. A conclusion from Tables 3 and 4 is that the test results are not invariant to data transformations, which complicates the issue. There is not a single country for which the tests

TABLE 4

Observed Significances or p-Values of Linearity Tests Against STAR and Other Nonlinearities Using the Logistic Transformation of Quarterly, Seasonally Unadjusted Unemployment Rates of 13 OECD Countries. 1960 (i)-1986 (iv)

Country	AR order $\hat{p}$	Test statistics S_1	S_3	BLT ($\hat{p}$; 1, 1)	BLT ($\hat{p}$; 2, 2)	Q(4)
Austria	2	0.220	0.178	0.066	0.370	0.896
Belgium	2	0.240	0.105	0.339	0.062	0.110
	(5)	(0.486)	(0.015)	(0.518)	(0.106)	(0.246)
Canada	2	0.477	0.263	0.930	0.554	0.072
	(6)	(0.166)	(0.085)	(0.696)	(0.049)	(0.817)
FR Germany	3	0.004	0.008	0.019	0.128	0.009
	(7)	(0.068)	(0.026)	(0.004)	(0.044)	(0.005)
Finland	4	0.656	0.646	0.226	0.207	0.023
	(7)	(0.051)	(0.090)	(0.280)	(0.098)	(0.113)
France*		<0.0001	<0.0001	0.477	0.0001	0.009
Italy	6	0.004	0.005	0.704	0.116	0.0004
Japan	4	0.165	0.317	0.476	0.044	0.139
The Netherlands*	6	0.0002	<0.0001	0.860	0.019	0.099
Norway	4	0.240	0.083	0.315	0.136	0.370
Sweden†	1	0.833	0.313	0.418	0.700	0.033
	(4)	0.682	0.159	0.992	0.678	<0.0001
United Kingdom	6	0.323	0.038	0.791	0.879	0.487
United States	2	0.406	0.390	0.876	0.691	0.756
	(7)	(0.546)	(0.276)	(0.801)	(0.876)	(0.548)

• The figures in parenthesis are related to tests in which the AR model has been selected by using AIC. If there are no values in parenthesis, SBIC and AIC yield the same result.

* Results are based on the logarithm of the number of unemployed; they are the same as in Table 3.

† Results are based on data from 1962 (i) to 1986 (iv).

against STAR consistently reject linearity. (U.K. can be an exception if we insist that the failure of S_1 to reject the null in Table 4 is just an indication of that the nonlinearity in the STAR model stems mainly from θ_0.) Only for the three countries with the lowest unemployment rates do the results for STAR tests (supporting linearity) seem to be unaffected by the logistic transformation.

7 International Industrial Production Series

Next we shall compare the results obtained so far with those obtained by studying output series. Because the manufacturing is likely to show more cyclical variation than GNP, we use international data on industrial production for this purpose. The observations are again quarterly, seasonally unadjusted values of the logarithmic index of industrial production in 13 OECD countries, published in the OECD Main Economic Indicators, and they cover the quarters 1960 (i) to 1986 (iv). The time series are made approximately stationary by seasonal differencing in the same way as the unemployment series. The specification of the AR order is carried out as before using SBIC and AIC. The test results appear in Table 5.

TABLE 5

Observed Significances or p-Values of Linearity Tests Against STAR and Other Nonlinearities Using Quarterly, Seasonally Unadjusted Logarithmic Indices of Industrial Production from 13 OECD Countries, 1960 (i)-1986 (iv)

Country	AR order $\hat{p}$	Test statistics S_1	S_3	BLT ($\hat{p}$; 1, 1)	BLT ($\hat{p}$; 2, 2)	Q(4)
Austria	5	0.048	0.072	0.020	0.096	0.046
Belgium	5	0.054	0.069	0.024	0.044	0.0002
Canada	5	0.138	0.107	0.266	0.096	0.889
FR Germany	4	0.027	0.096	0.514	0.544	0.998
Finland	1	0.891	0.793	0.459	0.927	0.720
	(4)	(0.717)	(0.866)	(0.880)	(0.653)	(0.873)
France	5	0.109	0.039	0.381	0.553	0.019
Italy	5	0.124	0.056	0.986	0.987	0.013
Japan	5	0.033	0.083	0.889	0.227	0.001
The Netherlands	1	0.197	0.102	0.255	0.436	0.004
	(5)	(0.676)	(0.307)	(0.650)	(0.632)	(0.049)
Norway	5	0.046	0.090	0.463	0.724	0.022
Sweden	4	0.998	0.890	0.820	0.934	0.989
	(5)	(0.584)	(0.384)	(0.932)	(0.974)	(0.945)
United Kingdom	5	0.140	0.181	0.332	0.818	0.521
	(7)	(0.278)	(0.28)	(0.37)	(0.808)	(0.286)
United States	2	0.033	0.036	0.201	0.110	0.082
	(6)	(0.017)	(0.062)	(0.170)	(0.092)	(0.061)

• The figures in parenthesis are related to tests in which the AR model has been selected by using AIC. If there are no values in parenthesis, SBIC and AIC yield the same result.

While the U.S. unemployment rate seems linear, at least after the logistic transformation, the hypothesis of the logarithmic industrial production being nonlinear and of STAR type receives support from the data. [3] The same is also true for Japan: linearity is rejected at the 0.05 level of significance in favour of STAR. However, for the Japanese data $Q(n)$ rejects linearity very strongly, so that ARCH cannot be excluded from consideration. There is evidence for heteroskedasticity in several other countries as well: this includes Austria, Belgium, France, Italy, The Netherlands and Norway. Thus the rejections or "near-rejections" of linearity

3. Note that ROTHMAN [1989] found cyclical asymmetry in the U.S. unemployment series (the sign process) emerging from the manufacturing sector.

against STAR for these countries might also arise from ARCH. An exception is West Germany: there is no evidence of ARCH but linearity is rejected against STAR. Of the four countries with linear unemployment series, Finland and Sweden still stand out: no test rejects linearity. A reason for this may be that both countries have mostly been enjoying sustained economic growth without violent swings of any kind during the observation period. The linearity of annual differences of logarithmic quarterly industrial production is also accepted for the UK and Canada.

These results do not fully accord with those of FALK [1986] who analyzed the asymmetry of business cycles using NEFTÇI's [1984] technique and industrial production data from Canada, France, Italy, United Kingdom and West Germany. His application differs from ours in so many respects that a reconciliation does not seem possible. Falk used seasonally adjusted quarterly data and applied linear detrending before dichotomizing the first differences. Thus his series are much more heavily manipulated than ours. His estimation period extended from 1951 (i) to 1983 (iv), and he interpreted the results as not making cyclical symmetry an unlikely possibility. As noticed above, we do obtain rejections against STAR at the 0.05 significance level with shorter time series already in studying the industrial output of West Germany and France.

Finally, in our application the probability values of tests against bilinearity are generally clearly higher than those of STAR tests. Many STAR probability values are around 0.1 or less, whereas that is rare for tests against bilinearity. Thus it is rather unlikely that amplitude changes or outliers have caused any low probability values in STAR tests. On the other hand, as noted above, in some cases the possibility of STAR tests responding to ARCH cannot be excluded.

8 Final Remarks

The test results call for caution in drawing definite conclusions. They seem not to be independent of the specification of the model. Transformations of variables also affect some of the results. This concerns not only nonlinearity in the mean but conditional heteroskedasticity as well. Selecting another variable to represent the business cycle may reverse conclusions as well as changing the observation period. Similar problems are likely to appear also in many other applications involving testing linearity of economic and other time series.

If, however, one wants to sort out countries in which the business cycles appear nonlinear of STAR type in 1960-1986, our investigation leaves us with two examples, West Germany and France. For the U.S. and Japan, nonlinearity of industrial production cannot be excluded, whereas the unemployment series appear symmetric. The caveat due to ARCH nevertheless has to be made in most cases. As to the U.S., some doubt remains for the untransformed unemployment series, but the main evidence for nonlinearity in general seems to come from the pre-1960 data. When the period 1948(i)-1959(iv) is excluded from consideration, the case for asymmetry or nonlinearity in general in unemployment series is weakened. At the other end of the scale, Finland and Sweden appear as examples of countries with linear business cycles. A broad, hardly disputable conclusion is that there are nonlinearities in quite a few international macroeconomic time series, and this fact may well warrant further consideration.

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